Pion and kaon decay constants: Lattice versus resonance chiral theory

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The Lattice results for the pion and kaon decay constants are analyzed within the Resonance Chiral Theory framework in the large N_C limit. The approximately linear behavior of the observable at large light-quark mass is explained through the interaction with the lightest multiplet of scalar resonances. The analysis of the Lattice results allows to obtain the resonance mass $M_S = 1049 \pm 25$ MeV and some of the Chiral Perturbation Theory parameters at leading order in $1/N_C$.

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I. INTRODUCTION

Quantum Chromodynamics (QCD) has been proven to be the proper theory to describe the strong interactions. However, in the low energy region the theory in terms of quarks and gluons becomes highly non perturbative. These degrees of freedom get confined within complex hadronic states. Below the first resonance multiplet ($E \ll M_\rho$), the spectrum contains just the light octet of pseudoscalars, the pseudo-Nambu-Goldstone bosons (pNGB) from the spontaneous chiral symmetry breaking. At low momenta and small pNGB masses one may describe their interactions through a chiral invariant effective field theory, Chiral Perturbation Theory (χPT) [1]. It establishes an expansion on powers of the external momenta and masses over a characteristic chiral scale $\Lambda_\chi \sim 4\pi F \sim 1.2$ GeV, being $F \simeq F_\pi = 92.4$ MeV the physical pion decay constant.

The chiral expansion breaks down when either the momenta or the pNGB masses become large, as they approach to the $\rho(770)$ mass. The mesonic resonances can be then produced and their effects cannot be any longer neglected.

Alternatively, it is possible to employ the $1/N_C$ expansion to describe the matrix elements—being N_C the number of colors in QCD—[2]. Resonance Chiral Theory (RχT) [3] incorporates the interactions between resonances and Goldstones at leading order in $1/N_C$, and also implements the chiral symmetry of the interaction. Likewise, χPT is fully recovered in the low energy limit.

At leading order in $1/N_C$ (LO) the observables are given by the amplitudes at tree-level, being the mesonic loops suppressed by $1/N_C$ [2]. In that situation, RχT is able to reproduce the short distance behavior required by QCD for the pion form factors, two-point Green functions and forward scattering amplitudes [4,5]. The $1/N_C$ counting can be also carried on to the next order (NLO) in a systematic way and quantum loop corrections might be calculated [6].

The present study is focused on how the variation of the quark masses affects the pion and kaon decay constants, F_π and F_K, under the RχT framework and the $1/N_C$ expansion. The Lattice calculations have provided information about QCD results for unphysical values of the *up*, *down* quarks [7]. The simulations are sometimes forced to work with u, d masses of nearly the size of the physical *strange* quark mass or higher. Thus, the usual χPT extrapolations break down and generate large unphysical chiral logarithms, with a large bending in the curves [7–9]. Nonetheless, more than purely numerical values, this work aims to provide a possible way to analyze the Lattice simulations at large quark masses, explaining why the usual linear extrapolations work so well and what are the underlying physical foundations of this behavior.

Through the inclusion of the first resonance multiplets, with masses $M_R \sim 1$ GeV, one expects to reproduce the physics for the Goldstones up to that range of momenta and masses. Moreover, at LO in $1/N_C$ only the scalar resonances contribute to F_π and F_K. Since the first multiplet of pseudoscalar resonances is at much higher masses, it will not be considered in the calculation and its mixing with the pNGB will be neglected.

II. RχT LAGRANGIAN AT LO IN $1/N_C$

The leading order resonance lagrangian was developed in Ref. [3] and it includes one multiplet of vector, axial-vector, scalar and pseudoscalar resonances. It has terms including only Goldstones—the $\mathcal{O}(p^2)$ χPT Lagrangian [1],

$$\mathcal{L}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle, \tag{1}$$

and pieces including as well resonance fields. The brackets $\langle \ldots \rangle$ mean trace of the flavor matrices. The chiral tensors u_μ, χ_+ including the Goldstone fields can be obtained in Refs. [3–5]. They contain the external fields v^μ, a^μ, s and p, and the nonlinear realization of the chiral symmetry $u = \exp(i\Phi/\sqrt{2}F)$, with the pNGB fields $\Phi = \sum_a \phi_a \lambda_a/\sqrt{2}$.

