Triple product correlations in top squark decays

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We propose several T-odd asymmetries in the decay chains of the top squarks $\tilde{t}_m \to t\tilde{\chi}^0_k$ and $t \to bW^+ \to bl\nu$ and $\tilde{\chi}^0_k \to l^\pm \tilde{l}^\mp_n \to l^\pm l^\mp \tilde{\chi}^0_1$, for $l = e, \mu, \tau$. We calculate the asymmetries within the Minimal Supersymmetric Standard Model with complex parameters M_1, μ, and A_t. We give the analytic formulae for the decay distributions. We present numerical results for the asymmetries and estimate the event rates necessary to observe them. The largest T-odd asymmetry can be as large as 40%.

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I. INTRODUCTION

In the Minimal Supersymmetric Standard Model (MSSM) [1,2] with complex parameters, there are new sources of CP violation in addition to the Cabibbo-Kobayashi-Maskawa phase of the standard model (SM). After redefining the fields these are the phase of the Higgsino mass parameter μ, two of the phases of the gaugino masses M_i, $i = 1, 2, 3$ (usually these are chosen to be the phases of M_1 and M_3), and the phases of the trilinear couplings A_f, ϕ_{A_f}. The latter ones for the third generations $f = \tau, t, b$ are rather unconstrained by the experimental upper bounds on the electric dipole moments of electron and neutron [3]. It is therefore especially interesting to search for observables which could be probed in forthcoming collider experiments in order to determine the phases $\phi_{A_{\tau,t,b}}$. The influence of the phases $\phi_{A_{\tau,t,b}}$ has been discussed in the literature before. Some examples of studies discussing CP sensitive observables in SM processes or in processes which might occur in the SM with an extended Higgs sector are in [4], where the influence of the phases $\phi_{A_{t,b}}$ arises due to loop corrections. Other studies focus on the $\phi_{A_{\tau,t,b}}$ dependence in supersymmetric processes. There the dependence of $\phi_{A_{\tau,t,b}}$ on either CP-odd observables [5] or on CP-even observables [6–8] have been discussed. A CP sensitive asymmetry in the 3-body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau\tau^+$ involving the transverse polarization of the τ lepton has been proposed in [9].

In this paper we investigate whether the search for aplanarities in the decay chain of the top squarks $\tilde{t}_{1,2}$ can give information on the CP phase ϕ_{A_t} or on other couplings of the MSSM Lagrangian. We consider the decay chain

$$\tilde{t}_m \to t\tilde{\chi}^0_k, \tag{1}$$

with the subsequent decays of t and $\tilde{\chi}^0_k$. We work in the approximation when both the top quark and the neutralino $\tilde{\chi}^0_k$ are produced on mass-shell. As the top quark does not form a bound state (because of its large mass), both the top quark and the neutralino $\tilde{\chi}^0_k$ decay with definite momentum and polarization. We consider two possibilities for the top quark decay:

$$t \to bW^+ \quad \text{and} \quad t \to bW^+ \to bl\nu(bcs), \tag{2}$$

and the following two-decay chains for $\tilde{\chi}^0_k$:

$$\begin{aligned} &\tilde{\chi}^0_k \to \tilde{l}^-_n l^+_1, \quad \tilde{l}^-_n \to l^-_2 \tilde{\chi}^0_1 \quad \text{and} \\ &\tilde{\chi}^0_k \to \tilde{l}^+_n l^-_1, \quad \tilde{l}^+_n \to l^+_2 \tilde{\chi}^0_1, \end{aligned} \tag{3}$$

where the label of the leptons indicates their origin and where both $l^\pm_1$ and $l^\mp_2$ are from the same lepton family. (Sometimes in literature $l^\pm_1$ and $l^\mp_2$ are called the near and far lepton, see e.g. [10].) We assume that the momenta of all ordinary particles in (1)–(3) can be measured or reconstructed, these are p_t, p_b, p_l, $p_{l^\pm_1}$, and $p_{l^\mp_2}$. The final state consists of two opposite signed leptons of the same family, l^+l^-, a b quark and $q\bar{q}'$ jets (or l) from the t quark decay and missing energy.

An useful tool for studying CP violation are triple product correlations $(\mathbf{q}_1 \times \mathbf{q}_2 \cdot \mathbf{q}_3) \equiv (\mathbf{q}_1\mathbf{q}_2\mathbf{q}_3)$ [11,12], where $\mathbf{q}_i$ can be any of the 3-momenta of the particles in the decay chain. Triple product correlations are an example of T-odd correlations that change sign under a flip of the 3-momenta $\mathbf{q}_i \to -\mathbf{q}_i$. The time reversal operation T implies not only reverse of the 3-momenta and polarizations of the particles but also an interchange of the initial and final states. Because of the antiunitary nature of the time reversal operation, a nonzero value of a T-odd observable would imply T-violation if loop amplitudes are neglected. Any triple product correlation would be direct evidence that T invariance is broken and as CPT invariance holds, CP conservation is broken as well. As the triple product correlations in the processes (1)–(3) are a tree-level effect, they do not contain the suppression factor due to radiative corrections that is always present when such correlations are considered in processes with ordinary particles.

In the top squark decays (1)–(3) no triple product correlations can arise solely from the decays of either t

or $\tilde{\chi}_k^0$. Triple products originate from the covariant products $\varepsilon(q_1q_2q_3q_4)$ written in the laboratory system. In order that $\varepsilon(q_1q_2q_3q_4) \neq 0$ leads to a CP asymmetry at tree-level we need both a CP violating phase and at least a 3-body decay mediated by a particle that is not a scalar. The top quark decay modes (2) proceed in the SM and at tree-level no CP violating phases occur, thus no correlations of the type $(\mathbf{p}_l\mathbf{p}_b\mathbf{p}_t)$ can appear. The $\tilde{\chi}_k^0$ decays (3) are 3-body decays mediated by the scalar lepton $\tilde{l}_n$ and, as $\tilde{l}_n$ does not transfer information about the spin of $\tilde{\chi}_k^0$ to its decay products, again no triple products can be formed.

Thus, the only correlations which occur are among the momenta of the decay products of both t and $\tilde{\chi}_k^0$. These correlations reflect the spin properties of t and $\tilde{\chi}_k^0$. In order to obtain analytic expressions for the distributions of the decay products we use the formalism of Kawasaki, Shirafuji, and Tsai [13]. We work in the narrow width approximation for t and $\tilde{\chi}_k^0$.

As T-odd observables we consider up-down asymmetries, which are defined by

$$A_T \equiv \frac{\int d\Omega sgn(\mathcal{O})d\Gamma/d\Omega}{\int d\Omega d\Gamma/d\Omega} = \frac{N[\mathcal{O}>0]-N[\mathcal{O}<0]}{N[\mathcal{O}>0]+N[\mathcal{O}<0]}, \tag{4}$$

where $d\Gamma$ stands for the differential decay width and $d\Omega$ involves the angles of integration. In Eq. (4) $\mathcal{O}$ represents the triple product correlation on which we focus and $N[\mathcal{O}>(<)0]$ is the number of events for which $\mathcal{O}>(<)0$. According to the decay channels of the top quark we consider two cases:

(i) If $t \to bW$, the possible triple products are

$$(\mathbf{p}_b\mathbf{p}_t\boldsymbol{l}_{1,2}^{\pm}) \quad \text{and} \quad (\mathbf{p}_b\boldsymbol{l}_1^{+}\boldsymbol{l}_2^{-}). \tag{5}$$

(ii) If a final leptonic (νl) or hadronic (cs) decay mode of W is measured, then possible triple product correlations are

$$(\mathbf{p}_t\mathbf{p}_{l,c}\boldsymbol{l}_{1,2}^{\pm}), \qquad (\mathbf{p}_b\mathbf{p}_{l,c}\boldsymbol{l}_{1,2}^{\pm}) \quad \text{and} \quad (\mathbf{p}_{l,c}\boldsymbol{l}_1^{+}\boldsymbol{l}_2^{-}). \tag{6}$$

In most of the asymmetries studied below b-tagging will be necessary. In those asymmetries which involve the decay $W \to cs$ also c-tagging will be necessary [14].

The decay $\tilde{\chi}_k^0 \to Z^0\tilde{\chi}_1^0, Z^0 \to l^+l^-$, leads to the same final state as decay (3) and also gives rise to the above triple product correlations. In this paper we will not consider triple product correlations in this decay, because due to the small $Z^0l^+l^-$ vector coupling one can expect that the corresponding T-odd asymmetries are much smaller than those following from the decay (3). (In this context see also [15–17].)

The paper is organized as follows: In the next section we give the relevant terms of the Lagrangian. In Sec. III we present the results of our calculation in compact form using the formalism of [13]. Section IV contains the formulae for various decay distributions. We propose several T-odd asymmetries in Sec. V. In Sec. VI we perform a numerical analysis of the T-odd asymmetries proposed. Finally, we summarize and conclude in Sec. VII.

II. LAGRANGIAN AND COUPLINGS

The terms of the Lagrangian necessary to calculate the T-odd asymmetries and the decay rates of $\tilde{t}_m \to \tilde{\chi}_k^0 t$ and $\tilde{\chi}_k^0 \to \tilde{l}_n^{\mp} l_1^{\pm} \to l_1^{\pm} l_2^{\mp} \tilde{\chi}_1^0$ in the presence of the CP phases are

$$\mathcal{L}_{l\tilde{l}\tilde{\chi}^0} = g\bar{l}(a_{nk}^{\tilde{l}}P_R + b_{nk}^{\tilde{l}}P_L)\tilde{\chi}_k^0\tilde{l}_n + \text{h.c.}, \tag{7}$$

$$\mathcal{L}_{t\tilde{t}\tilde{\chi}^0} = g\bar{t}(a_{mk}^{\tilde{t}}P_R + b_{mk}^{\tilde{t}}P_L)\tilde{\chi}_k^0\tilde{t}_m + \text{h.c.}, \tag{8}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, g is the $SU(2)_L$ gauge coupling constant and the couplings are defined as

$$\begin{aligned} a_{nk}^{\tilde{l}} &= (\mathcal{R}_{n1}^{\tilde{l}})^* f_{Lk}^l + (\mathcal{R}_{n2}^{\tilde{l}})^* h_{Rk}^l, \\ b_{nk}^{\tilde{l}} &= (\mathcal{R}_{n1}^{\tilde{l}})^* h_{Lk}^l + (\mathcal{R}_{n2}^{\tilde{l}})^* f_{Rk}^l, \end{aligned} \tag{9}$$

with $\mathcal{R}_{nj}^{\tilde{l}}$ being the scalar lepton mixing matrix and

$$f_{Lk}^l = \frac{1}{\sqrt{2}}(N_{k2} + \tan\theta_W N_{k1}), \qquad f_{Rk}^l = -\sqrt{2}\tan\theta_W N_{k1}^*,$$
$$h_{Rk}^l = (h_{Lk}^l)^* = Y_l N_{k3}, \tag{10}$$

and

$$a_{mk}^{\tilde{t}} = \sum_{n=1}^{2}(\mathcal{R}_{mn}^{\tilde{t}})^*\mathcal{A}_{kn}^t, \qquad b_{mk}^{\tilde{t}} = \sum_{n=1}^{2}(\mathcal{R}_{mn}^{\tilde{t}})^*\mathcal{B}_{kn}^t. \tag{11}$$

