"Forbidden" decays of hybrid mesons to $\pi\rho$ can be large

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The observation of $\pi_1(1600) \to \pi\rho$ is shown in the flux-tube model to be compatible with this state being a hybrid meson with branching ratio to this channel ~30%. The $\pi\rho$ widths of other hybrids are related by rather general arguments. These results enable cross sections for photoproduction of hybrids to be predicted.

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The "smoking gun" for hybrid mesons has been the possibility of them having combinations of J^{PC} that are forbidden to conventional $q\bar{q}$. Examples of such exotic states include 1^{-+}, which in lattice QCD and the flux-tube model is predicted to occur ~2 GeV in mass [1–3], with the further exotic states $0^{+-}, 2^{+-}$ occurring somewhat higher in mass .

A 1^{-+} state, $\pi_1(1600)$, has been claimed in independent experiments [4–6]. Its mass is somewhat lower than lattice computations had anticipated, but allowing for the lighter masses of $n\bar{n}(\equiv u\bar{u}$ or $d\bar{d})$ relative to the $s\bar{s}$ states studied in Ref. [1], a 1^{-+} with a mass ~1780(200) MeV is not implausible [7]. These observations are in different channels and it has not yet been established that they refer to a single state [8]. Observation in the channel $\pi\rho$ [4] has raised questions about its nature, given that the standard predictions of hadronic decays of hybrids have been that they decay into excited states [9,10] and are even forbidden into $\pi\rho$ [9–11].

The latter selection rule applies in a symmetry limit (specifically where the π and ρ have the same size) and in the case where the decay is triggered by breaking the flux-tube. It is not a general axiom. The approach described in the present paper severely breaks this symmetry by shrinking the π to a pointlike current. This follows a standard approach for calculating pion emission, which has been applied with reasonable success to conventional decays for over 30 years [12–14]. We are now able to apply it to the decays of hybrids following the recent development in Ref. [15,16]. This is built on an insight of Isgur [17], which in simplistic terms is that the flux-tube is a dynamical degree of freedom which can be excited by the action of a current on its ends - the quarks. (The application of this idea is described extensively in our paper [16]). It is the purpose of the present paper to apply these ideas to calculate $\pi\rho$ decays of hybrids. We shall see that they can be large. Our results reveal that existing calculations in the literature implicitly allow this, and that the J^{PC} dependence of our results is also found to occur in those calculations. Finally, given the empirical success of converting $\pi\rho$ amplitudes to $\pi\gamma$ for known states, we can predict the $\gamma\pi \to \mathcal{H}$ amplitudes, which are an essential requirement for estimating their photoproduction cross sections.

I. PION EMISSION FROM CONVENTIONAL MESONS

Emission of a π^+ by the quark in a meson $(q_i\bar{q})$ has matrix element

$$\begin{aligned}\mathcal{M}(q_i \to q_f + \pi^+)\\ &= \int d^3\vec{p} \int d^3\vec{p}\,'\phi_f^*(\vec{p})\phi_i(\vec{p})\left\langle \bar{q}\left(\frac{1}{2}\vec{P}_f + \vec{p}\,'\right) \middle| \bar{q}\left(\frac{1}{2}\vec{P}_i + \vec{p}\right)\right\rangle \\ &\quad\times \left\langle q_f\left(\frac{1}{2}\vec{P}_f - \vec{p}\,'\right) \middle| \frac{g}{2m}\int d^3\vec{r}\,\overline{\psi}(\vec{r})\gamma^\mu q_\mu \gamma^5 \frac{\tau^-}{\sqrt{2}}\psi(\vec{r})e^{-i\vec{q}\cdot\vec{r}} \middle| q_i\left(\frac{1}{2}\vec{P}_i - \vec{p}\right)\right\rangle,\end{aligned}$$

where the structure of the current is the divergence of the axial current, but to the order in the nonrelativistic reduction that we will work could equally well be a pseudoscalar form; $\frac{\tau^-}{\sqrt{2}}$ is the isospin lowering operator, and the $\phi(\vec{p})$ are the internal momentum wave functions. g is the pion-quark-quark coupling constant which we will determine by fitting conventional meson decay rates.

Expanding the Dirac spinors ψ in terms of quark creation and annihilation operators, retaining only those which will annihilate q_i and create q_f and performing the nonrelativistic reduction working in the rest frame of the initial meson we find

$$\begin{aligned}\mathcal{M}(q_i \to q_f + \pi^+)\\ &= -\frac{g}{2m}F(q_i, q_f)\int d^3\vec{p}\,\phi_f^*(\vec{p} + \vec{q}/2)\phi_i(\vec{p})\\ &\quad\times\left[\vec{\sigma}\cdot\vec{q}\left(1 + \frac{q^0}{2m}\right) + \frac{q^0}{m}\vec{\sigma}\cdot\vec{p} + \dots\right].\end{aligned}$$

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where $F(q_i, q_f) = \langle q_f | \frac{\tau^-}{\sqrt{2}} | q_i \rangle$ is a flavor factor accounting for isospin conservation at the pion-quark-quark vertex. Fourier transforming with $\phi(\vec{p}) = \int d^3\vec{r} e^{-i\vec{p}\cdot\vec{r}} \psi(\vec{r})$ gives

$$\mathcal{M}\left(\frac{q_i}{\bar{q}_i} \to \frac{q_f}{\bar{q}_f}\pi^+\right) = \mp \frac{g}{2m} F\left(\frac{q_i}{\bar{q}_i}, \frac{q_f}{\bar{q}_f}\right) \int d^3\vec{r} \psi_f^*(\vec{r}) \left[\vec{\sigma}\cdot\vec{q} \mp \frac{q^0}{m}\vec{\sigma}\cdot\vec{p}'\right] e^{\pm i\vec{q}\cdot\vec{r}/2} \psi_i(\vec{r}) \quad (1)$$

for emission by the quark or the antiquark and where the operator $\vec{p}'$ is $-i\overleftarrow{\nabla}_{\vec{r}}$ acting backwards onto the final state wave function.

