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Fermion masses in supersymmetric SO(10) with type II seesaw mechanism: A nonminimal predictive scenario

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A predictive framework for fermion masses and mixing is given by the supersymmetric SO(10) model with one 10_H, one $\overline{126}_H$, one 126_H, and one 210_H Higgs representations, and type-II seesaw dominating the neutrino mass matrix. We investigate the origin of the tension between this model and lepton mixing data and refine previous numerical analyses. We discuss an extension of the minimal model that includes one 120_H chiral superfield representation. This exhausts the possible renormalizable contributions to the Yukawa sector. In spite of the increase in the number of parameters the predictivity of the minimal setting is not spoiled. We argue that the contributions to fermion masses due to the doublet components of 120_H can be naturally small compared to those of 10_H and $\overline{126}_H$, thus acting as a perturbation in the fermion mass generation. The antisymmetric nature of the 120_H Yukawa coupling affects at leading order the determination of the mixing angles and it allows to remove the inconsistencies between predictions and data on the neutrino parameters. An improvement in the experimental bound on U_{e3} can tell this scenario from the minimal model.

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I. INTRODUCTION

Remarkably enough only a few years after the last run of LEP, which has marked the era of precision electroweak laboratory tests, neutrino physics is reaching the age of precision studies (for a recent review see Ref. [1]). Neutrinos being massive, flavor mixing is present in the leptonic sector as well, underlying, together with the known mass differences, the phenomenon of neutrino oscillations. The smallness of their masses compared to the other fermions and the evidence that the shape of the leptonic mixings differs substantially from the hierarchical structure of the quark mixings add a challenge in understanding the origin of the fermionic spectrum. Even more so when considering grand unified scenarios where lepton and quark Yukawa couplings are related by gauge symmetry.

Some interest has been revived in the recent years on a supersymmetric (SUSY) implementation of SO(10) which has a minimal number of parameters [2,3] [as many as the Minimal Supersymmetric Standard Model (MSSM) with massive neutrinos and exact R-parity] and exhibits a remarkable predictivity in the neutrino sector linking the large values of the atmospheric mixing to the $b - \tau$ Yukawa unification [4,5].

The minimal renormalizable SUSY SO(10) model [6,7] contains in addition to three generations of 16_F matter supermultiplets the following Higgs chiral supermultiplets: 10_H, $\overline{126}_H$, 126_H, and 210_H. The 10_H and $\overline{126}_H$ representations couple to the matter bilinear $16_{Fi}16_{Fj} = (10_S + 126_S + 120_A)_{ij}$ in the superpotential leading to the minimal set of Yukawa couplings needed for a realistic fermion mass spectrum [8] (S, A denote the symmetry property in the generation indices i, j). The 126_H representation is needed in order to preserve supersymmetry from D-term breaking, while the 210_H triggers the SO(10) gauge symmetry breaking and provides the needed mixings among the Higgs supermultiplets. Besides leading to a realistic matter mass spectrum, the model features exact R-parity conservation, due to the even congruency class ($B - L = 2$) of the 10 and 126 representations (120 shares the same property), with relevant implications for cosmology and proton decay [9–15].

The smallness of the neutrino masses naturally follows from the seesaw mechanism which is present in a twofold type:

$$M_\nu = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L}. \tag{1}$$

The first term represents the canonical (type I) seesaw [16]. The Majorana mass matrix M_{ν_R} is generated by the vacuum expectation value (VEV) of a $SU(2)_R$ triplet field in $\overline{126}_H$, while M_{ν_D} is the Dirac neutrino mass matrix. The second term (type II seesaw [17]) is present due to a very small VEV (proportional to the square of the electroweak scale over the GUT scale) induced by the weak scale breaking on the $SU(2)_L$ triplet component in $\overline{126}_H$.

Assuming the dominance of type II seesaw one finds an intriguing connection [4,5] between $b - \tau$ unification (which can be approximatively achieved even in the presence of $\overline{126}_H$) and the almost maximal atmospheric neutrino mixing angle ($\sin^2 2\theta_{23} \gtrsim 0.9$ at 90% C.L. [18]). On the other hand, the detailed numerical analyses which have been carried out in Refs. [19,20] show a number of

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possible shortcomings. First, the 1-3 lepton mixing is bound to be quite close to the present experimental upper bound ($U_{e3} \lesssim 0.2$ at 90% C.L. [21]). Second, the solar mixing angle θ_{12} is predicted too close to maximal, while the SNO result, $\theta_{12} = 32.5^{+2.4}_{-2.3}$ degrees [22], definitely excludes this possibility. Third, the deviation of atmospheric mixing θ_{23} from maximal is too large. When the effect of CP phases is taken into account, only the solar mixing is significantly affected and its fitted value can be in agreement with data, while U_{e3} still cannot be smaller than ≈ 0.16 [20]. On the other hand, the Cabibbo-Kobayashi-Maskawa (CKM) phase is predicted to be in the second or third quadrant requiring significant contributions to CP violation from other sources [23].

Type I seesaw has been also investigated within the SUSY SO(10) scenario [24–26]. Even though one may recover in some limit the type II seesaw relation between $b - \tau$ Yukawa unification and large neutrino atmospheric mixing, a dominant type I seesaw contribution is highly disfavored by the global fit of neutrino data, unless nonrenormalizable terms are added to the minimal model [23,27].

In the first part of this paper (Section II) we present an independent study of the minimal renormalizable SUSY SO(10) model with dominant type II seesaw. We work out simple analytic arguments to explain the origin of the tension between the fermion mass sum rules of the model and the experimental values of the lepton mixing. A new numerical fit is presented, including some experimental uncertainties previously neglected. We refine the results in the literature, albeit confirming some of the shortcomings of the model in reproducing the detailed structure of the neutrino parameters.

In order to improve the agreement with the data, one may certainly consider extensions of the minimal SO(10) model, that include additional 10_H and/or $\overline{126}_H$ representations. However, the presence of the new set of Yukawa couplings spoils the predictivity of the minimal model for fermion mass textures. A more interesting option is considering the presence of Planck-induced nonrenormalizable operators that make the SO(10) model an effective theory at the GUT scale. The presence of the new Yukawa terms leads to some additional (M_G/M_{Pl} suppressed) contributions in the fermion mass sum rules. On the other hand, even though the size of the corrections is under control, the breakdown of renormalizability allows for a number of possible effective scenarios whose effects generally overlap, thus weakening again the predictive feature of the minimal model. In Ref. [23] the authors choose to consider effective operators that transform as $\overline{1}26$ thus maintaining the symmetry property of the renormalizable Yukawa terms. Because of that the fermion mass relations present strong similarities with the minimal model, whose predictions remain to a large extent unmodified, while obtaining some of the desired improvements.

In this paper we take the standpoint of maintaining renormalizability, while considering the effects of adding to the minimal model content a 120_H supermultiplet. All possible renormalizable contributions to the Yukawa sector are present (an early discussion can be found in [28], more recently the case of type I seesaw was addressed in [29]). We argue (Section III) that the induced VEVs of the bidoublet components of 120_H can be naturally suppressed compared to the weak scale. This allows for treating the 120_H contributions to fermion masses as a perturbation, thus preserving most of the features of the minimal model. On the other hand, due to the different symmetry properties of the new Yukawa couplings, we show that to leading order in the fermion mass corrections an excellent fit of the neutrino parameters is obtained. Future neutrino data may provide a test of this scenario and discriminate it with respect to the minimal one.

The analytic tools for the study of the 120_H contributions to the fermion mass textures are developed and discussed in Section IV, while the numerical results and the experimental signature of the extended renormalizable SO(10) model are presented in Section V.

We limit the present analysis to real Yukawa coupling. This choice allows to clearly evaluate what is the weight of each quark (lepton) mass or mixing angle in the fit. In fact, previous studies [20,23] show that the effect of the phases is subleading and adds only a minor freedom in reproducing the data. Finally, the aim of the present discussion is to emphasize the role of symmetry and size of the 120_H corrections in predicting a realistic fermion mass spectrum. The investigation of CP violation and possible connections between the CKM phase and the CP phases in the lepton sector is left for future work.

