## Local duality and charge symmetry violation in quark distributions

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We use local quark-hadron duality to calculate the nucleon structure function as seen by neutrino and muon beams. Our result indicates a possible signal of charge symmetry violation at the parton level in the very large x region.

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There has been significant activity in the study of local quark-hadron duality in the last four years, most of it triggered by high quality data for the proton structure function, at the resonance region, obtained in the late 1990's at the Thomas Jefferson National Laboratory [1]. From this experimental result, it was possible to test quantitatively the Bloom and Gilman ideas [2] on the relation between the exclusive cross section at low  $Q^2$  and the inclusive cross section at high  $Q^2$ . Specifically, the data from the Thomas Jefferson National Laboratory shows that the equivalence between  $F_2$  calculated from electron quark scattering, and  $F_2^{\text{res}}$  calculated from averaging the resonance structure function, including the nucleon pole, holds for  $Q^2$  as low as 0.5 GeV<sup>2</sup>.

The QCD justification for local duality was supplied by de Rùjula, Georgi and Politzer [3]. Using the Nachtmann variable,  $\xi = 2x/(1 + \sqrt{1 + 4x^2M^2/Q^2})$ , they showed that the lower moments of a structure function  $F(\xi, Q^2)$ are independent of  $Q^2$ , up to perturbative QCD corrections, in the resonance and in the scaling region. For higher moments, however, higher twist contributions are fundamental. As the lower moments give the most important contributions when reconstructing the  $\xi$  (or x) dependence of the structure functions, it follows that  $F^{\text{res}}(x, Q^2) \approx F(\xi)$ . In reality what happens is that  $F^{\rm res}(x, Q^2)$  oscillates around the scaling function as we approach the resonance poles, the origin of these oscillations being higher twists contributing with alternated signs. However, these higher twist contributions cancel on the average. The scaling function, on the other hand, is calculated at very high  $Q^2$ , meaning that all the resonance peaks have moved to the large x region. Following [2-4] we relate the scaling structure function to the elastic part of the structure function calculated at the nucleon pole. This enables us to estimate the behavior of the scaling structure function in the large x region, as long as local duality holds.

Our particular interest is the isoscalar  $F_2^{\nu N}$  structure function measured in deep inelastic neutrino-nucleon scattering. At large x, assuming that charge symmetry holds at the parton level, we should have in leading order

$$F_2^{\nu N}(x \to 1) \simeq x [u(x) + \overline{u}(x) + d(x) + \overline{d}(x)], \quad (1)$$

where we did not write the contribution from the strange quarks because they are not expected to contribute in this region [5,6]. On the other hand, the isoscalar structure function  $F_2^{\mu N}$ , measured in muon scattering should be given, at large *x*, by

$$F_2^{\mu N}(x \to 1) \simeq \frac{5}{18} x [u(x) + \overline{u}(x) + d(x) + \overline{d}(x)].$$
(2)

In fact, both neutrino and muon structure functions of an isoscalar target have been measured already, but at intermediate x [7,8]. In this region the target mass corrections, along with the nucleon strange and antistrange quark distributions, are essential to reconcile both experiments with the assumption of universal parton distributions [8,9]. With the absence of strange quarks and antiquarks, we must have from Eqs. (1) and (2) that

$$\frac{5}{18}F_2^{\nu N}(x \to 1) \simeq F_2^{\mu N}(x \to 1).$$
(3)

A failure of Eq. (3) would suggest that either an unexpected strange distributions, perhaps intrinsic strangeness [10], at large x or that charge symmetry between the proton and the neutron affects the large x distributions [11]. Any of these two conclusions are very significant and justify a deeper study of the structure functions as probed by neutrinos and muons in this region. This is the main objective of this letter. We will use local quarkhadron duality to investigate relation (3).