In the large N_C limit the $q\bar{q}$ resonances form $U(3)$ multiplets. The fields of a multiplet can be put together in a 3×3 flavor matrix which transforms linearly under the chiral symmetry,

$$R = \sum_a R_a \frac{\lambda_a}{\sqrt{2}}, \tag{2}$$

containing one chiral singlet field R_0 and the remaining octet fields R_a, with $a = 1, \dots 8$, being all the states degenerated in the large N_C and massless quark limit.

The kinetic terms are then constructed with these flavor matrices:

$$\begin{aligned} \mathcal{L}_R^{\rm Kin}(R = V, A) &= -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu} \nabla_\nu R^{\nu\mu} - \frac{1}{2} M_R^2 R_{\mu\nu} R^{\mu\nu}\rangle, \\ \mathcal{L}_R^{\rm Kin}(R = S, P) &= \frac{1}{2}\langle \nabla^\mu R \nabla_\mu R - M_R^2 R^2\rangle. \end{aligned} \tag{3}$$

One has also an interaction lagrangian, linear in the resonance fields [3],

$$\begin{aligned} \mathcal{L}_{2V} &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle, \\ \mathcal{L}_{2A} &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle, \\ \mathcal{L}_{2S} &= c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle, \\ \mathcal{L}_{2P} &= i d_m\langle P\chi_-\rangle, \end{aligned} \tag{4}$$

where the vector and axial-vector resonances are given in the antisymmetric formalism.

III. SCALAR TADPOLE AND FIELD REDEFINITION

Analyzing the LO lagrangian one observes the presence of a term linear in the scalar fields, i.e., a scalar tadpole. It is given by the term of $\mathcal{L}_{2S}$ in Eq. (4) with the coupling c_m, which also provides the vertex for the scalar resonance production from a scalar quark current.

Chiral symmetry requires the quark masses to enter into the lagrangian only through the tensor $\chi = 2B_0\{s(x) + ip(x)\}$, which appears in the chiral covariant combinations $\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u$. In order to recover physical QCD, the external fields are evaluated at the end of the calculation as $\chi = 2B_0\mathcal{M}$, being $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ the diagonal matrix with the light-quark masses.

The terms of the lagrangian containing the scalar fields are

$$\begin{aligned} \mathcal{L}_S^{\rm Kin} + \mathcal{L}_{2S} = &\frac{1}{2}\langle \nabla^\mu S \nabla_\mu S\rangle - \frac{1}{2} M_S^2 \langle S^2\rangle + c_m\langle S\chi_+\rangle \\ &+ \mathcal{O}(S\Phi^2), \end{aligned} \tag{5}$$

with $\chi_+ = 4B_0\mathcal{M} + \mathcal{O}(\Phi^2)$. The scalar field has therefore a nonzero vacuum expectation value (v.e.v.). In order to define the quantum field theory around the minimum one needs to perform in the scalar field the shift:

$$S = \bar{S} + \frac{c_m}{M_S^2}\chi_+, \tag{6}$$

where the shifted fields $\bar{S}$ has a zero v.e.v. and the shift is not the same for all the scalar fields S in the multiplet, but proportional to the quark mass matrix $\mathcal{M}$, i.e., different for each resonance.

The part of the lagrangian containing the vector, axial-vector and pseudoscalar resonances, $\mathcal{L}_R[R = V, A, P]$, remains unchanged under the shift but the remaining $\mathcal{O}(p^2)$ chiral term $\mathcal{L}_{2\chi}$ and the terms $\mathcal{L}_S^{\rm Kin} + \mathcal{L}_{2S}$ containing scalar fields become

$$\mathcal{L}_S^{\rm Kin} + \mathcal{L}_{2S} + \mathcal{L}_{2\chi} = \mathcal{L}_{\bar{S}}^{\rm Kin'} + \mathcal{L}'_{2\bar{S}} + \mathcal{L}'_{2\chi}, \tag{7}$$

