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Chiral symmetry breaking and scalar string confinement

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We address the old difficulty in accommodating the scalar quark-antiquark confining potential together with chiral symmetry breaking. We develop a quark confining potential inspired in the QCD scalar flux tube. The coupling to quarks consists in a double vector vertex. We study the Dirac and spin structure of this potential. In the limit of massless quarks, the quark vertex is vector. Nevertheless, symmetry breaking generates a new scalar quark vertex. In the heavy quark limit, the coupling is mostly scalar. We solve the mass gap equation and find that this potential produces spontaneous chiral symmetry breaking for light quarks. The quantitative results of this model are encouraging.

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I. INTRODUCTION

Spontaneous chiral symmetry breaking (SχSB) is accepted to occur in low energy hadronic physics. Another important feature of hadronic physics, suggested by string models of confinement, by the spectroscopy of hadrons, and by lattice simulations, is scalar confinement. However, the coupling of a light quark to a string remains to be investigated. Moreover, SχSB and scalar confinement are apparently conflicting, since the first requires a chiral invariant coupling to the quarks, such as the vector coupling of QCD. Here we try to solve this old chiral symmetry versus scalar confinement conflict of hadronic physics, which remained open for many years. The interaction vertex plays a key role in this resolution.

The presently favored confinement picture in the literature is the flux tube or string picture, with a string tension of approximately 900 MeV/fm. Quantum mechanics suggests that a thin string, in its ground state, should be a scalar object [1]. Only higher energy excitations of the string would have angular momentum. This was capitalized on by Isgur and Paton in the flux tube model [2].

Nambu and Jona-Lasinio [3] showed that including chiral symmetry in fermionic systems provides a natural explanation for the small pion mass, which is much lighter than all the other isovector hadrons. The QCD Lagrangian is chiral invariant in the limit of vanishing quark masses, and the mechanism of SχSB is widely accepted to occur in low energy hadronic physics for the light flavors u, d, and s, where $m_u, m_d \ll m_s < \Lambda_{\mathrm{QCD}} < M_N/3$. The small pion mass is not the only implication of chiral symmetry. The axial Ward identities and current algebra led to beautiful theorems, the partially conserved axial current (PCAC) theorems. In what concerns the different variants of the quark model (QM), they are widely used as simplifications of QCD. The QM is convenient to study quark bound states and hadron-hadron scattering. Recently, it has been shown that the PCAC theorems, like the Weinberg theorem for $\pi - \pi$ scattering, are also reproduced in QMs with SχSB [4]. Another important benefit of having SχSB in the QM is the reduced number of parameters. The mass gap equation generates a dynamical constituent quark mass, which is no longer an independent parameter, even for quarks with a vanishing current mass. The quark-quark, quark-antiquark, antiquark-antiquark potentials, and the quark-antiquark annihilation and creation interactions all originated from the same chiral invariant Bethe-Salpeter kernel. Therefore, any QM for light quarks should comply with the SχSB.

The vector coupling, a simple coupling present in the quark-gluon vertex of QCD, is used in many chiral invariant QMs [5–8], and it cannot be ruled out yet. Nevertheless, although we also used many times the usual vector coupling [4], here we explore a different vertex for the coupling of the quark to the confining scalar string. To motivate our choice, it is interesting to review some criticisms to the vector confinement of quarks.

We can learn much by comparing the spectrum of the hydrogen atom with the masses of all hadron families. In a perturbative QCD scenario, the hadron spectroscopy would be dominated by the one-gluon exchange, which is qualitatively similar to the one-photon exchange interaction that explains in detail atomic physics. It turns out that all the hadronic families, say, of mesons or baryons, with light or heavy flavors, show similar differences with the hydrogen spectrum. It is remarkable that the spin-orbit potential (also called fine interaction in atomic physics) turns out to be suppressed in hadronic spectra. It is smaller than the spin-spin potential (hyperfine interaction). This constitutes evidence of nonperturbative QCD. Another evidence of nonperturbative QCD is present in the angular and radial excitations of hadrons, which fit linear trajectories in Regge plots, and suggest a long range, probably linear, confining potential for the quarks. This led Henriques, Kellett, and Moorhouse [9] to de-

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velop a QM where a short range vector potential and a long range scalar potential partly cancel the spin-orbit contribution. The short range potential is Coulomb-like (inspired in the one-gluon exchange) and its quark vertex has a vector coupling structure $\bar{\psi}\gamma^{\mu}\psi$. The long range potential has scalar coupling $\bar{\psi}\psi$ and is a linear potential [9–12].