For this purpose, we will need the hadronic tensor that enters in the quasi elastic neutrino-nucleon cross section,  $\nu_{\mu}(\overline{\nu}_{\mu}) + n(p) \rightarrow \mu^{-}(\mu^{+}) + p(n)$ . Keeping only the relevant and sufficient terms for this study [12], we start with the matrix elements of the charged current, which is given by

$$< p(P')|J_{+}^{\mu}(0)|n(P)> = < n(P')|J_{-}^{\mu}(0)|p(P)>$$

$$= \overline{u}(P') \bigg[ F_{1}^{V}(Q^{2})\gamma^{\mu} + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M}F_{2}^{V}(Q^{2}) - G_{A}(Q^{2})\gamma^{\mu}\gamma_{5} \bigg] u(P), \quad (4)$$

where  $F_1^V(Q^2)$  and  $F_2^V(Q^2)$  are, respectively, the isovector Dirac and Pauli form factors, and  $G_A(Q^2)$  the axial form

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factor. The elastic part of the hadronic tensor calculated from charged current is then

$$W^{el}_{\mu\nu} = -F^{el}_{1} \frac{g_{\mu\nu}}{M} + F^{el}_{2} \frac{P_{\mu}P_{\nu}}{2M^{3}\tau} + iF^{el}_{3} \varepsilon_{\mu\nu\alpha\beta} \frac{P^{\alpha}q^{\beta}}{4M^{3}\tau}, \quad (5)$$

with  $\tau = Q^2/4M^2$  and

$$F_1^{el} = \frac{M}{2} \,\delta\!\left(\nu - \frac{Q^2}{2M}\right) \! \left[\tau (G_M^V)^2 + (1+\tau)G_A^2\right], \quad (6)$$

$$F_2^{el} = M\tau \delta \left(\nu - \frac{Q^2}{2M}\right) \left[\frac{(G_E^V)^2 + \tau (G_M^V)^2}{1 + \tau} + G_A^2\right], \quad (7)$$

$$F_3^{el} = M\tau \delta \left(\nu - \frac{Q^2}{2M}\right) [2G_M^V G_A], \tag{8}$$

where  $\nu$  in Eqs. (6)–(8) is the energy transfer between the beam and the target. The isovector electric and magnetic form factors are given by  $G_{E,M}^V = G_{E,M}^p - G_{E,M}^n$ , with  $G_E^N = F_1^N - \tau F_2^N$  and  $G_M^N = F_1^N + F_2^N$ .

As previously discussed, local quark-hadron duality is translated into approximately equal low moments for the resonance and the scaling structure functions for each resonance. Hence, looking at the nucleon pole only, we will have the following equation relating the scaling structure function  $F_2^{\nu N}$  and the elastic contribution  $F_2^{el}$ 

$$\int_{\xi_{th}}^{1} F_2^{\nu N}(\xi) d\xi \simeq \int_{\xi_{th}}^{1} F_2^{el}(\xi, Q^2) d\xi, \qquad (9)$$

where  $\xi_{th} = 2x_{th}/(1 + \sqrt{1 + x_{th}^2/\tau})$  is the Nachtmann variable at the pion threshold, with  $x_{th} = Q^2/[Q^2 + m_{\pi}(2M + m_{\pi})]$ . The  $Q^2$  independence of the scaling  $F_2^{\nu N}$  means that we are not taking into account the perturbative QCD corrections to it. We now use Eq. (7) on the right hand side of Eq. (9)

$$\int_{\xi_{th}}^{1} F_{2}^{\nu N}(\xi) d\xi \simeq \frac{\xi_{0}^{2}}{4 - 2\xi_{0}} \bigg[ \frac{(G_{E}^{V})^{2} + \tau (G_{M}^{V})^{2}}{1 + \tau} + G_{A}^{2} \bigg],$$
(10)

where  $\xi_0$  is the Nachtmann variable at the nucleon pole. Taking the derivative of Eq. (10) with respect to  $x_{th}$ , with  $\xi_0$  fixed, we get

