

Spectator effects and lifetimes of heavy hadrons

Fabrizio Gabbiani,¹ Andrei I. Onishchenko,¹ and Alexey A. Petrov^{1,2}

¹*Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48201, USA*

²*Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, Michigan 48109, USA*

(Received 22 July 2004; published 17 November 2004)

We present a calculation of subleading $1/m_b$ corrections to spectator effects in the ratios of beauty hadron lifetimes in heavy-quark expansion. We find that these effects are sizable and should be taken into account in systematic analyses of heavy hadron lifetimes. In particular, the inclusion of $1/m_b$ corrections brings into agreement the theoretical predictions and experimental observations of the ratio of lifetimes of Λ_b baryon and B_d meson. We obtain $\tau(B_u)/\tau(B_d) = 1.06 \pm 0.02$, $\tau(B_s)/\tau(B_d) = 1.00 \pm 0.01$, $\tau(\Lambda_b)/\tau(B_d) = 0.86 \pm 0.05$.

DOI: 10.1103/PhysRevD.70.094031

PACS numbers: 12.39.Hg, 13.25.Hw, 13.30.Eg

I. INTRODUCTION

The hierarchy of lifetimes of heavy hadrons can be understood in the heavy-quark expansion (HQE), which makes use of the disparity of scales present in the decays of hadrons containing b quarks. HQE predicts the ratios of lifetimes of beauty mesons, which agree with the experimental observations well within experimental and theoretical uncertainties. Most recent analyses give [1,2]

$$\begin{aligned} \tau(B_u)/\tau(B_d)|_{ex} &= 1.085 \pm 0.017, \\ \tau(B_u)/\tau(B_d)|_{th} &= 1.07 \pm 0.03, \end{aligned} \quad (1)$$

$$\begin{aligned} \tau(B_s)/\tau(B_d)|_{ex} &= 0.951 \pm 0.038, \\ \tau(B_s)/\tau(B_d)|_{th} &= 1.00 \pm 0.02, \end{aligned} \quad (2)$$

which show evidence of excellent agreement of theoretical predictions and experimental measurements. This agreement also provides us with some confidence that quark-hadron duality, which states that smeared partonic amplitudes can be replaced by the hadronic ones, is expected to hold in inclusive decays of heavy flavors. The heavy-quark expansion also provides a qualitative understanding of the hierarchy of lifetimes of charmed hadrons. Since the expansion parameter there is much larger, only qualitative agreement is expected.

The only remaining problem appears in the ratios of lifetimes of heavy mesons and baryons. For instance, the low experimental value of the ratio $\tau(\Lambda_b)/\tau(B_d)$ has long been a puzzle for the theory. The latest compilation of experimental observations [2] suggests that

$$\tau(\Lambda_b)/\tau(B_d)|_{ex} = 0.797 \pm 0.053, \quad (3)$$

which is inconsistent with the first theoretical predictions of this ratio, 0.90 – 0.95 [3–5] at the level of approximately 2σ . Only recent next-to-leading-order (NLO) calculations of perturbative QCD [6] and $1/m_b$ corrections [7] to spectator effects significantly reduced the discrepancy, yielding, as reported in [8],

$$\tau(\Lambda_b)/\tau(B_d)|_{th} = 0.88 \pm 0.05, \quad (4)$$

which appears in some agreement with experimental data, mainly due to sizable theoretical and experimental uncertainties. The problem of $\tau(\Lambda_b)/\tau(B_d)$ ratio could reappear again if future measurements at Fermilab and CERN would find the mean value to stay the same with error bars shrinking. Upcoming Fermilab measurements of Λ_b lifetime could shed more light on the experimental side of this issue.

Here we expand our calculation of subleading contributions to spectator effects in the $1/m_b$ expansion [7] to study their impact on the ratios of lifetimes of heavy mesons. We include the full charm quark-mass dependence, generalizing our result of Ref. [7]. We also discuss the convergence of the $1/m_b$ expansion in the analysis of spectator effects by estimating next, $1/m_b^2$ -suppressed corrections to the spectator effects in factorization.

This paper is organized as follows. We set up the relevant formalism and argue for the need to compute subleading $1/m_b$ corrections to spectator effects in Sec. II. In Sec. III we discuss the impact of $1/m_b$ corrections to lifetime ratios of heavy mesons and baryons and assess the convergence of the $1/m_b$ expansion. We present the complete results from $1/m_b^2$ corrections to spectator effects. Finally, we present our conclusions in Sec. IV. Some of the relevant formulas are shown in the appendices.

II. FORMALISM

The inclusive decay rate of a heavy hadron H_b is most conveniently computed by employing the optical theorem to relate the decay width to the imaginary part of the forward matrix element of the transition operator:

$$\Gamma(H_b) = \frac{1}{2M_{H_b}} \langle H_b | \mathcal{T} | H_b \rangle, \quad (5)$$

$$\mathcal{T} = \text{Im}i \int d^4x T \{ H_{\text{eff}}(x) H_{\text{eff}}(0) \}.$$

Here H_{eff} represents an effective $\Delta B = 1$ Hamiltonian,

$$H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_{d'=d,s,u'=u,c} V_{u'd'}^* [C_1(\mu) Q_1^{u'd'}(\mu) + C_2(\mu) Q_2^{u'd'}(\mu)] + h.c., \quad (6)$$

where the four-quark operators Q_1 and Q_2 are given by

$$Q_1^{u'd'} = \bar{d}'_L \gamma_\mu u'_L \bar{c}_L \gamma^\mu b_L, \quad Q_2^{u'd'} = \bar{c}_L \gamma_\mu u'_L \bar{d}'_L \gamma^\mu b_L. \quad (7)$$

In the heavy-quark limit, the energy release is large, so the correlator in Eq. (5) is dominated by short-distance physics. An Operator Product Expansion (OPE) can be constructed for Eq. (5), which results in a prediction of decay widths of Eq. (5) as a series of matrix elements of local operators of increasing dimension suppressed by powers of $1/m_b$ ¹:

$$\begin{aligned} \Gamma(H_b) &= \frac{1}{2M_{H_b}} \sum_k \langle H_b | \mathcal{T}_k | H_b \rangle \\ &= \sum_k \frac{C_k(\mu)}{m_b^k} \langle H_b | \mathcal{O}_k^{\Delta B=0}(\mu) | H_b \rangle. \end{aligned} \quad (8)$$

In other words, the calculation of $\Gamma(H_b)$ is equivalent to computing the matching coefficients of the effective $\Delta B = 0$ Lagrangian with subsequent computation of its matrix elements. Indeed, at the end, the scale dependence of the Wilson coefficients in Eq. (8) should match the scale dependence of the computed matrix elements.

It is customary to make predictions for the ratios of lifetimes (widths), as many theoretical uncertainties cancel out in the ratio. In addition, since the differences of lifetimes should come from the differences in the “brown

mucks” of heavy hadrons, at the leading order in HQE all beauty hadrons with light spectators have the same lifetime.

The difference between meson and baryon lifetimes first occurs at order $1/m^2$ and is essentially due to the different structure of mesons and baryons. In other words, no lifetime difference is induced among the members of the meson multiplet. The ratio of heavy-meson and baryon lifetimes receives a shift which amounts to at most 1–2%, not sufficient to explain the observed pattern of lifetimes.

The main effect appears at the $1/m^3$ level and comes from the set of dimension-six four-quark operators. Their contribution is enhanced due to the phase-space factor $16\pi^2$, and induces corrections of order $16\pi^2(\Lambda_{\text{QCD}}/m_b)^3 = \mathcal{O}(5\text{--}10\%)$. These operators introduce a difference in lifetimes for both heavy mesons and baryons. The effects of matrix elements of these operators, commonly referred to as Weak Annihilation (WA), Weak Scattering (WS), and Pauli Interference (PI) have been computed in [4,9,11–16] at leading order in perturbative QCD, and, more recently, including NLO perturbative QCD corrections in [6] and $1/m_b$ corrections [7].

The contributions of these operators to the lifetime ratios are governed by the matrix elements of $\Delta B = 0$ four-fermion operators

$$\mathcal{T}_{\text{spec}} = \mathcal{T}_{\text{spec}}^u + \mathcal{T}_{\text{spec}}^{d'} + \mathcal{T}_{\text{spec}}^{s'}, \quad (9)$$

where the \mathcal{T}_i 's are

$$\begin{aligned} \mathcal{T}_{\text{spec}}^u &= \frac{G_F^2 m_b^2 |V_{bc}|^2 (1-z)^2}{2\pi} \{ (c_1^2 + c_2^2) O_1^u + 2c_1 c_2 \tilde{O}_1^u + \delta_{1/m}^u + \delta_{1/m^2}^u \}, \\ \mathcal{T}_{\text{spec}}^{d'} &= -\frac{G_F^2 m_b^2 |V_{bc}|^2 (1-z)^2}{4\pi} \left\{ c_1^2 \left[(1+z) O_1^{d'} + \frac{2}{3} (1+2z) O_2^{d'} \right] + (N_c c_2^2 + 2c_1 c_2) \left[(1+z) \tilde{O}_1^{d'} + \frac{2}{3} (1+2z) \tilde{O}_2^{d'} \right] \right. \\ &\quad \left. + \delta_{1/m}^{d'} + \delta_{1/m^2}^{d'} \right\}, \\ \mathcal{T}_{\text{spec}}^{s'} &= -\frac{G_F^2 m_b^2 |V_{bc}|^2 \sqrt{1-4z}}{4\pi} \left\{ c_1^2 \left[O_1^{s'} + \frac{2}{3} (1+2z) O_2^{s'} \right] + (N_c c_2^2 + 2c_1 c_2) \left[\tilde{O}_1^{s'} + \frac{2}{3} (1+2z) \tilde{O}_2^{s'} \right] + \delta_{1/m}^{s'} + \delta_{1/m^2}^{s'} \right\}. \end{aligned} \quad (10)$$