Moreover, the same scalar confinement picture is extracted from lattice simulations. In quenched lattices which simulate the quark-antiquark potential in the heavy quark limit, the pattern of spin-spin, tensor, and spin-orbit interactions is compatible with a scalar confinement [13–15].

It was also realized by Adler [16] that a linear confinement with vector couplings was not sufficient to provide the correct quark condensate. Moreover, the gluon propagator extracted from the lattice, when used in a one-gluon exchange truncation of the quark mass gap equation, falls short of providing the expected quark condensate [17]. Although some ansatz for the gluon propagator used in the truncated Schwinger-Dyson equations of QCD are able to produce a reasonable quark condensate [18,19], it is nevertheless interesting to investigate the result of a linear confinement with scalar couplings.

Importantly, these gluon propagators exhibit a nonperturbative mass [17,19–21]. This mass could be consistent with a Meissner effect in Yang-Mills fields, leading to the formation of electric flux tubes (confining string) for the quarks. The flux tube models of confinement have been introduced in the seminal papers of Mandelstam [22] and 't Hooft [23] in the dual superconductor. Here we will estimate the effect of the confining string on the quark condensate.

In this paper, we explore scalar string confinement from the perspective of chiral symmetry breaking. In Sec. II we define the vertex and the potential used in our QM. The self-consistent mass gap equation for the quarks is derived in Sec. III. In Sec. IV we solve numerically the mass gap equation and calculate the quark condensate. Finally, in Sec. V we present some conclusions.

II. THE DOUBLE VERTEX NONPERTURBATIVE CONFINING INTERACTION

In this paper, we will assume that the quarks are coupled to a scalar object which provides a linear confinement. On top of that, we want this coupling to use, as a microscopic building block, the vector gluon-quark which is present in QCD. Notice that to get a Lorentz scalar coupling a simple one-gluon vertex is not enough. The coupling needs at least two vertices. The simplest way to achieve this is to have the string emitting two effective gluons that couple to the same quark line. This double vertex, coupling the string to a quark line via two intermediate gluon propagators, is presented in Fig. 1. Effectively, this double vertex is similar to the vertices that couple a quark to a gluon ladder in the soft Pomeron models [24,25]. It is also related to the light quark vertex in the heavy-light quark bound states studied in the local gauge coordinate [26–28]. Such a double vertex was also introduced in the coupling of short strings to quarks [29]. In the cumulant expansion formalism in terms of gluon correlators [26,30], this means the quark-antiquark coupling will be dominated by four gluon correlators.

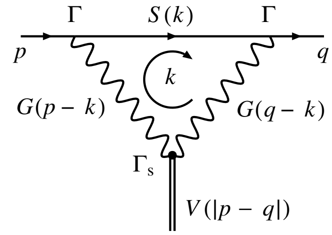


FIG. 1. The coupling of a quark to a string with a double gluon vertex.

We choose to construct the simplest possible coupling, simulating that of a quark line with a scalar string, using two vector couplings as building blocks. The most general coupling of this kind is presented in Fig. 1 and it is simply read

$$\int \frac{d^4k}{(2\pi)^4}(\Gamma V^a)S(k)(\Gamma V^b)G^{bc}(p-k)(\Gamma_S W^{cde})G^{ad}(q-k), \tag{1}$$

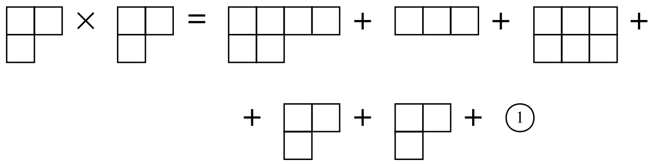
where Γ is the Dirac structure of the fermion-gluon interaction and V^a the usual color interaction $\lambda^a/2$, with λ^a the Gell-Mann matrices. We denote the quark propagator by $S(k)$ and the gluon propagator by $G^{ab}(k)$. Finally, $\Gamma_S W^{cde}$ represents the coupling of the gluon pair to the string.