yielding a kinetic term structure for $\bar{S}$,

$$\begin{aligned} \mathcal{L}_{\bar{S}}^{\rm Kin'} = &\frac{1}{2}\langle \nabla^\mu\Big(\bar{S} + \frac{c_m}{M_S^2}\chi_+\Big)\nabla_\mu\Big(\bar{S} + \frac{c_m}{M_S^2}\chi_+\Big)\rangle \\ &- \frac{1}{2}\langle M_S^2 \bar{S}^2\rangle, \end{aligned} \tag{8}$$

an interaction lagrangian without tadpoles,

$$\mathcal{L}'_{2\bar{S}} = c_d\langle \bar{S} u_\mu u^\mu\rangle \tag{9}$$

and a modified χPT term,

$$\begin{aligned} \mathcal{L}'_\chi = &\frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \frac{c_m^2}{2M_S^2}\langle \chi_+\chi_+\rangle \\ &+ \frac{c_d c_m}{M_S^2}\langle \chi_+ u^\mu u_\mu\rangle. \end{aligned} \tag{10}$$

It is important to notice that all the terms in $\mathcal{L}'_\chi$ are of the same order in $1/N_C$, since $F^2 \sim c_m^2 \sim c_d c_m \sim \mathcal{O}(N_C)$.

In order to convert the pNGB kinetic term to the canonical form, one needs to perform an $\mathcal{O}(N_C^0)$ rescaling on the pNGB fields $\Phi = C_\phi^{\frac{1}{2}}\Phi^{(\rm can)}$. At LO in $1/N_C$ one finds that the resonance couplings become fixed by the QCD short distance constraints [5,10]: $c_d = c_m = F/2$. Thus, considering $\mathcal{L}'_\chi$ at LO in $1/N_C$ one gets:

$$\begin{aligned} &\begin{cases} m_\pi^2 &= 2B_0 m_{u,d}, \\ C_\pi &= \Big[1 + \frac{2m_\pi^2}{M_S^2}\Big]^{-1}, \end{cases} \\ &\begin{cases} m_K^2 &= B_0(m_{u,d} + m_s), \\ C_K &= \Big[1 + \frac{2m_K^2}{M_S^2}\Big]^{-1}. \end{cases} \end{aligned} \tag{11}$$

IV. F_π AND F_K AT LEADING ORDER

Since the scalar tadpole has been removed, at LO there is only one diagram contributing to the pion decay constant: the tree-level production of the pNGB from the axial current,

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\phi^+(p)\rangle = i\sqrt{2}FC_\phi^{-1/2}p_\mu, \qquad (12)$$

and therefore the pion and kaon decay constants are

$$F_\pi = F\left(1+\frac{2m_\pi^2}{M_S^2}\right)^{1/2}, \qquad F_K = F\left(1+\frac{2m_K^2}{M_S^2}\right)^{1/2}. \quad (13)$$

When the pNGB masses become small enough ($m_\phi^2 \ll M_S^2$) the decay constants may be expanded in powers of m_ϕ^2, recovering then the tree-level χPT result

$$F_\phi = F\left[1+\frac{4L_5}{F^2}m_\phi^2 + \frac{4L_4}{F^2}(2m_K^2+m_\pi^2) + \mathcal{O}(m_\phi^4)\right], \qquad (14)$$

with the couplings $L_5 = F^2/4M_S^2$ and $L_4 = 0$ [5]. Moreover, the RχT expression in Eq. (13) deviates very slowly from linearity within the range of masses usually considered in the Lattice simulations. Thus, the large N_C limit explains in a natural way the suppression of the logarithms and why the linear extrapolations provide such good descriptions.

Only around the zero pNGB mass one has sizable corrections from one loop diagrams (NLO in $1/N_C$): $F_\pi = F[1+\frac{4L_5^r(\mu)}{F^2}m_\pi^2 - \frac{m_\pi^2}{16\pi^2F^2}\ln\frac{m_\pi^2}{\mu^2}+\ldots]$. The inclusion of this NLO effects within the framework of the $1/N_C$ expansion turns out then to be crucial to improve the precision of the chiral extrapolations; to include this logarithmic corrections the calculation should be taken up to the NLO in $1/N_C$ following the procedure developed in Ref. [6].