II. FERMION MASSES AND MIXING IN THE MINIMAL SUSY *SO*(10) WITH TYPE-II SEESAW

As mentioned in the previous section, the Higgs sector of the minimal renormalizable SUSY SO(10) model includes in addition to 10_H, the $\overline{126}_H$, 126_H, and 210_H representations. The 10_H and $\overline{126}_H$ representations couple to the matter bilinear $16_F 16_F$ in the superpotential leading to the minimal set of Yukawa couplings needed for a realistic fermion mass spectrum. In this case the fermion mass matrices are given by [8,30]

$$M_u = Y_{10} v_u^{10} + Y_{126} v_u^{126}, \qquad M_d = Y_{10} v_d^{10} + Y_{126} v_d^{126},$$
$$M_l = Y_{10} v_d^{10} - 3 Y_{126} v_d^{126}, \qquad M_\nu \propto Y_{126}, \tag{2}$$

where dominant type-II seesaw has been assumed for the neutrino mass matrix. In general, the two light Higgs isodoublets $H_{u,d}$ are a linear combination of the scalar isodoublets contained in 10 and $\overline{1}26$ representations as well as of those contained in the other Higgs representations not coupled to fermions. However, Eq. (2) is valid independently on the composition of $H_{u,d}$, that does not affect the following analysis. The only constraint is $\sum_i |v_i|^2 = (174\text{ GeV})^2$, where the sum runs over all the isodoublets of the model.

The choice of type II seesaw is motivated by the connection between approximate $b-\tau$ unification at the GUT scale and almost maximal atmospheric mixing [4,27]. In fact, Eq. (2) implies

$$k\tilde{M}_l = \tilde{M}_u + r\tilde{M}_d \qquad M_\nu \propto M_l - M_d, \tag{3}$$

where $\tilde{M}_l \equiv M_l/m_\tau$, $\tilde{M}_u \equiv M_u/m_t$, and $\tilde{M}_d \equiv M_d/m_b$, while k and r are functions of the VEVs in Eq. (2) and of m_τ, m_t, m_b. Considering only the 2-3 blocks and neglecting also second generation masses, one can extract the relation between 2-3 quark and lepton mixings [4]:

$$\tan 2\theta_{23} \approx \frac{2\sin\theta_{23}^q}{2\sin^2\theta_{23}^q - \frac{m_b - m_\tau}{m_b}} \qquad (\theta_{23}^q \approx 0.04).$$

Clearly, large atmospheric mixing requires cancellation between m_b and m_τ. However, a complete three generation fit of fermion masses and mixing at the GUT scale is highly nontrivial. In this paper we will limit our analysis to the case of no CP violation, where all mixing matrices and mass eigenvalues are real.

A. Understanding analytically the constraints on the lepton mixing

The standard approach [19,20] to decipher the predictions of this model is the following. Let us rewrite Eq. (3) on the basis where M_d is diagonal:

$$\tilde{M}'_l \equiv U_d^T \tilde{M}_l U_d = \frac{1}{k}(V_{CKM}^T \tilde{D}_u V_{CKM} + r\tilde{D}_d), \tag{4}$$

$$M'_\nu \equiv U_d^T M_\nu U_d = m_0\left[\tilde{M}'_l - \frac{m_b}{m_\tau}\tilde{D}_d\right], \tag{5}$$

where $\tilde{D}_u = \mathrm{diag}(\frac{m_u}{m_t}, \frac{m_c}{m_t}, 1)$, $\tilde{D}_d = \mathrm{diag}(\frac{m_d}{m_b}, \frac{m_s}{m_b}, 1)$, V_{CKM} is the Cabibbo-Kobayashi-Maskawa matrix and m_0 is an overall neutrino mass scale. All parameters are to be evaluated at the GUT scale, since the relations in Eq. (2) are derived at the scale where the unified gauge group is broken to the Standard Model (SM).

The uncertainties in quark masses and mixing as well as the parameters k and r can be used to fit the charged lepton masses via Eq. (4). The equality of the traces of the left-hand side (LHS) and the right-hand side (RHS) implies $k = 1 + r + \mathcal{O}(\lambda^2)$ (where $\lambda \approx 0.22$ is the Cabibbo angle). It turns out (see e-print of Ref. [19]) that a better fit of the atmospheric mixing is obtained for $m_s < 0$ and $m_\mu > 0$ (once third generation masses are chosen positive). In this case, the requirement of reproducing the correct value of m_μ/m_τ leads to $k \approx 0.25$ and $r \approx -0.75$. Given r and k the RHS of Eq. (5) is completely determined and defines the neutrino mass matrix up to an overall mass scale.

For the purpose of a simple analytical understanding of the predictions in the lepton sector, we assume for the time being $k = 0.25$ and $r = -0.75$ exactly. Detailed variations in k and r are taken into account in the numerical fit discussed in Section II B. Using the Wolfenstein parametrization of V_{CKM} and neglecting $\mathcal{O}(\lambda^4)$ terms we obtain

$$\tilde{M}'_l \equiv U_l \tilde{D}_l U_l^T = \begin{pmatrix} 0 & 0 & 4V_{td} \\ \dots & -3\frac{m_s}{m_b} & 4V_{ts} \\ \dots & \dots & 1 \end{pmatrix}, \qquad M'_\nu \equiv U_\nu D_\nu U_\nu^T = m_0 \begin{pmatrix} 0 & 0 & 4V_{td} \\ \dots & -\left(\frac{m_b}{m_\tau} + 3\right)\frac{m_s}{m_b} & 4V_{ts} \\ \dots & \dots & 1 - \frac{m_b}{m_\tau} \end{pmatrix}, \tag{6}$$

where we use the convention in which V_{ts} and m_s are negative and V_{td}, m_b, and m_τ positive. Notice that the "parallel" structure of $\tilde{M}'_l$ and M'_ν is "broken" by $b-\tau$ unification. It is impressive that the structure of these matrices reflects qualitatively the basic features of lepton masses and mixings: $\tilde{M}'_l$ is hierarchical with small mixing angles and

$$\frac{m_\mu}{m_\tau} \approx -3\frac{m_s}{m_b} - 16V_{ts}^2.$$

Both large 1-2 and 2-3 mixing should be contained in M'_ν: this is the case since the elements in the 2-3 block of M'_ν can be taken of the same order (dominant $\mu\tau$-block) and the 1-3 element is automatically smaller. As a consequence [31], the spectrum of neutrinos is predicted to be with normal hierarchy.

In Ref. [27] the exact computation of the leptonic 2-3 mixing is performed for the present model in the case of two generations. The authors find two solutions for large mixing. One corresponds to the scenario described above: $\tilde{M}'_l$ hierarchical and $b-\tau$ unification inducing large 2-3 mixing in M'_ν. The second solution corresponds to $r \approx -1$ (and $|k| \ll 1$), leading to a cancellation in the 33-entry of $\tilde{M}'_l$. However, it can be easily shown that this possibility is spoiled by a three generation analysis. In fact, we find that in this case the charged lepton mass matrix has the form

$$\tilde{M}'_l|_{r\approx -1} = -\frac{m_b}{m_s a}\begin{pmatrix} 0 & 0 & V_{td} \\ \dots & -\frac{m_s}{m_b} & V_{ts} \\ \dots & \dots & -\frac{V_{ts}^2 m_b}{m_s} \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{7}$$

where $a \approx (1 + V_{ts}^2 m_b^2/m_s^2)$ is of order unity. This structure can be suitable to generate a small m_μ/m_τ in the case of two generations, but since the determinant of $\tilde{M}'_l$ is of order λ^2, it is clear that the hierarchy $m_e \div m_\mu \div m_\tau \approx \lambda^5 \div \lambda^2 \div 1$ cannot be reproduced.

