## Electromagnetic form factors of the $\rho$ meson in light cone QCD sum rules

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We investigate the electromagnetic form factors of the  $\rho$  meson in light cone QCD sum rules. We find that the ratio of the magnetic and charge form factors is larger than 2 at all values of  $Q^2$ ,  $(Q^2 \ge 0.5 \text{ GeV}^2)$ . The values of the individual form factors at fixed values of  $Q^2$  predicted by the light cone QCD sum rules are quite different compared to the results of other approaches. These results can be checked in the future, when more precise data on  $\rho$  meson form factors is available.

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## I. INTRODUCTION

The QCD sum rules method [1] is one of the most powerful tools in studying low energy hadron physics. In this method, physically measurable quantities of hadrons are connected with QCD parameters, where hadrons are represented by their interpolating quark currents. The main idea of this method is to calculate the correlator functions of the interpolating quark currents in the deep Euclidean region with the help of operator product expansion (OPE) which allows one to take into account both perturbative and nonperturbative contributions. Relevant physical quantities are determined by matching the correlator to its phenomenological representation.

In the current literature, a new widely discussed alternative to traditional sum rules, namely, QCD light cone sum rules (QLCSR) is a convenient tool for the study of exclusive processes. This method is based on OPE on the light cone, which is an expansion over twists of the operators, rather than dimensions, as is the case in the traditional QCD sum rules. Moreover, in this method, all the nonperturbative dynamics encoded in the light cone distribution amplitudes determine the matrix elements of the nonlocal operators between the vacuum and the hadronic states (more about this method and its applications can be found in [2,3]).

In the present work, we study the  $\rho$  meson form factors in QLCSR. It should be mentioned here that the  $\rho$  meson form factors are calculated at intermediate momentum transfer by using the three-point QCD sum rules method in [4]. Recently,  $\rho$  meson form factors, including leading  $\mathcal{O}(\alpha_s)$  order QCD corrections, are analyzed within the same framework [5]. The  $Q^2 = 0$  point lies outside the applicability region of the three-point QCD sum rules. Consequently, extrapolating these form factors to  $Q^2 =$ 0, in principle, one can determine static characteristics of the  $\rho$  meson, such as magnetic and quadrupole moments. The  $\rho$  meson magnetic moment has already been investigated in the framework of the traditional three-point and QLCSR in [6,7], respectively. It should be noted here that the QLCSR is successfully applied to a wide range of problems of the hadron physics, for example, magnetic moments of the octet and decuplet baryons are calculated in [8,9], and magnetic moment of the nucleon in [10], respectively.

The paper is organized as follows. In Sec. II, we derive the sum rules for the  $\rho$  meson electromagnetic form factors. In Sec. III, our numerical results and a comparison of them with the results of various approaches existing in the literature is presented.

## **II. THEORETICAL FRAMEWORK**

In order to determine  $\rho$  meson electromagnetic form factors we will use the QLCSR method. For this purpose, we consider the following correlator function

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{iqx} \langle \rho(p) | J^{el}_{\mu}(x) J_{\nu}(0) | 0 \rangle, \quad (1)$$

where

$$J_{\nu}(x) = \bar{u}(x)\gamma_{\nu}d(x), \qquad J_{\mu}^{el}(x) = \sum_{q=u,d} e_{q}\bar{q}(x)\gamma_{\mu}q(x),$$

are the  $\rho$  meson interpolating and electromagnetic currents, respectively.

First, we calculate the phenomenological part of the correlator. By inserting the complete set of states which have the quantum numbers of the  $\rho$  meson between the currents in Eq. (1), we get

$$\Pi_{\mu\nu} = \frac{\langle \rho(p,\varepsilon) | J^{el}_{\mu}(x) | \rho(p',\varepsilon') \rangle \langle \rho(p',\varepsilon') | J_{\nu}(0) | 0 \rangle}{p'^2 - m_{\rho}^2} + \cdots,$$
(2)

where p' = p + q, q is the momentum of the electromagnetic current,  $\varepsilon$  is the polarization of the  $\rho$  meson, and  $\cdots$  describes contributions from higher states. The matrix element  $\langle \rho | j_{\nu} | 0 \rangle$  is determined as

$$\langle \rho | j_{\nu} | 0 \rangle = f_{\rho} m_{\rho} \varepsilon_{\nu}^{*}(p). \tag{3}$$