The RχT expressions for F_π and F_K are now compared with the Lattice results from the MILC Collaboration [7], which generates data for a wide range of values of the quark masses. They can be beheld in Fig. 1(a). The simulations handle two kinds of quark masses: The sea-quark masses of the fermions within closed loops, and the valence-quark masses of those which are not from the sea. In the MILC simulation the *strange* quark valence-mass $m_s^{\rm val}$ and the *strange* quark sea-mass $m_s^{\rm sea}$ are introduced as an input, taking both the same value $m_s^{\rm val} = m_s^{\rm sea} = m_s$. This input m_s was approximately tuned from the vector to pseudoscalar mass ratio in initial runs with fairly heavy quarks [11]. Their best determinations of the physical *strange* mass turned out to be lower, such that $m_s \simeq 1.14 m_s^{\rm Phys}$ [7], being $m_s^{\rm Phys}$ the physical mass of the *strange* quark. Within our framework, the *strange* quark mass may be related with the kaon mass through Eq. (11). The u/d quark valence-masses $m_{u,d}^{\rm val}$ (isospin limit is assumed) are varied continuously between nearly zero and the physical mass of the *strange* quark. Finally, the simulation is run for two values of the sea-masses of the quarks u/d: $m_{u,d}^{\rm sea} = m_s/2.3$ and $m_{u,d}^{sea} = m_s/4.5$ (squares and triangles, respectively, in Fig. 1(b)].

The modifications due to the sea-quark mass are much smaller than those from the valence-masses, as it is expected in the large N_C limit, since the closed quark loops would be suppressed by $1/N_C$. Thus, in this work I have generated the matrix elements for equal values of the sea and valence-masses, i.e., for $m_{u,d}^{\rm sea} = m_{u,d}^{\rm val}$, through a simple linear extrapolation from the samples $m_{u,d}^{\rm sea} = m_s/2.3$ and $m_{u,d}^{\rm sea} = m_s/4.5$ (error bars in Fig. 1(b)].

The theoretical expressions derived from RχT for F_π and F_K in Eqs. (13) were fitted to these extrapolated points. The relative error in my input data for the fit was 1%, a typical discretization error, but it did not account for the uncertainties in the $m_{u,d}^{\rm sea}$ extrapolation to the value $m_{u,d}^{\rm sea} = m_{u,d}^{\rm val}$. The variation of the decay constant when changing $m_{u,d}^{\rm sea}$ could be considered as a rough estimate of this uncertainty. In addition, the errors due to NLO contributions in $1/N_C$ have not been considered.