Here the terms $\delta_{1/m}^i$ and δ_{1/m^2}^i refer to $1/m_b$ and $1/m_b^2$ corrections to spectator effects, which we discuss below. Note that we include the full $z = m_c^2/m_b^2$ dependence, which is fully consistent only after the inclusion of higher $1/m_b$ corrections. The operators O_i and \tilde{O}_i in Eq. (10) are

defined as

$$\begin{aligned} O_1^q &= \bar{b}_i \gamma^\mu (1 - \gamma_5) b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) q_j, \\ O_2^q &= \bar{b}_i \gamma^\mu \gamma_5 b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) q_j, \\ \tilde{O}_1^q &= \bar{b}_i \gamma^\mu (1 - \gamma_5) b_j \bar{q}_i \gamma_\mu (1 - \gamma_5) q_j, \\ \tilde{O}_2^q &= \bar{b}_i \gamma^\mu \gamma_5 b_j \bar{q}_i \gamma_\mu (1 - \gamma_5) q_j. \end{aligned} \quad (11)$$

¹We choose the heavy-quark pole mass as an expansion parameter, which is consistent with most of the previous calculations. Of course, final predictions for the lifetime ratios are independent of that choice [10].

In addition, there is a dimension-six operator $O_3^q = \bar{b}_i \gamma^\mu (1 - \gamma_5) b_i \bar{q}_j \gamma_\mu \gamma_5 q_j$ whose contribution is always suppressed by powers of the light-quark mass. It gives a

negligible contribution to the ratios of lifetimes of heavy hadrons (see also Sec. III).

In order to assess the impact of these and other operators, parametrizations of their matrix elements must be introduced. The meson matrix elements are

$$\begin{aligned}\langle B_q | O_1^q | B_q \rangle &= f_{B_q}^2 m_{B_q}^2 \left(2\epsilon_1 + \frac{B_1}{N_c} \right), \\ \langle B_q | \tilde{O}_1^q | B_q \rangle &= f_{B_q}^2 m_{B_q}^2 B_1, \\ \langle B_q | O_2^q | B_q \rangle &= -f_{B_q}^2 m_{B_q}^2 \left[\frac{m_{B_q}^2}{(m_b + m_q)^2} \left(2\epsilon_2 + \frac{B_2}{N_c} \right) \right. \\ &\quad \left. + \frac{1}{2} \left(2\epsilon_1 + \frac{B_1}{N_c} \right) \right], \\ \langle B_q | \tilde{O}_2^q | B_q \rangle &= -f_{B_q}^2 m_{B_q}^2 \left[\frac{m_{B_q}^2}{(m_b + m_q)^2} B_2 + \frac{1}{2} B_1 \right].\end{aligned}\quad (12)$$

Here the parameters B_i and ϵ_i are usually referred to as ‘‘singlet’’ and ‘‘octet’’ parameters (see Appendix A for the explanation). Expressed in terms of these parameters, the lifetime ratios of heavy mesons can be written as

$$\begin{aligned}\tau(B_u)/\tau(B_d) &= 1 + 16\pi^2 \frac{f_B^2 m_B}{m_b^3 c_3(m_b)} [G_1^{ss}(m_b) B_1(m_b) \\ &\quad + G_1^{oo}(m_b) \epsilon_1(m_b) + G_2^{ss}(m_b) B_2(m_b) \\ &\quad + G_1^{oo}(m_b) \epsilon_2(m_b)] + \delta_{1/m},\end{aligned}\quad (13)$$

where the coefficients G were computed at NLO in [6,15]. $\delta_{1/m}$ represents spectator corrections of order $1/m_b$ and higher, which we present below in this paper.

Estimates of the matrix elements of four-fermion operators in baryon decays are not easy. Following [4], the parameter \tilde{B} is used to account for the deviation of the Λ_b wave function from being totally color-asymmetric,

$$\begin{aligned}\langle \Lambda_b | O_1^q | \Lambda_b \rangle &= -\tilde{B} \langle \Lambda_b | \tilde{O}_1^q | \Lambda_b \rangle = \frac{\tilde{B}}{6} f_{B_q}^2 m_{B_q} m_{\Lambda_b} r, \\ \langle \Lambda_b | O_2^q | \Lambda_b \rangle &= -\tilde{B} \langle \Lambda_b | \tilde{O}_2^q | \Lambda_b \rangle = \frac{\tilde{B}}{6} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \delta.\end{aligned}\quad (14)$$

Note that $\delta = \mathcal{O}(1/m_b)$, which follows from the heavy-quark spin symmetry. It needs to be included as we consider higher-order corrections in $1/m_b$. $\tilde{B} = 1$ in the valence approximation. While these parameters have not been computed model-independently, various quark-model arguments suggest that the meson and baryon matrix elements are quite different. Thus a meson-baryon lifetime difference can be produced. In general, one can

parametrize the meson-baryon lifetime ratio as

$$\begin{aligned}\tau(\Lambda_b)/\tau(B_d) &\simeq 0.98 - (d_1 + d_2 \tilde{B})r - (d_3 \epsilon_1 + d_4 \epsilon_2) \\ &\quad - (d_5 B_1 + d_6 B_2) \\ &\simeq 0.98 - m_b^2 (d'_1 + d'_2 \tilde{B})r - m_b^2 [(d'_3 \epsilon_1 + d'_4 \epsilon_2) \\ &\quad + (d'_5 B_1 + d'_6 B_2)],\end{aligned}\quad (15)$$

where in the last line we scaled out the coefficient m_b^2 emphasizing the fact that these corrections are suppressed by $1/m_b^3$ compared to the leading m_b^5 effect. The scale-dependent parameters $(d_i(m_b) = \{0.023, 0.028, 0.16, -0.16, 0.08, -0.08\}$ at NLO) are defined in [4]. The parameter $r = |\psi_{bq}^{\Lambda_b}(0)|^2 / |\psi_{bq}^{B_q}(0)|^2$ is the ratio of the wave functions at the origin of the Λ_b and B_q mesons.

It is interesting to note that in the absence of $1/m_b$ corrections to spectator effects, it would be equally correct to substitute the b -quark mass in Eq. (15) with the corresponding meson and baryon masses, so

$$\begin{aligned}\tau(\Lambda_b)/\tau(B_d) &\simeq 0.98 - m_{\Lambda_b}^2 (d'_1 + d'_2 \tilde{B})r \\ &\quad - m_{B_d}^2 [(d'_3 \epsilon_1 + d'_4 \epsilon_2) + (d'_5 B_1 + d'_6 B_2)],\end{aligned}\quad (16)$$

which reflects the fact that WS and PI effects occur for the heavy and light quarks initially bound in the B_d meson and Λ_b baryon, respectively. While correct up to the order $1/m_b^3$, these simple substitutions reduce the ratio of lifetimes by approximately 3–4%! We take this as an indication of the importance of bound-state effects on the spectator corrections, represented by subleading $1/m_b$ corrections to spectator operators.

III. SUBLEADING CORRECTIONS TO SPECTATOR EFFECTS

We computed the higher-order corrections, including charm quark-mass effects, to Eq. (10) in the heavy-quark expansion, denoted below as $\delta_{1/m}^q$ and δ_{1/m^2}^q .

A. $1/m_b$ corrections

The $1/m_b$ corrections to the spectator effects are computed, as in Ref. [7], by expanding the forward scattering amplitude of Eq. (5) in the light-quark momentum and matching the result onto the operators containing derivative insertions (see Fig. 1). The $\delta_{1/m}^q$ contributions can be written in the following form:

$$\begin{aligned}
\delta_{1/m}^u &= -2(c_1^2 + c_2^2) \frac{1+z}{1-z} R_1^u - 4c_1c_2 \frac{1+z}{1-z} \tilde{R}_1^u, \\
\delta_{1/m}^{d'} &= c_1^2 \left[\frac{8z^2}{1-z} R_0^{d'} + \frac{2}{3} \frac{1+z+10z^2}{1-z} R_1^{d'} + \frac{2}{3} (1+2z)(R_2^{d'} - R_3^{d'}) \right] + (N_c c_2^2 + 2c_1c_2) \left[\frac{8z^2}{1-z} \tilde{R}_0^{d'} + \frac{2}{3} \frac{1+z+10z^2}{1-z} \tilde{R}_1^{d'} \right. \\
&\quad \left. + \frac{2}{3} (1+2z)(\tilde{R}_2^{d'} - \tilde{R}_3^{d'}) \right], \\
\delta_{1/m}^{s'} &= c_1^2 \left[\frac{16z^2}{1-4z} R_0^{s'} + \frac{2}{3} \frac{1-2z+16z^2}{1-4z} R_1^{s'} + \frac{2}{3} (1+2z)(R_2^{s'} - R_3^{s'}) \right] + (N_c c_2^2 + 2c_1c_2) \left[\frac{16z^2}{1-4z} \tilde{R}_0^{s'} \right. \\
&\quad \left. + \frac{2}{3} \frac{1-2z+16z^2}{1-4z} \tilde{R}_1^{s'} + \frac{2}{3} (1+2z)(\tilde{R}_2^{s'} - \tilde{R}_3^{s'}) \right],
\end{aligned} \tag{17}$$

where the following operators contribute

$$\begin{aligned}
R_0^q &= \frac{1}{m_b^2} \bar{b}_i \gamma^\mu \gamma_5 \vec{D}^\alpha b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) \vec{D}_\alpha q_j, \\
R_1^q &= \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1 - \gamma_5) \vec{D}^\alpha b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) \vec{D}_\alpha q_j, \\
R_2^q &= \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1 - \gamma_5) \vec{D}^\nu b_i \bar{q}_j \gamma_\nu (1 - \gamma_5) \vec{D}_\mu q_j, \\
R_3^q &= \frac{m_q}{m_b} \bar{b}_i (1 - \gamma_5) b_i \bar{q}_j (1 - \gamma_5) q_j.
\end{aligned} \tag{18}$$

Here \tilde{R}_i^q denote the color-rearranged operators that follow from the expressions for R_i^q by interchanging color indices of b_i and q_j Dirac spinors. Note that the above result contains *full* QCD b fields, thus there is no immediate power counting available for these operators. The power counting becomes manifest at the level of the matrix elements.