Here $\mathcal{R}_{mn}^{\tilde{t}}$ is the mixing matrix of the top squarks and

$$\mathcal{A}_k^t = \begin{pmatrix} f_{Lk}^t \\ h_{Rk}^t \end{pmatrix}, \qquad \mathcal{B}_k^t = \begin{pmatrix} h_{Lk}^t \\ f_{Rk}^t \end{pmatrix}, \tag{12}$$

with

$$\begin{aligned} f_{Lk}^t &= -\frac{1}{\sqrt{2}}\left(N_{k2} + \frac{1}{3}\tan\theta_W N_{k1}\right), \\ f_{Rk}^t &= \frac{2\sqrt{2}}{3}\tan\theta_W N_{k1}^*, \qquad h_{Lk}^t = (h_{Rk}^t)^* = -Y_t N_{k4}^*. \end{aligned} \tag{13}$$

The unitary 4×4 neutralino mixing matrix N is defined in Appendix A, Eq. (A2), $Y_t = m_t/(\sqrt{2}m_W\sin\beta)$ and $Y_l = m_l/(\sqrt{2}m_W\cos\beta)$. The top squark mixing matrix $\mathcal{R}^{\tilde{t}}$ is given in Appendix B, the scalar lepton mixing matrix $\mathcal{R}^{\tilde{l}}$ can be found, for instance, in [6].

III. FORMALISM

According to the formalism of [13] the differential decay rate of (1)–(3), when spin-spin correlations are taken into account, is

$$d\Gamma = d\Gamma(\tilde{t}_m \to t\tilde{\chi}_k^0)\frac{E_t}{m_t\Gamma_t}d\Gamma(t\to ...)\frac{E_{\chi_k}}{m_{\chi_k}\Gamma_{\chi_k}}d\Gamma(\tilde{\chi}_k^0 \to ...), \tag{14}$$

where the factors $E_{\chi_k}/m_{\chi_k}\Gamma_{\chi_k}$ and $E_t/m_t\Gamma_t$ stem from the used narrow width approximation for t and $\tilde{\chi}_k^0$, Γ_t and Γ_{χ_k} are the total widths of the particles and m_{χ_k} and m_t are their masses. We have

$$d\Gamma(\tilde{t}_m \to t\tilde{\chi}_k^0) = \frac{4}{2m_{\tilde{t}_m}}|A|^2 d\Phi_{\tilde{t}}, \tag{15}$$

where

$$d\Phi_{\tilde{t}} = \frac{(2\pi)^4}{(2\pi)^6}\delta(p_{\tilde{t}_m} - p_t - p_{\chi_k})\frac{d\mathbf{p}_t}{2E_t}\frac{d\mathbf{p}_{\chi_k}}{2E_{\chi_k}} \tag{16}$$

is the differential decay rate of the top squark $\tilde{t}_m$ into a top quark with polarization 4-vector ξ_t^α, and a neutralino $\tilde{\chi}_k^0$ with polarization 4-vector $\xi_{\chi_k}^\alpha$. For the matrix element A we have

$$A = g\bar{u}(p_t)(b_{mk}^{\tilde{t}}P_L + a_{mk}^{\tilde{t}}P_R)v(p_{\chi_k}). \tag{17}$$

In evaluating $|A|^2$ we use the spin density matrices of t and $\tilde{\chi}_k^0$

$$\begin{aligned}\rho(p_t) &= \Lambda(p_t)\frac{1+\gamma_5 \not{\xi}_t}{2},\\ \rho(-p_{\chi_k}) &= -\Lambda(-p_{\chi_k})\frac{1+\gamma_5\not{\xi}_{\chi_k}}{2},\end{aligned} \tag{18}$$

with

$$\Lambda(p_t) = \not{p}_t + m_t, \qquad \Lambda(p_{\chi_k}) = \not{p}_{\chi_k} + m_{\chi_k}, \tag{19}$$

where p_t and p_{χ_k} are the momentum 4-vectors of the top quark and the neutralino $\tilde{\chi}_k^0$. We have

$$\begin{aligned}|A|^2 = \frac{g^2}{2}\{&(|a_{mk}^{\tilde{t}}|^2 + |b_{mk}^{\tilde{t}}|^2)[(p_{\chi_k}p_t) + m_{\chi_k}m_t(\xi_{\chi_k}\xi_t)]\\ &- (|a_{mk}^{\tilde{t}}|^2 - |b_{mk}^{\tilde{t}}|^2)[m_t(p_{\chi_k}\xi_t) + m_{\chi_k}(\xi_{\chi_k}p_t)]\\ &- 2\Re e(a_{mk}^{\tilde{t}*}b_{mk}^{\tilde{t}})[m_{\chi_k}m_t - (p_{\chi_k}\xi_t)(\xi_{\chi_k}p_t)\\ &+ (p_{\chi_k}p_t)(\xi_{\chi_k}\xi_t)] + 2\Im m(a_{mk}^{\tilde{t}*}b_{mk}^{\tilde{t}})\varepsilon(p_{\chi_k}\xi_{\chi_k}\xi_t p_t)\},\end{aligned} \tag{20}$$

where $\varepsilon^{0123} = 1$. The polarization 4-vectors ξ_t^α and $\xi_{\chi_k}^\alpha$ are determined through the decay processes of the top quark and the neutralino. $d\Gamma(t \to ...)$ and $d\Gamma(\tilde{\chi}_k^0 \to ...)$ are the differential decay rates of the unpolarized top and unpolarized neutralino.

Next we shall consider the decays of $\tilde{\chi}_k^0$ and t. According to the chosen decay mode of the top quark, Eq. (2), we have to distinguish two cases. We consider them separately.

A. Decay rates for $\tilde{\chi}_k^0 \to l_1^\pm \tilde{l}_n^\mp$

For the width of the neutralino decay into a lepton l_1^+ and a scalar lepton $\tilde{l}_n$ we write

$$d\Gamma(\tilde{\chi}_k^0 \to l_1^+\tilde{l}_n^-) = \frac{1}{2\cdot 2E_{\chi_k}}\mathrm{Tr}\{\bar{B}[-\Lambda(-p_{\chi_k})]B\}d\Phi_{\chi_k}, \tag{21}$$

with

$$d\Phi_{\chi_k} = \frac{(2\pi)^4}{(2\pi)^6}\delta(p_{\chi_k} - l_1^+ - p_{\tilde{l}})\frac{d\mathbf{p}_{\tilde{l}}}{2E_{\tilde{l}}}\frac{d\boldsymbol{l}_1^+}{2E_+}. \tag{22}$$

Here B is defined through the decay matrix element

$$\bar{v}_\sigma(p_{\chi_k})B^\sigma = g\bar{v}(p_{\chi_k})(b_{nk}^{\tilde{l}*}P_R + a_{nk}^{\tilde{l}*}P_L)v(l_1^+), \tag{23}$$

p_{χ_k} and l_1^+ are the momentum 4-vectors of the neutralino and the lepton, $E_{\tilde{l}}$ and E_+ are the energies of $\tilde{l}_n$ and l_1^+. For the distribution of the decay products we obtain

$$d\Gamma(\tilde{\chi}_k^0 \to l_1^+\tilde{l}_n^-) = \frac{g^2}{2E_{\chi_k}}(|a_{nk}^{\tilde{l}}|^2 + |b_{nk}^{\tilde{l}}|^2)(p_{\chi_k}l_1^+)d\Phi_{\chi_k}. \tag{24}$$

For the polarization vector $\xi_{\chi_k}^\alpha$ of the neutralino $\tilde{\chi}_k^0$, determined through the $\tilde{\chi}_k^0$-decay, we have

$$\begin{aligned}\xi_{\chi_k}^\alpha &= \left(g^{\alpha\beta} - \frac{p_{\chi_k}^\alpha p_{\chi_k}^\beta}{m_{\chi_k}^2}\right)\frac{\mathrm{Tr}\{\bar{B}[-\Lambda(-p_{\chi_k})]\gamma_5\gamma_\beta B\}}{\mathrm{Tr}\{\bar{B}[-\Lambda(-p_{\chi_k})]B\}}\\ &= \alpha_+\frac{m_{\chi_k}}{(p_{\chi_k}l_1^+)}Q_+^\alpha,\end{aligned} \tag{25}$$

with

$$Q_+^\alpha = \left[(l_1^+)^\alpha - \frac{(p_{\chi_k}l_1^+)}{m_{\chi_k}^2}p_{\chi_k}^\alpha\right], \qquad \alpha_+ = \frac{|b_{nk}^{\tilde{l}}|^2 - |a_{nk}^{\tilde{l}}|^2}{|b_{nk}^{\tilde{l}}|^2 + |a_{nk}^{\tilde{l}}|^2}. \tag{26}$$

Respecting the condition $(\xi_{\chi_k}p_{\chi_k}) = 0$, the vector Q_+^α is orthogonal to the momentum 4-vector of $\tilde{\chi}_k^0$. This is the only orthogonal 4-vector composed of the available momenta p_{χ_k} and l_1^+. As it can be seen from (25) and (26), ξ_{χ_k} is in the $\tilde{\chi}_k^0$-decay plane. Further we shall assume that $\tilde{l}_n$ is produced on mass-shell, $p_{\tilde{l}}^2 = m_{\tilde{l}}^2$, then $(p_{\chi_k}l_1^+) = (m_{\chi_k}^2 - m_{\tilde{l}}^2)/2$, where we neglect the lepton mass in the kinematics, i.e., $m_l = 0$. The prefactor α_+ determines the sensitivity to the polarization of $\tilde{\chi}_k^0$.

Note that the polarization vector of $\tilde{\chi}_k^0$, Eq. (25), does not change if we take the subsequent decay $\tilde{l}_n^- \to \tilde{\chi}_1^0 l_2^-$ into account. Note further, that the polarization vector of the C-conjugated decay $\tilde{\chi}_k^0 \to l_1^-\tilde{l}_n^+$ changes sign compared to ξ_{χ_k} in Eq. (25).