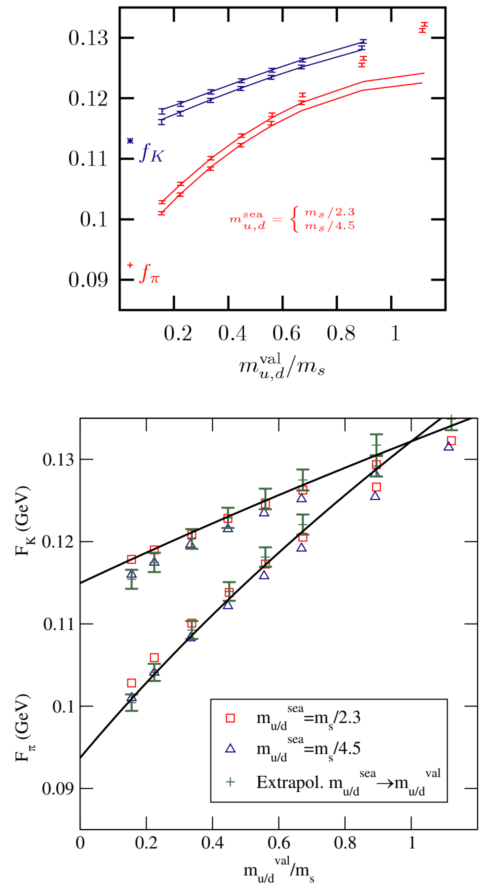

FIG. 1 (color online). (a) Lattice results for F_π and F_K and χPT extrapolations taken from MILC Collaboration . (b) Comparison of the Lattice results [7] with the RχT extrapolations for the values from the fit, $F = 93.7$ MeV, $M_S = 1020$ MeV and the splitting $e_m^S = -0.02$. The kaon mass varies as $m_K^2 = (m_K^{\rm Phys})^2 + [m_\pi^2 - (m_\pi^{\rm Phys})^2]/2$ due to its light-quark content. They are shown together with the data for $m_{u,d}^{\rm sea} = m_s/2.3$ (squares), $m_{u,d}^{\rm sea} = m_s/4.5$ (triangles) and the linear extrapolation to the value $m_{u,d}^{\rm sea} = m_{u,d}^{\rm val}$ (error bars).

The fit yields the values $F = 94.1 \pm 0.9$ MeV and $M_S = 1049 \pm 25$ MeV, with $\chi^2/\mathrm{dof} = 11.0/13$. This gives the physical pion and kaon decay constants, $F_\pi = 95.8 \pm 0.9$ MeV and $F_K = 113 \pm 1.4$ MeV, within an acceptable agreement with the experimental values $F_{\pi^+} = 92.4 \pm 0.07 \pm 0.3$ MeV and $F_{K^+} = 113.0 \pm 1.0 \pm 0.3$ MeV [12]. One may also estimate the χPT coupling at LO in $1/N_C$, $L_5 = F^2/4M_S^2 = (2.01 \pm 0.10) \cdot 10^{-3}$.

The chiral logarithms are not present in this calculation. In χPT they produce an important nonanalytic effect and a large bending in the F_π curve, since its slope becomes large at small pion mass due to $\frac{dF_\pi}{dm_\pi^2} \sim \ln\frac{m_\pi^2}{\mu}$. The effect of the logs on F_K is much more reduced since the value of the kaon mass does not become small when $m_{u,d} \rightarrow 0$. That is the reason for the better agreement of the F_K result. Eventually the NLO calculation in $1/N_C$ (one loop) would introduce this extra nonanalytic curvature and the usual one loop result for F_π in χPT would be recovered.

The mass splitting between the two isoscalar resonances can also be studied since their contributions to F_π and F_K are different. The full discussion is developed in the appendix. At LO in $1/N_C$ one finds two mass eigenstates with square masses $\bar{M}_{S_n}^2 \equiv M_S^2 - 8e_m^S B_0 m_{u,d}$ and $\bar{M}_{S_s}^2 \equiv M_S^2 - 8e_m^S B_0 m_s$, with quark contents $(\frac{1}{\sqrt{2}}\bar{u}u + \frac{1}{\sqrt{2}}\bar{d}d)$ and $\bar{s}s$ respectively [13]. The pion and kaon decay constants depend now on the splitting parameter e_m^S, which rules the relation between the resonance masses and the quark masses at LO in $1/N_C$. His value is not fixed by chiral symmetry and it changes from one multiplet to another. It depends on the internal dynamics of that particular hadron and, therefore, the presence of massive quarks does not modify the resonance masses by just the naive addition of a pion or kaon mass. However, our fit to the former data is not sensitive to this coupling, yielding: $F = 93.7 \pm 1.5$ MeV, $M_S = 1020 \pm 80$ MeV and $e_m^S = -0.02 \pm 0.05$, with similar $\chi^2/\mathrm{dof} = 10.9/12$. It provides an estimate of the scalar masses at large N_C: One gets the values $\bar{M}_{S_n} = 1020 \pm 80$ MeV and $\bar{M}_{S_s} = 1040 \pm 90$ MeV, highly correlated, and their splitting, equal to $\bar{M}_{S_s} - \bar{M}_{S_n} = 20 \pm 40$ MeV. One must be cautious interpreting this LO result. The NLO corrections from the loops and the OZI-rule violations may introduce relevant effects. Thus, whereas at LO in $1/N_C$ it is described by just the LO masses, at NLO it might gain radiative corrections and shifts in the physical masses even of the order of $1/N_C \simeq 33\%$.