Compare Eq. (1) with Eq. (19) in [13], which has the opposite sign definition for $\vec{r}$. Their decay widths are in terms of two independent form-factor parameters, g, h which they fit to data. Our approach determines for example h in terms of g and we hence have less freedom to fit, but more predictive power.

Using the representation

$$u \sim \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad d \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \bar{u} \sim \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \bar{d} \sim \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

and the explicit form

$$\frac{\tau^-}{\sqrt{2}} \sim \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

we see that the only nonzero flavor factors are $F(d, u) = +1$ and $F(\bar{u}, \bar{d}) = -1$. We find that Eq. (1) with these charge factors correctly conserves isospin and G-parity.

With our normalizations, the partial width of a meson of spin-J is given by

$$\Gamma(M_J \to \pi V) = \frac{C}{2J+1} \frac{|\vec{q}|}{2\pi} \sum |\mathcal{M}|^2, \quad (2)$$

where the sum is over quark and antiquark emission and either helicities or partial waves. C is the number of end-state charge possibilities (e.g., $C = 2$ for isovector to $\rho\pi$, $C = 1$ for isovector to $\omega\pi$).

We now have all the tools required to calculate decay widths. We will demonstrate the method with the important channel $b_1 \to \omega\pi$.

II. $b_1 \to \omega\pi$ IN THE PION EMISSION MODEL

Considering first the spin and spatial dependence and leaving for now the flavor dependence we can write for the amplitude, $A_{q,\bar{q}} =$

$$\int r^2 dr \int d\Omega \frac{R_f^*(r)}{\sqrt{4\pi}} \left\langle S = 1, m_s \left| \left[\vec{\sigma}_{q,\bar{q}} \cdot \vec{q} \mp \frac{q^0}{m} \vec{\sigma}_{q,\bar{q}} \cdot \vec{p}' \right] \right| S = 0 \right\rangle e^{\pm i\vec{q}\cdot\vec{r}/2} Y_1^{m_L}(\Omega) R_i(r).$$

Evaluating the angular integrals and the spin matrix element gives us the helicity amplitudes,

$$A_{q,\bar{q}}(0) = i\frac{g}{2m}\frac{|\vec{q}|}{\sqrt{3}}\left[3\langle j_1\rangle + \frac{\langle\overleftarrow{\partial} j_0\rangle}{m} - 2\frac{\langle\overleftarrow{\partial} j_2\rangle}{m}\right],$$
$$A_{q,\bar{q}}(\pm) = i\frac{g}{2m}\frac{|\vec{q}|}{\sqrt{3}}\left[\frac{\langle\overleftarrow{\partial} j_0\rangle}{m} + \frac{\langle\overleftarrow{\partial} j_2\rangle}{m}\right],$$

where $\langle\overleftarrow{\partial} j_L\rangle$ is shorthand for $\int r^2 dr \frac{dR_f^*}{dr} j_L(|\vec{q}|r/2) R_i$.

Partial-wave amplitudes can be constructed according to Table XI of [13]. We find

$$A_{q,\bar{q}}(S) = i\frac{g}{2m}|\vec{q}|\left[\langle j_1\rangle + \frac{\langle\overleftarrow{\partial} j_0\rangle}{m}\right],$$
$$A_{q,\bar{q}}(D) = -i\sqrt{2}\frac{g}{2m}|\vec{q}|\left[\langle j_1\rangle - \frac{\langle\overleftarrow{\partial} j_2\rangle}{m}\right].$$

Including the flavor factors for the decay of b_1^+ we obtain a nonzero matrix element for the end-state $\omega\pi^+$ only, and not, for example, $\rho^+\pi^0$, in line with conservation of isospin and G-parity. The D/S amplitude ratio then for this decay is

$$\frac{D}{S}(b_1 \to \omega\pi) = -\sqrt{2}\frac{\langle j_1\rangle - \frac{\langle\overleftarrow{\partial} j_2\rangle}{m}}{\langle j_1\rangle + \frac{\langle\overleftarrow{\partial} j_0\rangle}{m}}, \quad (3)$$

A sensitive test of hadron decay models comes from comparing the D/S amplitude ratios for the decays $b_1 \to \omega\pi, a_1 \to \rho\pi$ with the experimental world averages $+0.277(27)$, $-0.108(16)$ [18]. For the $a_1 \to \rho\pi$ decay in this model we obtain $\frac{D}{S}(a_1 \to \rho\pi) = -\frac{1}{2}\frac{D}{S}(b_1 \to \omega\pi)$, which is also found in the $^3P_0$ model and the "sKs" and "j^0Kj^0" models discussed in [19].

Using wave functions obtained variationally from the Isgur-Paton meson Hamiltonian (IP) (see Appendix D of [16]) we obtain ratios $+0.45$ and -0.22 for $\frac{D}{S} \times (b_1 \to \omega\pi), \frac{D}{S}(a_1 \to \rho\pi)$, which have the right sign and relative size, but are roughly a factor of 2 too large in magnitude. A standard approximation [19] is to describe the mesons by harmonic oscillator wave functions with a single β value for all states. If we do this and fit to the experimental b_1 ratio we find $\beta = 0.39(3)$ GeV, which is considerably larger than $\bar{\beta} \approx 0.31$ GeV in IP but is in good agreement with $\beta \approx 0.4$ in [19].