In the $z \rightarrow 0$ limit the result of Eq. (17) reduces to the set of $1/m_b$ corrections obtained by us in Ref. [7]. We complete the calculation presented there by computing the contribution of Eq. (17) to the ratios of heavy-meson lifetimes. As before, we use factorization to guide our parametrizations of Λ_b and meson matrix elements, but,

contrary to Ref. [7], we elect to keep corrections that would vanish in the vacuum-insertion approximation. Our parametrizations for meson and baryon matrix elements are presented in Appendix B. Inserting Eqs. (B2) and (B5) of Appendix B into Eq. (17) we obtain an estimate of the effect. We postpone the presentation of the answer, complete with our estimate of its uncertainties, until the end of this section, after we assess the convergence of the $1/m_b$ expansion for the spectator corrections.

Numerically, the set of $1/m_b$ corrections does not markedly affect the ratios of meson lifetimes, changing the $\tau(B_u)/\tau(B_d)$ and the $\tau(B_s)/\tau(B_d)$ ratios by less than half a percent. The effect is more pronounced in the ratio of Λ_b and B_d lifetimes, where it constitutes 40–45% of the leading spectator contribution represented by WS and PI effects [see Eqs. (15) and (16)], or an overall correction of about $-(3-4)\%$ to the $\tau(\Lambda_b)/\tau(B_d)$ ratio. While such a sizable effect is surprising, the main source of such a large correction can be readily identified. While the individual $1/m_b$ corrections to the matrix elements representing WS and PI are of order 20%, as expected from the naive power counting, they contribute to the Λ_b lifetime with the same (negative) sign, instead of destructively interfering as do the contributions to Eqs. (15) and (16) representing WS and PI [5], as explained in [7]. This conspiracy of two $\sim 20\%$ effects produces a sizable shift in the ratio of the Λ_b - and B -meson lifetimes.

B. $1/m_b^2$ corrections

In order to see how well the $1/m_b$ expansion converges in the calculation of lifetime ratios, we compute a set of δ_{1/m_b^2} corrections to spectator effects. It is expected that at this order even more operators will contribute, so the exact prediction of lifetime differences becomes unfeasible. Therefore, we shall use our calculation only to see to what extent we can trust our results for the $1/m_b$ corrections. We will parametrize $1/m_b^2$ corrections similar to our parametrization of $1/m_b$ effects above and will use the factorization approximation to assess their contributions to the lifetime ratios.

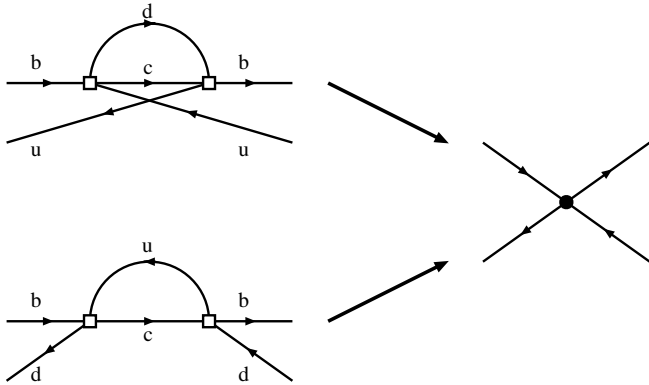


FIG. 1. Kinetic corrections to spectator effects. The operators of Eqs. (17) and (19) are obtained by expanding the diagrams in powers of spectator's momentum.

There are two classes of corrections that arise at this order. One class involves kinetic corrections which can be computed in a way analogous to the previous case by expanding the forward scattering amplitudes in the

powers of spectator momentum (see Fig. 1). A second class involves corrections arising from an interaction with background gluon fields. The kinetic corrections can be written as:

$$\begin{aligned}
\delta_{1/m^2}^u &= (c_1^2 + c_2^2) \left[\frac{m_u^2}{m_b^2} \frac{1+z}{1-z} O_1^u + \frac{8z^2}{(1-z)^2} W_1^u \right] + 2c_1 c_2 \left[\frac{m_u^2}{m_b^2} \frac{1+z}{1-z} \tilde{O}_1^u + \frac{8z^2}{(1-z)^2} \tilde{W}_1^u \right], \\
\delta_{1/m^2}^{d'} &= c_1^2 \left[\frac{m_{d'}^2}{m_b^2} \frac{1+z+2z^2}{1-z} O_1^{d'} + \frac{m_{d'}^2}{m_b^2} \frac{4z^2}{1-z} O_2^{d'} + \frac{m_{d'}^2}{m_b^2} \frac{2(1+2z)}{3} O_3^{d'} + \frac{4z^2(7z-5)}{(1-z)^2} W_1^{d'} + \frac{8z^2(4z-3)}{(1-z)^2} W_2^{d'} \right. \\
&\quad \left. + \frac{8z^2}{1-z} (W_3^{d'} - W_4^{d'}) \right] + (N_c c_2^2 + c_1 c_2) \left[\frac{m_{d'}^2}{m_b^2} \frac{1+z+2z^2}{1-z} \tilde{O}_1^{d'} + \frac{m_{d'}^2}{m_b^2} \frac{4z^2}{1-z} \tilde{O}_2^{d'} + \frac{m_{d'}^2}{m_b^2} \frac{2(1+2z)}{3} \tilde{O}_3^{d'} \right. \\
&\quad \left. + \frac{4z^2(7z-5)}{(1-z)^2} \tilde{W}_1^{d'} + \frac{8z^2(4z-3)}{(1-z)^2} \tilde{W}_2^{d'} + \frac{8z^2}{1-z} (\tilde{W}_3^{d'} - \tilde{W}_4^{d'}) \right], \\
\delta_{1/m^2}^{s'} &= c_1 \left[\frac{m_{s'}^2}{m_b^2} \frac{1-2z}{1-4z} O_1^{s'} + \frac{m_{s'}^2}{m_b^2} \frac{8z^2}{1-4z} O_2^{s'} + \frac{m_{s'}^2}{m_b^2} \frac{2(1+2z)}{3} O_3^{s'} + \frac{8z^2(16z-5)}{(1-4z)^2} W_1^{s'} + \frac{16z^2(10z-3)}{(1-4z)^2} W_2^{s'} \right. \\
&\quad \left. + \frac{16z^2}{1-4z} (W_3^{s'} - W_4^{s'}) \right] + (N_c c_2^2 + 2c_1 c_2) \left[\frac{m_{s'}^2}{m_b^2} \frac{1-2z}{1-4z} \tilde{O}_1^{s'} + \frac{m_{s'}^2}{m_b^2} \frac{8z^2}{1-4z} \tilde{O}_2^{s'} + \frac{m_{s'}^2}{m_b^2} \frac{2(1+2z)}{3} \tilde{O}_3^{s'} \right. \\
&\quad \left. + \frac{8z^2(16z-5)}{(1-4z)^2} \tilde{W}_1^{s'} + \frac{16z^2(10z-3)}{(1-4z)^2} \tilde{W}_2^{s'} + \frac{16z^2}{1-4z} (\tilde{W}_3^{s'} - \tilde{W}_4^{s'}) \right].
\end{aligned} \tag{19}$$

We again retain the dependence on quark masses in the above expression, including the terms proportional to light-quark masses (and thus multiplied by the operators O_i). The operators in Eq. (19) are defined as

$$\begin{aligned}
W_1^q &= \frac{1}{m_b^4} \bar{b}_i \gamma^\mu (1 - \gamma_5) \tilde{D}^\alpha \tilde{D}^\beta b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) \tilde{D}_\alpha \tilde{D}_\beta q_j, \\
W_2^q &= \frac{1}{m_b^4} \bar{b}_i \gamma^\mu \gamma_5 \tilde{D}^\alpha \tilde{D}^\beta b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) \tilde{D}_\alpha \tilde{D}_\beta q_j, \\
W_3^q &= \frac{1}{m_b^4} \bar{b}_i \gamma^\mu (1 - \gamma_5) \tilde{D}^\nu \tilde{D}^\alpha b_i \bar{q}_j \gamma_\nu (1 - \gamma_5) \tilde{D}_\mu \tilde{D}_\alpha q_j, \\
W_4^q &= \frac{m_q}{m_b} \bar{b}_i (1 - \gamma_5) \tilde{D}^\alpha b_i \bar{q}_j (1 - \gamma_5) \tilde{D}_\alpha q_j,
\end{aligned} \tag{20}$$

where, as before, \tilde{W}_i^q denote the color-rearranged operators that follow from the expressions for W_i^q by interchanging the color indices of b_i and q_j Dirac spinors. The parametrization of matrix elements of these operators can be found in Appendix B.