Assuming parity and time-reversal invariance, the electromagnetic vertex of the  $\rho$  meson can be written in terms of three Lorentz invariant form factors [11]

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$$\begin{split} \langle \rho(p,\varepsilon) | J^{el}_{\mu} | \rho(\rho',\varepsilon') \rangle &= \varepsilon^{*\alpha} \varepsilon'^{\beta} \bigg\{ -G_1(Q^2) g_{\alpha\beta}(p+p')_{\mu} \\ &- G_2(Q^2) (q_{\beta}g_{\mu\alpha} - q_{\alpha}g_{\mu\beta}) \\ &+ \frac{1}{2m_{\rho}^2} G_3(Q^2) q_{\alpha}q_{\beta}(p+p')_{\mu} \bigg\}, \end{split}$$

$$\end{split}$$

$$(4)$$

where  $Q^2 = -q^2$  is the square of the momentum transfer. It should be mentioned here that, in practical computation, instead of calculating the Lorentz invariant form factors  $G_i(Q^2)$ , the physical charge  $G_C$ , the magnetic  $G_M$ , and quadrupole  $G_Q$  form factors are often used.

The Lorentz invariant form factors  $G_i(Q^2)$  are related to the charge, magnetic, and quadrupole form factors through the following relations

$$G_{C} = G_{1} + \frac{2}{3}\eta G_{Q}, \qquad G_{M} = G_{2}, G_{Q} = G_{1} - G_{2} + (1 + \eta)G_{3},$$
(5)

where  $\eta = Q^2/4m_{\rho}^2$ . At zero momentum transfer, these form factors are proportional to the usual static quantities of charge *e*, magnetic moment  $\mu$ , and quadrupole moment  $\mathcal{D}$ 

$$eG_C(0) = e, \qquad eG_M(0) = 2m_\rho \mu, eG_Q(0) = m_\rho^2 \mathcal{D}.$$
(6)

Substituting Eqs. (3) and (4) into Eq. (2), the phenomenological part of the correlator takes the following form

$$\Pi_{\mu\nu} = \frac{f_{\rho}m_{\rho}}{p'^2 - m_{\rho}^2} \Big( G_1(p + p')_{\mu} \Big[ \varepsilon_{\nu} - \frac{(\varepsilon q)p'_{\nu}}{m_{\rho}^2} \Big] \\ + G_2 \Big\{ \varepsilon_{\mu}q_{\nu} - (\varepsilon q)g_{\mu\nu} + \frac{1}{m_{\rho}^2} \Big[ \frac{1}{2}Q^2 \varepsilon_{\mu}p'_{\nu} \\ + (q\varepsilon)p'_{\mu}p'_{\nu} \Big] \Big\} - G_3 \frac{(p + p')_{\mu}}{2m_{\rho}^2} (q\varepsilon) \Big( q_{\nu} \\ + \frac{Q^2}{2m_{\rho}^2}p'_{\nu} \Big) \Big).$$
(7)

It is apparent from Eq. (7) that there are many different structures, each of which can be used to extract the abovementioned form factors.

In order to find out the  $\rho$  meson electromagnetic form factors, we pick the following three structures:  $\varepsilon_{\nu}p_{\mu}$ ,  $\varepsilon_{\mu}q_{\nu}$ , and  $(\varepsilon q)p_{\mu}q_{\nu}$ . Hence we can write

$$\Pi_{\mu\nu}(p,q) = \Pi_1(p,q)\varepsilon_{\nu}p_{\mu} + \Pi_2(p,q)\varepsilon_{\mu}q_{\nu} + \Pi_3(p,q)(\varepsilon q)p_{\mu}q_{\nu} + \cdots,$$
(8)

where

$$\Pi_{1}(p,q) = -G_{1}(Q^{2}) \frac{2f_{\rho}m_{\rho}}{m_{\rho}^{2} - (p+q)^{2}},$$

$$\Pi_{2}(p,q) = -G_{2}(Q^{2})(1+2\eta) \frac{f_{\rho}m_{\rho}}{m_{\rho}^{2} - (p+q)^{2}},$$

$$\Pi_{3}(p,q) = [2G_{1}(Q^{2}) - G_{2}(Q^{2}) + G_{3}(Q^{2})(1+2\eta)] \times \frac{f_{\rho}m_{\rho}}{m_{\rho}^{2}[m_{\rho}^{2} - (p+q)^{2}]}.$$
(9)