That the effective β is larger than our IP value may be due to our pointlike pion approximation. Experimentally the pion is not pointlike, it has a charge radius comparable with other light mesons and our radial wave function overlap should really take some account of this. Quite possibly we are feeling this in the increased β, which has subsumed the effect of the pion wave function. However it is worth noting that the effective β values (presented in [9]) for the meson wave functions computed in the model of Godfrey and Isgur [13] are much larger than for the IP Hamiltonian, and much closer to the $\beta = 0.4$ GeV preferred above. Godfrey and Isgur use a partially relativized Hamiltonian and consider spin-dependent terms, so it may be that the simple IP Hamiltonian as applied to conventional mesons is missing some important effects which set the size of meson states.

III. CONVENTIONAL MESON DECAYS TO πV

The $\pi(\rho/\omega)$ decays of the spin-triplet $L = 1, 2$ mesons can be computed in this model, the results being tabulated in Table I. Another precision test of the decay model is the F/P ratio of the $\pi_2(1670) \to \rho\pi$ decay, which has recently been measured for the first time by the E852 Collaboration who find $F/P = -0.72 \pm 0.07 \pm 0.14$ [20]. In the pion emission model we find

$$\frac{F}{P} = -\sqrt{\frac{3}{2}}\frac{\langle j_2\rangle - \frac{\langle\overleftrightarrow{\partial} j_3\rangle}{m}}{\langle j_2\rangle + \frac{\langle\overleftrightarrow{\partial} j_1\rangle}{m}}.$$

With IP, $\beta = 0.4$ wave functions this would equal $+0.57$, $+0.31$, neither of which is compatible with the experimental value. Since this is an $L = 2 \to L = 0$ transition we might expect there to be a different effective β, which we can fit using the experimental value. With equal β harmonic oscillator wave functions we find

$$\frac{F}{P} = -\sqrt{\frac{3}{2}}\frac{1 + \frac{|\vec{q}|}{4m}}{1 + \frac{|\vec{q}|}{4m} - \frac{10\beta^2}{|\vec{q}|m}}, \tag{4}$$

which cannot be satisfied by any real β with the experimental numbers. So we see that the pion emission model as formulated here cannot accommodate the E852 value for F/P. The $^3P_0$ decay model successfully accommodates the D/S ratios discussed earlier with parameter values which do a good job of describing a wide range of hadron decays. With the same parameters it predicts $F/P \sim +0.4$ [21] which is in the same region as the pion emission predictions and not compatible with experiment. If the E852 result is confirmed it casts doubt over the validity of these commonly used nonrelativistic hadron decay models for high partial waves.

We present in Table II the partial widths for a number of meson decays computed in the model using $g = 2, 3$ for $\beta = 0.4$ wave functions and $g = 3$ for IP wave functions. Also shown are the predictions of the $^3P_0$ model taken from [19,22] and experimental values taken from [18].

Unfortunately there is little precision data on $\pi(\rho/\omega)$ decays available, so the best we can say is that the pion emission model does a reasonable job of describing the data. The qualitative pattern of large, small and intermediate empirical widths is faithfully reproduced which suggests that the underlying group transformation properties of the states and the pion transition operator are valid. We cannot accurately pin down the value of the coupling using the experimental data. Note also that despite their failure to predict the precision D/S ratios, the IP wave functions do as good a job overall of describing the data as the $\beta = 0.4$ wave functions.

TABLE I. πV decay widths for $L = 1, 2$ conventional light-quark mesons in the pion emission model.

S-waves	D-waves
$\Gamma_S(a_1 \to \rho\pi) = \frac{8}{3}\Sigma$	$\Gamma_D(a_1 \to \rho\pi) = \frac{4}{3}\Delta$
$\Gamma_S(b_1 \to \omega\pi) = \frac{2}{3}\Sigma$	$\Gamma_D(b_1 \to \omega\pi) = \frac{4}{3}\Delta$
...	$\Gamma_D(a_2 \to \rho\pi) = \frac{12}{5}\Delta$
P-waves	F-waves
$\Gamma_P(\pi_2 \to \rho\pi) = \frac{8}{5}\Pi$	$\Gamma_F(\pi_2 \to \rho\pi) = \frac{12}{5}\Phi$
...	$\Gamma_F(\omega_3 \to \rho\pi) = \frac{24}{7}\Phi$
...	$\Gamma_F(\rho_3 \to \omega\pi) = \frac{8}{7}\Phi$

$$\Sigma \equiv \frac{|\vec{q}|^3}{2\pi}\left(\frac{g}{2m}\right)^2\left(\langle j_1\rangle + \frac{\langle\overleftrightarrow{\partial} j_0\rangle}{m}\right)^2$$

$$\Delta \equiv \frac{|\vec{q}|^3}{2\pi}\left(\frac{g}{2m}\right)^2\left(\langle j_1\rangle - \frac{\langle\overleftrightarrow{\partial} j_2\rangle}{m}\right)^2$$

$$\Pi \equiv \frac{|\vec{q}|^3}{2\pi}\left(\frac{g}{2m}\right)^2\left(\langle j_2\rangle + \frac{\langle\overleftrightarrow{\partial} j_1\rangle}{m}\right)^2$$

$$\Phi \equiv \frac{|\vec{q}|^3}{2\pi}\left(\frac{g}{2m}\right)^2\left(\langle j_2\rangle - \frac{\langle\overleftrightarrow{\partial} j_3\rangle}{m}\right)^2$$

IV. HYBRID MESON DECAYS TO πV

We can derive the matrix element for decay in the same manner as we did for conventional decays, but now explicitly including the flux-tube degrees-of-freedom. The essential change lies in the modification of the quark positions and momenta as detailed in [15,16],

TABLE II. Numerical estimates of πV decay widths in MeV for $L = 1, 2$ conventional light-quark mesons in the pion emission model using wave function parametrizations as described in the text.