For the coupling of the string to a light quark, we use the vertex obtained in the heavy-light quark system, computed in the local coordinate gauge [27]. This model interpolates between the heavy-light meson in the local gauge and the effective QM. So, for the Dirac structure of the quark-gluon subvertices Γ, we have a γ^0 matrix, which is also compatible with the Coulomb gauge.

In the color sector, as already stated, the coupling of the same subvertices has a $\lambda^a/2$ structure. The remaining subvertex includes the coupling of two color octets; see Fig. 2. The string is also a colored object; it contains a flux of color-electric field. For a scalar coupling, which is symmetric, we use the symmetric structure function d^{abc} defined by

$$\{\lambda^a, \lambda^b\} = \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda^c. \tag{2}$$

This will result in a color contribution for the effective vertex of



$$8 + 8 = 27 + 10 + \overline{10} + 8 + 8 + 1$$

FIG. 2. String color.

$$d^{abc}\frac{\lambda^b}{2}\frac{\lambda^c}{2} = C\frac{\lambda^a}{2}, \qquad C = \frac{5}{6}. \tag{3}$$

In QMs the string usually couples to the quark line with a $\lambda^a/2$. In our case it couples with two $\lambda^a/2$, one for each subvertex. But as we can see from (3) the effective result is the same.

The gluon propagators and the different subvertices result in a distribution in the loop momentum k. Here different choices would be possible. For simplicity, we assume that the relative momentum $p - q$ flows equally in the two effective gluon lines. This amounts to considering that the momentum k distribution is a Dirac delta

$$(2\pi)^3\delta^3\left(\boldsymbol{k} - \frac{\boldsymbol{p}+\boldsymbol{q}}{2}\right). \tag{4}$$

We also remark that the distribution in k is normalized to unity once the correct string tension is included in the relative potential $V(p - q)$. The above assumptions can be summarized in the formula for the loop vertex

$$\int\frac{d^4k}{(2\pi)^4}\left(\gamma^0\frac{\lambda^a}{2}\right)S(k)\left(\gamma^0\frac{\lambda^b}{2}\right)d^{abc}(2\pi)^3\delta^3\left(\boldsymbol{k} - \frac{\boldsymbol{p}+\boldsymbol{q}}{2}\right). \tag{5}$$

The equal-time approximation, which is standard in QMs, allows the computation of the double vertex as a functional of the running quark mass. The quark mass will be computed self-consistently in the next sections. The most general expression for the full quark propagator can be written as

$$S(k) = \frac{iZ(k^2)}{\not{k} - m(k^2) + i\epsilon}. \tag{6}$$

Since the interaction is instantaneous, we lose covariance, and the most general propagator becomes

$$S(k) = \frac{iZ_k}{k_0\gamma^0 - A_k\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma} - B_k + i\epsilon}. \tag{7}$$

We will assume Z_k, A_k, and B_k functions of the trimomentum $\boldsymbol{k}$. The propagator can be decomposed in a particle and an antiparticle propagators in the following way:

$$S(k) = \frac{iZ_k\Lambda^+(\boldsymbol{k})\beta}{k_0 - E_k + i\epsilon} - \frac{iZ_k\Lambda^-(\boldsymbol{k})\beta}{-k_0 - E_k + i\epsilon}, \tag{8}$$

with the quark energy projectors

$$\Lambda^+(\boldsymbol{k}) = \frac{1 + s_k\beta + c_k\hat{\boldsymbol{k}}\cdot\boldsymbol{\alpha}}{2} = \sum_s u_s(k)u_s^\dagger(k),$$
$$\Lambda^-(\boldsymbol{k}) = \frac{1 - s_k\beta - c_k\hat{\boldsymbol{k}}\cdot\boldsymbol{\alpha}}{2} = \sum_s v_s(k)v_s^\dagger(k), \tag{9}$$

and where $E_k = \sqrt{A_k^2 + B_k^2}$, $s_k = \sin\varphi_k = B_k/E_k$, $c_k = \cos\varphi_k = A_k/E_k$, with φ_k the chiral angle, a convenient function for algebraic and numerical computations. The energy loop integral in Eq. (5) can be easily calculated

$$\int\frac{dk^0}{2\pi}\frac{i}{k^0 \mp E \pm i\epsilon}\Lambda^\pm\beta = \pm\Lambda^\pm\beta, \tag{10}$$

$$\int\frac{dk^0}{2\pi}S(k) = Z_k(\Lambda^+ - \Lambda^-)\beta = Z_k(s_k\beta + c_k\hat{\boldsymbol{k}}\cdot\boldsymbol{\alpha})\beta. \tag{11}$$