Let us analyze in some detail Eq. (6). The matrices $\tilde{M}_l^l$ and M_ν^l depend only on four quark parameters (V_{td}, V_{ts}, m_s/m_b, m_b/m_τ) and they are required to reproduce five lepton parameters (m_μ/m_τ, $\Delta m_\odot^2/\Delta m_A^2$, θ_{12}, θ_{13}, and θ_{23}), where θ_{ij} are the mixing angles in the lepton mixing matrix $U_{PMNS} \equiv U_l^T U_\nu$. The first generation masses m_e and m_1 are sensitive also to subleading terms neglected in Eq. (6). Since the quark parameters are known with small uncertainties, there is very small freedom to fit lepton data. Notice that, in good approximation, we can compare directly the neutrino masses and mixing angles obtained from Eq. (6) at the GUT scale with the experimental values at the electroweak scale. As a matter of fact, in the case of normal hierarchy, the running of the neutrino mass matrix has a negligible effect on mass squared differences and mixings [32–34].

For the mixing angles in $\tilde{M}_l^l$ and M_ν^l we use the notation $c_{ij}^{l,\nu} \equiv \cos\theta_{ij}^{l,\nu}$ and $s_{ij}^{l,\nu} \equiv \sin\theta_{ij}^{l,\nu}$. Since $4m_s/m_b \sim 4V_{ts} \lesssim \lambda$, the 2-3 mixing in M_ν^l is generically of order unity if $b-\tau$ unification is realized to $\lambda \div \lambda^2$ accuracy. In general, the deviation from maximal mixing increases with $\Delta m_\odot^2/\Delta m_A^2$. The other two mixing angles in M_ν^l are given approximately by

$$\begin{aligned} s_{13}^\nu &\approx (c_{23}^\nu)^3 \frac{4V_{td}}{1-\frac{m_b}{m_\tau}} = \mathcal{O}(\lambda), \\ \sin 2\theta_{12}^\nu &\approx s_{23}^\nu (c_{23}^\nu)^2 \frac{8V_{td}}{1-\frac{m_b}{m_\tau}} \sqrt{\frac{\Delta m_A^2}{\Delta m_\odot^2}} = \mathcal{O}(1), \end{aligned} \tag{8}$$

where we used $\sqrt{\Delta m_\odot^2/\Delta m_A^2} \sim \lambda$. If one neglects the small mixing in $\tilde{M}_l^l$, all oscillation data can be reproduced. For example, taking $m_b/m_\tau = 0.89$, $V_{ts} = -0.035$, $V_{td} = 0.011$, $m_s/m_b = -0.028$, one obtains $s_{13}^\nu \approx 0.12$, $\tan\theta_{23}^\nu \approx 0.97$, $\tan^2\theta_{12}^\nu \approx 0.43$, and $\Delta m_\odot^2/\Delta m_A^2 \approx 0.038$.

However, it turns out that the small mixing angles in $\tilde{M}_l^l$ add up to those in M_ν^l in such a way to spoil the agreement with data. In fact, we find

$$\begin{aligned} \theta_{23}^l &\approx 4V_{ts} \approx -0.14 \sim -\lambda, \\ \theta_{13}^l &\approx 4V_{td} \approx 0.04 \sim \lambda^2, \\ \theta_{12}^l &\approx -\frac{16V_{ts}V_{td}}{3m_\mu/m_\tau} \approx 0.10 \sim \lambda, \end{aligned} \tag{9}$$

where the numerical estimates are obtained from the input values at the end of the previous paragraph. The effect of the two $O(\lambda)$ rotations in U_l modifies the physical mixing angles in $U_{PMNS} \equiv U_l^T U_\nu$ as follows:

$$\begin{aligned} \theta_{23} &\approx \theta_{23}^\nu + \theta_{23}^l, \\ s_{13} &\approx s_{13}^\nu + s_{12}^l s_{23}^\nu, \\ \sin 2\theta_{12} &\approx \sin 2\theta_{12}^\nu \left(1 + s_{12}^l \frac{2c_{23}^\nu}{\tan 2\theta_{12}^\nu}\right). \end{aligned} \tag{10}$$

As a consequence, to reproduce data one needs θ_{23}^ν larger than $\pi/4$, s_{13}^ν significantly below the experimental upper bound and $\sin 2\theta_{12}^\nu$ smaller than the solar mixing value. Both the deviation from $\theta_{23}^\nu = \pi/4$ and the suppression of $\sin 2\theta_{12}^\nu$ tend to increase the ratio $\Delta m_\odot^2/\Delta m_A^2$ above the allowed range, producing a tension between predictions and experimental data, as confirmed by the numerical study that follows.

B. Discussion of the numerical results

Previous numerical analysis of fermion masses and mixing in SUSY SO(10) with 10 and $\overline{1}26$ Higgs fields coupled to matter and type II seesaw dominance are given in Refs. [19,20]. All studies find a tension between lepton mixings and quark parameters: s_{13} turns out to be close to the present upper bound (≈ 0.16), the atmospheric mixing can be hardly close enough to maximal ($\sin^2 2\theta_{23} \lesssim 0.9$) and the solar mixing is too large to fit the Large Mixing Angle MSW solution ($\sin^2 2\theta_{12} \gtrsim 0.9$). The last drawback can be relaxed tuning CP violating phases, but in disagreement with the known value of the CKM phase [20,23].

We have run an independent fit of the experimental data paying particular attention to the uncertainties in the input parameters. It is in fact crucial to determine how far the minimal SO(10) scenario can be pushed in reproducing the known fermion spectrum and mixings. Because of the complexity of the numerical analysis, we have taken advantage of the analytical results derived in the previous section to elaborate an efficient approach to the fit, while obtaining a rationale for the emerging patterns.

We input the GUT scale values of quark masses given in Ref. [35] for two typical values of $\tan\beta$, namely $\tan\beta = 10$ and $\tan\beta = 55$, and consider both 1- and 2-σ ranges.

Our numerical fit confirms the results of Ref. [20] for the central values of the quark mixing angles and $\Delta m_\odot^2/\Delta m_A^2 \lesssim 0.05$ there considered. In this case we find $\sin^2 2\theta_{23} \lesssim 0.93$, $|U_{e3}| \approx 0.16$, and $\sin^2 2\theta_{12} \gtrsim 0.92$, the latter being excluded at the 90% C.L. When we include the 1-σ uncertainties in the V_{CKM} entries [36] and allow for $0.019 \leq \Delta m_\odot^2/\Delta m_A^2 \leq 0.069$ (the 90% C.L. experimental range [18,22,37]) we do not find any major deviation due to the V_{CKM} entries, the largest effects being related to the extended $\Delta m_\odot^2/\Delta m_A^2$ range. For $\tan\beta = 10$ larger values (albeit not maximal) of the atmospheric neutrino mixing angle are allowed, the upper bound being $\sin^2 2\theta_{23} \lesssim 0.97$, together with a reduced solar mixing, namely $\sin^2 2\theta_{12} \gtrsim 0.85$ (the extreme values are obtained for $\Delta m_\odot^2/\Delta m_A^2$ close to the 90% C.L. upper bound). The predictions for $|U_{e3}|$ are not significantly modified: $|U_{e3}| \approx 0.16$. For $\tan\beta = 55$ the results are quite similar: the upper bound for the atmospheric mixing is reduced to $\sin^2 2\theta_{23} \lesssim 0.95$, while the solar angle lower bound is relaxed to $\sin^2 2\theta_{12} \gtrsim 0.82$. The $|U_{e3}|$ parameter is bound to be about 0.15.

Only when the scan is performed over the 2-σ ranges of the quark sector parameters, maximal atmospheric mixing is allowed, while the lower bound for $|U_{e3}|$ can be at most reduced to about 0.12 and the solar mixing angle can be lowered to 0.75 (such values are reached for large $\tan\beta$ and for $\Delta m^2_{\odot}/\Delta m^2_A$ close to the 90% C.L. upper bound).