The RχT extrapolation is shown in Fig. 1(b) (solid line). Since our resonance lagrangian does not distinguish between valence and sea quarks, the RχT expressions are compared with an emulation of the Lattice data for $m_{u,d}^{\mathrm{sea}} = m_{u,d}^{\mathrm{val}}$, obtained by linear extrapolation, as it was explained before.

V. CONCLUSIONS

RχT has been shown to be an interesting tool to analyze the Lattice data, which are usually generated for nonphysical values of the *up*, *down* quark masses. The present study hints that the light mesonic resonances may play an important role in the extrapolations at large u, d masses. This work explores the pion and kaon decay constants, providing successful results. Its importance and aim is not just the determination of the decay constants but to present an alternative idea about how to interpret the Lattice simulations for large unphysical values of the masses, giving a clear explanation of why the usual linear extrapolations yield such a good result. Likewise, this provides solid theoretical foundations based on the underlying QCD for these techniques. Thus, the $1/N_C$ expansion might be as well a suitable framework to describe the heavy quark matrix elements $(f_B, B_B \ldots)$ at large values of the u, d quark masses, where a similar linear behavior has been also observed.

The fact that at low energies RχT recovers χPT ensures that we are introducing the proper low mass behavior [3,4,6]. Former works [8,9] noticed the necessity of a separation scale Λ_{cut} where the χPT loops become irrelevant. The resonance masses provide a 'natural' scale where the χPT extrapolations break down and where the dynamics of the observable changes drastically.

The fits to the simulations were done for an emulation of the Lattice data, obtained by extrapolating $m_{u,d}^{\mathrm{sea}}$ to the value $m_{u,d}^{\mathrm{sea}} = m_{u,d}^{\mathrm{val}}$, that varied in a wide and continuous range between zero and the *strange* quark mass. For a more proper analysis one would need a simulation with equal sea and valence-masses. However, the main dependence comes from the valence-quarks and the sea-quark effects are small, since the closed quark loops are suppressed by $1/N_C$. Therefore, the present calculation can be considered an adequate estimate of the hadronic parameters. The values $F = 94.1 \pm 0.9$ MeV and $M_S = 1049 \pm 25$ MeV were obtained from that fit, together with the χPT coupling estimate at LO in $1/N_C$, $L_5 = (2.01 \pm 0.10) \times 10^{-3}$. The scalar mass splitting showed large uncertainties.

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APPENDIX: SCALAR MASS SPLITTING

The scalar resonance multiplet has been considered at first as degenerate in mass. Nonetheless the resonance masses can gain contributions due to the quark masses.

In the large N_C limit, chiral symmetry requires that at order $\mathcal{O}(m_q)$ the mass splitting comes only through a chiral invariant term [13]:

$$\Delta \mathcal{L}_{m_q} = e_m^S \langle \chi_+ S^2 \rangle, \tag{15}$$

being e_m^S an $\mathcal{O}(N_C^0)$ dimensionless constant, independent of the quark masses.

The shift in the scalar field is now slightly different:

$$S = \bar{S} + c_m \chi_+ [M_S^2 - 2e_m^S \chi_+]^{-1}. \tag{16}$$

The mass eigenvalues for the $I = 0$ scalars are not M_S^2 anymore, but $\bar{M}_{S_n} \equiv M_S^2 - 8e_m^S B_0 m_{u,d}$, for the state $(\frac{1}{\sqrt{2}} \bar{u}u + \frac{1}{\sqrt{2}} \bar{d}d)$, and $\bar{M}_{S_s} \equiv M_S^2 - 8e_m^S B_0 m_s$, for $\bar{s}s$. Nonetheless, the physical scalar states will separate from this ideal mixing and their masses will gain contributions due to NLO effects in $1/N_C$.

At LO in $1/N_C$ the pNGB masses still remain as given in Eq. (11). However, the rescaling factors for pions and kaons change accordingly:

$$\begin{aligned} C_\pi &= \left[1 + \frac{2m_\pi^2}{M_S^2 - 4e_m^S m_\pi^2} \right]^{-1}, \\ C_K &= \left[1 + \frac{2m_K^2}{M_S^2 - 4e_m^S m_K^2} \right]^{-1} \quad , \end{aligned} \tag{17}$$

and, therefore, the pion and kaon decay constants get modified.

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