B. Decay rate for $t \to bW^+$

When the top quark decays according to $t \to bW^+$ we have

$$d\Gamma(t \to bW^+) = \frac{1}{2 \cdot 2E_t} \mathrm{Tr}[\bar{C}_b \Lambda(p_t) C_b] d\Phi_t^b, \quad (27)$$

with

$$d\Phi_t^b = \frac{(2\pi)^4}{(2\pi)^6} \delta(p_t - p_b - p_W) \frac{d\mathbf{p}_b}{2E_b} \frac{d\mathbf{p}_W}{2E_W}, \quad (28)$$

where C_b is defined by the decay matrix element as follows:

$$\bar{C}_b^\sigma u_\sigma(p_t) = \frac{g}{\sqrt{2}} \bar{u}(p_b) \gamma_\alpha P_L u(p_t) \epsilon^{\alpha *}(p_W), \quad (29)$$

where p_b, p_t, and p_W are the momentum 4-vectors of the bottom quark, the top quark and the W boson. Then for the distribution of the decay products we obtain

$$d\Gamma(t \to bW^+) = \frac{g^2}{8E_t} \frac{(m_t^2 - m_W^2)(2m_W^2 + m_t^2)}{m_W^2} d\Phi_t^b. \quad (30)$$

We denote by ξ_b the polarization 4-vector of the top quark, determined by the decay $t \to bW$. Its expression is given by the formula

$$\xi_b^\alpha = \left(g^{\alpha\beta} - \frac{p_t^\alpha p_t^\beta}{m_t^2}\right) \frac{\mathrm{Tr}[\bar{C}_b \Lambda(p_t) \gamma_5 \gamma_\beta C_b]}{\mathrm{Tr}[\bar{C}_b \Lambda(p_t) C_b]}. \quad (31)$$

From (29) and (31) we obtain the polarization vector

$$\xi_b^\alpha = \alpha_b \frac{m_t}{(p_t p_b)} Q_b^\alpha, \quad (32)$$

with

$$\alpha_b = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2}, \qquad (p_t p_b) = \frac{m_t^2 - m_W^2}{2},$$
$$Q_b^\alpha = \left[p_b^\alpha - \frac{(p_t p_b)}{m_t^2} p_t^\alpha\right], \quad (33)$$

where in the kinematics we have set $m_b = 0$. Here Q_b^α is the 4-vector orthogonal to p_t^α and α_b determines the sensitivity to the polarization of the top quark.

C. Decay rates for $t \to bl\nu$ and for $t \to bcs$

We consider here the decay $t \to bl\nu$. For the decay $t \to bcs$ one has to make the replacements $p_\nu \to p_s$, $p_l \to p_c$ in the equations below. For the inclusion of QCD corrections to the decay of a polarized top quark we refer to [18]. When the top quark decays according to $t \to bl\nu$ we have

$$d\Gamma(t \to bl\nu) = \frac{1}{2 \cdot 2E_t} \mathrm{Tr}[\bar{C}_l \Lambda(p_t) C_l] d\Phi_t^l, \quad (34)$$

with

$$d\Phi_t^l = \frac{(2\pi)^4}{(2\pi)^9} \delta(p_t - p_b - p_l - p_\nu) \frac{d\mathbf{p}_b}{2E_b} \frac{d\mathbf{p}_l}{2E_l} \frac{d\mathbf{p}_\nu}{2E_\nu}, \quad (35)$$

where C_l is defined through the decay matrix element as follows:

$$\bar{C}_l^\sigma u_\sigma(p_t) = -i\left(\frac{g}{\sqrt{2}}\right)^2 \bar{u}(p_\nu) \gamma_\alpha P_L v(p_l) \times \frac{g^{\alpha\beta} - p_W^\alpha p_W^\beta / m_W^2}{D_W} \bar{u}(p_b) \gamma_\beta P_L u(p_t), \quad (36)$$

with

$$D_W = (p_W^2 - m_W^2) + i m_W \Gamma_W, \qquad p_W^\alpha = p_t^\alpha - p_b^\alpha. \quad (37)$$

Then we obtain:

$$d\Gamma(t \to bl\nu) = \frac{g^4}{2E_t |D_W|^2} (p_t p_l)[m_t^2 - 2(p_t p_l)] d\Phi_t^l. \quad (38)$$

From (36), for the polarization vector of the top, that we denote by ξ_l, we have

$$\xi_l^\alpha = \left(g^{\alpha\beta} - \frac{p_t^\alpha p_t^\beta}{m_t^2}\right) \frac{\mathrm{Tr}[\bar{C}_l \Lambda(p_t) \gamma_5 \gamma_\beta C_l]}{\mathrm{Tr}[\bar{C}_l \Lambda(p_t) C_l]} = \alpha_l \frac{m_t}{(p_t p_l)} Q_l^\alpha, \quad (39)$$

with

$$Q_l^\alpha = \left[p_l^\alpha - \frac{(p_t p_l)}{m_t^2} p_t^\alpha\right], \qquad \alpha_l = -1, \quad (40)$$

Q_l^α is orthogonal to p_t^α and lays in the top quark decay plane (in the rest frame of the top quark). In general, with the available vectors in the decay, p_t, p_b and p_l, one can form three independent combinations orthogonal to p_t^α: two in the decay plane, Q_b^α and Q_l^α, and one transverse to it, $\varepsilon(\alpha p_t p_b p_l)$. As CP invariance holds in the top quark decay, there is no contribution to the transverse component, and because of the vector-axialvector structure of the interaction, there is no contribution to Q_b^α either.

Inserting (15), (21), and (27) or (34) into (14) we obtain $d\Gamma$ in terms of the polarization vectors:

$$d\Gamma^{b,l} = \frac{1}{2m_{\tilde{t}_m}} \frac{1}{2m_t \Gamma_t} \frac{1}{2m_{\chi_k} \Gamma_{\chi_k}} |A|^2 \mathrm{Tr}\{\bar{B}[-\Lambda(-p_{\chi_k})]B\} \times \mathrm{Tr}[\bar{C}_{b,l} \Lambda(p_t) C_{b,l}] \frac{1}{2m_{\tilde{l}} \Gamma_{\tilde{l}}} g^2(|a_{n1}^{\tilde{l}}|^2 + |b_{n1}^{\tilde{l}}|^2) \times (m_{\tilde{l}}^2 - m_{\chi_1}^2) d\Phi^{b,l}, \quad (41)$$

where we have used the narrow width approximation for the scalar lepton propagator. $m_{\tilde{l}}$ and $\Gamma_{\tilde{l}}$ is the mass and the total decay width of $\tilde{l}$, $|A|^2$ is given by (20) and $d\Phi^{b,l}$ denotes the phase space for the two different decay modes of the top quark:

$$d\Phi^{b,l} = d\Phi_{\tilde{t}} \cdot d\Phi_t^{b,l} \cdot d\Phi_{\chi_k} \cdot d\Phi_{\tilde{l}}. \tag{42}$$

In order to obtain the angular distributions of the ordinary particles in (1)–(3) we have to use the explicit expressions for ξ_{χ_k} and ξ_t, and carry the integration over the phase space of the supersymmetric particles.

IV. DECAY DISTRIBUTIONS

In this section we derive the analytical expressions for the decay distributions of $\tilde{t}_m \to t\tilde{\chi}_k^0 \to bW^+\tilde{\chi}_1^0 l_1^\pm l_2^\mp$ and $\tilde{t}_m \to t\tilde{\chi}_k^0 \to bl\nu\tilde{\chi}_1^0 l_1^\pm l_2^\mp$. We consider separately the two decays (2) of the top quark.

A. Decay distribution for $\tilde{t}_m \to t\tilde{\chi}_k^0 \to bW^+\tilde{\chi}_1^0 l_1^\pm l_2^\mp$

We choose $\mathbf{p}_t$ in the direction of the Z-axis and $\mathbf{p}_t$ and $\mathbf{p}_b$ determine the YZ-plane:

$$\begin{aligned} p_{\tilde{t}_m} &= (m_{\tilde{t}_m}, \overrightarrow{0}), \qquad \mathbf{p}_t = |\mathbf{p}|(0,0,1), \\ \mathbf{p}_{\chi_k} &= |\mathbf{p}|(0,0,-1), \qquad \mathbf{p}_b = E_b(0, s_b, c_b), \\ \boldsymbol{l}_1^+ &= E_+(s_+c_{\phi_+}, s_+s_{\phi_+}, c_+), \\ \boldsymbol{l}_2^- &= E_-(s_-c_{\phi_-}, s_-s_{\phi_-}, c_-), \end{aligned} \tag{43}$$

where we have used the brief notation $c_b = \cos\theta_b$, $s_{\phi_+} = \sin\phi_+$, etc., The ranges of the angles are $0 \le \theta_b, \theta_+, \theta_- \le \pi$; $0 \le \phi_+, \phi_- \le 2\pi$. Then we can carry out part of the phase space integration. Using (43), Eq. (42) is given by

$$d\Phi^b = \frac{|\mathbf{p}|(m_t^2 - m_W^2)(m_{\chi_k}^2 - m_{\tilde{l}}^2)}{2m_{\tilde{t}_m} 8^2(2\pi)^4 E_t^2 E_{\chi_k}^2} \times \frac{dc_b dc_+ d\phi_+}{(1-\beta_t c_b)^2(1+\beta_{\chi_k}c_+)^2} \cdot d\Phi_{\tilde{l}}, \tag{44}$$

where

$$d\Phi_{\tilde{l}} = \frac{1}{8(2\pi)^2} \frac{m_{\tilde{l}}^2 - m_{\chi_1}^2}{E_{\tilde{l}}^2(1-\beta_{\tilde{l}}c_{\tilde{l}l^-})^2} d\Omega_-, \tag{45}$$

and

$$\begin{aligned} \beta_t &= \frac{|\mathbf{p}|}{E_t}, \qquad \beta_{\chi_k} = \frac{|\mathbf{p}|}{E_{\chi_k}}, \qquad \beta_{\tilde{l}} = \frac{|\mathbf{p}_{\tilde{l}}|}{E_{\tilde{l}}}, \\ |\mathbf{p}| &= \frac{\lambda^{1/2}(m_{\tilde{t}_m}^2, m_t^2, m_{\chi_k}^2)}{2m_{\tilde{t}_m}}, \\ E_t &= \sqrt{|\mathbf{p}|^2 + m_t^2} = \frac{m_{\tilde{t}_m}^2 + m_t^2 - m_{\chi_k}^2}{2m_{\tilde{t}_m}}, \\ E_{\chi_k} &= \sqrt{|\mathbf{p}|^2 + m_{\chi_k}^2} = \frac{m_{\tilde{t}_m}^2 - m_t^2 + m_{\chi_k}^2}{2m_{\tilde{t}_m}}, \\ c_{\tilde{l}l^-} &= (\hat{\mathbf{p}}_{\tilde{l}} \cdot \hat{\mathbf{p}}_{l_2^-}), \end{aligned} \tag{46}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$. Then from (41), using the explicit expressions for the polarization vectors (25) and (32) and the decay distributions (24) and (30), for the angular distributions of the b-quark and the leptons l_1^+ and l_2^-, we obtain