Finally, using the Dirac delta distribution for the remaining integrals over the three momentum $\boldsymbol{k}$, and summing in color indices, we get the following effective vertex:

$$\mathcal{V}_{\rm eff} = C\frac{\lambda^c}{2}Z_k\gamma^0(s_k - c_k\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma})\gamma^0|_{k=(p+q)/2}. \tag{12}$$

In the remainder of the paper. we will follow the convention $k = (p + q)/2$.

Equation (12) shows that the double vertex actually solves the problem of matching chiral symmetry breaking and scalar confinement. In the chiral limit of a vanishing quark mass, the effective vertex $\mathcal{V}_{\rm eff} \to C\lambda^c/2\boldsymbol{k}\cdot\boldsymbol{\gamma}$ is proportional to the γ^μ and is therefore chiral invariant as it should be, whereas in the heavy quark limit, $\mathcal{V}_{\rm eff} \to C\lambda^c/2$ is simply a scalar vertex. The Gell-Mann matrix λ^c provides the usual color vector coupling as expected in a QM. We anticipate that the dynamical generation of a quark mass will also generate a scalar coupling for light quarks, and this results in an effective vertex, including a chiral invariant vertex $\not{p}$ and the standard scalar vertex 1.

The dependence in the relative momentum must comply with the linear confinement which is derived from the string picture,

$$\mathbf{V}_\varepsilon(\boldsymbol{x}) = \frac{16}{3C^2}V_\varepsilon(\boldsymbol{x}) = \frac{16}{3C^2}\sigma|\boldsymbol{x}|e^{-\varepsilon|\boldsymbol{x}|}, \tag{13}$$

where $\sigma \simeq 120$ MeV/fm is the string constant, corresponding to a string tension $(16\sigma/3C^2)$ of approximately 900 MeV/fm, and C is the algebraic color factor defined in Eq. (3). The damping factor ε regularizes the Fourier transform,

$$\mathbf{V}_{\varepsilon}(\boldsymbol{p}) = -\frac{16}{3C^2} 8\pi\sigma\left[\frac{1}{(|\boldsymbol{p}|^2 + \varepsilon^2)^2} - \frac{4\varepsilon^2}{(|\boldsymbol{p}|^2 + \varepsilon^2)^3}\right], \tag{14}$$

and in the limit $\varepsilon \to 0$ we have

$$-iV_0(\boldsymbol{p} - \boldsymbol{q}) = -i\frac{-8\pi\sigma}{|\boldsymbol{p} - \boldsymbol{q}|^4}. \tag{15}$$

III. MASS GAP EQUATION

We solve the mass gap equation using the Schwinger-Dyson formalism

$$S^{-1} = S_0^{-1} - \Sigma, \tag{16}$$

where the dressed propagator is defined in Eq. (7) and the free propagator has a similar definition with the quark bare mass m_0,

$$\begin{aligned} S^{-1}(p) &= (-i)Z_p^{-1}(p_0\gamma^0 - B_p\hat{\boldsymbol{p}}\cdot\boldsymbol{\gamma} - A_p + i\epsilon), \\ S_0^{-1}(p) &= (-i)(p_0\gamma^0 - \boldsymbol{p}\cdot\boldsymbol{\gamma} - m_0 + i\epsilon). \end{aligned} \tag{17}$$