Mode	IP ($g = 3$)	$\beta = 0.4$($g = 2, 3$)	$^3P_0$	Data
$\Gamma_S(a_1 \to \rho\pi)$	280	(255, 580)	530	$150 \to 360$
$\Gamma_D(a_1 \to \rho\pi)$	14	(5,10)	15	$3 \to 8$
$\Gamma_S(b_1 \to \omega\pi)$	70	(64, 145)	132	<132
$\Gamma_D(b_1 \to \omega\pi)$	14	(5,10)	11	<10
$\Gamma_D(a_2 \to \rho\pi)$	52	(18,40)	54	75 ± 7
$\Gamma_{F+P}(\pi_2 \to \rho\pi)$	162	(131, 297)	118	81 ± 11
$\Gamma_F(\omega_3 \to \rho\pi)$	77	(16,36)	50	< 74
$\Gamma_F(\rho_3 \to \omega\pi)$	19	(5,12)	19	26 ± 13

$$\vec{r}_{q,\bar{q}} = \pm\frac{1}{2}\vec{r} - \sqrt{\frac{2b}{\pi^3}}\frac{\beta_1 r}{m}\vec{a}; \tag{5}$$

$$\vec{p}_{q,\bar{q}} = \pm\vec{p} - i\sqrt{\frac{2b}{\pi^3}}\pi\beta_1\vec{a}; \tag{6}$$

where $\vec{r}$ is the internal "longitudinal" relative coordinate through the c.m. and parallel to the $q\bar{q}$ axis, and $\vec{a}$ is the transverse Fourier mode of the flux-tube associated with the transverse displacement of the $q\bar{q}$ relative to the $\vec{r}$ [16,17]. We have already used the fact that $\vec{r}$ can cause transitions between S and P wave conventional $q\bar{q}$ states; $\vec{a}$ can analogously generate transitions between conventional and hybrid (excited flux-tube) states. The parameter β_1 is effectively the measure of the transverse extent of the flux-tube wave function which will not appear in state widths; if described by Gaussian wave functions [16,17] one effectively has

$$\vec{p}_a|g.s.\rangle = i\beta_1^2\vec{a}|g.s.\rangle$$

which has been used in deriving the expression in Eq. (6).

The matrix element for the lightest hybrid multiplet (one phonon in the $p = 1$ mode) to decay to $\pi\rho$, retaining only the required terms linear in $\vec{a}$ is,

$$\begin{aligned}\mathcal{M}_{q,\bar{q}} = {}& \mp i\frac{g}{2m}\sqrt{\frac{2b}{\pi}}\beta_1 F\left(\frac{q_i}{q_i}, \frac{q_f}{q_f}\right)\int d^3\vec{r}\int d^2\vec{a}\,\psi_\rho^{(C)*}(\vec{r})\chi_0^*(\vec{a}) \\ & \times\left\langle S=1, m_\rho\,\middle|\left(\vec{\sigma}_{q,\bar{q}}\cdot\vec{q} \mp \frac{q^0}{m}\vec{\sigma}_{q,\bar{q}}\cdot\vec{p}'\right)\frac{r}{\pi m}\vec{q}.\vec{a} - \frac{q^0}{m}\vec{\sigma}_{q,\bar{q}}\cdot\vec{a}\,\middle|\,S, m_s'\right\rangle e^{\pm i\vec{q}\cdot\vec{r}/2}\psi_{m'}^{(\mathcal{H})}(\vec{r})\chi_1(\vec{a}).\end{aligned} \tag{7}$$

The $\vec{\sigma}.\vec{p}'\vec{q}.\vec{a}$ term is formally suppressed at order $|\vec{q}|/m$ relative to the leading term and as such we will neglect it initially. We will return at the end of this paper to consider the effect it and other neglected terms might have.

The calculation of matrix elements can be performed immediately using the techniques of Ref. [16] (see especially Eqs. (18,19) of that paper). We present the results in Table III.

We show in Fig. 1, $\langle j_L\rangle = \langle C|j_L|\mathcal{H}\rangle$ as a function of $m_{\mathcal{H}}$ for IP and $\beta = 0.4$ wave functions. Note that only $\langle j_0\rangle$ differs considerably between the two wave function choices and as such we expect that while P- and D-wave predictions will be rather robust with respect to wave function parametrizations, S-wave rates will be quite sensitive. We will quote rates predicted with both wave function choices where they differ considerably, and we will use $g = 3$ throughout.

TABLE III. πV decay widths for $p = 1$ light-quark hybrid mesons in the pion emission/flux-tube model.