$$\begin{aligned} \frac{d^5\Gamma_b}{dc_b d\Omega_+ d\Omega_-} &= N_{\tilde{l}} \frac{1}{E_{\tilde{l}}^2(1-\beta_{\tilde{l}}c_{\tilde{l}l^-})^2} N_b \frac{1}{(1-\beta_t c_b)^2(1+\beta_{\chi_k}c_+)^2} \Big\{(|a_{mk}^{\tilde{t}}|^2 + |b_{mk}^{\tilde{t}}|^2)\Big[(p_{\chi_k}p_t) + \alpha_b\alpha_+ \frac{m_t^2}{(p_t p_b)} \frac{m_{\chi_k}^2}{(p_{\chi_k}l_1^+)} \\ &\times (Q_+Q_b)\Big] - (|a_{mk}^{\tilde{t}}|^2 - |b_{mk}^{\tilde{t}}|^2)\Big[\alpha_b \frac{m_t^2}{(p_t p_b)}(Q_b p_{\chi_k}) + \alpha_+ \frac{m_{\chi_k}^2}{(p_{\chi_k}l_1^+)}(Q_+p_t)\Big] - 2\Re e(a_{mk}^{\tilde{t}*}b_{mk}^{\tilde{t}})m_{\chi_k}m_t \\ &\times \Big[1 - \frac{\alpha_b}{(p_t p_b)} \frac{\alpha_+}{(p_{\chi_k}l_1^+)}((p_{\chi_k}Q_b)(Q_+p_t) - (p_{\chi_k}p_t)(Q_+Q_b))\Big] + 2\Im m(a_{mk}^{\tilde{t}*}b_{mk}^{\tilde{t}})\alpha_b\alpha_+ \frac{m_t}{(p_t p_b)} \frac{m_{\chi_k}}{(p_{\chi_k}l_1^+)} \\ &\times m_{\tilde{t}_m}(\boldsymbol{l}_1^+\mathbf{p}_b\mathbf{p}_t)\Big\}, \end{aligned} \tag{47}$$

where

$$\begin{aligned} N_{\tilde{l}} &= \frac{1}{m_{\tilde{l}}\Gamma_{\tilde{l}}} \frac{\alpha_w}{8} \frac{1}{4\pi}(|a_{n1}^{\tilde{l}}|^2 + |b_{n1}^{\tilde{l}}|^2)(m_{\tilde{l}}^2 - m_{\chi_1}^2)^2, \\ N_b &= \left(\frac{\alpha_w}{8}\right)^3 \frac{(2m_W^2 + m_t^2)(m_t^2 - m_W^2)^2(m_{\chi_k}^2 - m_{\tilde{l}}^2)^2(|a_{nk}^{\tilde{l}}|^2 + |b_{nk}^{\tilde{l}}|^2)|\mathbf{p}|}{2\pi m_{\tilde{t}_m}^2 m_W^2 m_t \Gamma_t m_{\chi_k}\Gamma_{\chi_k}E_t^2E_{\chi_k}^2}, \\ E_b &= \frac{m_t^2 - m_W^2}{2E_t(1-\beta_t c_b)}, \qquad E_+ = \frac{m_{\chi_k}^2 - m_{\tilde{l}}^2}{2E_{\chi_k}(1+\beta_{\chi_k}c_+)}, \qquad E_- = \frac{m_{\tilde{l}}^2 - m_{\chi_1}^2}{2E_{\tilde{l}}(1-\beta_{\tilde{l}}c_{\tilde{l}l^-})}, \qquad (\boldsymbol{l}_1^+\mathbf{p}_b\mathbf{p}_t) = E_+E_b|\mathbf{p}|s_b s_+ c_{\phi_+}. \end{aligned} \tag{48}$$

B. Decay distribution of $\tilde{t}_m \to t\tilde{\chi}_k^0 \to bl\nu\tilde{\chi}_1^0 l_1^{\pm} l_2^{\mp}$

The angular distribution of the final b-quark and leptons l, l_1^+ and l_2^- is obtained from the previous results if we fix the coordinate system so that $\mathbf{p}_t$ and $\mathbf{p}_l$ determine the YZ-plane as follows

$$\mathbf{p}_t = |\mathbf{p}|(0,0,1), \qquad \mathbf{p}_{\chi_k} = |\mathbf{p}|(0,0,-1), \qquad \mathbf{p}_l = E_l(0, s_l, c_l), \qquad \mathbf{p}_b = E_b(s_b c_{\phi_b}, s_b s_{\phi_b}, c_b), \tag{49}$$

where the ranges of the angles are $0 \le \theta_b, \theta_l \le \pi$; $0 \le \phi_b \le 2\pi$. Then the dependence on the b-quark momentum is only in the phase space. We obtain

$$d\Phi^l = \frac{|\mathbf{p}| m_W^2 (m_t^2 - m_W^2)(m_{\chi_k}^2 - m_{\tilde{l}}^2)}{2 m_{\tilde{t}_m} 8^3 (2\pi)^7 E_t^2 E_{\chi_k}^2} \frac{dc_l d\Omega_b d\Omega_+}{(1-\beta_t c_b)^2 (1+\beta_{\chi_k} c_+)^2 [E_t(1-\beta_t c_l) - E_b(1-c_{bl})]^2} \cdot \frac{ds_W}{2\pi} \cdot d\Phi_{\tilde{l}}, \tag{50}$$

where $s_W = p_W^2$. We obtain the angular distribution by a replacement of the phase space $d\Phi^b \to d\Phi^l$ and the following replacements in the curly brackets of (47): $\alpha_b \to \alpha_l$, $Q_b \to Q_l$ and $p_b \to p_l$. The angular decay rate distribution of l_1^+, l_2^-, l and b is:

$$\begin{aligned}\frac{d^7\Gamma_l}{dc_l d\Omega_b d\Omega_+ d\Omega_-} = {}& N_{\tilde{l}} \frac{1}{E_{\tilde{l}}^2 (1-\beta_{\tilde{l}} c_{\tilde{l}l^-})^2} N_l \frac{(p_t p_l)[m_t^2 - 2(p_t p_l)]}{(1-\beta_t c_b)^2 (1+\beta_{\chi_k} c_+)^2 [E_t(1-\beta_t c_l) - E_b(1-c_{bl})]^2} \Bigg\{ (|a_{mk}^{\tilde{t}}|^2 + |b_{mk}^{\tilde{t}}|^2) \\ & \times \left[(p_{\chi_k} p_t) + \alpha_l \alpha_+ \frac{m_t^2}{(p_t p_l)} \frac{m_{\chi_k}^2}{(p_{\chi_k} l_1^+)} (Q_+ Q_l) \right] - (|a_{mk}^{\tilde{t}}|^2 - |b_{mk}^{\tilde{t}}|^2) \Bigg[\alpha_l \frac{m_t^2}{(p_t p_l)} (Q_l p_{\chi_k}) + \alpha_+ \frac{m_{\chi_k}^2}{(p_{\chi_k} l_1^+)} \\ & \times (Q_+ p_t) \Bigg] - 2\Re e(a_{mk}^{\tilde{t}*} b_{mk}^{\tilde{t}}) m_{\chi_k} m_t \left[1 - \frac{\alpha_l}{(p_t p_l)} \frac{\alpha_+}{(p_{\chi_k} l_1^+)} ((p_{\chi_k} Q_l)(Q_+ p_t) - (p_{\chi_k} p_t)(Q_+ Q_l)) \right] \\ & + 2\Im m(a_{mk}^{\tilde{t}*} b_{mk}^{\tilde{t}}) \alpha_l \alpha_+ \frac{m_t}{(p_t p_l)} \frac{m_{\chi_k}}{(p_{\chi_k} l_1^+)} m_{\tilde{t}_m} (\boldsymbol{l}_1^+ \mathbf{p}_l \mathbf{p}_t) \Bigg\},\end{aligned} \tag{51}$$

where

$$N_l = \left(\frac{\alpha_w}{8}\right)^4 \times \frac{m_W^2 (m_t^2 - m_W^2) |\mathbf{p}| (m_{\chi_k}^2 - m_{\tilde{l}}^2)^2 (|a_{nk}^{\tilde{l}}|^2 + |b_{nk}^{\tilde{l}}|^2)}{\pi^2 m_{\tilde{t}_m}^2 m_t \Gamma_t m_{\chi_k} \Gamma_{\chi_k} m_W \Gamma_W E_t^2 E_{\chi_k}^2}, \tag{52}$$

$$E_l = \frac{m_W^2}{2[E_t(1-\beta_t c_l) - E_b(1-c_{bl})]}, \qquad c_{bl} = (\hat{\mathbf{p}}_b \cdot \hat{\mathbf{p}}_l), \tag{53}$$

$$(\boldsymbol{l}_1^+ \mathbf{p}_l \mathbf{p}_t) = E_+ E_l |\mathbf{p}| s_l s_+ c_{\phi_+}. \tag{54}$$

The distribution of l_1^- and l_2^+ from the C-conjugate decay $\tilde{\chi}_k^0 \to l_1^- \tilde{l}_n^+ \to l_1^- l_2^+ \tilde{\chi}_1^0$ is obtained from (47) and (51) by the replacements $l_1^+ \to l_1^-$, $l_2^- \to l_2^+$ and $\alpha_+ \to -\alpha_+$.

As can be seen from the angular distributions, Eqs. (47) and (51), the prefactor of the triple product correlations [last term in Eqs. (47) and (51)] is $\Im m(a_{mk}^{\tilde{t}*} b_{mk}^{\tilde{t}})$ and consequently the T-odd asymmetries (to be defined in the next section) are proportional to this prefactor. Therefore, in order to study the dependence of the T-odd asymmetries on the MSSM parameters, it is useful to give the explicit expression for $\Im m(a_{mk}^{\tilde{t}*} b_{mk}^{\tilde{t}})$ for $m = 1$ using Eqs. (11)–(13):

$$\begin{aligned}\Im m(a_{1k}^{\tilde{t}*} b_{1k}^{\tilde{t}}) = {}& -\cos^2\theta_{\tilde{t}} Y_t \Im m(f_{Lk}^{t*} N_{k4}^*) \\ & - \sin^2\theta_{\tilde{t}} \frac{2\sqrt{2}}{3} Y_t \tan\theta_W \Im m(N_{k1}^* N_{k4}^*) \\ & + \cos\theta_{\tilde{t}} \sin\theta_{\tilde{t}} \left[\frac{2\sqrt{2}}{3} \tan\theta_W \Im m(f_{Lk}^{t*} N_{k1}^* e^{i\phi_{\tilde{t}}}) \right. \\ & \left. + Y_t^2 \Im m(N_{k4}^* N_{k4}^* e^{-i\phi_{\tilde{t}}}) \right].\end{aligned} \tag{55}$$

We can see from (55) that if CP violation is solely due to $\phi_{A_t} \neq 0$, the T-odd asymmetries are proportional to $\sin 2\theta_{\tilde{t}} \sin\phi_{\tilde{t}}$, which can be naturally large because of the large top squark mixing [see Eqs. (B1) and (B4)]. Moreover, one can see from (55) that the term $\propto \sin 2\theta_{\tilde{t}}$ can be sizable also in a Higgsinolike scenario ($|\mu| < M_2$) because of the large top Yukawa coupling.