As pointed out in Ref. [27], the two neutrino analysis suggests the possible relevance of the parameter region characterized by $r \approx -1$ (corresponding to the solution $\sigma = +1$ in the notation of Ref. [27]), where the atmospheric mixing may be *naturally* large. However, we have checked that in this domain one cannot recover a good fit of the electron mass, in full agreement with our argument after Eq. (7). In Section V we display some graphics of the results here discussed compared to those of the extended model introduced in the next section.

In conclusion our numerical analysis confirms the patterns found by previous authors and analytically discussed in the previous subsection. The minimal renormalizable SUSY SO(10) model, while providing a suggestive and appealing framework for understanding the main features of the quark and lepton spectra, fails in reproducing the data at the present 1-σ experimental accuracy. When considering the 2-σ experimental ranges, agreement with the data is obtained in limited regions of the parameter space. This motivated us to study the effects of extending the minimal model to include one 120_H representation.

III. 120_H-EXTENSION OF THE MINIMAL RENORMALIZABLE SUSY SO(10)

In this section we discuss how the inclusion of one 120_H representation in the minimal renormalizable SUSY SO(10) may affect the electroweak breaking pattern. Using a simplified and self-explanatory notation the superpotential of the extended model reads:

$$\begin{aligned} W_Y &= 16_F(Y_{10}10_H + Y_{126}\overline{126}_H + Y_{120}120_H)16_F, \\ W_H &= M_{10}10_H^2 + M_{126}\overline{126}_H 126_H + M_{210}210_H^2 + M_{120}120_H^2 + \lambda 210_H^3 + \eta 210_H\overline{126}_H 126_H + 10_H 210_H(\alpha 126_H \\ &\quad + \beta\overline{126}_H) + \eta' 210_H 120_H 120_H + \gamma 10_H 210_H 120_H + 120_H 210_H(\alpha' 126_H + \beta'\overline{126}_H), \end{aligned} \tag{11}$$

where the 3×3 matrices (in general complex) Y_{10} and Y_{126} are symmetric while Y_{120} is antisymmetric.

We do not report in detail the Higgs potential derived from Eq. (11), the D-terms and the scalar soft breaking terms. Since 120_H does not contribute to SO(10) breaking, we may assume that the correct breaking pattern to the minimal supersymmetric standard model (MSSM) is achieved [2,3]. SO(10) can be broken spontaneously down to the MSSM either directly or via one or more intermediate steps [38,39]. For the discussion that follows it is convenient to write down the explicit decomposition of the SO(10) Higgs representations under the Pati-Salam (PS) subgroup $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \equiv G_{422}$:

$$\begin{aligned} 10 &= (1,2,2) \oplus (6,1,1)\overline{1}26 = (15,2,2) \oplus (\overline{1}0,3,1) \oplus (10,1,3) \oplus (6,1,1)120 \\ &= (1,2,2) \oplus (15,2,2) \oplus (\overline{1}0,1,1) \oplus (10,1,1) \oplus (6,3,1) \oplus (6,1,3)210 \\ &= (15,1,3) \oplus (15,3,1) \oplus (\overline{1}0,2,2) \oplus (10,2,2) \oplus (6,2,2) \oplus (15,1,1) \oplus (1,1,1) \end{aligned} \tag{12}$$

It is also helpful to recall the $SU(3)_c \otimes U(1)_{B-L}$ decomposition of the $SU(4)_{PS}$ multiplets (with standard model $B-L$ normalization):

$$\begin{aligned} 6 &= (3,-2/3) \oplus (\overline{3},+2/3), \\ 10 &= (6,+2/3) \oplus (3,-2/3) \oplus (1,-2), \\ 15 &= (8,0) \oplus (3,+4/3) \oplus (\overline{3},-4/3) \oplus (1,0). \end{aligned} \tag{13}$$

A nonvanishing VEV of the $(1,1,1)_{210}$ ($(15,1,1)_{210}$) component of 210_H triggers the breaking of SO(10) down to G_{422} (G_{3221}). We denote the scale of this spontaneous breaking by M_G. The subsequent left-right (LR) symmetry breaking step to the MSSM group G_{321} is achieved at the scale $M_R \leq M_G$ by VEVs of $(15,1,3)_{210}$, $(\overline{1}0,1,3)_{126}$, and $(10,1,3)_{\overline{1}26}$. Since the $B-L$ charge of the color singlets contained in 10 (and 15) of $SU(4)_{PS}$ is even, R-parity is preserved. The study of gauge coupling unification in SUSY SO(10) favors the scenario of the direct $SO(10) \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_Q$ breaking chain (see, e.g., Ref. [3]). As a consequence we will henceforth take $M_R \approx M_G \approx 10^{16}$ GeV.

The final electroweak breaking step is obtained by the VEVs induced by weak scale SUSY soft-breaking terms on the light left-handed (LH) doublets obtained from the colorless components of the bidoublets present in Eq. (12). Since 210_H mixes 126_H, $\overline{126}_H$, as well as 120_H, with 10_H one expects that all the color singlet LH doublets mix to give (via fine tuning) the two light Higgs doublet superfields of the MSSM, leaving the other states heavy.

In this respect the bidoublet components in the 120_H representation may exhibit a specific feature. Since no 120_H component participates at the spontaneous breaking of SO(10) down to the MSSM group we may consider the value of its mass parameter M_{120} in Eq. (11) as taken at the cutoff scale of the model, i.e., the Planck mass M_{Pl} [40], without requiring any fine tuning among the mass and the couplings in the potential. As a consequence of this assumption, one expects decoupling effects of 120_H proportional to $M_G/M_{Pl} \approx 10^{-3}$. In particular the 120_H colorless $SU(2)_L$ doublet components acquire an induced VEV suppressed by the above factor with respect to the doublets contained in the other representations.

Relations among the VEVs of the relevant components can be obtained, neglecting explicit soft SUSY breaking, from the F-term (and D-term) flatness of the supersymmetric vacuum, i.e., by requiring $\langle F_X\rangle = \langle \partial W/\partial X\rangle = 0$ for any superfield X in the superpotential, replaced by its scalar component.

Considering the SO(10) superpotential in Eq. (11) and its decomposition in terms of G_{422}, of which some relevant terms are

$$\begin{aligned}
126_H\overline{126}_H210_H &= (\overline{10},1,3)_{126}(10,1,3)_{\overline{126}}(15,1,1)_{210} + \ldots,\\
10_H120_H210_H &= (15,2,2)_{120}(1,2,2)_{10}(15,1,1)_{210} + (1,2,2)_{120}(1,2,2)_{10}(1,1,1)_{210} + \ldots,\\
10_H126_H210_H &= (15,2,2)_{126}(1,2,2)_{10}(15,1,1)_{210} + (10,3,1)_{126}(1,2,2)_{10}(\overline{10},2,2)_{210} + \ldots,\\
10_H\overline{126}_H210_H &= (10,1,3)_{\overline{126}}(1,2,2)_{10}(\overline{10},2,2)_{210} + \ldots,
\end{aligned}$$

the vacuum F-flatness in the 120_H bidoublet directions yields

$$\begin{aligned}
\langle(15,2,2)_{120}\rangle &\sim \frac{M_R^2}{M_{120}M_{210}}\langle(1,2,2)_{10}\rangle,\\
\langle(1,2,2)_{120}\rangle &\sim \frac{M_G}{M_{120}}\langle(1,2,2)_{10}\rangle,
\end{aligned} \tag{14}$$

where $O(1)$ couplings are assumed and $\langle(\overline{10},1,3)_{126}\rangle = \langle(10,1,3)_{\overline{126}}\rangle \sim M_R$ (as required by D-flatness at M_R).

In an analogous way for the colorless $\overline{126}_H$ LH components one obtains

$$\begin{aligned}
\langle(15,2,2)_{\overline{126}}\rangle &\sim \frac{M_R^2}{M_{126}M_{210}}\langle(1,2,2)_{10}\rangle,\\
\langle(\overline{10},3,1)_{\overline{126}}\rangle &\sim \frac{M_R}{M_{210}}\frac{\langle(1,2,2)_{10}\rangle^2}{M_{126}},
\end{aligned} \tag{15}$$

Notice in Eq. (15) the very small VEV induced on the $\overline{126}_H$ LH triplet by the weak breaking, leading in the SUSY case [10,11,41] to the type II seesaw term in Eq. (1).