$$F_{2}^{\nu N}(x_{th}) \simeq -\beta \left[ \frac{(G_{M}^{V})^{2} - (G_{E}^{V})^{2}}{4M^{2}(1+\tau)^{2}} + \frac{1}{1+\tau} \left( \frac{d(G_{E}^{V})^{2}}{dQ^{2}} + \tau \frac{d(G_{M}^{V})^{2}}{dQ^{2}} \right) + \frac{dG_{A}^{2}}{dQ^{2}} \right],$$
(11)

with  $\beta = (Q^4/M^2)(\xi_0^2/\xi_{th}^3)(2 - \xi_{th}/x_{th})/(4 - 2\xi_0)$ . The same calculation for the  $F_1^{\nu N}$  and  $F_3^{\nu N}$  structure functions gives

$$F_{1}^{\nu N}(x_{th}) \simeq -\frac{\beta}{2} \left[ \frac{-G_{A}^{2}}{4M^{2}\tau^{2}} + \frac{d(G_{M}^{V})^{2}}{dQ^{2}} + \frac{1+\tau}{\tau} \times \frac{dG_{A}^{2}}{dQ^{2}} \right], F_{3}^{\nu N}(x_{th})$$
$$\simeq -\beta \frac{d(2G_{M}^{V}G_{A})}{dQ^{2}}.$$
(12)

Notice that  $F_1^{\nu N}(x_{th})$  and  $F_2^{\nu N}(x_{th})$  have the same behavior in the  $\tau \to \infty$  region.  $F_3^{\nu N}(x_{th})$ , on the other hand, is associated with an interference between the vector and axial parts of the charged current. Finally, a similar calculation can be made for the eletromagnetic structure functions. We quote here our result for the  $F_2^{\mu p}$  case

$$F_{2}^{\mu p}(x_{th}) \simeq -2\beta \left[ \frac{(G_{M}^{p})^{2} - (G_{E}^{p})^{2}}{4M^{2}(1+\tau)^{2}} + \frac{1}{1+\tau} \left( \frac{d(G_{E}^{p})^{2}}{dQ^{2}} + \tau \frac{d(G_{M}^{p})^{2}}{dQ^{2}} \right) \right],$$
(13)

which agrees with [4], except for a  $1/x_{th}$  in the  $\beta$  factor and an overall minus sign. As only ratios are shown in [4], the conclusions presented in that work are unaffected.

Using Eqs. (11) and (13) in Eq. (3) we have, in the large  $Q^2$  limit, that

$$\frac{5}{18}F_2^{\nu N}(x=x_{th}\to 1) - F_2^{\mu N}(x=x_{th}\to 1) \simeq +\frac{13}{18}\beta \left(\frac{d(G_M^p)^2}{dQ^2} + \frac{d(G_M^n)^2}{dQ^2}\right) + \frac{5}{9}\beta \frac{d(G_M^pG_M^n)}{dQ^2} - \frac{5}{18}\beta \frac{dG_A^2}{dQ^2},\tag{14}$$

which is not zero, unless some numerical coincidence happens. Equation (14) incorporates the main point of this work. To understand how large the deviation of  $5F_2^{\nu N}/18F_2^{\mu N}$  is from 1, when using quark-hadron duality to calculate the scaling functions, we used a world data parametrization [13] to calculate the form factors appearing in (11) and (13). The result, shown in Fig. 1, is clearly different from 1. The calculation should not be trusted for  $x = x_{th} \leq 0.78$ , where  $Q^2 \leq 1$  GeV<sup>2</sup>. However, at  $x = x_{th} \sim 0.9$ ,  $Q^2 \sim 2.5$  GeV<sup>2</sup> and  $W^2 \sim 1.25$  GeV<sup>2</sup>, a region where local quark-hadron duality has more chances of being respected, although its validity, mainly for  $Q^2 < 1.5$  GeV<sup>2</sup>, is still controversial [14,15]. In any case, according to Ref. [16] the extraction of the elastic form factors from the scaling structure functions gives a reasonable agreement with the experimental data, indicating that the calculation of the large x structure functions, in the present kinematical regime, may be justified at some extent. Finally, another source of error that could affect the result of Fig. 1 is the experimental uncertainty on the elastic form factors, which are found to be around 10% in the  $Q^2 \sim 2$  GeV<sup>2</sup> region [17].