V. T-ODD ASYMMETRIES

We shall distinguish three classes of asymmetries according to the lepton momentum (stemming from the decay chain $\tilde{\chi}_k^0 \to \tilde{l}_n^{\mp} l_1^{\pm} \to l_1^{\pm} l_2^{\mp} \tilde{\chi}_1^0$) involved in the triple product: (i) when the momentum vector of lepton $l_1^{\pm}$ from the decay $\tilde{\chi}_k^0 \to \tilde{l}_n^{\mp} l_1^{\pm}$ enters; (ii) when the lepton momentum vector of lepton $l_2^{\mp}$ from the decay $\tilde{l}_n^{\mp} \to l_2^{\mp} \tilde{\chi}_1^0$ enters; and (iii) when both momentum vectors of $l_1^{\pm}$ and $l_2^{\mp}$ from the decay $\tilde{\chi}_k^0 \to l_1^{\pm} l_2^{\mp} \tilde{\chi}_1^0$ enter.

The first class involves the asymmetries

$$A_1^{\pm} = \frac{N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_1^{\pm}) > 0] - N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_1^{\pm}) < 0]}{N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_1^{\pm}) > 0] + N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_1^{\pm}) < 0]}, \quad (56)$$

$$A_2^{\pm} = \frac{N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_1^{\pm}) > 0] - N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_1^{\pm}) < 0]}{N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_1^{\pm}) > 0] + N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_1^{\pm}) < 0]}, \quad (57)$$

$$A_3^{\pm} = \frac{N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_1^{\pm}) > 0] - N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_1^{\pm}) < 0]}{N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_1^{\pm}) > 0] + N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_1^{\pm}) < 0]}, \quad (58)$$

where $\mathbf{p}_l$ is the lepton momentum in the decay $t \to bl\nu$.

In the second class of the asymmetries l_1 is replaced by l_2

$$A_1'^{\pm} = \frac{N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_2^{\mp}) > 0] - N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_2^{\mp}) < 0]}{N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_2^{\mp}) > 0] + N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_2^{\mp}) < 0]}, \quad (59)$$

$$A_2'^{\pm} = \frac{N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_2^{\mp}) > 0] - N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_2^{\mp}) < 0]}{N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_2^{\mp}) > 0] + N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}_2^{\mp}) < 0]}, \quad (60)$$

$$A_3'^{\pm} = \frac{N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_2^{\mp}) > 0] - N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_2^{\mp}) < 0]}{N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_2^{\mp}) > 0] + N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}_2^{\mp}) < 0]}. \quad (61)$$

The third class of asymmetries is

$$A_4^{\pm} = \frac{N[(\mathbf{p}_b \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) > 0] - N[(\mathbf{p}_b \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) < 0]}{N[(\mathbf{p}_b \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) > 0] + N[(\mathbf{p}_b \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) < 0]}, \quad (62)$$

$$A_5^{\pm} = \frac{N[(\mathbf{p}_l \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) > 0] - N[(\mathbf{p}_l \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) < 0]}{N[(\mathbf{p}_l \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) > 0] + N[(\mathbf{p}_l \boldsymbol{l}_1^{\pm} \boldsymbol{l}_2^{\mp}) < 0]}. \quad (63)$$

Since the polarization vectors of $\tilde{\chi}_k^0$ for the two C-conjugate decay modes of (3) differ only by a sign [see Eq. (25)] the value of the asymmetries with upper indices + and − are related by

$$A_i^+ = -A_i^-(i = 1, \ldots, 5) \quad \text{and} \quad A_i'^+ = -A_i'^-(i = 1, 2, 3). \quad (64)$$

In order to measure all of the listed asymmetries it is necessary to distinguish the lepton $l_1^{\pm}$, originating from the decay $\tilde{\chi}_k^0 \to \tilde{l}_n^{\mp} l_1^{\pm}$, and the lepton $l_2^{\mp}$ from the subsequent decay $\tilde{l}_n^{\mp} \to \tilde{\chi}_1^0 l_2^{\mp}$. This can be accomplished by measuring the energies of the leptons and making use of their different energy distributions, when the masses of the particles involved are known. $l_1^{\pm}$ and $l_2^{\mp}$ can be distinguished if their measured energies do not lie in the overlapping region of their energy distributions.

We define the fourth class of asymmetries as follows:

$$A_1 = \frac{N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}^+) > 0] - N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}^+) < 0]}{N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}^+) > 0] + N[(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}^+) < 0]}, \quad (65)$$

$$A_2 = \frac{N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}^+) > 0] - N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}^+) < 0]}{N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}^+) > 0] + N[(\mathbf{p}_l \mathbf{p}_t \boldsymbol{l}^+) < 0]}, \quad (66)$$

$$A_3 = \frac{N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}^+) > 0] - N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}^+) < 0]}{N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}^+) > 0] + N[(\mathbf{p}_l \mathbf{p}_b \boldsymbol{l}^+) < 0]}, \quad (67)$$

where $\boldsymbol{l}^+$ stands for the momentum 3-vector of either l_1^+ or l_2^+. Evidently we have

$$A_i = \frac{A_i^+ + A_i'^+}{2}, \quad i = 1, 2, 3. \quad (68)$$

A measurement of A_i, $i = 1, 2, 3$, does not require one to distinguish between the leptons $l_1^{\pm}$ and $l_2^{\mp}$, it requires only a measurement of their charges. Analogous asymmetries can be defined for l^- as well.

It should also be noted that the asymmetries given in Eqs. (56)–(58) do not depend on the mass of $\tilde{l}_n$. The asymmetries above are written down for the leptonic decay $W^+ \to l\nu$. For the hadronic decay $W^+ \to cs$ the analogous asymmetries are obtained by replacing $\mathbf{p}_l \to \mathbf{p}_c$.

VI. NUMERICAL RESULTS

All proposed T-odd asymmetries depend on $\Im m(a_{mk}^{\tilde{t}*} b_{mk}^{\tilde{t}})$, Eq. (55), and measure therefore the same combination of CP phases in the MSSM, but they have different magnitude. In this section we present numerical results for the asymmetries $A_i^{\pm}, A_i'^{\pm}(i = 1, 2, 3)$, Eqs. (56)–(61), $A_4^{\pm}, A_5^{\pm}$, Eqs. (62) and (63), and $A_i(i = 1, 2, 3)$, Eqs. (65)–(67). In order not to vary too many parameters we fix $m_{\tilde{t}_1} = 400$ GeV, $m_{\tilde{t}_2} = 800$ GeV, and $\tan\beta = 10$.[1] We take $m_t = 178$ GeV and we also use the grand unified theory relation $|M_1| = 5/3 \tan^2\Theta_W M_2$. We take $m_{\tilde{l}_1} = 130$ GeV, $m_{\tilde{l}_2} = 300$ GeV, where we assume $\tilde{l}_1 \approx \tilde{l}_R$ for $l = e, \mu$, which is suggested in mSugra models. In the scalar tau sector we take into account scalar tau mixing choosing $A_\tau = 500$ GeV. In our numerical study we take $|A_t|$, ϕ_{A_t}, M_2, ϕ_{M_1}, $|\mu|$, ϕ_μ as input parameters. Note that for a given set of input parameters we obtain two solutions for $(M_{\tilde{Q}}, M_{\tilde{U}})$ corresponding to the cases $M_{\tilde{t}_{LL}}^2 > M_{\tilde{t}_{RR}}^2$ and $M_{\tilde{t}_{LL}}^2 < M_{\tilde{t}_{RR}}^2$ in Eqs. (B2) and (B3) which we will treat separately. In the plots we impose the phenomenological constraints: $m_{\tilde{\chi}_1^{\pm}} > 103$ GeV,$m_{\tilde{\chi}_1^0} > 50$ GeV and $\tilde{\chi}_1^0$ is the lightest supersymmetric particle (LSP).

In Fig. 1 we plot the various asymmetries (56)–(67) for the decay $\tilde{t}_1 \to t\tilde{\chi}_2^0$ as a function of ϕ_{A_t} for the case $M_{\tilde{Q}} < M_{\tilde{U}}$. The MSSM parameters are $M_2 = 250$ GeV, $|\mu| = 200$ GeV, $|A_t| = 1200$ GeV, and $\phi_{M_1} = \phi_\mu = 0$. As can be seen in Fig. 1(a) the absolute value of the asymmetry A_2^+ (dashed line) is much larger than the absolute value of A_1^+ (solid line), which can be attributed to the sensitivity factor of the top quark polarization for the two asymmetries. For A_1^+ this factor is $|\alpha_b| \simeq 0.4$ (for $m_t = 178$ GeV),

[1] The prefactor $\Im m(a_{mk}^{\tilde{t}*} b_{mk}^{\tilde{t}})$ of the asymmetries does not depend very much on the value of $\tan\beta$ if $|A_t| \gg |\mu|/\tan\beta$.

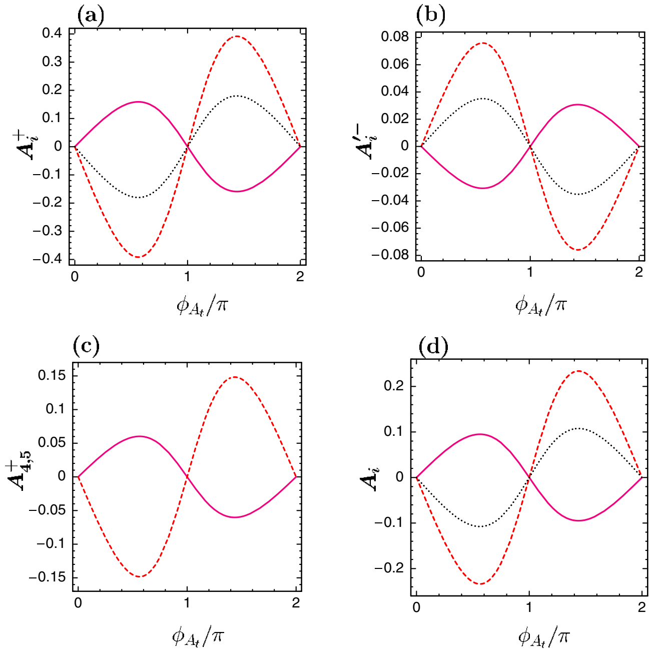

FIG. 1 (color online). T-odd asymmetries (a) A_i^+, $i = 1, 2, 3$, Eq. (56)–(58), (b) $A_i'^-$, $i = 1, 2, 3$, Eq. (59)–(61), (c) A_i^+, $i = 4, 5$, Eqs. (62) and (63), and (d) A_i, $i = 1, 2, 3$, Eqs. (65)–(67) for $\tilde{t}_1 \to t\tilde{\chi}_2^0$ as a function of ϕ_{A_t}. In (a), (b), (d) the solid (dashed, dotted) lines correspond to the indices $i = 1(2, 3)$, in (c) the solid (dashed) line corresponds to $i = 4(5)$. The MSSM parameters are chosen as $|A_t| = 1200$ GeV, $M_2 = 250$ GeV, $|\mu| = 200$ GeV, $\tan\beta = 10$, $\phi_{M_1} = \phi_\mu = 0$, $m_{\tilde{t}_1} = 400$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $M_{\tilde{Q}} < M_{\tilde{U}}$, for $l = e, \mu$.