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For the invariant amplitudes  $\prod_i (p, q)$ , one can write a general dispersion relation in  $(p + q)^2$  in the form

$$\Pi_i(p,q) = \int ds \frac{\rho_i(s,q)}{s - (p+q)^2} + \text{subtr.}, \quad (10)$$

where the spectral density corresponding to Eq. (7) is

$$\rho_i(s,q) = AF_i(Q^2)\delta(s-m_\rho^2), \qquad (11)$$

where

$$\begin{split} F_1 &= -2G_1(Q^2) f_\rho m_\rho, \\ F_2 &= -G_2(Q^2)(1+2\eta) f_\rho m_\rho, \\ F_3 &= \left[ 2G_1(Q^2) - G_2(Q^2) + G_3(Q^2)(1+2\eta) \right] \frac{f_\rho m_\rho}{m_\rho^2}. \end{split}$$

According to QCD sum rules philosophy, in constructing sum rules for the form factors we need representation of the correlator function from QCD side. After contracting quark fields, the correlator function takes the form:

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle \rho(p) | e_u \bar{u}(x) \gamma_\mu S_d(x,0) \gamma_\nu d(0) + e_d \bar{u}(0) \gamma_\nu S_u(0,x) \gamma_\mu d(x) | 0 \rangle, \qquad (12)$$

where  $S_q(x)$  is the full quark operator of u (or d) quark. Imposing SU(2) symmetry and neglecting u and d quark masses, we have  $S_u(x, 0) = S_d(x, 0) = S(x)$ , and hence the full light quark propagator takes the form

$$S(x) = \frac{i \not t}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2 m_0^2}{192} \langle \bar{q}q \rangle$$
$$- i g_s \int_0^1 dv \bigg[ \frac{\not t \sigma_{\alpha\beta}}{16\pi^2 x^2} G_{\alpha\beta}(vx)$$
$$- \frac{i}{4\pi^2 x^2} v x_\alpha G_{\alpha\beta} \gamma_\beta \bigg]. \tag{13}$$

Note that the second and third terms of S(x) do not give any contribution to the considered problem after Borel transformation is carried out (see below). Rewriting Eq. (13) in momentum representation and substituting it in Eq. (12), we get ELECTROMAGNETIC FORM FACTORS OF THE  $\rho$  MESON...

$$\Pi_{\mu\nu} = \frac{i^2}{4} \int d^4x d^4k e^{i(q-k)x} \Big\{ e_u \langle \rho(p) | \bar{u}(x) \Gamma_i d(0) | 0 \rangle \operatorname{Tr} \gamma_\mu \frac{\not{k}}{k^2} \gamma_\nu \Gamma_i - e_u \langle \rho(p) | \bar{u}(x) g G_{\alpha\beta}(\nu x) \Gamma_i d(0) | 0 \rangle \operatorname{Tr} \gamma_\mu \Big[ \frac{1}{2} \frac{\not{k}}{k^4} \sigma_{\alpha\beta} - \frac{\upsilon}{k^2} x_\alpha \gamma_\beta \Big] \gamma_\nu \Gamma_i + e_d \langle \rho(p) | \bar{u}(0) \Gamma_i d(x) | 0 \rangle \operatorname{Tr} \gamma_\nu \frac{\not{k}}{k^2} \gamma_\mu \Gamma_i - e_d \langle \rho(p) | \bar{u}(0) g G_{\alpha\beta}(\nu x) \Gamma_i d(x) | 0 \rangle \operatorname{Tr} \gamma_\nu \Big[ \frac{1}{2} \frac{\not{k}}{k^4} \sigma_{\alpha\beta} - \frac{\upsilon}{k^2} x_\alpha \gamma_\beta \Big] \gamma_\mu \Gamma_i \Big\},$$
(14)

where  $\Gamma_i = (I, \gamma_5, \gamma_{\alpha}, \gamma_{\alpha}\gamma_5, \sigma_{\alpha\beta})$  is the full set of the Dirac matrices. It follows from this expression that, in order to calculate the correlator from QCD side one needs to know the matrix element of the nonlocal operators between vacuum and the  $\rho$  meson, i.e.,  $\rho$  meson distribution amplitudes (or wave functions). We can see from Eq. (14) that main contribution to the correlator comes only from the wave functions that contain an odd number of  $\gamma$ -matrices that are defined in the following way [12]:

$$\langle \rho(p,\varepsilon) | \bar{u}(x) \gamma_{\mu} d(0) | 0 \rangle = f_{\rho} m_{\rho} \bigg\{ \frac{\varepsilon x}{px} \int_{0}^{1} dv e^{ivpx} \bigg[ \phi_{\parallel}(v,\mu) + \frac{m_{\rho}^{2} x^{2}}{16} A(v,\mu) \bigg] p_{\mu} + \bigg( \varepsilon_{\mu} - \frac{\varepsilon x}{px} p_{\mu} \bigg) \int_{0}^{1} dv e^{ivpx} g_{\perp}^{v}(v,x) - \frac{1}{2} x_{\mu} \frac{\varepsilon x}{(px)^{2}} m_{\rho}^{2} \int_{0}^{1} dv e^{ivpx} C(v,\mu) \bigg],$$

$$(15)$$

$$\langle \rho(p,\varepsilon) | \bar{u}(x) \gamma_{\mu} \gamma_{5} d(0) | 0 \rangle = \frac{1}{4} f_{\rho} m_{\rho} \epsilon_{\mu\alpha\beta\delta} \varepsilon_{\alpha} p_{\beta} x_{\delta} \int_{0}^{1} dv e^{ivpx} g_{\perp}^{a}(v,\mu), \tag{16}$$

$$\langle \rho(p,\varepsilon) | \bar{u}(x) g G_{\mu\nu} \gamma_{\alpha} d(0) | 0 \rangle = -i f_{\rho} m_{\rho} P_{\alpha} (P_{\nu} \varepsilon_{\mu}^{\perp} - P_{\mu} \varepsilon_{\nu}^{\perp}) \mathcal{V} - i f_{\rho} m_{\rho}^{3} \frac{\varepsilon x}{px} (P_{\mu} g_{\alpha\nu}^{\perp} - P_{\nu} g_{\alpha\mu}^{\perp}) \Phi$$
$$- i f_{\rho} m_{\rho}^{3} \frac{\varepsilon x}{(px)^{2}} P_{\alpha} (P_{\mu} x_{\nu} - P_{\nu} x_{\mu}) \Psi, \tag{17}$$

$$\langle \rho(p,\varepsilon) | \bar{u}(x) g \widetilde{G}_{\mu\nu} \gamma_{\alpha} \gamma_{5} d(0) | 0 \rangle = f_{\rho} m_{\rho} P_{\alpha} (P_{\nu} \varepsilon_{\mu}^{\perp} - P_{\mu} \varepsilon_{\nu}^{\perp}) A + i f_{\rho} m_{\rho}^{3} \frac{\varepsilon x}{px} (P_{\mu} g_{\alpha\nu}^{\perp} - P_{\nu} g_{\alpha\mu}^{\perp}) \widetilde{\Phi} + f_{\rho} m_{\rho}^{3} \frac{\varepsilon x}{(px)^{2}} P_{\alpha} (P_{\mu} x_{\nu} - P_{\nu} x_{\mu}) \widetilde{\Psi},$$
(18)

where  $P_{\mu}$ ,  $\varepsilon_{\mu}^{\perp}$ , and  $g_{\mu\nu}^{\perp}$  are defined as

$$P_{\mu} = p_{\mu} - \frac{m_{\rho}^2}{2px} x_{\mu}, \qquad \varepsilon_{\mu}^{\perp} = \varepsilon_{\mu} - \frac{\varepsilon x}{px} \left( P_{\mu} - \frac{m_{\rho}^2}{px} x_{\mu} \right), \qquad g_{\mu\nu}^{\perp} = \left( g_{\mu\nu} - \frac{P_{\mu} x_{\nu} + P_{\nu} x_{\nu}}{px} \right),$$

where px = Px has been used.