In addition to the set of kinetic corrections considered above, the effects of the interactions of the intermediate quarks with background gluon fields should also be included at this order. The contribution of those operators can be computed from the diagrams of Fig. 2, resulting in

$$\mathcal{T}_{\text{spec},G} = \mathcal{T}_{\text{spec},G}^u + \mathcal{T}_{\text{spec},G}^{d'} + \mathcal{T}_{\text{spec},G}^{s'}, \tag{21}$$

where

$$\mathcal{T}_{\text{spec},G}^u = 0, \tag{22}$$

$$\begin{aligned}
\mathcal{T}_{\text{spec},G}^{d'} &= -\frac{G_F^2 |V_{bc}|^2}{4\pi} \{ c_1^2 [(1-z^2)P_1^{d'} - (1-z^2)P_2^{d'} \\
&\quad + 2z(1-z)P_3^{d'} + 4z^2P_4^{d'}] + 2c_1 c_2 z [(1-z)P_5^{d'} + (1-z)P_6^{d'} + 2zP_7^{d'} + 2zP_8^{d'}] \},
\end{aligned} \tag{23}$$

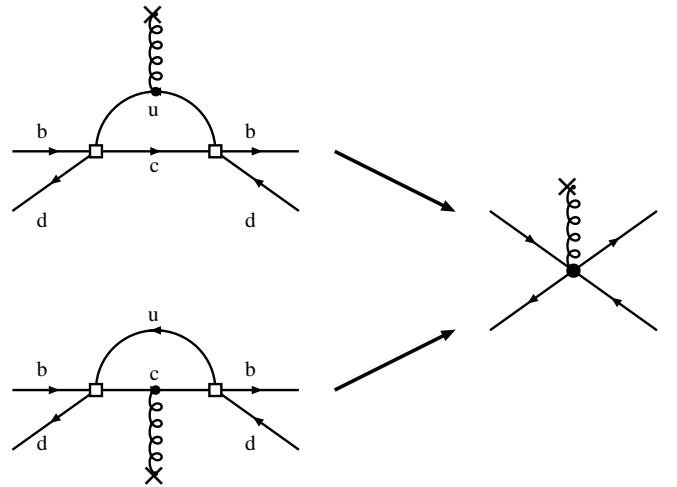


FIG. 2. Corrections to spectator contribution coming from the interactions with background gluon fields.

TABLE I. Coefficients k_i appearing in Eq. (27) for $\tau(\Lambda_b)/\tau(B_d)$.

μ	Coefficients k_i for the parameters listed below. All entries are $\times 10^{-3}$.											
	B_1	B_2	ϵ_1	ϵ_2	r	δ	$\tilde{B}r$	$\tilde{B}\delta$	β_0	$\tilde{\beta}_0$	β_1	$\tilde{\beta}_1$
$m_b/2$	-7.82	9.19	-158	183	-27.4	3.76	-33.3	11.1	0.001	1.95	-0.014	-24.0
m_b	-7.96	9.31	-160	196	-22.4	3.39	-27.8	12.4	0.031	1.78	-0.382	-21.9
$2m_b$	-7.45	8.63	-160	201	-18.9	3.33	-24.6	13.0	0.079	1.69	-0.976	-20.9
μ	β_2	$\tilde{\beta}_2$	β_3	$\tilde{\beta}_3$	β_0^Λ	$\tilde{\beta}_0^\Lambda$	β_1^Λ	$\tilde{\beta}_1^\Lambda$	β_2^Λ	$\tilde{\beta}_2^\Lambda$	β_3^Λ	$\tilde{\beta}_3^\Lambda$
$m_b/2$	-0.002	-3.68	0.002	3.68	-0.144	-0.048	-12.7	-6.85	-0.818	-0.272	-0.818	-0.272
m_b	-0.058	-3.36	0.058	3.36	-0.132	-0.039	-11.2	-4.59	-0.747	-0.223	-0.747	-0.223
$2m_b$	-0.150	-3.20	0.150	3.20	-0.125	-0.030	-10.4	-3.09	-0.711	-0.170	-0.711	-0.170

$$\mathcal{T}_{\text{spec},G}^{s'} = -\frac{G_F^2 |V_{bc}|^2}{4\pi\sqrt{1-4z}} \{c_1^2 [(1-2z)P_1^{s'} - (1-4z)P_2^{s'} + 2zP_3^{s'} + 4zP_4^{s'}] + 4c_1c_2z[P_7^{s'} + P_8^{s'} + P_9^{s'} + P_{10}^{s'}]\}. \quad (24)$$

The local four-quark operators in the above formulas are shown in Eq. (25).

$$\begin{aligned}
P_1^q &= \bar{b}_k \gamma^\mu (1 - \gamma_5) t_{kl}^a \tilde{G}_{\mu\nu}^a b_l \bar{q}_i \gamma^\nu (1 - \gamma_5) q_i, & P_2^q &= \bar{b}_i \gamma^\mu (1 - \gamma_5) b_i \bar{q}_k \gamma^\nu (1 - \gamma_5) t_{kl}^a \tilde{G}_{\mu\nu}^a q_l, \\
P_3^q &= \bar{b}_k \gamma^\mu (1 + \gamma_5) t_{kl}^a \tilde{G}_{\mu\nu}^a b_l \bar{q}_i \gamma^\nu (1 - \gamma_5) q_i, & P_4^q &= \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1 - \gamma_5) \vec{D}^\nu \vec{D}^\alpha b_i \bar{q}_k \gamma_\alpha (1 - \gamma_5) t_{kl}^a \tilde{G}_{\mu\nu}^a q_l, \\
P_5^q &= \bar{b}_k \gamma^\mu (1 + \gamma_5) b_i t_{kl}^a \tilde{G}_{\mu\nu}^a \bar{q}_i \gamma^\nu (1 - \gamma_5) q_l, & P_6^q &= \bar{b}_i \gamma^\mu (1 + \gamma_5) b_k t_{kl}^a \tilde{G}_{\mu\nu}^a \bar{q}_l \gamma^\nu (1 - \gamma_5) q_i, \\
P_7^q &= \frac{1}{m_b^2} \bar{b}_k \gamma^\mu (1 - \gamma_5) \vec{D}^\nu \vec{D}^\alpha b_i t_{kl}^a \tilde{G}_{\mu\nu}^a \bar{q}_i \gamma_\alpha (1 - \gamma_5) q_l, & P_8^q &= \frac{1}{m_b^2} \bar{b}_i \gamma^\mu (1 - \gamma_5) \vec{D}^\nu \vec{D}^\alpha b_k t_{kl}^a \tilde{G}_{\mu\nu}^a \bar{q}_l \gamma_\alpha (1 - \gamma_5) q_i, \\
P_9^q &= \bar{b}_k \gamma^\mu b_i t_{kl}^a \tilde{G}_{\mu\nu}^a \bar{q}_i \gamma^\nu (1 - \gamma_5) q_l, & P_{10}^q &= \bar{b}_i \gamma^\mu b_k t_{kl}^a \tilde{G}_{\mu\nu}^a \bar{q}_l \gamma^\nu (1 - \gamma_5) q_i.
\end{aligned} \quad (25)$$

Analogously to the previous section, we parametrize the matrix elements as

$$\begin{aligned}
\langle B_q | P_i^q | B_q \rangle &= \frac{1}{4} f_{B_q}^2 m_{B_q}^2 \alpha_i^q \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right)^2, \\
\langle \Lambda_b | P_i^q | \Lambda_b \rangle &= \frac{1}{4} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \alpha_i^\Lambda \left(\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right)^2.
\end{aligned} \quad (26)$$

We set $\alpha_i^q = \alpha_i^\Lambda = 1 \text{ GeV}^2$ to obtain a numerical estimate of this effect. It is clear that no precise prediction is possible with so many operators contributing to the ratios of lifetimes. This, of course, is expected, as the number of contributing operators always increases significantly with each order in OPE. Fortunately, at least in factorization, the effect of $1/m_b^2$ corrections is tiny, a fraction of a percent. The contribution of $1/m_b^2$ -suppressed effect is included into the coefficient k_0 defined in the next section. We can evaluate the contribution of both $1/m_b$ and $1/m_b^2$

by performing two analyses. First, we can study the perturbative scale dependence of the final result using ‘‘factorized’’ values for the matrix elements.² We shall see that the scale dependence of the resulting effect is very mild. Second, we shall randomly vary the parameters describing the matrix elements by $\pm 30\%$ around their factorized values to see the uncertainty of our result.

C. Phenomenology of heavy hadron lifetimes

Let us now discuss the phenomenological implications of the results presented in the previous sections. As usual in the OPE-based calculation, next-order corrections bring new unknown coefficients. In our numerical results we assume the value of the b -quark pole mass to be $m_b = 4.8 \pm 0.1 \text{ GeV}$ and $f_B = 200 \pm 25 \text{ MeV}$.

The resulting lifetime ratios are parametrized according to Appendix B. For instance, in the case of $\tau(\Lambda_b)/\tau(B_d)$ we can write

$$\left. \frac{\tau(\Lambda_b)}{\tau(B_d)} \right|_{h.o.} = k_0 + \sum_{i=1} k_i p_i, \quad (27)$$

where p_i are the parameters defined in Appendix B,

²In the absence of model-independent calculations of the matrix elements described above, there is a number of ways to estimate them. We chose to use the quark model to relate the matrix elements of R_i and R_i operators to the matrix elements of dimension-six operators O_i , which are parametrized in terms of B_i and ϵ_i . We then use lattice QCD predictions for B_i and ϵ_i to obtain numerical estimates of our corrections. We shall refer to this procedure as ‘‘factorization.’’