Considering the one-step breaking of SO(10) ($M_R \sim M_G$) from Eqs. (14)–(15) and the assumption $M_{120} \sim M_{Pl}$ one obtains that the 120_H LH doublet VEVs are suppressed by $O(M_G/M_{Pl})$ with respect to those in 10_H and $\overline{126}_H$. Since this result is controlled by the decoupling of the 120_H representation, the suppression of the 120_H VEVs is not spoiled by the soft SUSY breaking potential which triggers at the weak scale the $SU(2)_L \times U(1)_Y$ breaking. After the needed minimal fine tuning, the two light Higgs doublets have 120_H components suppressed by $O(M_G/M_{Pl})$ so that on the broken vacuum Eq. (14) is reproduced. This feature represents the basic ingredient in the following discussion of fermion masses and mixings.

The role of 120_H can be replaced by Planck-scale induced nonrenormalizable operators which transform accordingly. On the other hand, *ad hoc* assumptions on the ultraviolet completion of the model and on the symmetry properties of the effective couplings are generally needed in order to reproduce the minimal (renormalizable) setup here discussed.

IV. 120_H-CORRECTIONS TO FERMION MASSES AND MIXING

The most general structure of the fermion mass matrices in the renormalizable SO(10) model with all possible types of Higgs fields coupled to fermions is given by [28]

$$\begin{aligned}
M_u &= Y_{10}v_u^{10} + Y_{126}v_u^{126} + Y_{120}v_u^{120},\\
M_d &= Y_{10}v_d^{10} + Y_{126}v_d^{126} + Y_{120}v_d^{120},\\
M_l &= Y_{10}v_d^{10} - 3Y_{126}v_d^{126} + Y_{120}v_l^{120},\\
M_\nu &\propto Y_{126},
\end{aligned} \tag{16}$$

where the VEVs v_x^{120} are three independent linear combinations of the four 120_H isodoublet VEVs, and type II seesaw is assumed to dominate in M_ν. Motivated by the discussion in the previous section, we take $v_x^{120}/v_x^{10\,126} \sim M_G/M_{Pl}$, such that the 120_H contributions to the mass matrices can be treated as a small perturbation.

The analogue of Eq. (3) now reads

$$\begin{aligned}
k\tilde{M}_l &= \tilde{M}_u + r\tilde{M}_d + Y_{120}(k\varepsilon_l - \varepsilon_u - r\varepsilon_d),\\
M_\nu &\propto [M_l - M_d + Y_{120}(m_b\varepsilon_d - m_\tau\varepsilon_l)],
\end{aligned} \tag{17}$$

with the shorthand notation

$$\varepsilon_u \equiv \frac{v_u^{120}}{m_t}, \qquad \varepsilon_d \equiv \frac{v_d^{120}}{m_b}, \qquad \varepsilon_l \equiv \frac{v_l^{120}}{m_\tau}.$$

The mass matrices of charged fermions are asymmetric and can be diagonalized by means of a biorthogonal transformation ($M_x = V_x^R D_x V_x^{LT}$, $x = u, d, l$), so that

$$kV_d^{RT}\tilde{M}_lV_d^L = W^T\tilde{D}_uV_{CKM} + r\tilde{D}_d + Y'_{120}(k\varepsilon_l - \varepsilon_u - r\varepsilon_d), \tag{18}$$

where

$$W \equiv V_u^{RT}V_d^R, \quad V_{CKM} \equiv V_u^{LT}V_d^L, \quad Y'_{120} \equiv V_d^{RT}Y_{120}V_d^L. \tag{19}$$

The missing ingredient needed to perform the fitting procedure of charged lepton masses, in analogy to the minimal model case, is the right-handed quark mixing matrix W. Since for $\varepsilon_x = 0$ one has $W = V_{CKM}$, it is convenient to write W as V_{CKM} plus order ε_x corrections. One obtains (see Appendix A)

$$W = V_{CKM} + 2(-\varepsilon_u Z'_u V_{CKM} + \varepsilon_d V_{CKM} Z'_d) + \mathcal{O}(\varepsilon_x^2), \tag{20}$$

where the antisymmetric matrices $Z'_{u,d}$ are given by

$$(Z'_x)_{ij} = \frac{(Y'_x)_{ij}}{(\tilde{D}_x)_{ii} + (\tilde{D}_x)_{jj}} \tag{21}$$

and $Y'_u \equiv V_{CKM} Y'_{120} V^T_{CKM}$, $Y'_d \equiv Y'_{120}$.

As shown in Appendix A, the antisymmetry of Y_{120} implies that the eigenvalues of the symmetric mass matrices are unmodified up to $\mathcal{O}(\varepsilon_x^2)$ corrections. This suggests that the 120_H induced mass corrections may affect at the leading order the determination of the mixing angle in such a way not to destabilize the good fit of the mass eigenvalues obtained in the minimal model. This feature is relevant for understanding qualitatively the numerical discussion presented in the next section (see also Appendix B).

The type II neutrino mass matrix now reads

$$\begin{aligned} V_d^{LT} M_\nu V_d^L = m_0 V_d^{LT} V_d^R \bigg[& V_d^{RT} \tilde{M}_l V_d^L - \tfrac{m_b}{m_\tau} \tilde{D}_d \\ & + Y'_{120}\Big(\tfrac{m_b}{m_\tau}\varepsilon_d - \varepsilon_l\Big)\bigg], \end{aligned} \tag{22}$$

where m_0 is an overall neutrino mass scale. Using $V_d^{LT} V_d^R \approx 1 + 2\varepsilon_d Z'_d$ and $V_d^{LT} V_u^R = V^T_{CKM}(1 + 2\varepsilon_u Z'_u)$ (see Appendix A), as well as Eq. (21), one obtains

$$V_d^{LT} M_\nu V_d^L = M'_\nu + \Delta M'_\nu, \tag{23}$$

where M'_ν is given in Eq. (5) and

$$\begin{aligned} \Delta M'_\nu = \tfrac{m_0}{k}\bigg[& \varepsilon_u V^T_{CKM}(2Z'_u\tilde{D}_u - Y'_u)V_{CKM} \\ & + \varepsilon_d\Big(r - \tfrac{m_b}{m_\tau}k\Big)(2Z'_d\tilde{D}_d - Y'_d)\bigg]. \end{aligned} \tag{24}$$

Using Eq. (21) and taking into account the hierarchy among quark masses, one obtains, up to order ε_x corrections ($x = u, d$),

$$(2Z'_x\tilde{D}_x - Y'_x)_{ij} \approx (Y'_x)_{ij}\mathrm{sign}(j - i) \equiv (Y^s_x)_{ij}. \tag{25}$$

Neglecting for simplicity $\mathcal{O}(\lambda)$ terms one can write $V^T_{CKM} Y^s_u V_{CKM} \sim Y^s_d \equiv Y^s_{120}$ which finally leads to

$$\Delta M'_\nu \approx \frac{m_0}{k}\bigg[\varepsilon_u + \varepsilon_d\Big(r - \frac{m_b}{m_\tau}k\Big)\bigg]Y^s_{120}. \tag{26}$$

The form of $\Delta M'_\nu$ allows for a direct and simple assessment of the effects of the Y'_{120}-matrix entries on the minimal model neutrino mass spectrum and lepton mixings. It is interesting that $\Delta M'_\nu$ does not depend on ε_l and therefore originates entirely from the quark sector corrections.

Once we have reconstructed the matrices on the left-hand side of Eq. (18) and Eq. (23), the lepton mixing matrix U_{PMNS} is given by

$$U_{PMNS} \equiv U_l^T U_\nu, \tag{27}$$

where

$$V_d^{LT} M_l^T M_l V_d^L \equiv U_l D_l^2 U_l^T, \qquad V_d^{LT} M_\nu V_d^L \equiv U_\nu D_\nu U_\nu^T.$$

Notice that Eq. (27) does not depend on V_d^L.