$\mathcal{P}_{\mathcal{H}} = +$	
S-waves	D-waves
$\Gamma_S(a_{1H}\to\rho\pi) = \frac{1}{3}\sum_{\mathcal{H}}$	$\Gamma_D(a_{1H}\to\rho\pi) = \frac{1}{6}\Delta_{\mathcal{H}}$
$\Gamma_S(b_{1H}\to\omega\pi) = \frac{1}{3}\sum_{\mathcal{H}}$	$\Gamma_D(b_{1H}\to\omega\pi) = \frac{1}{24}\Delta_{\mathcal{H}}$
$\cdots$	$\Gamma_D(b_{2H}\to\omega\pi) = \frac{3}{40}\Delta_{\mathcal{H}}$
$\mathcal{P}_{\mathcal{H}} = -$	
P-waves	F-waves
	$\Gamma_D(b_{2H}\to\omega\pi) = \frac{3}{40}\Delta_{\mathcal{H}}$
$\Gamma_P(\rho_H\to\omega\pi) = \frac{1}{4}\Pi_{\mathcal{H}}$	$\cdots$
$\Gamma_P(\pi_H\to\rho\pi) = \Pi_{\mathcal{H}}$	$\cdots$
$\Gamma_P(\pi_{1H}\to\rho\pi) = \frac{1}{4}\Pi_{\mathcal{H}}$	$\cdots$
$\Gamma_P(\pi_{2H}\to\rho\pi) = \frac{1}{4}\Pi_{\mathcal{H}}$	$\cdots$
$\Sigma_{\mathcal{H}} \equiv g^2\frac{\lvert\vec{q}\rvert^3}{m^2}\frac{b}{\pi^2 m^2}\lvert\frac{2}{\pi}\langle j_1\rangle - \langle j_0\rangle\rvert^2$	
$\Delta_{\mathcal{H}} \equiv g^2\frac{\lvert\vec{q}\rvert^3}{m^2}\frac{b}{\pi^2 m^2}\lvert\frac{4}{\pi}\langle j_1\rangle - \langle j_2\rangle\rvert^2$	
$\Pi_{\mathcal{H}} \equiv g^2\frac{\lvert\vec{q}\rvert^3}{m^2}\frac{b}{\pi^2 m^2}\lvert\langle j_1\rangle\rvert^2$	

A. Negative Parity Hybrids

1^{--} The spin-singlet negative parity hybrid, $\rho_H\to\pi\omega$ and may be compared with the states $\rho(1460)$ and $\rho(1700)$ [18], which have been suggested to have hybrid vector meson content [23].

	Γ/MeV	b.r.
$\rho(1460)\to\omega\pi$	29	9%
$\rho(1700)\to\omega\pi$	93	33%

Although $\omega\pi$ decays are seen for these states, the branching fractions have not been accurately determined. If the states are mixtures of hybrid and

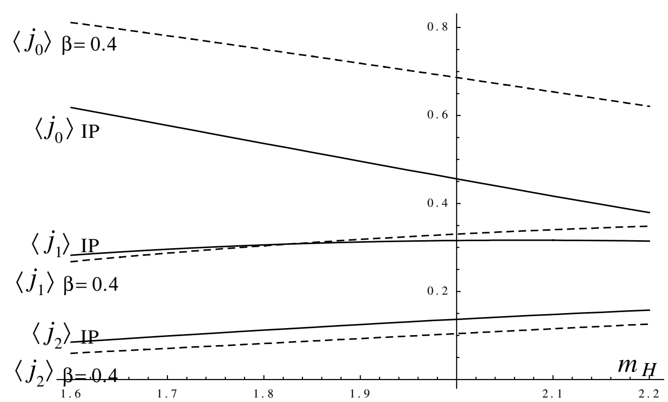

FIG. 1. Transition matrix elements of the spherical Bessel functions, $\langle R_\rho(r)|j_L(|\vec{q}|r/2)|R_{\mathcal{H}}(r)\rangle$, using wave function parametrizations as described in the text.

conventional, as proposed in [23], the $\pi\omega$ widths become rather model dependent and no clear conclusions can be drawn.

1^{-+}

$$\Gamma[\pi_1(1600) \to \rho\pi] \approx 57\text{MeV} \qquad (8)$$

This width for the 1^{-+} corresponds to a branching ratio [24] $\sim 34^{+6}_{-17}\%$ if we identify with the state in [4] with $m = 1600$ MeV and $\Gamma = 168 \pm 20^{+150}_{-12}$ MeV. This is encouraging; had the branching ratio been $\sim 1\%$ it would be implausible for the state to have been seen in this mode; conversely had the branching ratio been predicted to be $\sim 100\%$ the required absence of other channels would have disagreed with the experimental observation of this state in various channels. This result is consistent within errors with:

(1) an experimental limit $b.r.[1^{-+}(1600) \to \pi\rho] \leq 40\%$ [25];
(2) with the relative branching ratios of Ref. [26]: $br(b_1\pi){:}br(\eta'\pi){:}br(\rho\pi) = 1{:}1.0 \pm 0.3{:}1.6 \pm 0.4$;
(3) with an analysis of the E852 data, assuming purely ρ exchange in the production mechanism which gave a branching ratio of $20 \pm 2\%$ [27].

This is an appropriate point to compare with the flux-tube breaking model, where $\rho\pi$ decays come about only when one allows different radial wave functions for the ρ and the π. In [10] the authors consider a particular realization of this symmetry breaking and find a width for $\pi_1 \to \rho\pi$ of only 8 MeV, where this assumes the hybrid is at 2 GeV; a state at 1600 MeV will have reduced phase-space and a further reduction in width.

0^{-+}

$$\Gamma[\pi(1800) \to \rho\pi] \approx 480 \text{ MeV}$$

In this model the pseudoscalar hybrid has a width significantly larger than the 1^{-+} state. We will show later that this is a rather general prediction of the flux-tube model. This numerical prediction, which would signal a very broad state indeed, may be better considered an upper limit. When considering conventional decays in this model we found that with $\beta = 0.4$ wave functions there was little difference in overall fit quality between $g = 2, 3$. Using $g = 2$ here would reduce the partial width to ~ 210 MeV.