TABLE II. Same as Table I for $\tau(B_u)/\tau(B_d)$.

μ	Coefficients k_i for the parameters listed below. All entries are $\times 10^{-3}$.											
	B_1	B_2	ϵ_1	ϵ_2	β_0	$\tilde{\beta}_0$	β_1	$\tilde{\beta}_1$	β_2	$\tilde{\beta}_2$	β_3	$\tilde{\beta}_3$
$m_b/2$	82.1	18.8	-782	206	0.001	1.95	-17.6	124	-0.002	-3.68	0.002	3.68
m_b	60.8	13.3	-727	211	0.031	1.78	-6.58	107	-0.058	-3.36	0.058	3.36
$2m_b$	46.5	9.98	-693	213	0.079	1.69	0.661	98.9	-0.150	-3.20	0.150	3.20

$$p_i = \{B_1, B_2, \epsilon_1, \epsilon_2, r, \delta, \tilde{B}r, \tilde{B}\delta, \beta_0, \tilde{\beta}_0, \beta_1, \tilde{\beta}_1, \beta_2, \tilde{\beta}_2, \beta_3, \tilde{\beta}_3, \beta_0^\Lambda, \tilde{\beta}_0^\Lambda, \beta_1^\Lambda, \tilde{\beta}_1^\Lambda, \beta_2^\Lambda, \tilde{\beta}_2^\Lambda, \beta_3^\Lambda, \tilde{\beta}_3^\Lambda\}, \quad (28)$$

and k_i are their coefficients. $k_0 = 1.00$ in the case of mesons, while for Λ_b $k_0 = \{0.991, 0.981, 0.975\}$ for $\mu = \{m_b/2, m_b, 2m_b\}$, respectively. They appear in Table I, and a similar procedure is followed for $\tau(B_u)/\tau(B_d)$ and $\tau(B_s)/\tau(B_d)$, whose coefficients are shown in Tables II and III, respectively. Since all three heavy mesons belong to the same SU(3) triplet, their lifetimes are the same at order $1/m_b^2$. The computation of the ratios of heavy-meson lifetimes is equivalent to the computation of U -spin or isospin-violating corrections. Both $1/m_b^3$ -suppressed spectator effects and our corrections computed in the previous sections arise from the spectator interactions and thus provide a source of U -spin or isospin-symmetry breaking. We shall, however, assume that the matrix elements of both $1/m_b^3$ and $1/m_b^4$ operators respect isospin. The ratio of lifetimes of B_s and B_d mesons involves a breaking of U -spin symmetry, so the matrix elements of dimension-six operators could differ by about 30%. We partially take this breaking into account by retaining light quark-mass dependence. Nevertheless, we still introduce different B and ϵ parameters to describe B_s and B_d lifetimes. We will also assume that, apart from the explicit light quark-mass effects, matrix elements of $1/m_b^4$ operators respect U -spin symmetry.

Since the computation of $1/m_b$ corrections to spectator effects was performed at leading order in QCD, we also studied the scale dependence of the result. A comparison between the results computed with leading logarithmic accuracy (LL), next-to-leading log accuracy (NLL), and the total NLL including $1/m_b$ and $1/m_b^2$ corrections is plotted for $\tau(\Lambda_b)/\tau(B_d)$ in Fig. 3 for a renormalization

scale parameter interval $m_b/2 \leq \mu \leq 2m_b$, assuming “factorization,” i.e., setting all the β parameters in Eqs. (B2) and (B5) to the value calculated using our factorization ansatz. Figures 4 and 5 show analogous plots for $\tau(B_u)/\tau(B_d)$ and $\tau(B_s)/\tau(B_d)$. As can be seen from these graphs, the scale dependence of the predictions is very mild. Next, we fix the scale in our calculations and vary the values of parameters of the matrix elements. We adopt the statistical approach for presenting our results and generate 20 000-point probability distributions of the ratios of lifetimes obtained by randomly varying our parameters within a $\pm 30\%$ interval around their factorization values, for three different scales μ . The decay constants f_{B_q} and b -quark pole mass m_b are taken to vary within 1σ interval indicated above. The results are presented in Figs. 6–8. These figures represent the main result of this paper. They show a nice agreement between the experimental results and theoretical predictions for all of the ratios of lifetimes of heavy hadrons. In fact, it is a relatively “large” ratio of $\tau(B_s)/\tau(B_d)$ that can pose a potential problem if future measurements at Fermilab and CERN would find the mean value to stay the same with its error bars shrinking. Upcoming Fermilab and CERN measurements would again clarify this issue.

It also appears that the theoretical errors in the ratio of $\tau(B_s)/\tau(B_d)$ are much smaller than the other two. We believe that the main reason for it is the fact that $1/m_b^3$ and higher spectator corrections to both B_s and B_d inclusive widths are given by the same type of diagrams. In other words, apart from the different charm quark-mass dependence (which comes at the level of m_c^2/m_b^2 correc-

TABLE III. Same as Table I for $\tau(B_s)/\tau(B_d)$.

μ	Coefficients k_i for the parameters listed below. All entries are $\times 10^{-3}$.							
	B_1	B_2	ϵ_1	ϵ_2	B_1^s	B_2^s	ϵ_1^s	ϵ_2^s
$m_b/2$	-7.82	9.19	-158	184	11.2	-12.4	180	-230
m_b	-7.96	9.31	-160	196	13.5	-16.8	182	-243
$2m_b$	-7.45	8.63	-160	201	14.7	-19.1	183	-249
μ	β_0	$\tilde{\beta}_0$	β_1	$\tilde{\beta}_1$	β_2	$\tilde{\beta}_2$	β_3	$\tilde{\beta}_3$
$m_b/2$	-0.003	-5.72	0.011	18.9	0.0013	2.20	-0.0013	-2.20
m_b	-0.091	-5.23	0.300	17.3	0.0349	2.00	-0.0349	-2.00
$2m_b$	-0.233	-4.97	0.768	16.4	0.0892	1.91	-0.0892	-1.91

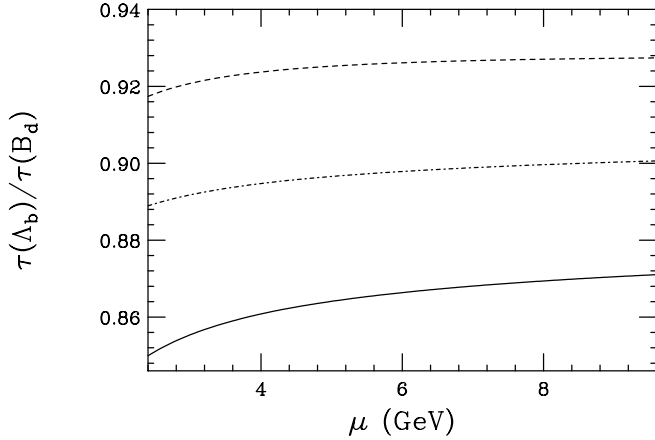


FIG. 3. LL (dashed line), NLL (dash-dotted line), and NLL + $1/m_b$ (solid line) contributions to $\tau(\Lambda_b)/\tau(B_d)$ are plotted vs the scale parameter μ in GeV.

tions to $1/m_b^3$ effects), both widths of B_s and B_d are affected by the weak scattering contributions in the same way. Thus, besides possible SU(3)-breaking effects in the matrix elements of the four-fermion operators, the contributions of spectator interactions to the numerator and denominator of the ratio of B_s and B_d widths cancel each other to a large degree. This effect persists for the class of $1/m_b$ corrections to spectator effects discussed here as well.

IV. CONCLUSIONS

We computed subleading $1/m_b$ and $1/m_b^2$ corrections to the spectator effects driving the difference in the lifetimes of heavy mesons and baryons. Thanks to the same $16\pi^2$ phase-space enhancement as $1/m_b^3$ -suppressed spectator effects, these corrections constitute the most important set of $1/m_b^4$ -suppressed corrections.

We showed that a set of $1/m_b$ corrections to spectator effects can be parametrized by eight new parameters β_i and $\tilde{\beta}_i$ ($i = 0, \dots, 3$) for heavy mesons and eight new

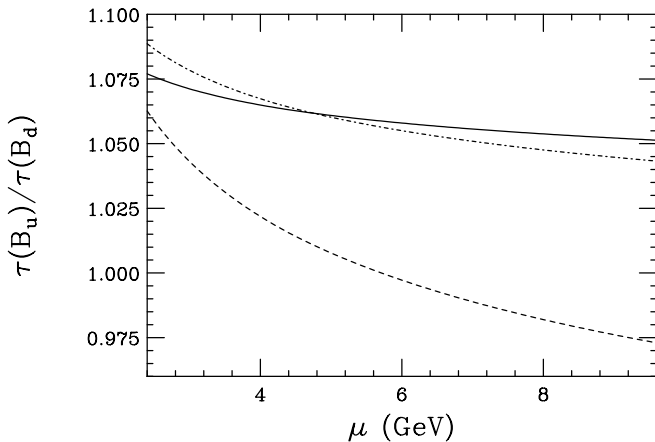


FIG. 4. Same as Fig. 3 for $\tau(B_u)/\tau(B_d)$.

parameters β_i^Λ and $\tilde{\beta}_i^\Lambda$ for heavy baryons. Although model-independent values of these parameters will not be known until dedicated lattice simulations are performed, we present an estimate of these parameters based on quark-model arguments. Compared to [7], we kept all matrix elements, including those vanishing in the factorization approximation. This is important, as the Wilson coefficients of those operators are larger than the ones multiplying the operators whose matrix elements survive in the $N_c \rightarrow \infty$ limit. We also perform studies of convergence of this expansion by computing a set of $1/m_b^2$ corrections to spectator effects and estimating their size in factorization. The expansion appears to be well convergent.