V. NUMERICAL STUDY OF SUSY $SO(10)$ WITH 120_H-CORRECTIONS

We are now ready to present the results of the numerical analysis that accounts for the effects of the Y_{120} contributions on the fermion mass matrices. Together with the GUT scale quark mass ranges and mixings given in Ref. [35] (for $\tan\beta = 10$, 55), we need to input the following additional set of parameters: Y'_{120}, ε_l, ε_u, ε_d. For simplicity in the present discussion all CP-phases are set to zero.

We perform an extensive scan within the allowed quark mass and mixing ranges. For each point within the scanned region, W is given by Eq. (20). Using this input, we search for values of r and k such that the charged lepton masses obtained from Eq. (18) fit the charged lepton data. For each r there remains the freedom to shift m_b (and/or m_t) together with m_d, m_s, v_d^{120} (m_u, m_c, v_u^{120}) within the allowed ranges, while keeping $\tilde{D}_{u,d}$ and $\varepsilon_{u,d}$ constant. For different values of m_b, one set of parameters fitting Eq. (18) is mapped into another fitting set with different solutions of the neutrino mass matrices in Eq. (22). This procedure generates as a numerical artifact the "chains" of solutions that are visible in the figures.

For illustration purposes we present our results for $\varepsilon_l = 0$ ($\Delta M'_\nu$ does not depend on ε_l) and for the following form of the antisymmetric matrix Y'_{120}:

$$Y'_{120} = a\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}. \tag{28}$$

As shown in Appendix B, thanks to the reduced values of $\Delta m^2_\odot/\Delta m^2_A$ that are obtained, the texture in Eq. (28) allows for a substantial suppression of the solar mixing angle with respect to the corresponding minimal model solutions ($a = 0$) as well as for reduced values of $|U_{e3}|$.

The parameters $v_{u,d}^{120}$ are given by

$$v_{u,d}^{120} \sim \frac{M_G}{M_{Pl}} v_{u,d}^{10} \approx 10^{-1}(\sin\beta, \cos\beta)\ \mathrm{GeV}.$$

Since $(Z'_{u,d})_{12} \approx (Y'_{120})_{12} m_{t,b}/m_{c,s}$ (see Eq. (21)), the ex-

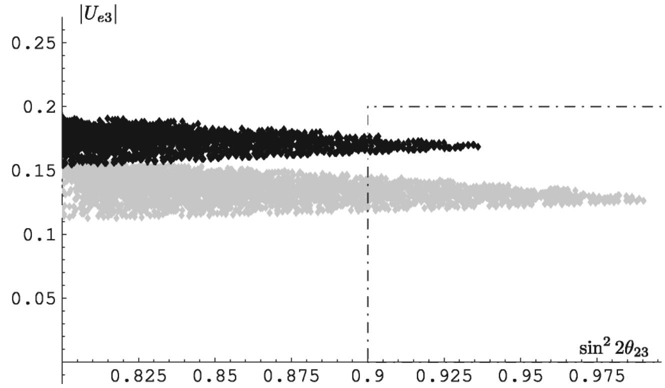

FIG. 1. $|U_{e3}|$ as a function of $\sin^2 2\theta_{23}$ in the minimal model (in black) and for the 120_H-extension with the set of parameters specified in the text (in gray). The dot-dashed contour encloses the experimentally allowed region at the 90% C.L.

pansion in Eq. (20) of W to leading order in $\varepsilon_{u,d}$ turns out to be a good approximation for $a \approx 0.1$. The typical size of the $\varepsilon_{u,d} Y^{l,s}_{120}$ terms in Eqs. (18) and (26) evaluated at the GUT scale and for $\tan\beta = 10$ is then given by

$$\begin{aligned} \varepsilon_u (Y^{l,s}_{120})_{ij} &\approx a \frac{10^{-1}\ \mathrm{GeV}}{m_t} \approx \mathcal{O}(10^{-4}), \\ \varepsilon_d (Y^{l,s}_{120})_{ij} &\approx a \frac{10^{-2}\ \mathrm{GeV}}{m_b} \approx \mathcal{O}(10^{-3}). \end{aligned} \tag{29}$$

To compare in an effective way the deviations obtained in the extended $SO(10)$ scenario with respect to the minimal model results, we present some of relevant allowed parameter planes for $\tan\beta = 10$ and 1-σ ranges of the quark masses, while taking the central values of the quark mixing angles. Considering the 90% C.L. range $0.019 \leq \Delta m^2_\odot / \Delta m^2_A \leq 0.069$, the allowed area for the U_{e3} parameter as a function of $\sin^2 2\theta_{23} > 0.8$ is shown in Fig. 1. The minimal model value $|U_{e3}| \approx 0.16$ is re-

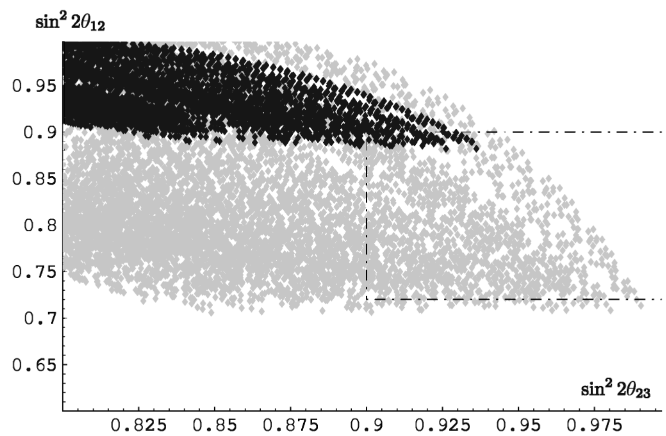

FIG. 2. $\sin^2 2\theta_{12}$ as a function of $\sin^2 2\theta_{23}$ in the minimal model (black) and for the 120_H-extension with the set of parameters specified in the text (gray). The dot-dashed contour encloses the experimentally allowed region at the 90% C.L.

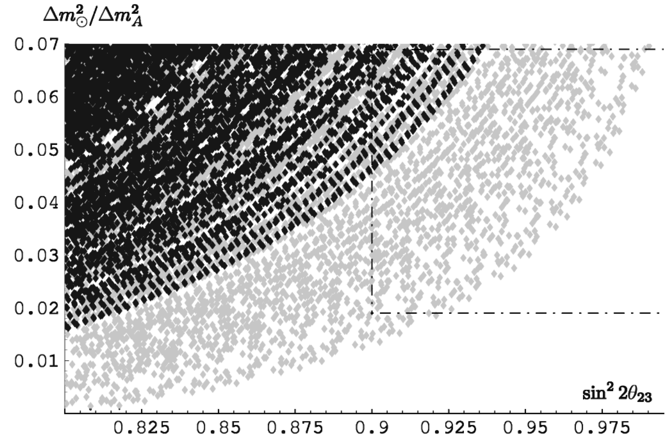

FIG. 3. $\Delta m^2_\odot / \Delta m^2_A$ as a function of $\sin^2 2\theta_{23}$ in the minimal model (black) and for the 120_H-extension with the set of parameters specified in the text (gray). The dot-dashed contour encloses the experimentally allowed region at 90% C.L.

duced by the 120_H corrections to $0.11 < |U_{e3}| < 0.14$. Even within such constrained setup the atmospheric mixing is allowed to be well within the 90% C.L. experimental region ($0.90 \leq \sin^2 2\theta_{23} \leq 1$) and in fact can be close to maximal.