Unfortunately we cannot really use our result to make any statement about the hybrid character or otherwise of the $\pi(1800)$ state. As well as our considerable theoretical uncertainties, we have no experimental measurement of the state's $\rho\pi$ branching ratio. The experiment that we would look to for such data is E852 but they note in [20] that the pseudoscalar partial-wave suffers badly from the required $(\pi\pi)_S$ parametrization uncertainties and as such no reliable data can be extracted.

2^{-+} The isovector 2^{-+} state in this model has the same partial width as the 1^{-+} state and hence should be as prominent in experiment. There is a very broad ($\Gamma \sim 600$ MeV) candidate π_2 state seen at 2100 MeV in $\rho\pi$ and $f_2\pi$ modes [18], which may correspond to the broad, unparametrized enhancement in $\rho\pi$ above 2 GeV seen by E852 [20]. The observation in $\omega\rho$ of $\pi_2(1950)$ [5] casts some doubt over a hybrid interpretation.

B. Positive Parity Hybrids

1^{++}

$$\Gamma_S[a_{1H}(2100) \to \rho\pi] \approx \begin{matrix}160\\660\end{matrix} \text{ MeV}$$

$$\Gamma_D[a_{1H}(2100) \to \rho\pi] \approx \begin{matrix}110\\170\end{matrix} \text{ MeV}$$

The upper/lower values are with IP, $\beta = 0.4$ wave functions and with $g = 3$. We can probably consider these values to be lower and upper limits on the S-wave width—the $\beta = 0.4$ wave functions are the optimum choice for $L = 1 \to L = 0$ conventional transitions, the IP solution shows that hybrids have a slightly smaller β than $L = 1$ states and hence we expect $\langle j_0 \rangle$ to fall faster with $|\vec{q}|$ than in the $\beta = 0.4$ case. The effective β in the IP solution is too small in the conventional sector and hence probably too small for hybrids too—hence our upper/lower assignment for $\beta = 0.4$, IP. Furthermore it has been noted in [28] that the quark model has a tendency not to describe well decays in which the end-state is in an S-wave, while the quarks in the initial meson are in a higher angular momentum eigenstate.

What is clear is that we have a considerable partial width into $\rho\pi$ for the axial hybrid. This is at odds with the claim that the $a_1(2096)$ state seen by E852 has hybrid character as it is not seen at all in $\rho\pi$ by E852, who see only the dominant $a_1(1260)$ in 1^{++}.

2^{+-} In the positive parity sector there are exotics $(0, 2)^{+-}$. The spin-0 state has no decay into πV, but we have some hope of seeing the spin-2 state if this model is correct. Normalizing against the exotic 1^{-+} we have,

$$\frac{\Gamma(b_{2H} \to \omega\pi)}{\Gamma(\pi_{1H} \to \rho\pi)} \approx \frac{72}{5\pi^2}\left(\frac{|\vec{q}_2|}{|\vec{q}_1|}\right)^3 \left(1 - \frac{\pi}{4}\frac{\langle j_2\rangle}{\langle j_1\rangle}\right)^2,$$

where we have neglected the slow mass dependence of $\langle j_{1,2}\rangle$. This ratio suggests a similar partial width for the two states.

Normalizing against the conventional 2^{++} decay gives

$$\frac{\Gamma(b_{2H}\to\omega\pi)}{\Gamma(a_2\to\rho\pi)} = \left(\frac{|\vec{q}_{\mathcal{H}}|}{|\vec{q}_C|}\right)^3 \frac{4b}{\pi^3 m^2} \times \left|\frac{\langle j_1\rangle_{\mathcal{H}} - \frac{\pi}{4}\langle j_2\rangle_{\mathcal{H}}}{\langle j_1\rangle_C - \frac{\langle\overleftarrow{\partial} j_2\rangle_C}{m}}\right|^2$$

where there is some suppression for hybrids from $\frac{4b}{\pi^3 m^2}\approx 0.2$ but which is compensated by the increase in phase-space. For a b_{2H} at 1600 MeV we anticipate a partial width around 50% of the $a_2\to\rho\pi$ partial width. A heavier b_{2H} at 2100 MeV, with the increase in phase-space would be around 150% of a_2. Thus we expect $b_{2H}\to\omega\pi$ with a partial width of tens of MeV.

The possible sighting of such a state around 1650 MeV by E852 in $\pi^- p\to(\omega\pi^-)p$ [29] is, in light of our estimates, rather interesting and a dedicated study of this observation would be enlightening.

V. HIGHER ORDER EFFECTS

The reader will recall that in Eq. (7) we chose to neglect a term transforming as $\vec{\sigma}.\vec{p}'\vec{q}.\vec{a}$ on the grounds that it is subleading by one power of $|\vec{q}|/m$, or equivalently v/c. Unfortunately, for the states we are considering, $|\vec{q}|/m$ is not necessarily small and our truncation appears artificial. A common approach in quark model treatments of hadron decays is to truncate the nonrelativistic expansion of the operator at the highest order in v/c for which we know all possible terms. We do not include the subset of effects at the next order that we are able to calculate as they may be negated by other effects at this order that we are unable to calculate.