The main result of this paper are Figs. 6–8, which represent the effects of subleading spectator effects on the ratios of lifetimes of heavy mesons and baryons. We see that subleading corrections to spectator effects affect the ratio of heavy-meson lifetimes only modestly, at the level of a fraction of a percent. On the other hand, the effect on the Λ_b - B_d lifetime ratio is quite substantial, at the level of -3% . This can be explained by the partial cancellation of WS and PI effects in Λ_b baryon and constructive interference of $1/m_b$ corrections to the spectator effects.

There is no theoretically consistent way to translate the histograms of Figs. 6–8 into numerical predictions for the lifetime ratios. As a useful estimate it is possible to fit the histograms to Gaussian distributions and extract theoretical predictions for the mean values and deviations of the ratios of lifetimes. Predictions obtained this way should be treated with care, as it is not expected that the theoretical predictions are distributed according to the Gaussian distribution. This being said, we proceed by fitting the distributions to Gaussian and, correcting for the scale uncertainty [varied in the interval $(m_b/2, 2m_b)$] by inflating the error bars, extracting the ratios $\tau(B_u)/\tau(B_d) = 1.06 \pm 0.02$, $\tau(B_s)/\tau(B_d) = 1.00 \pm 0.01$, and $\tau(\Lambda_b)/\tau(B_d) = 0.86 \pm 0.05$. In these equations the

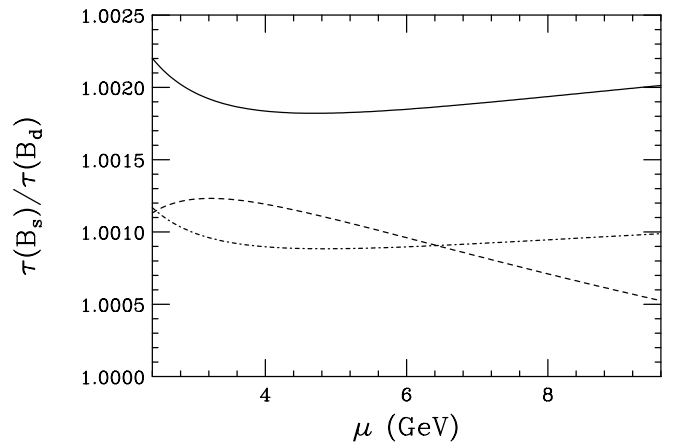


FIG. 5. Same as Fig. 3 for $\tau(B_s)/\tau(B_d)$.

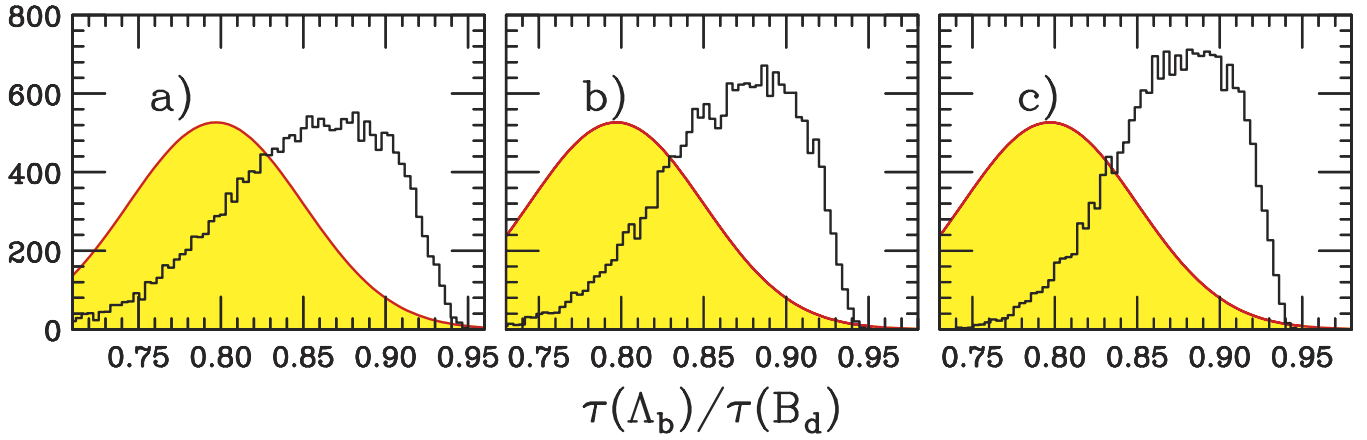


FIG. 6 (color online). Histograms showing the random distributions around the central values of the f_{B_q} , m_b , B , δ , ϵ , β_i , and $\tilde{\beta}_i$ parameters of Eqs. (B2) and (B5) contributing to $\tau(\Lambda_b)/\tau(B_d)$. Three histograms are shown for the scales $\mu = m_b/2$ (a), $\mu = m_b$ (b), and $\mu = 2m_b$ (c). The shaded curves, presented for the convenience, represent current experimental result. Their normalization is arbitrary.

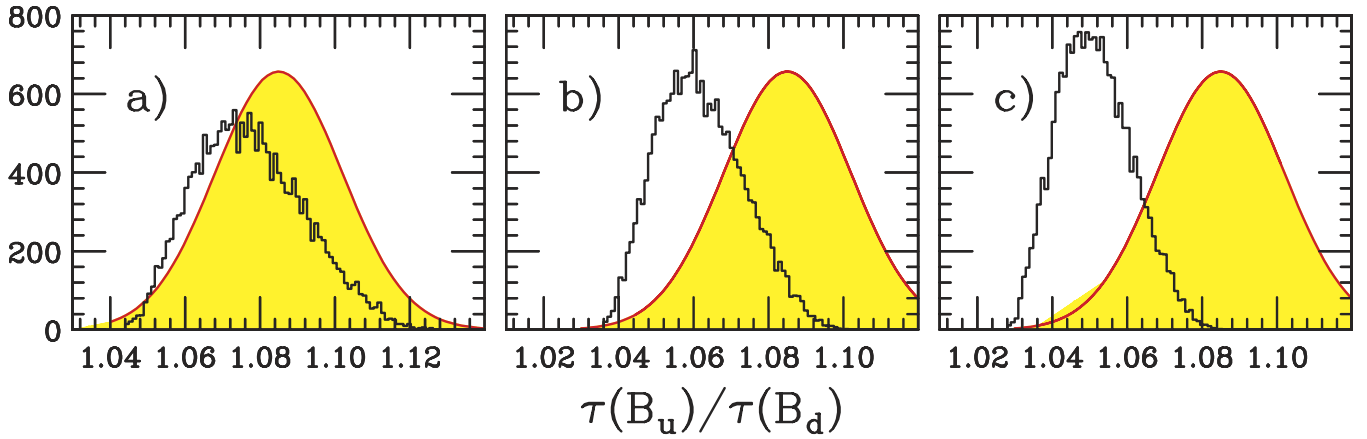


FIG. 7 (color online). Same as Fig. 6 for $\tau(B_u)/\tau(B_d)$.

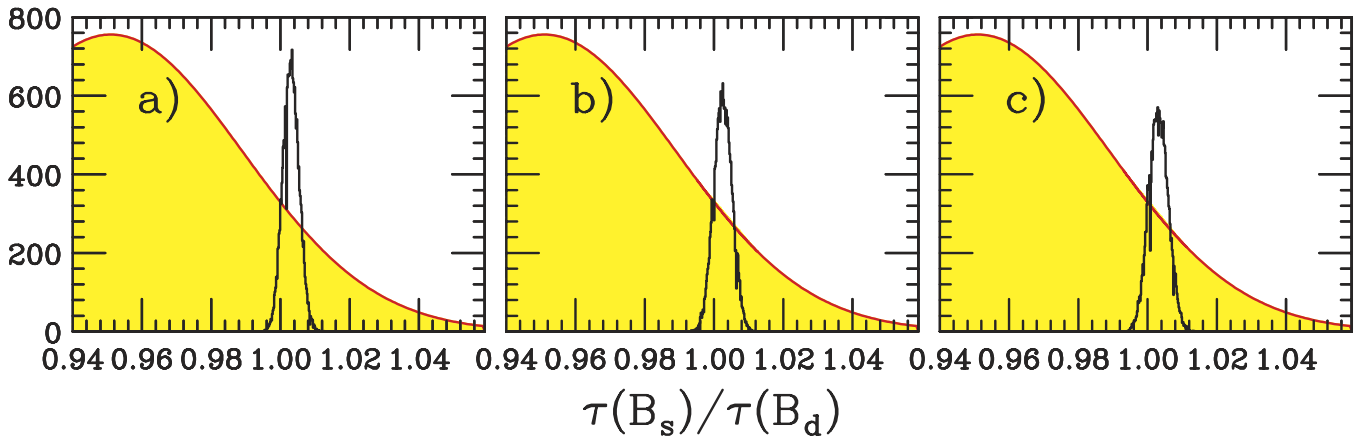


FIG. 8 (color online). Same as Fig. 6 for $\tau(B_s)/\tau(B_d)$.

mean values are taken at the scale $\mu = m_b$. This brings the experimental and theoretical ratios of baryon and meson lifetimes into agreement.

ACKNOWLEDGMENTS

We would like to thank N. Uraltsev and M. Voloshin for helpful discussions and A. Lenz for pointing out a numerical mistake in the first version of the manuscript. A. A. P. thanks the Fermilab Theory Group for hospitality where part of this work was completed. This work was supported in part by the U.S. National Science Foundation under Grant No. PHY-0244853, and by the U.S. Department of Energy under Contract No. DE-FG02-96ER41005.