The self-energy term, in this model, is given by the three loop diagram of Fig. 3. Technically, other diagrams with crossed gluon legs could exist but we assume that this is the dominant diagram. The double vertex loop $\mathcal{V}_{\text{eff}}$ was already obtained in Eq. (12) and it will be used to calculate the self-energy,

$$\begin{aligned} \Sigma(p) &= \int\frac{d^3q}{(2\pi)^3}Z_k(s_k + c_k\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma})Z_q(s_q - c_q\hat{\boldsymbol{q}}\cdot\boldsymbol{\gamma})\times \\ &\quad\times Z_k(s_k + c_k\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma})C^2\frac{3}{16}(-i)\mathbf{V}_{\varepsilon}(|\boldsymbol{p} - \boldsymbol{q}|) \\ &= \int\frac{d^3q}{(2\pi)^3}[(s_k^2 - c_k^2)s_q + 2s_kc_kc_q\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}} - c_q\hat{\boldsymbol{q}}\cdot\boldsymbol{\gamma} \\ &\quad + 2(c_k^2c_q\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}} + s_kc_ks_q)\hat{\boldsymbol{k}}\cdot\boldsymbol{\gamma}]Z_k^2Z_q(-i) \\ &\quad\times V_{\varepsilon}(|\boldsymbol{p} - \boldsymbol{q}|), \end{aligned} \tag{18}$$

where we used the properties of the $\boldsymbol{\gamma}$ matrices. Since the self-energy does not have a γ_0 component, we can conclude from Eqs. (16) and (17) that

$$Z_p = 1. \tag{19}$$

Now the mass gap equation needs only to fix two other momentum functions, either A_p and B_p or E_p and φ_p.

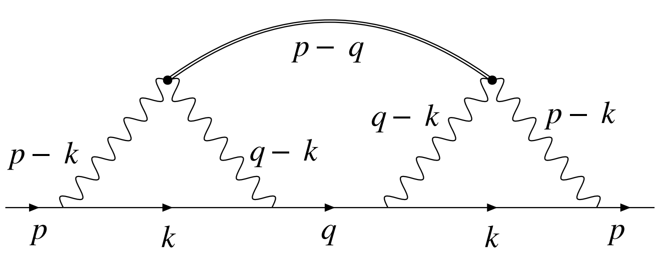


FIG. 3. The mass gap equation self-energy term.

We want to determine the quark condensate $\langle\bar{\psi}\psi\rangle$ solving the self-consistent mass gap equation. For the calculation of this physical quantity, we need to know only the chiral angle since

$$-\langle\bar{\psi}\psi\rangle = \int\frac{d^4p}{(2\pi)^4}\text{Tr}S(p) = 6\int\frac{d^3p}{(2\pi)^3}\sin\varphi_p, \tag{20}$$

where the factor 3 comes from the color trace.

With an ultraviolet finite potential, and in the chiral limit, the quark condensate is finite. In this case, the quark condensate is the order parameter that produces a simple measure of the extent of the spontaneous breaking of chiral symmetry. This quark condensate can be compared with the ones computed in similar models [16]. In the different class of models with ultraviolet divergent interactions, such as the one-gluon exchange $1/r$ Coulomb interaction, we would have an UV logarithmic divergence in the mass gap equation. For instance, in the approach of Bloch, the quark condensate is then divergent like the logarithm of the UV cutoff to the power of the quark mass anomalous dimension [31]. Nevertheless, the IR part of chiral symmetry breaking that we address here remains relevant for the hadron spectrum and for low energy hadronic physics.

A way to get an equation that depends only on the chiral angle φ_p is to use a projection of the mass gap. In the spin formalism we have some useful relations we will use for this purpose,

$$\begin{aligned} \bar{u}_s(p)v_{s'}(p) &= c_p\hat{\boldsymbol{p}}\cdot[\boldsymbol{\sigma}(i\sigma_2)]_{ss'}, \\ \bar{u}_s(p)\gamma^0v_{s'}(p) &= \mathbf{0}\cdot[\boldsymbol{\sigma}(i\sigma_2)]_{ss'}, \\ \bar{u}_s(p)\gamma^iv_{s'}(p) &= -[\delta^{ij} - (1 - s_p)\hat{p}^i\hat{p}^j][\sigma^j(i\sigma_2)]_{ss'}. \end{aligned} \tag{21}$$