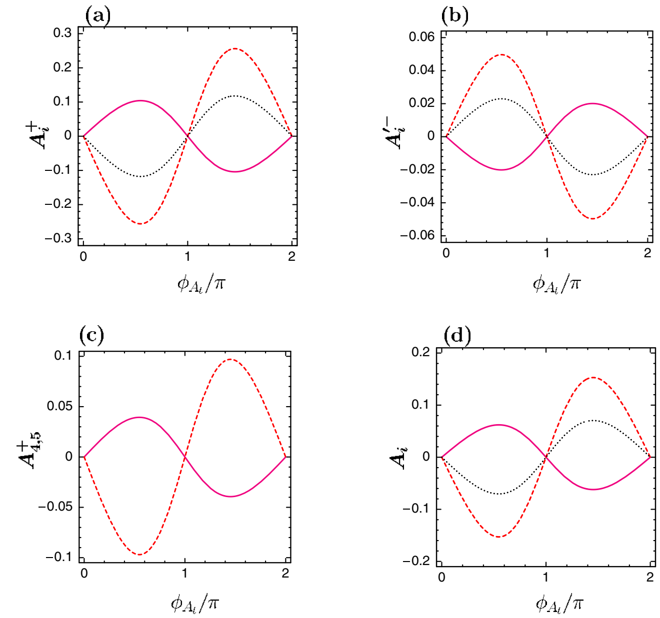

FIG. 2 (color online). T-odd asymmetries (a) A_i^+, $i = 1, 2, 3$, Eq. (56)–(58), (b) $A_i'^-$, $i = 1, 2, 3$, Eq. (59)–(61), (c) A_i^+, $i = 4, 5$, Eqs. (62) and (63), and (d) A_i, $i = 1, 2, 3$, Eqs. (65)–(67) for $\tilde{t}_1 \to t\tilde{\chi}_2^0$ as a function of ϕ_{A_t}. In (a), (b), (d) the solid (dashed, dotted) lines correspond to the indices $i = 1(2, 3)$, in (c) the solid (dashed) line corresponds to $i = 4(5)$. The MSSM parameters are chosen as $|A_t| = 1200$ GeV, $M_2 = 250$ GeV, $|\mu| = 200$ GeV, $\tan\beta = 10$, $\phi_{M_1} = \phi_\mu = 0$, $m_{\tilde{t}_1} = 400$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $M_{\tilde{Q}} > M_{\tilde{U}}$, for $l = e, \mu$.

Eq. (33), whereas for A_2^+ it is $|\alpha_l| = 1$, Eq. (40) (see also [18] where QCD corrections are included). This difference can also be seen in Fig. 1(c) by comparing A_4^+, Eq. (62), with A_5^+, Eq. (63). In Fig. 1(d) the asymmetries $A_i (i = 1, 2, 3)$, Eqs. (65)–(67), are displayed for which we have to distinguish the leptons in the decay chain $\tilde{\chi}_2^0 \to \tilde{l}_1^\mp l_1^\pm \to \tilde{\chi}_1^0 l_2^\mp l_1^\pm$ only by their charge. It is interesting to note that the asymmetry A_2 can be as large as 24%. In Fig. 2 we plot the same asymmetries as in Fig. 1, but now for the case $M_{\tilde{Q}} > M_{\tilde{U}}$. As can be seen also for this case the largest asymmetry is A_2^+, Eq. (57), which is however somewhat reduced compared to the case $M_{\tilde{Q}} < M_{\tilde{U}}$.

In Fig. 3 we plot the contours of the asymmetry A_2^+ for the decay $\tilde{t}_1 \to t\tilde{\chi}_2^0$ in the $|\mu| - M_2$ plane where we have taken $\phi_{A_t} = \frac{\pi}{2}$ and the other parameters as in the previous figures. One sees in Figs. 3(a)–3(d) that the asymmetry is largest for large gaugino-Higgsino mixing ($|\mu| \sim M_2$). Figures 3(a) and 3(b) correspond to the case where $\tilde{\chi}_2^0$ decays into $l = e, \mu$, whereas Figs. 3(c) and 3(d) correspond to the case $\tilde{\chi}_2^0 \to \tilde{\tau}_1 \tau^+$. The asymmetries in Figs. 3(a) and 3(b) are larger than those in Figs. 3(c) and 3(d), because of the effect of scalar tau mixing which leads to $|\alpha_+| < 1$ [see Eq. (26)], while for $l = e, \mu$, Figs. 3(a) and 3(b), we have $|\alpha_+| = 1$. Moreover, in Figs. 3(a) and 3(b) there is a sign change of the asymmetries because of a sign change of the prefactor $\Im m(a_{12}^{\tilde{t}*} b_{12}^{\tilde{t}})$. This sign change does not appear in Figs. 3(c) and 3(d), because it is compensated by a simultaneous sign change of α_+, which occurs due to a level crossing of the states $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$. In Figs. 3(c) and 3(d) there is a different sign change of the asymmetries in the lower right part of the $|\mu| - M_2$ plane because α_+, Eq. (26), changes sign there.

In Fig. 4 we show the contours of the asymmetry A_2^+ for the decay $\tilde{t}_1 \to t\tilde{\chi}_2^0 \to bW^+ \tilde{l}_1^- l_1^+ (l = e, \mu)$ in the $\phi_{A_t} - \phi_{M_1}$ plane taking $A_t = 1200$ GeV, $M_2 = 250$ GeV, $|\mu| = 200$ GeV, and $\phi_\mu = 0$ for the two cases $M_{\tilde{Q}} < M_{\tilde{U}}$ [Fig. 4(a)] and $M_{\tilde{Q}} > M_{\tilde{U}}$ [Fig. 4(b)]. In Fig. 4(a) the largest value of about 39% (-39%) for the asymmetry A_2^+ is obtained for $\phi_{A_t} = 1.4\pi$ ($\phi_{A_t} = 0.6\pi$). For $\phi_{A_t} = 0(\pi)$ and $\phi_{M_1} = 0.5\pi$ the asymmetry A_2^+ is about $-3.5(17.3)\%$. In Fig. 4(b) one can see that the largest value of the asymmetry A_2^+ is obtained if $\phi_{M_1} \neq 0, \pi$ and the asymmetry varies from about 25% for $\phi_{A_t} \approx 1.5\pi, \phi_{M_1} = 0$ to about 34% for $\phi_{A_t} \approx 1.5\pi, \phi_{M_1} \approx 1.4\pi$. For $\phi_{A_t} = 0(\pi)$ and $\phi_{M_1} = 0.5\pi$ the asymmetry A_2^+ is about $-10.6(3.2)\%$.

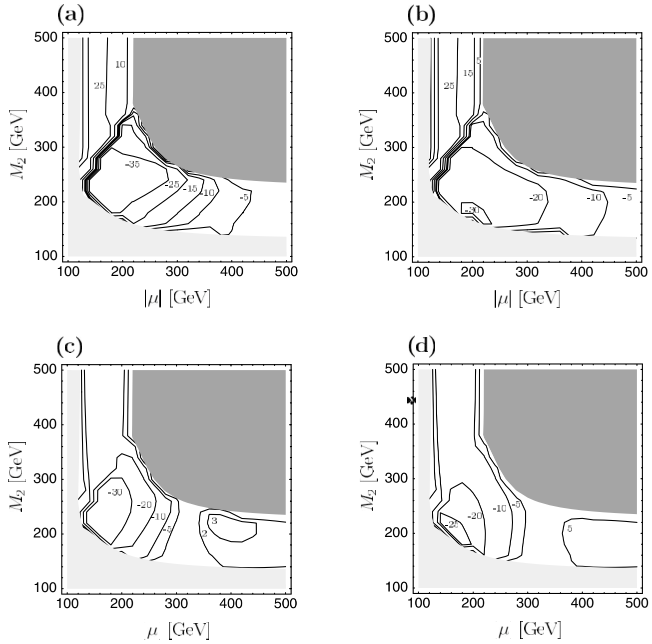

FIG. 3. Contours of the T-odd asymmetry A_2^+ in % for $\tilde{t}_1 \to t\tilde{\chi}_2^0 \to bW^+\tilde{l}_1^- l_1^+$ for $l = e, \mu$, (a) $M_{\tilde{Q}} < M_{\tilde{U}}$ and (b) $M_{\tilde{Q}} > M_{\tilde{U}}$ and for $l = \tau$ (c) $M_{\tilde{Q}} < M_{\tilde{U}}$ and (d) $M_{\tilde{Q}} > M_{\tilde{U}}$. The MSSM parameters are chosen as $|A_t| = 1200$ GeV, $\phi_{A_t} = 0.5\pi$, $\tan\beta = 10$, $\phi_{M_1} = \phi_\mu = 0$, $m_{\tilde{t}_1} = 400$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $|A_\tau| = 500$ GeV, $\phi_{A_\tau} = 0$, $m_{\tilde{\tau}_1} = 130$ GeV and $m_{\tilde{\tau}_2} = 300$ GeV. The light gray region is excluded because there $m_{\tilde{\chi}_1^\pm} < 103$ GeV and/or $m_{\tilde{\chi}_2^0} < m_{\tilde{l}_1}$. In the dark gray area the two-body decay $\tilde{t}_1 \to t\tilde{\chi}_2^0$ is kinematically forbidden.