APPENDIX A: FOUR-QUARK OPERATORS

This paper uses the basis of $1/m_b^3$ operators consistent with Ref. [3], but different from Refs. [4,6], given the relative simplicity of the higher-order operators in our basis. In the above calculations we used the following notation for local four-quark operators:

$$\begin{aligned} O_1^q &= \bar{b}_i \gamma^\mu (1 - \gamma_5) b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) q_j, \\ O_2^q &= \bar{b}_i \gamma^\mu \gamma_5 b_i \bar{q}_j \gamma_\mu (1 - \gamma_5) q_j, \\ O_3^q &= \bar{b}_i \gamma^\mu (1 - \gamma_5) b_i \bar{q}_j \gamma_\mu \gamma_5 q_j, \end{aligned} \quad (\text{A1})$$

as well as \tilde{O}_i^q operators that follow from the expressions for O_i^q by interchanging the color indices of b_i and q_j Dirac spinors. References [4,6] use more familiar ‘‘color-singlet’’ and ‘‘color-octet’’ operators

$$\begin{aligned} Q^q &= \bar{b}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu b_L, & Q_S^q &= \bar{b}_R q_L \bar{q}_L b_R, \\ T^q &= \bar{b}_L \gamma_\mu t^a q_L \bar{q}_L \gamma^\mu t^a b_L, & T_S^q &= \bar{b}_R t^a q_L \bar{q}_L t^a b_R. \end{aligned} \quad (\text{A2})$$

These operators can be expressed in our basis as

$$\begin{aligned} Q^q &= \frac{1}{4} \tilde{O}_1^q, & Q_S^q &= -\frac{1}{8} \tilde{O}_1^q - \frac{1}{4} \tilde{O}_2^q, \\ T^q &= -\frac{1}{8N_c} \tilde{O}_1^q + \frac{1}{8} O_1^q, \\ T_S^q &= \frac{1}{8N_c} \left(\frac{1}{2} \tilde{O}_1^q + \tilde{O}_2^q \right) - \frac{1}{8} \left(\frac{1}{2} O_1^q + O_2^q \right). \end{aligned} \quad (\text{A3})$$

Note that in the predictions for lifetime ratios, large matrix elements of the color-singlet operators Q^q and

Q_S^q are multiplied by a relatively small combination of Wilson coefficients, while matrix elements of color-octet operators come with large Wilson coefficients [4,14]. Nevertheless, the interpretation of matrix elements obtained in the vacuum-insertion approximation (i.e., representing the matrix elements of four-fermion operators as products of two matrix elements of current operators separated by a vacuum state) is relatively clear in this basis. For the dimension-six operators this procedure introduces four new scale-dependent parameters $B_i(\mu)$ and $\epsilon_i(\mu)$,

$$\begin{aligned} \frac{1}{2m_{B_q}} \langle B_q | Q^q | B_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} B_1(\mu), \\ \frac{1}{2m_{B_q}} \langle B_q | Q_S^q | B_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} B_2(\mu), \\ \frac{1}{2m_{B_q}} \langle B_q | T^q | B_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} \epsilon_1(\mu), \\ \frac{1}{2m_{B_q}} \langle B_q | T_S^q | B_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} \epsilon_2(\mu). \end{aligned} \quad (\text{A4})$$

These parameters can be computed using QCD sum rules, quark models, or on the lattice. A compilation of various estimates of these parameters can be found in [17].

APPENDIX B: ESTIMATE OF MATRIX ELEMENTS

We parametrize relevant matrix elements using factorization results as a guiding principle (a similar approach was used in [4,7,18]). Of course, these matrix elements would eventually have to be computed model-independently, using lattice QCD or QCD sum rules. In the absence of model-independent results, we obtain numerical predictions for the ratios of lifetimes, by relating matrix elements of the computed operators constituting $1/m_b$ and $1/m_b^2$ effects to the matrix elements of the operators of dimension six ($O_{1,2}$) using quark model. We then use lattice QCD predictions for the parameters B_i and ϵ_i and look at the ratio of the subleading- and leading-order spectator effects. We believe that some errors introduced by our use of the quark model should partially cancel in these ratios. The leading-order meson matrix elements are

$$\begin{aligned} \langle B_q | O_1^q | B_q \rangle &= f_{B_q}^2 m_{B_q}^2 \left(2\epsilon_1 + \frac{B_1}{N_c} \right), & \langle B_q | \tilde{O}_1^q | B_q \rangle &= f_{B_q}^2 m_{B_q}^2 B_1, \\ \langle B_q | O_2^q | B_q \rangle &= -f_{B_q}^2 m_{B_q}^2 \left[\frac{m_{B_q}^2}{(m_b + m_q)^2} \left(2\epsilon_2 + \frac{B_2}{N_c} \right) + \frac{1}{2} \left(2\epsilon_1 + \frac{B_1}{N_c} \right) \right], \\ \langle B_q | \tilde{O}_2^q | B_q \rangle &= -f_{B_q}^2 m_{B_q}^2 \left[\frac{m_{B_q}^2}{(m_b + m_q)^2} B_2 + \frac{1}{2} B_1 \right], \\ \langle B_q | O_3^q | B_q \rangle &= \langle B_q | O_2^q | B_q \rangle, & \langle B_q | \tilde{O}_3^q | B_q \rangle &= \langle B_q | \tilde{O}_2^q | B_q \rangle. \end{aligned} \quad (\text{B1})$$

The parameters B_i and ϵ_i represent matrix elements of ‘‘color-singlet’’ and ‘‘color-octet’’ operators. The above parameters can be computed in QCD sum rules, quark models, or on the lattice. Naively, one expects that in the large- N_c limit $B_i \sim \mathcal{O}(1)$, $\epsilon_i \sim \mathcal{O}(1/N_c)$.

A parametrization of $1/m_b$ -suppressed corrections could, in principle, be quite simple: after scaling out the

proper m_b dependence each new matrix element brings in a new parameter. This parametrization could be useful for lattice studies of these matrix elements. Here we shall choose a slightly different approach. Using the quark model to *guide* our parametrization, we relate our matrix elements to the matrix elements of singlet and octet operators using Fierz relations. We find

$$\begin{aligned}
\langle B_q | R_0^q | B_q \rangle &= -\frac{1}{2} f_{B_q}^2 m_{B_q}^2 \left(2\tilde{\beta}_0 + \frac{\beta_0}{N_c} \right) \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right), & \langle B_q | \tilde{R}_0^q | B_q \rangle &= -\frac{1}{2} f_{B_q}^2 m_{B_q}^2 \beta_0 \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right), \\
\langle B_q | R_1^q | B_q \rangle &= \frac{1}{2} f_{B_q}^2 m_{B_q}^2 \left(\frac{\beta_1}{N_c} + 2\tilde{\beta}_1 \right) \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right), & \langle B_q | \tilde{R}_1^q | B_q \rangle &= \frac{1}{2} f_{B_q}^2 m_{B_q}^2 \beta_1 \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right), \\
\langle B_q | R_2^q | B_q \rangle &= \frac{1}{12} f_{B_q}^2 m_{B_q}^2 \left(\frac{\beta_2}{N_c} + 2\tilde{\beta}_2 \right) \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right), & \langle B_q | \tilde{R}_2^q | B_q \rangle &= \frac{1}{12} f_{B_q}^2 m_{B_q}^2 \beta_2 \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right), \\
\langle B_q | R_3^q | B_q \rangle &= \frac{1}{12} f_{B_q}^2 m_{B_q}^2 \left(\frac{\beta_3}{N_c} + 2\tilde{\beta}_3 \right) \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right), & \langle B_q | \tilde{R}_3^q | B_q \rangle &= \frac{1}{12} f_{B_q}^2 m_{B_q}^2 \beta_3 \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right).
\end{aligned} \tag{B2}$$

In the large- N_c limit $\beta_i \sim \mathcal{O}(1)$ and $\tilde{\beta}_i \sim \mathcal{O}(1/N_c)$. We can use the quark model to relate $\beta_0 = \beta'_0(B_2 + B_1/2)$, $\beta_1 = \beta'_1 B_1$, $\beta_{2,3} = \beta'_{2,3}[(B_1 - 4B_2)(1 \pm m_b^2/m_{B_q}^2) + (B_1 + 2B_2)]$. The same relations hold for $\tilde{\beta}_i$ with $\tilde{\beta}_i \rightarrow \epsilon_i$. Here β'_i are used to parametrize the ratios of the matrix elements of the four-fermion operators and $1/m_b$ -suppressed operators. In our numerical studies we took the parameters B_i , ϵ_i from the lattice calculations of

matrix elements of dimension-six operators [6] and varied β'_i around its quark-model value of unity by $\pm 30\%$.