With Eqs. (17) and (21) we arrive at the relation [32]

$$\bar{u}_s(p)S^{-1}(p)v_{s'}(p) = 0, \tag{22}$$

which we already expected because the propagator is diagonal. The mass gap equation becomes

$$\bar{u}_s(p)S_0^{-1}(p)v_{s'}(p) - \bar{u}_s(p)\Sigma(p)v_{s'}(p) = 0. \tag{23}$$

The free propagator term for a zero bare mass, $m_0 = 0$, is simply

$$\bar{u}_s(p)(-i)\not{p}v_{s'}(p) = ips_p\hat{\boldsymbol{p}}\cdot[\boldsymbol{\sigma}(i\sigma_2)]_{ss'}, \tag{24}$$

and the mass gap self-energy term is

$$\overline{u}_s(p)\Sigma(p)v_{s'}(p) = \int \frac{d^3q}{(2\pi)^3}(-i)V_\varepsilon(|\boldsymbol{p}-\boldsymbol{q}|) \times \{[(s_k^2 - c_k^2)s_q + 2s_k c_k c_q \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}}]c_p\hat{\boldsymbol{p}} - 2(s_k c_k s_q + c_k^2 c_q \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}})[\hat{\boldsymbol{k}} - (1-s_p) \times \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}}\,\hat{\boldsymbol{p}}] + c_q[\hat{\boldsymbol{q}} - (1-s_p)\hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{p}}\,\hat{\boldsymbol{p}}]\} \cdot \boldsymbol{\sigma}(i\sigma_2). \tag{25}$$

As we can see from (24) and (25), both terms of the mass gap equation are proportional to $\hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma}(i\sigma_2)$. Since the Pauli matrices σ are linearly independent, we can substitute $\boldsymbol{\sigma}(i\sigma_2)$ by $\hat{\boldsymbol{p}}$ and still have a mass gap condition. With this simplification the mass gap equation reduces to

$$ips_p - \int \frac{d^3q}{(2\pi)^3}[(c_k^2 - s_k^2)s_q c_p - 2s_k c_k c_q c_p \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{q}} - c_q s_p \hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{p}} + 2s_k c_k s_q s_p \hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}} + 2c_k^2 c_q s_p \hat{\boldsymbol{k}}\cdot \hat{\boldsymbol{q}}\,\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{p}}]iV_\varepsilon(|\boldsymbol{p}-\boldsymbol{q}|) = 0. \tag{26}$$

Notice that if we take the integrand and set $\boldsymbol{q} = \boldsymbol{p}$ we will get $0 \times V_\varepsilon(0)$. In Sec. IV we will deal numerically with this IR behavior.

If we projected Eq. (16) with a particle-particle projector, we would obtain an independent equation that would permit us to calculate E_p [using the result from Eq. (26)].

IV. NUMERICAL SOLUTION OF THE MASS GAP EQUATION

The mass gap Eq. (26) is a difficult nonlinear integral equation that does not converge with the usual iterative methods. We developed a method to solve the mass gap equation with a differential equation, using a convergence parameter λ. This parameter is the radius of a sphere centered in $\boldsymbol{u} = \boldsymbol{p} - \boldsymbol{q} = 0$. It allows us to separate the integral into an integral inside ($\circ$) the sphere and another outside of it ($R^3 - \circ$),

$$\int \frac{d^3u}{(2\pi)^3} f(\boldsymbol{p},\boldsymbol{u})V_\varepsilon(u) = \int_\circ \frac{d^3u}{(2\pi)^3} f(\boldsymbol{p},\boldsymbol{u})V_\varepsilon(u) + \int_{R^3-\circ} \frac{d^3u}{(2\pi)^3} f(\boldsymbol{p},\boldsymbol{u})V_\varepsilon(u). \tag{27}$$

In our model $f(\boldsymbol{p},\boldsymbol{u})$ is the function dependent on the chiral angle presented in Eq. (26).