Explicit calculation of the "suppressed" $\vec{\sigma}.\vec{p}'\vec{q}.\vec{a}$ term shows that it has a considerable effect on our predictions, especially in the negative parity sector. Its net effect is to modify the P- and D-wave amplitudes according to

$$\Pi_{\mathcal{H}}\to\Pi'_{\mathcal{H}} = g^2\frac{|\vec{q}|^3}{m^2}\frac{b}{\pi^2 m^2}\left|\langle j_1\rangle + \frac{2}{\pi}\frac{\langle\overleftarrow{\partial} j_1\rangle}{m}\right|^2,$$

$$\Delta_{\mathcal{H}}\to\Delta'_{\mathcal{H}} = g^2\frac{|\vec{q}|^3}{m^2}\frac{b}{\pi^2 m^2}\left|\frac{4}{\pi}\langle j_1\rangle - \langle j_2\rangle - \frac{6}{\pi}\frac{\langle\overleftarrow{\partial} j_2\rangle}{m}\right|^2.$$

With harmonic oscillator wave functions $\overleftarrow{\partial}\to-\beta_C^2 r$ and $\Pi'_{\mathcal{H}}\sim|\langle(1-\frac{2\beta_C^2 r}{\pi m})j_1\rangle|^2$ which is approximately zero for light quarks due to an accidental cancellation. In [30], nonadiabatic corrections to this simplest flux-tube model are investigated and this approximate zero does not survive, hence we cannot trust that the cancellation is physically relevant. Unfortunately this also means that we must associate a considerable theoretical error with our predictions.

In light of this disturbing sensitivity to the order of truncation used, we should ask if there are any *more general* results to be extracted from this study. We find that there are and we discuss them in the next section.

VI. GENERAL CURRENT STRUCTURE ARGUMENTS

Making only the assumptions that the pion current should transform as a pseudoscalar, be linear in $\vec{a}$ and at most first order in $\vec{\sigma}$ we can have only the following possible structures,

$$j_\pi(\mathcal{P}_{\mathcal{H}}=+) = \alpha\sigma_z\vec{a}.\hat{z} + \beta[\sigma_-\vec{a}.\hat{x}_+ + \sigma_+\vec{a}.\hat{x}_-],$$
$$j_\pi(\mathcal{P}_{\mathcal{H}}=-) = \gamma[\sigma_-\vec{a}.\hat{x}_+ + \sigma_+\vec{a}.\hat{x}_-].$$

This form is consistent with that following from the Melosh transformation for π induced transitions [31]. As such it is more general, though less predictive, than any particular model where specific values for the parameters α,β,γ also would obtain[32]. Coupling the internal L,S by the Clebsch-Gordan coefficient, $\langle L=1,m';S,m_S|J,m_J\rangle$ and using the flux-tube matrix elements from [16], we find helicity amplitudes [33]

$\mathcal{P}=-$	$\mathcal{M}_0$	$\mathcal{M}_\pm$	$\mathcal{P}=+$	$\mathcal{M}_0$	$\mathcal{M}_\pm$
1^{--}	0	$\mp\sqrt{2}\gamma$	1^{++}	α	$-\sqrt{2}\beta$
0^{-+}	$\sqrt{\frac{8}{3}}\gamma$	0	0^{+-}	0	0
1^{-+}	0	$\pm\gamma$	1^{+-}	2β	$-\frac{1}{\sqrt{2}}\alpha+\beta$
2^{-+}	$\frac{2}{\sqrt{3}}\gamma$	γ	2^{+-}	0	$\pm(\frac{1}{\sqrt{2}}\alpha+\beta)$

Using the conversion from helicity to partial-wave amplitudes in Table XI of [13], these can be succinctly expressed as follows,

$\mathcal{P}=+$	1^{++}	0^{+-}	1^{+-}	2^{+-}	
$A_S=$	$\sqrt{3}$	0	$\sqrt{6}$	0	$\times S$
$A_D=$	$\sqrt{6}$	0	$\sqrt{3}$	3	$\times D$

$\mathcal{P}=-$	1^{--}	0^{-+}	1^{-+}	2^{-+}	
$A_P=$	$\frac{3}{\sqrt{2}}$	$\sqrt{3}$	$\frac{3}{2}$	$\frac{\sqrt{15}}{2}$	$\times P$

where $S=\frac{1}{3}(\alpha-2\sqrt{2}\beta)$, $P=\frac{\sqrt{8}}{3}\gamma$ and $D=-\frac{1}{3}(\alpha+\sqrt{2}\beta)$.

These correlate with the relative amplitudes in Table 6 of [10] who computed these decays in the assumption that the flux-tube breaks with creation of a new $q\bar{q}$ in $^3P_0$ state.

With the assumption only that the partial-wave amplitudes $S,D;(P)$ are common to the supermultiplets of hybrids with $\mathcal{P} = +(-)$ respectively, then for equal masses where $\Gamma \sim \frac{C}{2J+1}\sum_L |A_L|^2$ we have the following constraint on the widths for the isoscalar/isovector $\mathcal{P} = +$ states to πV.

$$3\Gamma(1^{+-}\{I = 0 \rightarrow \rho\pi\}) + 5\Gamma(2^{+-}\{I = 0 \rightarrow \rho\pi\}) = 9\Gamma(1^{++}\{I = 1 \rightarrow \rho\pi\})$$

For the $\mathcal{P} = -$ states we have,

$$[\Gamma(1^{-+}) = \Gamma(2^{-+}) = \frac{1}{4}\Gamma(0^{-+})]\{I = 1 \rightarrow \rho\pi\} = \frac{1}{3}\Gamma(1^{--})\{I = 0 \rightarrow \rho\pi\}.$$

It is trivial to check that the expressions we have derived explicitly satisfy these rules. As we have already mentioned, the predictions of Close and Page [10] satisfy these rules, as do the predictions of an alternative flux-tube breaking model [34] (which has different quantum numbers at the breaking point). These "sum-rules" appear to be a rather general property depending only upon the spin-orbit coupling structure of the states, consequently we expect them to be good if the flux-tube model is a good description of hybrid meson structure.