In Fig. 2 we illustrate the change of the predicted values of $\sin^2 2\theta_{12}$ as a function of $\sin^2 2\theta_{23}$. The lower bounds on $\sin^2 2\theta_{12}$, which can be clearly seen both for the minimal model and for the 120_H-extension, are determined by the 90% C.L. experimental upper bound $\Delta m^2_\odot / \Delta m^2_A \leq 0.069$ that we have here considered. In the extended model one obtains $\sin^2 2\theta_{12} \gtrsim 0.71$, thus covering the whole 90% C.L allowed range, while in the minimal model $\sin^2 2\theta_{12} \gtrsim 0.88$. For $\Delta m^2_\odot / \Delta m^2_A \lesssim 0.05$ one obtains $\sin^2 2\theta_{12} \gtrsim 0.92$ and 0.77 for the minimal and extended models, respectively. The presence of the 120_H-corrections allows, by reducing the values of $\Delta m^2_\odot / \Delta m^2_A$ (see Appendix B), for lower (larger) values of the solar (atmospheric) angle.

The atmospheric mixing angle as a function of $\Delta m^2_\odot / \Delta m^2_A$ is shown in Fig. 3. Notice that, for the central value of the mass squared ratio ($\Delta m^2_\odot / \Delta m^2_A \approx 0.035$), a significant deviation from maximal atmospheric mixing is present ($\sin^2 2\theta_{23} \lesssim 0.96$) in the extended model.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have investigated the predictions of the renormalizable supersymmetric SO(10) GUT for the masses and mixing angles of quarks and leptons. Only two symmetric Yukawa coupling matrices, Y_{10} and Y_{126}, determine M_u, M_d, M_l, and M_ν in the minimal model. We assumed that the low energy neutrino mass matrix is generated via type-II seesaw, so that $M_\nu \propto Y_{126}$. The model gives insight on the physics of flavor, the rationale being the following: there exists a weak basis in which

Y_{10} is almost diagonal with hierarchical eigenvalues and dominates the charged fermion mass matrices; at the same time Y_{126} contains two large mixings that show up in neutrino oscillations and provides the subleading corrections necessary to explain the CKM mixing and the differences among M_u, M_d, and M_l eigenvalue ratios. The simplicity of this framework suggests that such weak basis could be identified with the basis in which a flavor symmetry is realized.

We have reanalyzed the predictions of the minimal SUSY SO(10) model for the leptonic sector, once the quark data are considered. The neutrino mass matrix has a dominant 2-3 block, which implies that: (i) neutrino spectrum has normal hierarchy and (ii) the contribution of neutrino masses to neutrinoless 2β decay rate is negligible compared to near future experimental sensitivity. We have shown analytically that the 2-3 mixing is generically of order unity, but that, in order to keep the ratio of Y_{126} eigenvalues ($\approx \sqrt{\Delta m^2_\odot/\Delta m^2_A}$) small, a deviation from maximal mixing of order λ is required, which is inconsistent with atmospheric data. A similar problem (but due to different constraints among the model parameters) affects the 1-2 mixing: $\sin 2\theta_{12} \propto (\Delta m^2_\odot/\Delta m^2_A)^{-1/2}$ and the suppression of the mass ratio leads to a too large mixing (order λ above the experimental value). Finally, the 1-3 mixing is enhanced with respect to the quark sector by the approximate $b-\tau$ unification and by a sizable ($\sim \lambda^2$) 1-3 entry in the charged lepton sector: one finds $|U_{e3}| \approx 0.16$ for large θ_{23}. When complex phases are considered, a partially destructive interference between the neutrino and charged lepton contribution to θ_{12} can be realized [20], thus reconciling the model with solar data. However the other shortcomings of the minimal model fit (namely $|U_{e3}|$ close to the upper bound and large deviation from maximal 2-3 mixing) persist.

In summary, including 1-σ uncertainties in quark masses and mixings and taking $\tan\beta$ in the interval 10–55, the predicted ranges for the lepton mixing angles in the minimal model (for $\Delta m^2_\odot/\Delta m^2_A \lesssim 0.05$) are: $\sin^2 2\theta_{12} \gtrsim 0.89$, $\sin^2 2\theta_{23} \lesssim 0.97$, $|U_{e3}| \gtrsim 0.15$. Considering 2-σ uncertainties in the quark input data and the 90% C.L. range for $\Delta m^2_\odot/\Delta m^2_A$, the solar angle can be as low as $\sin^2 2\theta_{12} \gtrsim 0.75$ and θ_{23} is allowed to be maximal (for large $\tan\beta$ and $\Delta m^2_\odot/\Delta m^2_A$ close to the upper bound), while $0.12 \lesssim |U_{e3}| \lesssim 0.20$.

In spite of the apparent tension with the neutrino data at the 1-σ level, it is nevertheless remarkable that the gross features of fermion masses and mixing are reproduced within the minimal renormalizable framework. This suggests that even small perturbations of the mass matrices may be relevant to a better fit of the data. We have considered a renormalizable extension of the minimal model which includes the antisymmetric component of $16_F \otimes 16_F$, that is, the 120_H representation. We argued that the VEVs induced by soft SUSY breaking on the 120_H Higgs doublet components can be naturally suppressed by $O(M_G/M_{Pl})$ with respect to those of 10_H and $\overline{126}_H$, thus preserving to large extent the minimal model predictivity. In addition, we showed that the asymmetry of the 120_H Yukawa coupling plays a key role in fitting the flavor mixing data.

We developed a perturbation method to describe the small asymmetry induced in the quark mass matrices and to estimate its effect on the charged lepton mixing. We found that the 120_H corrections to the neutrino mass matrix affect to leading order only the off diagonal entries. The 1-2 mixing in U_{PMNS} and $\Delta m^2_\odot/\Delta m^2_A$ are more sensitive to the correction because, in the case of normal hierarchy, these parameters are related to small quantities in the neutrino mass matrix, which is dominated by the atmospheric sector. We showed how the 120_H contribution can be used to decrease the predicted mass squared ratio thus relaxing the phenomenological problems of the minimal model.

Even assuming for simplicity zero CP violating phases, we have shown that this framework allows already at 1-σ level for a consistent fit of all present data on fermion masses and mixing. In particular, it is possible to reproduce values of $\sin^2 2\theta_{23} \approx 1$, $\sin^2 2\theta_{12} \approx 0.8$, and $0.11 \lesssim |U_{e3}| \lesssim 0.15$, in complete agreement with the current observed leptonic mixing data.

A positive evidence of $|U_{e3}| < 0.15$, that is within the reach of forthcoming experiments, can reject the minimal renormalizable SO(10) model (at 1-σ) and quantify the size of the 120_H correction. We find that, for natural choices of the 120_H VEVs and couplings, a lower bound $|U_{e3}| \gtrsim 0.11$ is expected at 1-σ. Moreover, a deviation from maximal 2-3 mixing is expected ($\sin^2 2\theta_{23} \lesssim 0.97$), unless $\Delta m^2_\odot/\Delta m^2_A$ is large (close to the present upper bound).

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APPENDIX A: ANTISYMMETRIC PERTURBATION TO A SYMMETRIC MATRIX

Consider a real symmetric matrix S normalized so that the magnitude of its largest eigenvalue is 1. There exists an orthogonal matrix U such that $S = US^dU^T$ where S^d is diagonal. If one adds a (real) antisymmetric matrix εA with $|A_{ij}| \leq 1$ and $\varepsilon \ll 1$, a pair of orthogonal matrices can be found such that $S + \varepsilon A = V_1(\varepsilon)X^d(\varepsilon)V_2(\varepsilon)^T$. Up to $\mathcal{O}(\varepsilon^2)$ terms one gets

$$V_1(\varepsilon) = (1 + \varepsilon Z)U, \qquad V_2(\varepsilon) = (1 - \varepsilon Z)U, \\ X^d(\varepsilon) = S^d,$$

where the antisymmetric matrix Z satisfies

$$\{S^d, U^T Z U\} = U^T A U.$$

Denoting $U^T Z U \equiv Z'$ and $U^T A U \equiv A'$, one obtains

$$Z'_{ij} = \frac{A'_{ij}}{S^d_{ii} + S^d_{jj}}.$$

Proof: from $(S + \varepsilon A)^T = S - \varepsilon A$ we get $V_1(-\varepsilon) = V_2(\varepsilon)$ and $X^d(-\varepsilon) = X^d(\varepsilon)$ which yields $X^d(\varepsilon) = S^d + \mathcal{O}(\varepsilon^2)$. Expanding now $S + \varepsilon A = V_1(\varepsilon) X^d(\varepsilon) V_1(-\varepsilon)^T$ with the ansatz $V_1(\varepsilon) \equiv (1 + \varepsilon Z)U$ (where Z is antisymmetric by orthogonality of V_1) one obtains, to the leading order in ε, $A = \{Z, S\}$. The last step is to rewrite this relation in the diagonal basis for S.