In Eqs. (15)–(18),  $\phi_{\parallel}(v, \mu)$  is the leading twist-2 wave function, while  $g_{\perp}^v$ ,  $g_{\perp}^a$ , and  $\mathcal{V}$  are the twist-3, and all the remaining ones are the twist-4 wave functions. In Eqs. (17) and (18) the following relation is used

$$\mathcal{V}(v, px) = \int \mathcal{D}\alpha e^{ipx(\alpha_1 + v\alpha_3)} \mathcal{V}(\alpha_1, \alpha_2, \alpha_3),$$

where

$$\mathcal{D}\alpha = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3).$$

In order to suppress the contributions of higher states and continuum and further eliminate the subtraction terms in the dispersion relation, it is necessary to perform Borel transformation with respect to  $-(p + q)^2$ . The contributions of higher states and continuum are subtracted using the quark hadron duality (for details, see [2,3], [8–10], [13,14]).

The sum rules for form factors can be obtained after applying Borel transformation to the two different representations of the invariant functions  $F_i$  and then matching these results, and in doing so, we get T. M. ALIEV AND M. SAVCI\_

$$G_{1}(Q^{2}) = \frac{1}{2} e^{m_{\rho}^{2}/M^{2}} \bigg( \int_{v_{0}}^{1} dv e^{-s(v)/M^{2}} \bigg\{ -\frac{m_{\rho}^{2}}{4} \frac{1}{v^{2}M^{2}} [e_{u}A^{i}(v) - e_{d}A^{i}(\bar{v})] + \frac{1}{v} [e_{u}\Phi^{i}(v) - e_{d}\Phi^{i}(\bar{v})] \\ -\frac{2m_{\rho}^{2}}{vM^{2}} [e_{u}C^{ii}(v) - e_{d}C^{ii}(\bar{v})] + [e_{u}g_{\perp}^{v}(v) - e_{d}g_{\perp}^{v}(\bar{v})] + \frac{1}{4} \bigg( \frac{1}{v} - \frac{Q^{2} + m_{\rho}^{2}v^{2}}{v^{2}M^{2}} \bigg) [e_{u}g_{\perp}^{a}(v) + e_{d}g_{\perp}^{a}(\bar{v})] \bigg\} \bigg), \quad (19)$$

$$G_{2}(Q^{2}) = \frac{2m_{\rho}^{2}}{Q^{2} + 2m_{\rho}^{2}} e^{m_{\rho}^{2}/M^{2}} \bigg( \int_{v_{0}}^{1} dv e^{-s(v)/M^{2}} \bigg\{ -\frac{2m_{\rho}^{2}}{v^{2}M^{2}} [e_{u}C^{ii}(v) - e_{d}C^{ii}(\bar{v})] + \frac{1}{v} [e_{u}g_{\perp}^{v}(v) - e_{d}g_{\perp}^{v}(\bar{v})] - \frac{1}{4v^{2}} \bigg( 1 - \frac{Q^{2} + m_{\rho}^{2}v^{2}}{vM^{2}} \bigg) [e_{u}g_{\perp}^{a}(v) + e_{d}g_{\perp}^{a}(\bar{v})] \bigg\} \bigg),$$

$$(20)$$

$$-2G_{1} + G_{2} - G_{3}(1+2\eta) = m_{\rho}^{2} e^{m_{\rho}^{2}/M^{2}} \bigg( \int_{v_{0}}^{1} dv e^{-s(v)/M^{2}} \bigg\{ -\frac{m_{\rho}^{2}}{2v^{3}M^{4}} [e_{u}A^{i}(v) - e_{d}A^{i}(\bar{v})] - \frac{1}{2v^{2}M^{2}} [e_{u}g_{\perp}^{a}(v) + e_{d}g_{\perp}^{a}(\bar{v})] + \frac{2}{v^{2}M^{2}} [e_{u}\Phi^{i}(v) - e_{d}\Phi^{i}(\bar{v})] - \frac{4m_{\rho}^{2}v}{v^{3}M^{4}} [e_{u}C^{ii}(v) - e_{d}C^{ii}(\bar{v})] \bigg\} \bigg),$$
(21)

where  $M^2$  is the Borel parameter and

$$s(v) = \frac{(Q^2 + m_\rho^2 v)\bar{v}}{v}, \qquad \bar{v} = 1 - v,$$
  
$$v_0 = -\frac{1}{2m_\rho^2} [(s_0 + Q^2 - m_\rho^2)^2 - \sqrt{(s_0 + Q^2 - m_\rho^2)^2 + 4m_\rho^2 Q^2}],$$

where  $s_0$  is the continuum threshold.