A similar logic can be used to estimate $1/m_b^2$ -suppressed matrix elements. We simplify the result by using the factorization approximation, i.e., neglecting all the matrix elements of octet operators, which should suffice for our purposes of studying the convergence of the series. We obtain

$$\begin{aligned}
\langle B_q | W_1^q | B_q \rangle &= \frac{f_{B_q}^2 m_{B_q}^2}{4N_c} \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right)^2, & \langle B_q | \tilde{W}_1^q | B_q \rangle &= \frac{f_{B_q}^2 m_{B_q}^2}{4} \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right)^2, \\
\langle B_q | W_2^q | B_q \rangle &= -\frac{3f_{B_q}^2 m_{B_q}^2}{8N_c} \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right)^2, & \langle B_q | \tilde{W}_2^q | B_q \rangle &= -\frac{3f_{B_q}^2 m_{B_q}^2}{8} \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right)^2, \\
\langle B_q | W_3^q | B_q \rangle &= -\frac{f_{B_q}^2 m_b^2}{8N_c} \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right)^2, & \langle B_q | \tilde{W}_3^q | B_q \rangle &= -\frac{f_{B_q}^2 m_b^2}{8} \left(\frac{m_{B_q}^2}{m_b^2} - 1 \right)^2, & \langle B_q | W_4^q | B_q \rangle &= \langle B_q | W_3^q | B_q \rangle, \\
\langle B_q | \tilde{W}_4^q | B_q \rangle &= \langle B_q | \tilde{W}_3^q | B_q \rangle.
\end{aligned} \tag{B3}$$

In a similar way, we used the quark-diquark valence-quark model to guide our parametrizations of baryon matrix elements. The leading-order matrix elements are [4,7]

$$\begin{aligned}
\langle \Lambda_b | O_1^q | \Lambda_b \rangle &= -\tilde{B} \langle \Lambda_b | \tilde{O}_1^q | \Lambda_b \rangle = \frac{\tilde{B}}{6} f_{B_q}^2 m_{B_q} m_{\Lambda_b} r, & \langle \Lambda_b | O_2^q | \Lambda_b \rangle &= -\tilde{B} \langle \Lambda_b | \tilde{O}_2^q | \Lambda_b \rangle = \frac{\tilde{B}}{6} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \delta, \\
\langle \Lambda_b | O_3^q | \Lambda_b \rangle &= -\tilde{B} \langle \Lambda_b | \tilde{O}_3^q | \Lambda_b \rangle = \langle \Lambda_b | O_2^q | \Lambda_b \rangle,
\end{aligned} \tag{B4}$$

where $r = |\psi_{bq}^{\Lambda_b}(0)|^2 / |\psi_{bq}^{B_q}(0)|^2$ is the ratio of the wave functions at the origin of the Λ_b and B_q mesons, and $\tilde{B} = 1$ in the valence-quark model. Estimates of r vary from 0.1 to 1.8 and can potentially be larger [4]. Note that $\delta = \mathcal{O}(1/m_b)$, which follows from the heavy-quark spin symmetry. We use the same principles to parametrize the $1/m_b$ -suppressed contributions,

$$\begin{aligned}
\langle \Lambda_b | R_0^q | \Lambda_b \rangle &= -\frac{\beta_0^\Lambda}{\tilde{\beta}_0^\Lambda} \langle \Lambda_b | \tilde{R}_0^q | \Lambda_b \rangle = -\frac{\beta_0^\Lambda}{24} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \left(\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right), \\
\langle \Lambda_b | R_1^q | \Lambda_b \rangle &= -\frac{\beta_1^\Lambda}{\tilde{\beta}_1^\Lambda} \langle \Lambda_b | \tilde{R}_1^q | \Lambda_b \rangle = -\frac{\beta_1^\Lambda}{24} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \left(\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right), \\
\langle \Lambda_b | R_2^q | \Lambda_b \rangle &= -\frac{\beta_2^\Lambda}{\tilde{\beta}_2^\Lambda} \langle \Lambda_b | \tilde{R}_2^q | \Lambda_b \rangle = -\frac{\beta_2^\Lambda}{48} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \left(\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right), \\
\langle \Lambda_b | R_3^q | \Lambda_b \rangle &= -\frac{\beta_3^\Lambda}{\tilde{\beta}_3^\Lambda} \langle \Lambda_b | \tilde{R}_3^q | \Lambda_b \rangle = \frac{\beta_3^\Lambda}{48} f_{B_q}^2 m_{B_q} m_{\Lambda_b} \left(\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right).
\end{aligned} \tag{B5}$$

Note that the matrix elements contain the parameters β_0^Λ and $\tilde{\beta}_0^\Lambda$, which are of the order of $1/m_b$. Thus, these matrix elements only contribute at the order $1/m_b^2$ corrections to spectator effects (see below). In the valence-quark approximation these parameters are proportional to δ . Analogously to the meson case, the quark model relates $\beta_0^\Lambda = \beta_0^{\Lambda'} B \delta$, $\tilde{\beta}_0^\Lambda = \tilde{\beta}_0^{\Lambda'} \delta$, $\beta_1^\Lambda = \beta_1^{\Lambda'} \tilde{B} r$, $\tilde{\beta}_1^\Lambda = \tilde{\beta}_1^{\Lambda'} r$, $\beta_{2,3}^\Lambda = \beta_{2,3}^{\Lambda'} B [(r + 4\delta/3)(1 + m_b^2/m_{\Lambda_b}^2) - 2\delta/3]$, $\tilde{\beta}_{2,3}^\Lambda = \tilde{\beta}_{2,3}^{\Lambda'} [(r + 4\delta/3) \times (1 + m_b^2/m_{\Lambda_b}^2) - 2\delta/3]$. We use the valence-quark approximation for $1/m_b^2$ corrections. We obtain

$$\begin{aligned}
\langle \Lambda_b | W_1^q | \Lambda_b \rangle &= -\langle \Lambda_b | \tilde{W}_1^q | \Lambda_b \rangle = \frac{f_{B_q}^2 m_{B_q} m_{\Lambda_b}}{96} \left(\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right)^2, & \langle \Lambda_b | W_2^q | \Lambda_b \rangle &= -\langle \Lambda_b | \tilde{W}_2^q | \Lambda_b \rangle = 0 \\
\langle \Lambda_b | W_3^q | \Lambda_b \rangle &= -\langle \Lambda_b | \tilde{W}_3^q | \Lambda_b \rangle = \frac{f_{B_q}^2 m_{B_q} m_{\Lambda_b}}{192} \left(\frac{m_{\Lambda_b}^2}{m_b^2} - 1 \right)^2 \left(1 + \frac{m_b^2}{m_{\Lambda_b}^2} \right), \\
\langle \Lambda_b | W_4^q | \Lambda_b \rangle &= -\langle \Lambda_b | \tilde{W}_4^q | \Lambda_b \rangle = -\langle \Lambda_b | W_3^q | \Lambda_b \rangle,
\end{aligned} \tag{B6}$$

where we set all the terms of order $\mathcal{O}(1/m_b^3)$ in the matrix elements to zero.

-
- [1] S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004); E. Barberio, presented at the Workshop on the CKM Unitary Triangle, <http://ckm-workshop.web.cern.ch>; M. Battaglia *et al.*, hep-ph/0304132.
- [2] CDF Collaboration, J. Rademacker, hep-ex/0406021; See also: LEP B Lifetime Working Group, <http://lepbose.web.cern.ch/LEPBOSC/lifetimes/lepblife.html>.
- [3] I. I. Bigi *et al.*, hep-ph/9401298; M. B. Voloshin, hep-ph/0004257.
- [4] M. Neubert and C. T. Sachrajda, Nucl. Phys. **B483**, 339 (1997).
- [5] J. L. Rosner, Phys. Lett. B **379**, 267 (1996).
- [6] M. Ciuchini, E. Franco, V. Lubicz, and F. Mescia, Nucl. Phys. **B625**, 211 (2002); E. Franco, V. Lubicz, F. Mescia, and C. Tarantino, *ibid.* **B633**, 212 (2002).
- [7] F. Gabbiani, A. I. Onishchenko, and A. A. Petrov, Phys. Rev. D **68**, 114006 (2003).
- [8] C. Tarantino, Eur. Phys. J. C **33**, S895 (2004).
- [9] I. Bigi, M. Shifman, N. Uraltsev, and A. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993).
- [10] I. I. Y. Bigi, M. A. Shifman, N. G. Uraltsev, and A. I. Vainshtein, Phys. Rev. D **50**, 2234 (1994).
- [11] B. Guberina, S. Nussinov, R. D. Peccei, and R. Ruckl, Phys. Lett. **89B**, 111 (1979).
- [12] N. Bilic, B. Guberina, and J. Trampetic, Nucl. Phys. **B248**, 261 (1984).
- [13] B. Guberina, R. Ruckl, and J. Trampetic, Z. Phys. C **33**, 297 (1986).
- [14] M. A. Shifman and M. B. Voloshin, Yad. Fiz. **41**, 187 (1985) [Sov. J. Nucl. Phys. **41**, 120 (1985)].
- [15] M. Beneke, G. Buchalla, C. Greub, A. Lenz, and U. Nierste, Nucl. Phys. **B639**, 389 (2002).
- [16] B. Guberina, B. Melic, and H. Stefancic, Phys. Lett. B **484**, 43 (2000).
- [17] J. Chay, A. F. Falk, M. E. Luke, and A. A. Petrov, Phys. Rev. D **61**, 034020 (2000); see also [1] and , UKQCD Collaboration, M. Di Pierro and C. T. Sachrajda, Nucl. Phys. **B534**, 373 (1998); UKQCD Collaboration, M. Di Pierro, C. T. Sachrajda, and C. Michael, Phys. Lett. B **468**, 143 (1999); P. Colangelo and F. De Fazio, *ibid.* **387**, 371 (1996); M. S. Baek, J. Lee, C. Liu, and H. S. Song, Phys. Rev. D **57**, 4091 (1998); J. G. Korner, A. I. Onishchenko, A. A. Petrov, and A. A. Pivovarov, Phys. Rev. Lett. **91**, 192002 (2003).
- [18] M. Beneke, G. Buchalla, and I. Dunietz, Phys. Rev. D **54**, 4419 (1996); M. Beneke, G. Buchalla, A. Lenz, and U. Nierste, Phys. Lett. B **576**, 173 (2003).