These results allow us to estimate the form of the right-handed quark mixing matrix W in the presence of 120_H-perturbation. The quark mass matrices in Eq. (16) can be written as

$$\tilde{M}_u = \frac{1}{m_t} M^s_u + \varepsilon_u Y_{120}, \qquad \tilde{M}_d = \frac{1}{m_b} M^s_d + \varepsilon_d Y_{120}.$$

Here $M^s_{u,d}$ are the minimal model symmetric mass matrices, i.e., the pieces $Y_{10} v^{10}_{u,d} + Y_{126} v^{126}_{u,d}$. If the antisymmetric pieces $\varepsilon_i Y_{120}$ are very small compared to the symmetric part, the eigenvalues of $M^s_{u,d}$ coincide with those of the full $M_{u,d}$ up to $\mathcal{O}(\varepsilon^2)$ terms (while such corrections can be relevant for first generation masses they are negligible for the estimate of mixing angles). This implies, up to $\mathcal{O}(\varepsilon^2)$ terms,

$$\tilde{M}_x = U_x \tilde{D}_x U_x^T + \varepsilon_x Y_{120} = V^R_x \tilde{D}_x V^{LT}_x,$$

for $x = u, d$. The orthogonal matrices $V^{R,L}_x$ are given by

$$V^L_x = (1 - \varepsilon_x Z_x) U_x, \qquad V^R_x = (1 + \varepsilon_x Z_x) U_x.$$

and the antisymmetric Z_x satisfy

$$\{\tilde{D}_x, U^T_x Z_x U_x\} = U^T_x Y_{120} U_x.$$

Using Eq. (19) and $Z'_x \equiv U^T_x Z_x U_x$, one obtains

$$W = (1 - \varepsilon_u Z'_u) U^T_u U_d (1 + \varepsilon_d Z'_d), \\ V_{CKM} = (1 + \varepsilon_u Z'_u) U^T_u U_d (1 - \varepsilon_d Z'_d).$$

This proves Eq. (20).

APPENDIX B: EXAMPLE OF 120_H-EFFECT ON NEUTRINO PARAMETERS

In the minimal model without 120_H-contribution, the possibility of increasing the atmospheric mixing and decreasing the solar one to get into the allowed region is prevented by the upper bound on $\Delta m^2_\odot / \Delta m^2_A$. As shown in Section V, this problem can be removed in our scenario. We give a simple analytical argument to prove that the 120_H-correction to the neutrino mass matrix, Eq. (26), can be used to decrease the predicted value of $\Delta m^2_\odot / \Delta m^2_A$.

The minimal model neutrino mass matrix M'_ν in Eq. (6) can be written as

$$M'_\nu = m_0 \lambda \begin{pmatrix} X\lambda^2 & Y\lambda^3 & D\lambda \\ \ldots & A & C \\ \ldots & \ldots & B \end{pmatrix}, \tag{B1}$$

where X, Y, A, B, C, D are $\mathcal{O}(1)$ parameters. This texture generates the following neutrino spectrum hierarchy: $m_1 \div m_2 \div m_3 \sim \lambda \div \lambda \div 1$, the sign of m_2 being opposite to that of m_3 and m_1. Assuming the setup defined by Eqs. (28) and (29), we can estimate the leading contribution to $\Delta M'_\nu$ using Eq. (26):

$$\Delta M'_\nu \approx -\frac{m_0}{k} \varepsilon_d Y^s_{120} \sim -m_0 \lambda^3 \begin{pmatrix} 0 & 1 & 1 \\ \ldots & 0 & -1 \\ \ldots & \ldots & 0 \end{pmatrix}.$$

The three independent quantities $\mathrm{Tr}M$, $\mathrm{Tr}M^2$, and $\det M$ characterize completely the spectrum of a generic 3×3 real symmetric matrix M. Using the parametrization (B1) one obtains

$$\begin{aligned} \mathrm{Tr}M'_\nu &= m_0 \lambda [A + B + X\lambda^2], \\ \mathrm{Tr}(M'_\nu)^2 &= m_0^2 \lambda^2 [A^2 + B^2 + 2C^2 + 2D^2\lambda^2 \\ &\quad + X^2\lambda^4 + 2Y^2\lambda^6], \\ \det M'_\nu &= m_0^3 \lambda^3 [XAB\lambda^2 - AD^2\lambda^2 - C^2 X\lambda^2 \\ &\quad + 2DCY\lambda^4 - BY^2\lambda^6]. \end{aligned} \tag{B2}$$

The addition of $\Delta M'_\nu$ corresponds to the replacements $Y \to Y - \mathcal{O}(\lambda^{-1})$, $D \to D - \mathcal{O}(\lambda)$, and $C \to C + \mathcal{O}(\lambda^2)$, so that

$$\delta \mathrm{Tr}M'_\nu = 0, \ \delta \mathrm{Tr}(M'_\nu)^2 \approx m_0^2 \lambda^2 (4C\lambda^2) < 0, \\ \delta \det M'_\nu \approx m_0^3 \lambda^3 \cdot 2D\lambda^3 (A - C) > 0.$$

Therefore $\delta(m_1 + m_2 + m_3) = 0$, $\delta(m_1^2 + m_2^2 + m_3^2) \sim -m_0^2 \lambda^4$, and $\delta(m_1 m_2 m_3) \sim m_0^3 \lambda^6$. By writing the neutrino masses m_i as the sum of the minimal model value m_i^0 plus the 120_H-correction δ_i, one obtains

$$\delta_3(m_3^0 - m_1^0) + \delta_2(m_2^0 - m_1^0) \sim -m_0^2 \lambda^4, \\ \delta_3 m_2^0 (m_1^0 - m_3^0) + \delta_2 m_3^0 (m_1^0 - m_2^0) \sim +m_0^3 \lambda^6.$$

which after some algebra yields $\delta_2 / m_2^0 \sim -\lambda$ and $\delta_3 / m_3^0 \sim -\lambda^2$. The ratio of mass squared differences is shifted as follows:

$$\frac{\Delta m^2_\odot}{\Delta m^2_A} \to \frac{\Delta m^2_\odot}{\Delta m^2_A} \left[1 + 2\frac{\delta_2}{m_2^0} + \ldots \right]. \tag{B3}$$

Therefore the predicted value of $\Delta m^2_\odot / \Delta m^2_A$ is reduced with respect to the minimal model by a factor $\sim (1 - 2\lambda)$.

Note added.—After the completion of this work, a more precise determination of the solar mass squared difference became available thanks to the release of a new data set from the KamLAND experiment [42], that gives $\Delta m^2_\odot = (8.2^{+0.8}_{-0.6})10^{-5}$ eV^2 at 90% C.L. (raising the best fit value by about 20%). An updated combined analysis of SuperKamiokande and K2K data on atmospheric neutrinos [43] implies $\Delta m^2_A = (2.3^{+0.7}_{-0.9})10^{-3}$ eV^2 at 90% C.L. The mass squared ratio allowed range at 90% C.L. is thus given by $0.025 \lesssim \Delta m^2_\odot/\Delta m^2_A \lesssim 0.064$ with best fit value ≈ 0.036. Notice that the uncertainty is still dominated by the atmospheric mass squared difference. The reduced upper bound further restricts the corner in the parameter space where the minimal model may be viable (see Fig. 3). The predicted lower bound on $\sin^2 2\theta_{12}$ becomes larger and therefore the tension with the smaller preferred experimental value increases (see Fig. 2). These considerations strengthen the case for the 120_H extension of the model here proposed.

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