Let us first focus on the integral inside the sphere, where we have $u < \lambda$. Eventually, we will take the limit where $\lambda \to 0$ and this term will vanish. But for now we will expand the function f around $u = 0$ and take only the first nonvanishing term

$$\int_\circ \frac{d^3u}{(2\pi)^3} f(p,u,\omega)V_\varepsilon(u) \approx \int_\circ \frac{d^3u}{(2\pi)^3} \frac{\partial^2 f}{\partial u^2}\bigg|_{u=0} V_\varepsilon(u) \overset{\varepsilon\to 0}{=} \frac{\lambda(-8\pi\sigma)}{8\pi^2}\int_{-1}^{1} d\omega \frac{\partial^2 f}{\partial u^2}\bigg|_{u=0}, \tag{28}$$

where ω is the cosine of the angle between $\boldsymbol{p}$ and $\boldsymbol{u}$. For our particular model we have

$$\int_{-1}^{1} d\omega \frac{\partial^2 f}{\partial u^2}\bigg|_{u=0} = \frac{\sin(2\varphi_p) - 2p\cos(2\varphi_p)\varphi_p' + p^2\varphi_p''}{3p^2}. \tag{29}$$

In what concerns the integral outside the sphere ($u > \lambda$), we can take from the beginning $\varepsilon = 0$ since this integral is already regulated by λ. In this case we have

$$\int_{R^3-\circ} \frac{d^3u}{(2\pi)^3} f(\boldsymbol{p},\boldsymbol{u})V_0(u) = \frac{-8\pi\sigma}{4\pi^2}\int_\lambda^\infty du \times \int_1^{-1} d\omega \frac{1}{u^2} f(p,u,\omega). \tag{30}$$

Placing the two terms in the mass gap equation, we finally get the equation that we can iterate to find the solution,

$$3ps_p + \frac{\sigma}{\pi}\bigg\{\lambda\bigg[\varphi_p'' - \frac{2}{p}\cos(2\varphi_p)\varphi_p' + \frac{1}{p^2}\sin(2\varphi_p)\bigg] + 6\int_\lambda^\infty du \int_1^{-1} d\omega \frac{1}{u^2} f(p,u,\omega)\bigg\} = 0. \tag{31}$$

Our technique consists in starting with a large infrared cutoff λ, where the integral term of Eq. (31) is negligible. In this case Eq. (31) for the chiral angle φ_p becomes essentially a differential equation which can be solved with the standard shooting method [33]. It turns out that this equation possesses several solutions, and we specialize in the larger one, with no nodes, that corresponds to the stable vacuum [33–35]. Then one decreases step by step the λ parameter, using as an initial guess for the evaluation of the integral the φ_p determined for the previous value of λ. In this way, the integral is a simple function of the momentum p and we again have to solve a nonhomogeneous differential equation. Eventually, we are able to solve the mass gap equation for a λ parameter which is much smaller than the scale of the interaction. Finally, we extrapolate the set of obtained φ_p to the limit of $\lambda \to 0$.

We test the convergence of the method by computing the quark condensate $\langle \bar{\psi}\psi \rangle$—see Eq. (20). The evolution of the solution as a function of the infrared parameter λ is clearer when we display the quark condensate; see Fig. 4.

The solution of the mass gap equation is presented in Fig. 5, where we compare it with the single vertex model solution.

V. RESULTS AND CONCLUSION

In this paper, we build a QM for the coupling of quark to a scalar string. The quark confining interaction has a single parameter σ. This QM matches the apparently conflicting vector coupling of QCD with a scalar confinement. Our model can be interpreted as a double vertex that couples the quark to the string or, alternatively, as a $\gamma_0 S(k) \gamma_0$ vertex. Either way, this vertex decomposes in the sum of a scalar vertex 1 and a chiral invariant $\hat{\boldsymbol{k}} \cdot \boldsymbol{\gamma}$ vertex weighed by simple functions of the dynamical quark mass. In the chiral limit the scalar vertex vanishes, while in the heavy quark limit the confining potential is essentially scalar. Our results for the weighing factors of the scalar vertex and the remaining chiral invariant vertex are shown in Fig. 6.