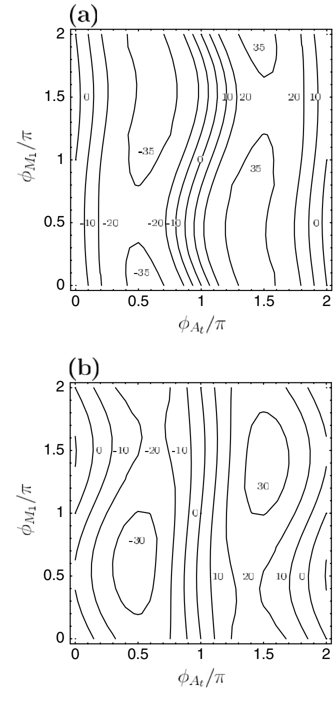

FIG. 4. Contours of the CP asymmetry A_2^+ in % for $\tilde{t}_1 \to t\tilde{\chi}_2^0 \to bW^+\tilde{l}_1^- l_1^+$. The MSSM parameters are chosen as $|A_t| = 1200$ GeV, $M_2 = 250$ GeV, $|\mu| = 200$ GeV, $\phi_\mu = 0$, $\tan\beta = 10$, $m_{\tilde{t}_1} = 400$ GeV, $m_{\tilde{t}_2} = 800$ GeV, for $l = e, \mu$, (a) $M_{\tilde{Q}} < M_{\tilde{U}}$ and (b) $M_{\tilde{Q}} > M_{\tilde{U}}$.

In Fig. 5 we show the asymmetry A_2^+, Eq. (57), as a function of ϕ_{A_t} for the decay of the heavier top squark. The MSSM parameters chosen are $|A_t| = 1200$ GeV, $M_2 = |\mu| = 200$ GeV and $\phi_{M_1} = \phi_\mu = 0$. Figure 5(a) displays the asymmetry for the decay $\tilde{t}_2 \to t\tilde{\chi}_3^0$ and Fig. 5(b) shows the asymmetry for $\tilde{t}_2 \to t\tilde{\chi}_4^0$. Figures 5(a) and 5(b) demonstrate that also for the decay of $\tilde{t}_2$ into the heavier neutralinos the asymmetry A_2^+ can be quite large. In Fig. 5(a) the two cases $M_{\tilde{Q}} < M_{\tilde{U}}$ and $M_{\tilde{Q}} > M_{\tilde{U}}$ give nearly the same curve for A_2^+, because the value of the decay width is nearly the same for the two cases. Note that an observation of a T-odd asymmetry would lead to a twofold ambiguity in the extraction of the CP phases, which can be seen in Figs. 1, 2, 4, and 5.

Next we give a theoretical estimate of the number of top squarks $\tilde{t}_1$ necessary to observe the T-odd asymmetries (56)–(67) in the decay $\tilde{t}_1 \to \tilde{\chi}_2^0 t$, where subsequently the neutralino $\tilde{\chi}_2^0$ decays into $\tilde{l}_1 l$ ($l = e, \mu, \tau$). This number can be estimated by

$$N_{\tilde{t}_1} \gtrsim \frac{\sigma^2}{(A_T)^2 B(W \to f) B(\tilde{t}_1 \to \tilde{\chi}_2^0 t) B(\tilde{\chi}_2^0 \to \tilde{l}_1 l)}, \quad (69)$$

where σ denotes the number of standard deviations and A_T stands for any of the above asymmetries. The value of the branching ratios of the W boson is given by $B(W \to f) = (32, 68, 32)\%(f = \sum_l \nu l, \sum_q q\bar{q}', cs)$ [19] corresponding to the asymmetry for which the estimate is made. For instance, for the asymmetry $A_1^\pm$, which is based on the triple product $(\mathbf{p}_b \mathbf{p}_t \boldsymbol{l}_1^\pm)$, we take the value for $B(W \to \sum_q q\bar{q}')$. The estimate is taken for two scenarios which we define in Table I. The scenarios chosen imply that $\sum_l B(\tilde{\chi}_2^0 \to \tilde{l}_1 l) = 1 (l = e, \mu, \tau)$ and that the energy distributions of the two final leptons (from the decay $\tilde{\chi}_k^0 \to \tilde{l}_1^- l_1^+$ and from the decay $\tilde{l}_1^- \to \tilde{\chi}_1^0 l_2^-$) do not overlap. This means that in all decays it is possible to decide from which decay the two leptons originate. For simplicity we will assume that the T-odd asymmetries (56)–(67) are equal for the three flavors in the subsequent decay $\tilde{\chi}_2^0 \to \tilde{l}_1 l$. This means that we neglect scalar tau mixing ($|\alpha_+| = 1$ for $l = e, \mu, \tau$) and in addition we take $m_{\tilde{l}_1} = m_{\tilde{e}_1} = m_{\tilde{\mu}_1} = m_{\tilde{\tau}_1}$. For the calculation of the branching ratios of the $\tilde{t}_1$ we use the formulae given in [7]. For scenario one (scenario 2) we obtain $B(\tilde{t}_1 \to \tilde{\chi}_2^0 t) = 22\%(2.1\%)$ where we assume that the bosonic decays $\tilde{t}_1 \to \tilde{b}_1 W^+$ and $\tilde{t}_1 \to \tilde{b}_1 H^+$ are kinematically not accessible.

In Table II we display the values of the asymmetries, Eqs. (56)–(67), and the numbers $N_{\tilde{t}_1}$ needed for a 3σ

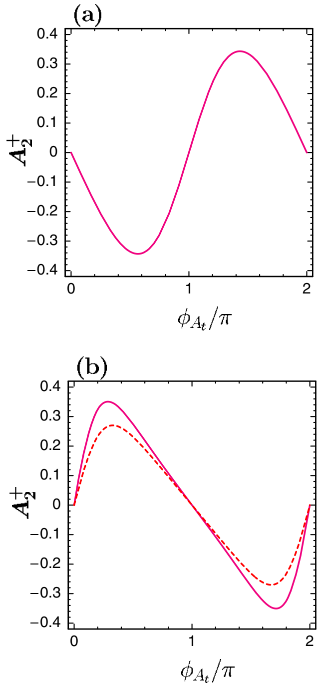

FIG. 5 (color online). T-odd asymmetry A_2^+, Eq. (57), for the processes (a) $\tilde{t}_2 \to t\tilde{\chi}_3^0 \to bW^+\tilde{l}_1^- l_1^+$ and (b) $\tilde{t}_2 \to t\tilde{\chi}_4^0 \to bW^+\tilde{l}_1^- l_1^+$ ($l = e, \mu$) as a function of ϕ_{A_t}. The MSSM parameters are chosen as $|A_t| = 1200$ GeV, $M_2 = |\mu| = 200$ GeV, $\tan\beta = 10$, $\phi_{M_1} = \phi_\mu = 0$, $m_{\tilde{t}_1} = 400$ GeV, $m_{\tilde{t}_2} = 800$ GeV for $M_{\tilde{Q}} < M_{\tilde{U}}$ (solid line) and $M_{\tilde{Q}} > M_{\tilde{U}}$ (dashed line).

evidence of these asymmetries. From Table II it can be seen that in order to have a 3σ evidence for some of the T-odd asymmetries in scenario one about $O(10^3)$ produced $\tilde{t}_1$'s are necessary. For scenario two $O(10^4)$ produced $\tilde{t}_1$'s are necessary for a 3σ evidence of some of the T-odd asymmetries. Assuming that $O(10^6)$ $\tilde{t}_1$'s can be produced at the LHC and $O(10^5)$ $\tilde{t}_1$'s at a future linear collider, there are good prospects to measure some of the asymmetries. It is however clear that detailed Monte Carlo studies taking into account background and detector simulation are necessary to predict the expected accuracy. This is, however, beyond the scope of the present paper.

TABLE I. The two scenarios used for the estimate of the necessary event rates.

Scenario one	Scenario two
$m_{\tilde{l}_1} = 129$ GeV	$m_{\tilde{l}_1} = 115$ GeV
$M_2 = 500$ GeV	$M_2 = 200$ GeV
$\|\mu\| = 150$ GeV	$\|\mu\| = 300$ GeV
$\tan\beta = 3$	$\tan\beta = 6$
$\phi_{M_1} = \phi_\mu = 0$	$\phi_{M_1} = 0,\ \phi_\mu = \pi$
$\|A_t\| = 1200$ GeV	$\|A_t\| = 1200$ GeV
$\phi_{A_t} = \frac{\pi}{2}$	$\phi_{A_t} = \frac{\pi}{6}$
$M_{\tilde{Q}} < M_{\tilde{U}}$	$M_{\tilde{Q}} > M_{\tilde{U}}$

TABLE II. The values of the T-odd asymmetries defined in Eqs. (56)–(67) and the number $N_{\tilde{t}_1}$ of top squarks required to measure these asymmetries with a 3σ evidence in the two considered scenarios (see Table I).

	Scenario one		Scenario two	
A_T	Value [%]	$N_{\tilde{t}_1} \cdot 10^{-3}$	Value [%]	$N_{\tilde{t}_1} \cdot 10^{-3}$
A_1^+	-11.5	4.5	15.9	24.8
A_2^+	28.3	1.6	-39.0	8.7
A_3^+	13.8	6.8	-16.3	50.4
$A_1'^+$	-1.3	355.9	5.1	242.2
$A_2'^+$	3.2	124.9	-12.6	84.3
$A_3'^+$	1.6	499.2	-5.3	476.8
A_4^+	-4.7	18.5	6.1	115.0
A_5^+	11.4	9.8	-14.9	60.2
A_1	-6.4	14.8	10.5	57.1
A_2	15.8	5.2	-25.8	20.1
A_3	7.7	21.6	-10.8	114.8

VII. SUMMARY AND CONCLUSION

We have proposed a set of T-odd asymmetries in the decay $\tilde{t}_m \to t\tilde{\chi}_k^0$ with the subsequent decays $t \to bW^+ \to bl\nu$ and $\tilde{\chi}_k^0 \to l^\pm \tilde{l}_n^\mp \to l^\pm l^\mp \tilde{\chi}_1^0$, for $l = e, \mu, \tau$. The asymmetries are based on triple product correlations involving the polarizations of the top quark and the $\tilde{\chi}_k^0$ and arise already at tree-level. All the proposed T-odd asymmetries probe CP violation in the $t - \tilde{t}_m - \tilde{\chi}_k^0$ couplings and are proportional to the product of left- and right-couplings. Since top squark mixing is naturally large due to the large top Yukawa coupling these asymmetries may be large and will allow to determine the CP violating phase ϕ_{A_t}, which is not easily accessible otherwise.

In a numerical study of the T-odd asymmetries we have found that the asymmetry $A_2^\pm$, which is based on the triple product $(\mathbf{p}_t \mathbf{p}_t \boldsymbol{l}_1^\pm)$, is the largest one and its magnitude can go up to 40%, while the others are smaller. We have also found that the asymmetry A_2, Eq. (65), based on $(\mathbf{p}_t \mathbf{p}_t \boldsymbol{l}^\pm)$, where $l^\pm$ can be any of the final leptons $l_1^\pm$ and $l_2^\mp$, distinguished only by their charges, is $\lesssim 26\%$. Moreover, we have made a theoretical estimate of the number of $\tilde{t}_1$ necessary to observe the T-odd asymmetries for two scenarios. Depending on the MSSM parameters, we have found that a $\tilde{t}_1$ production rate of $O(10^3)$ may be sufficient to observe some of the proposed T-odd asymmetries, which could be possible at the LHC or at a future linear collider.