Their practical use is that once we have a candidate in $(\rho/\omega)\pi$ we can estimate the partial widths of other hybrid states - even the nonexotic ones - in a relatively model-independent way. For example if the $\pi_1(1600)$ is a hybrid, then $\pi_{0H} \rightarrow \pi\rho$ must also be prominent.

VII. RADIATIVE DECAYS BY VECTOR MESON DOMINANCE (VMD)

For the conventional hadrons, the widths into πV may be used to give estimates for the widths into $\pi\gamma$ by converting the $V \rightarrow \gamma$ as in vector dominance. The basic premise is that the photon has some hadronic character—it can fluctuate into an off-shell vector meson with some amplitude and interact strongly with the hadron target. Our phenomenological treatment will follow that of Babcock & Rosner [35]. We find satisfactory agreement with experiment in the conventional sector for the radiative decays of the $a_{1,2}, b_1$ [30], which motivates our application of the method to hybrid states.

For the exotic hybrid candidate $\pi_1(1600)$, using the $\rho\pi$ partial width prediction of 57 MeV we find a $\gamma\pi$ partial width of $\sim$170 keV. This is a healthy width, comparable to the conventional $b_1 \rightarrow \gamma\pi$ width of 230 ± 60 keV. The expectation of the flux-tube breaking model supplemented with VMD is of a maximum width of $\sim$70 keV with a more realistic prediction of 20% of this [36].

The nonexotic 2^{-+} hybrid, according to the sum-rules of the previous section will have a partial width equal to this with modification only for the potentially different state mass.

The exotic b_2, if it has a mass $\sim$2100 MeV has $\gamma\pi$ width $\sim$50 keV. Much of the suppression relative to the π_1 width is down to the factor of 1/9 caused by $g_\omega = 3g_\rho$. This does not appear for the isosinglet 2^{+-} hybrid and there we expect $\Gamma[h_2(2100) \rightarrow \gamma\pi] \sim 450$ keV.

The axial hybrid a_{1H} was predicted to have a potentially very large $\rho\pi$ width which was rather sensitive to wave function parametrization. This large width unsurprisingly leads in VMD to a large radiative width $\sim 550 \rightarrow 1600$ keV which is comparable with our prediction in [15,16] on the basis of an $E1$ photon current exciting the flux-tube in a pion.

Photoproduction of hybrids through pion exchange.—At high photon energies and low momentum transfer, t, photoproduction of mesons is believed to have a significant contribution from the pion exchange mechanism. There is very little data available, but what exists is consistent with one pion exchange expectations (see, e.g., [37]). If this is really a significant mechanism then we should expect the hybrids we have identified as having large $\gamma\pi$ partial widths to be produced prominently in photoproduction.

In particular we noted that an isoscalar 2^{+-} would be large in $\gamma\pi$. A recent analysis of photoproduction of $\pi^+\pi^-\pi^0$ by the CLAS Collaboration[38] finds a large bump in the 2^{+-} partial-wave near 2 GeV. Given that we also expect this state to have a partial width into $\pi\rho$ comparable to $\Gamma(a_2 \rightarrow \rho\pi)$ this signal is most interesting.

As well as photoproduction of isoscalar/isovector 2^{+-} in pion exchange, there is also the possibility of diffractive photoproduction where the photon fluctuates into a ω/ρ which fuses in a P-wave with the Pomeron. We do not currently have the tools to calculate a rate for this process, but we have no reason to expect it to be small—the vector meson is already in the required spin-triplet so the Pomeron current could either excite the flux-tube by oscillating the quark or by interacting directly with the tube.

Hence the 2^{+-} exotic should be especially favored in photoproduction: the $I = 0$ coming from π exchange and the $I = 1$ from Pomeron/gluon exchange.

For the case of pion exchange dominance, a study similar to that in [37], using the full theoretical formalism of photoproduction modeling could be carried out using the matrix elements found in this model.

VIII. SUMMARY

We have used the formalism in which currents acting on quarks can excite the flux-tube to consider the pionic decays of hybrid mesons. The pion is considered to act as a pointlike current which couples only to the quarks (isospin ensures that it cannot couple directly to the flux-tube) and reasonable success in describing conventional meson decays is observed.

Previous studies of hadronic hybrid meson decays had assumed the need for pair production at some point on the flux-tube and had found that decays to a pair of mesons with identical spatial wave functions were forbidden. In the quark model with no spin-dependent effects, the pion and the rho have identical wave functions and this has lead to the expectation that the $\pi\rho$ channel for hybrid decay should be suppressed. The model outlined here maximally breaks the π/ρ symmetry and finds that $\pi\rho$ rates can be large, which seems to be required by the data showing a 1^{-+} resonance in the $\pi\rho$ channel.

Detailed consideration of the model shows that the v/c expansion of the pionic current is not under control and hence that numerical predictions may not be robust. In light of this, more general results were extracted which link $\pi\rho$ partial widths of different hybrid states and which appear to be dependent only on the spin-orbit structure of the hybrid meson states.

Radiative decay widths of hybrids were discussed under the assumption of VMD converting $\rho \to \gamma$ and were found to be not necessarily small. This offers hope to the experiments intending to produce hybrids via photoproduction, especially for $J^{PC} = 2^{+-}$.

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