We solve a very nonlinear mass gap equation for the dynamical generation of the constituent quark mass with SχSB. We show that SχSB not only generates a quark mass, but generates also a scalar vertex for the confinement. The results are encouraging because the quark condensate indeed increases when compared with the simpler, single vertex, vector confining potential. We should only compare these quark condensate results

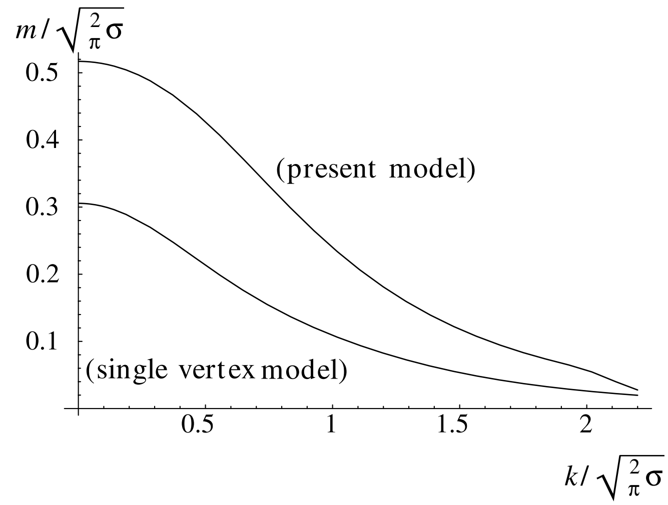


FIG. 5. The m_k solutions of the mass gap equation in units of $\sqrt{(2/\pi)\sigma}$. The larger mass is obtained with our double vertex, while the lower mass is obtained with the single one-gluon-exchangelike vertex. Both potentials use the same string tension.

with the four-fermion type models, i.e., models which have UV convergent results.

It is clear that the next step of this work will consist in adding the shorter range one-gluon exchange potential to the confining potential. The resulting model will have two parameters, one for the short range potential and another one for the confining potential. These parameters will be determined in the fit of the hadron spectrum. If we add a Coulomb-type potential, we will have UV divergences that need to be properly regularized and renormalized, using one of the prescriptions found in the literature [5,31,36,37]. In what concerns the mass gap equation, we expect that this will further enhance the quark condensate, possibly up to the expected $-(230 \text{ MeV})^3$.

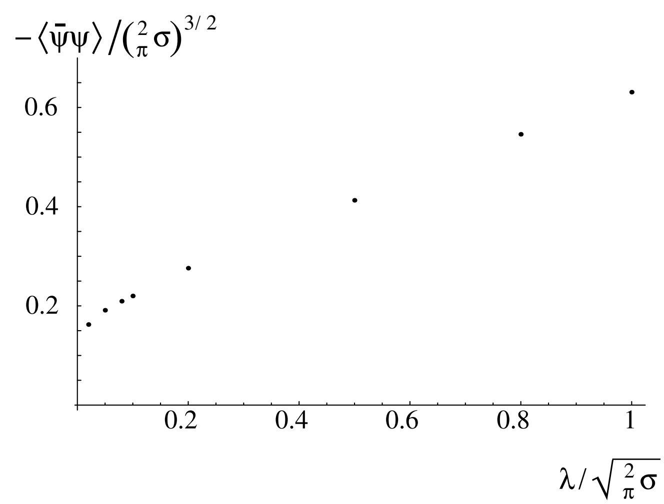


FIG. 4. Testing the convergence of the numerical method with the quark condensate $\langle \bar{\psi}\psi \rangle$.

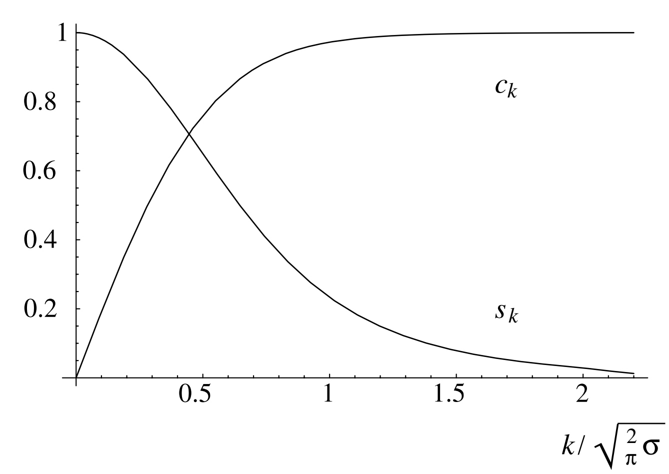


FIG. 6. Form factors for the scalar vertex (s_k) and for the remaining chiral invariant vertex (c_k).

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