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APPENDIX A: NEUTRALINO MASSES AND MIXING

At tree-level the neutralino mass matrix in the weak basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ is given as [1,2]:

$$\mathcal{M}_N = \begin{pmatrix} |M_1|e^{i\phi_{M_1}} & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -|\mu|e^{i\phi_\mu} \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -|\mu|e^{i\phi_\mu} & 0 \end{pmatrix}, \tag{A1}$$

where ϕ_{M_1} is the phase of M_1, and c_W and s_W are $\cos\theta_W$ and $\sin\theta_W$, respectively. This symmetric complex mass matrix is diagonalized by the unitary 4×4 matrix N:

$$N^* \mathcal{M}_N N^\dagger = \mathrm{diag}(m_{\tilde{\chi}_1^0}, \ldots, m_{\tilde{\chi}_4^0}), \quad 0 \le m_{\tilde{\chi}_1^0} \le \ldots \le m_{\tilde{\chi}_4^0}. \tag{A2}$$

APPENDIX B: MASSES AND MIXING IN SQUARK SECTOR

The left-right mixing of the top squarks is described by a Hermitian 2×2 mass matrix which in the basis $(\tilde{t}_L, \tilde{t}_R)$ reads

$$\mathcal{L}_M^{\tilde{t}} = -(\tilde{t}_L^\dagger, \tilde{t}_R^\dagger)\begin{pmatrix} M^2_{\tilde{t}_{LL}} & e^{-i\phi_{\tilde{t}}}|M^2_{\tilde{t}_{LR}}| \\ e^{i\phi_{\tilde{t}}}|M^2_{\tilde{t}_{LR}}| & M^2_{\tilde{t}_{RR}} \end{pmatrix}\begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \tag{B1}$$

where

$$M^2_{\tilde{t}_{LL}} = M^2_{\tilde{Q}} + (\frac{1}{2} - \frac{2}{3}\sin^2\Theta_W)\cos 2\beta m_Z^2 + m_t^2, \tag{B2}$$

$$M^2_{\tilde{t}_{RR}} = M^2_{\tilde{U}} + \frac{2}{3}\sin^2\Theta_W \cos 2\beta m_Z^2 + m_t^2, \tag{B3}$$

$$M^2_{\tilde{t}_{RL}} = (M^2_{\tilde{t}_{LR}})^* = m_t(A_t - \mu^* \cot\beta), \tag{B4}$$

$$\phi_{\tilde{t}} = \arg[A_{\tilde{t}} - \mu^* \cot\beta], \tag{B5}$$

where $\tan\beta = v_2/v_1$ with $v_1(v_2)$ being the vacuum expectation value of the Higgs field $H_1^0(H_2^0)$, m_t is the mass of the top quark and Θ_W is the weak mixing angle, μ is the Higgs-Higgsino mass parameter, and $M_{\tilde{Q}}$, $M_{\tilde{U}}$, A_t are the soft SUSY-breaking parameters of the top squark system. The mass eigenstates $\tilde{t}_i$ are $(\tilde{t}_1, \tilde{t}_2) = (\tilde{t}_L, \tilde{t}_R)\mathcal{R}^{\tilde{t}T}$ with

$$\mathcal{R}^{\tilde{t}} = \begin{pmatrix} e^{i\phi_{\tilde{t}}}\cos\theta_{\tilde{t}} & \sin\theta_{\tilde{t}} \\ -\sin\theta_{\tilde{t}} & e^{-i\phi_{\tilde{t}}}\cos\theta_{\tilde{t}} \end{pmatrix}, \tag{B6}$$

with

$$\cos\theta_{\tilde{t}} = \frac{-|M^2_{\tilde{t}_{LR}}|}{\sqrt{|M^2_{\tilde{t}_{LR}}|^2 + (m^2_{\tilde{t}_1} - M^2_{\tilde{t}_{LL}})^2}}, \quad \sin\theta_{\tilde{t}} = \frac{M^2_{\tilde{t}_{LL}} - m^2_{\tilde{t}_1}}{\sqrt{|M^2_{\tilde{t}_{LR}}|^2 + (m^2_{\tilde{t}_1} - M^2_{\tilde{t}_{LL}})^2}}. \tag{B7}$$

The mass eigenvalues are

$$m^2_{\tilde{t}_{1,2}} = \frac{1}{2}\left[(M^2_{\tilde{t}_{LL}} + M^2_{\tilde{t}_{RR}}) \mp \sqrt{(M^2_{\tilde{t}_{LL}} - M^2_{\tilde{t}_{RR}})^2 + 4|M^2_{\tilde{t}_{LR}}|^2}\right]. \tag{B8}$$

Note here that for $|A_t| \gg |\mu|\cot\beta$ we have $\phi_{\tilde{t}} \approx \phi_{A_t}$.

[1] H. P. Nilles, Phys. Rep. **110**, 1 (1984); H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985); R. Barbieri, Riv. Nuovo Cimento **11**, 1 (1988).
[2] J. F. Gunion and H. E. Haber, Nucl. Phys. **B272**, 1 (1986); **402**, 567(E) (1993); **B278**, 449 (1986).
[3] D. Chang, W. Y. Keung, and A. Pilaftsis, Phys. Rev. Lett. **82**, 900 (1999);**83**, 3972(E) (1999); A. Pilaftsis, Phys. Lett. B **471**, 174 (1999); D. Chang, W. F. Chang, and W. Y. Keung, Phys. Lett. B **478**, 239 (2000); A. Pilaftsis, Nucl. Phys. **B644**, 263 (2002).

[4] W. Bernreuther and P. Overmann, Z. Phys. C **72**, 461 (1996); B. Grzadkowski, Phys. Lett. B **305**, 384 (1993); E. Christova and M. Fabbrichesi, Phys. Lett. B **320**, 299 (1994); B. Grzadkowski and W. Y. Keung, Phys. Lett. B **316**, 137 (1993); **319**, 526 (1993); A. Bartl, E. Christova, and W. Majerotto, Nucl. Phys. **B460**, 235 (1996); **465**, 365(E) (1996); A. Bartl, E. Christova, T. Gajdosik, and W. Majerotto, Phys. Rev. D **58**, 074007 (1998); S. Bar-Shalom, D. Atwood, and A. Soni, Phys. Rev. D **57**, 1495 (1998); D. Atwood, S. Bar-Shalom, G. Eilam, and A. Soni, Phys. Rep. **347**, 1 (2001); A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer, and O. Vives, Phys. Rev. D **64**, 076009 (2001); E. Christova, S. Fichtinger, S. Kraml, and W. Majerotto, Phys. Rev. D **65**, 094002 (2002); F. Browning, D. Chang, and W. Y. Keung, Phys. Rev. D **64**, 015010 (2001); E. Christova, H. Eberl, W. Majerotto, and S. Kraml, Nucl. Phys. **B639**, 263 (2002);**B647**, 359 (2002).E. Christova, H. Eberl, W. Majerotto, and S. Kraml, J. High Energy Phys. 12 (2002) 021; E. Christova, E. Ginina, and M. Stoilov, J. High Energy Phys. 11 (2003) 027; J. R. Ellis, J. S. Lee, and A. Pilaftsis, Phys. Rev. D **70**, 075010 (2004).

[5] S. Y. Choi and M. Drees, Phys. Lett. B **435**, 356 (1998); Phys. Rev. Lett. **81**, 5509 (1998); M. Aoki and N. Oshimo, Mod. Phys. Lett. A **13**, 3225 (1998); S. Y. Choi, M. Drees, B. Gaissmaier, and J. S. Lee, Phys. Rev. D **64**, 095009 (2001); W. M. Yang and D. S. Du, Phys. Rev. D **65**, 115005 (2002); **67**, 055004 (2003); A. Bartl, T. Kernreiter and O. Kittel, Phys. Lett. B **578**, 341 (2004); S. Y. Choi, M. Drees, B. Gaissmaier, and J. Song, Phys. Rev. D **69**, 035008 (2004).

[6] A. Bartl, K. Hidaka, T. Kernreiter, and W. Porod, Phys. Rev. D **66**, 115009 (2002).

[7] A. Bartl, S. Hesselbach, K. Hidaka, T. Kernreiter and W. Porod, Phys. Rev. D **70**, 035003 (2004).

[8] A. Bartl, K. Hidaka, T. Kernreiter, and W. Porod, Phys. Lett. B **538**, 137 (2002); A. Bartl, S. Hesselbach, K. Hidaka, T. Kernreiter, and W. Porod, hep-ph/0306281; Phys. Lett. B **573**, 153 (2003); T. Gajdosik, R. M. Godbole, and S. Kraml, J. High Energy Phys. 09 (2004) 051.

[9] A. Bartl, T. Kernreiter, and W. Porod, Phys. Lett. B **538**, 59 (2002).

[10] A. J. Barr, Phys. Lett. B **596**, 205 (2004).

[11] W. Bernreuther and O. Nachtmann, Phys. Rev. Lett. **63**, 2787 (1989); **64**, 1072 (1990).

[12] G. Valencia, hep-ph/9411441, and references therein; G. C. Branco, L. Lavoura, and J. P. Silva, *CP violation* (Oxford University Press, New York, 1999).

[13] S. Kawasaki, T. Shirafuji, and S. Y. Tsai, Prog. Theor. Phys. **49**, 1656 (1973).

[14] C. J. Damerell and D. J. Jackson, *1996 DPF / DPB Summer Study on New Directions for High-Energy Physics (Snowmass 96), Snowmass, Colorado, 25 Jun - 12 Jul 1996* (unpublished); SLD Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **88**, 151801 (2002).

[15] S. Y. Choi, H. S. Song, and W. Y. Song, Phys. Rev. D **61**, 075004 (2000).

[16] A. Bartl, H. Fraas, T. Kernreiter, and O. Kittel, Eur. Phys. J. C **33**, 433 (2004).

[17] A. Bartl, H. Fraas, O. Kittel, and W. Majerotto, Phys. Rev. D **69**, 035007 (2004); Eur. Phys. J. C **36**, 233 (2004).

[18] A. Czarnecki, M. Jezabek, and J. H. Kuhn, Nucl. Phys. **B351**, 70 (1991); M. Jezabek, Nucl. Phys. (Proc. Suppl.) **37B**, 197 (1994); M. Fischer, S. Groote, J. G. Korner, and M. C. Mauser, Phys. Rev. D **65**, 054036 (2002); A. Brandenburg, Z. G. Si, and P. Uwer, Phys. Lett. B **539**, 235 (2002).

[19] Particle Data Group Collaboration, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).