The effect shown in Fig. 1 is larger than the known limitations of local quark-hadron duality. If taken seriously, they show an effect at large x that is not marginal. To understand it, let us look at the following ratio between

the scaling functions, where it is assumed that there is a charge symmetry violation at the parton level [18]

$$R_{c}(x) \equiv \frac{F_{2}^{\mu N}(x)}{\frac{5}{18}F_{2}^{\nu N}(x) - x[s(x) + \overline{s}(x)]/6}$$
  

$$\approx 1 - \frac{s(x) - \overline{s}(x)(x)}{\overline{Q}(x)} + \frac{4\delta u(x) - \delta \overline{u}(x) - 4\delta d(x) + \delta \overline{d}(x)}{5\overline{Q}(x)}, \quad (15)$$

where  $\overline{Q}(x) = \sum_{q=u,d,s} [q(x) + \overline{q}(x)] - 3[s(x) + \overline{s}(x)]/5$ , and the charge symmetry breaking terms are  $\delta u(x) = u^p(x) - d^n(x)$ ,  $\delta d(x) = d^p(x) - u^n(x)$ , and similar for the antiquarks. In the large x region, any charge symmetry breaking coming from antiquarks should be negligible, and assuming that the strange and antistrange distributions do not contribute, we will have

$$R_c(x \to 1) \simeq \frac{F_2^{\mu N}(x \to 1)}{\frac{5}{18}F_2^{\nu N}(x \to 1)}$$
  
$$\approx 1 + \frac{4[\delta u(x \to 1) - \delta d(x \to 1)]}{5\overline{Q}(x \to 1)}.$$
 (16)

Therefore, we can explain the results encapsulated in Eq. (14) and in Fig. 1, if we use Eq. (16) and allow for charge symmetry breaking in the quark distributions in the large x region. Our result requires that  $\delta u(x \rightarrow 1) > \delta d(x \rightarrow 1)$ , which is the same sign as obtained in the bag model calculation of Rodionov *et al.* [11] in the very large x region, although vanishingly small, as this theoretical calculation predicts a significant effect at intermediate x only.

It is also useful to look at the size of the effect given by Eq. (14) relative to the total magnitude of the



structure functions,  $2[(5/6)F_2^{\nu N}(x \rightarrow 1) - 3F_2^{\mu N}(x \rightarrow 1)]/[(5/6)F_2^{\nu N}(x \rightarrow 1) + 3F_2^{\mu N}(x \rightarrow 1)]$ . This is shown in Fig. 2. As before, there is a definite signal in the region around  $x = x_{th} \sim 0.9$ , of about 27%, which indicates a possible charge symmetry breaking in the quark distributions even if we allow a 20% error coming from the uncertainty in the local quark-hadron duality relations and the experimental determination of the elastic form factors. Of course, our assumption of a vanishing strange distribution at around  $x \sim 0.9$  may be questionable. However, fits of the world data, including the ones that allow for an asymmetric strange distribution at large x, corroborate this assumption [6,19].

The charge symmetry violation in the large x valence quark distributions calculated here can have significant effects in other areas of particle physics. For instance, recently Londergan and Thomas [20] analyzed the impact of such violation on the determination of the Weinberg angle as measured by NuTeV [21]. According to their analysis and our Fig. 1, the NuTeV anomaly would become larger because we have  $R_c(x \rightarrow 1) - 1 > 0$ , although nuclear corrections still have to be taken into account. In any case, the need for physics beyond the standard model would be more pressing. In summary, we have used local quark-hadron duality to study the relation between the isoscalar structure function as probed by neutrino and muon beams. Our result indicates a possibly sizeable violation of charge symmetry in the valence quark distributions at very large x.

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FIG. 2. The difference between  $F_2^{\nu N}(x_{th})$  and  $F_2^{\mu N}(x_{th})$  normalized by their total contribution.

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