In Eqs. (19)–(21), functions  $A^i$ ,  $\Phi^i$ , and  $C^{ii}$  are defined as follows

$$A^{i} = -\int_{0}^{v} du A(u), \qquad \Phi^{i}(v) = -\int_{0}^{v} du \Phi(u),$$
  

$$C^{i}(v) = -\int_{0}^{v} du C(u), \qquad C^{ii}(v) = -\int_{0}^{v} du C^{i}(u).$$

Note that the terms involving three-particle  $\rho$  meson wave functions are not presented in Eqs. (19)–(21) since their expressions are rather lengthy, however, their contributions are taken into account in the numerical analysis, which constitute about 5% of the total result.



FIG. 1. The dependence of the charge form factor  $G_C$  on  $M^2$  at six fixed values of  $Q^2$ , and at  $s_0 = 2.2 \text{ GeV}^2$ .

At the end of this section we would like to make the following remark. In the present work, leading  $\mathcal{O}(\alpha_s)$  QCD corrections to the correlation function are not calculated. These corrections are calculated in the framework of three-point QCD sum rules in [5] and it is obtained that they give ~25% contribution to the final result (after transforming the form factors used in [5] to our parametrization). We expect that QCD corrections to the correlator in the light cone QCD sum rules approach is of the same order as in three-point sum rule method and in determination of the form factors they bring ~25% uncertainty.

## **III. NUMERICAL ANALYSIS**

In this section we present our numerical calculations on charge, magnetic, and quadrupole form factors. The main input parameters of the QLCSR in regard to the abovementioned form factors are the  $\rho$  meson wave functions, whose explicit expressions are given in [12,13], and we use them in our analysis.



FIG. 2. The same as in Fig. 1, but for the magnetic form factor  $G_M$ .



FIG. 3. The same as in Fig. 1, but for the quadrupole form factor  $G_O$ .

Apart from the wave functions, the sum rules for the form factors depend on the value of the continuum threshold  $s_0$  and Borel parameter  $M^2$ . In the present work, we calculate the form factors at three different values of the threshold, i.e.,  $s_0 = 1.8 \text{ GeV}^2$ ,  $s_0 = 2.0 \text{ GeV}^2$ , and  $s_0 = 2.2 \text{ GeV}^2$ .

In Figs. 1–3 we present the dependence of  $G_C$ ,  $G_M$ , and  $G_D$  on  $M^2$  at six different values  $Q^2 = 0.5 \text{ GeV}^2$ ,  $Q^2 = 1.0 \text{ GeV}^2$ ,  $Q^2 = 2.0 \text{ GeV}^2$ ,  $Q^2 = 3.0 \text{ GeV}^2$ ,  $Q^2 = 4.0 \text{ GeV}^2$ , and  $Q^2 = 5.0 \text{ GeV}^2$  of the momentum transfer, respectively. The Borel parameter  $M^2$  in the sum rule is an auxiliary parameter and therefore physical quantities must be independent of it. For this reason we must determine the region of  $M^2$  in which the form factors are independent of its value. In determining the working region of the Borel parameter  $M^2$ , the following two conditions must be satisfied:

- (i)  $M^2$  should be large enough to suppress the contributions coming from higher twists, and,
- (ii)  $M^2$  should be small enough to suppress the continuum and higher state contributions.

In the present analysis, both conditions are satisfied for the three form factors when  $M^2$  varies in the region  $1.0 \text{ GeV}^2 \le M^2 \le 2.5 \text{ GeV}^2$ .



FIG. 4. The dependence of the charge form factor  $G_C$  on  $Q^2$  at three fixed values of  $s_0$ , and at  $M^2 = 1.0 \text{ GeV}^2$ .



FIG. 5. The same as in Fig. 4, but for the magnetic form factor  $G_M$ .

In Figs. 4–6 we present the dependence of the charge  $G_C$ , magnetic  $G_M$ , and quadrupole  $G_Q$  form factors on  $Q^2$ , at three different values  $s_0 = 1.8 \text{ GeV}^2$ ,  $s_0 = 2.0 \text{ GeV}^2$ , and  $s_0 = 2.2 \text{ GeV}^2$  of the threshold, and at the fixed value  $M^2 = 1.0 \text{ GeV}^2$  of the Borel parameter. We observe from these figures that the dependence of the form factors on  $s_0$  is rather weak when  $s_0$  varies from  $s_0 = 1.8 \text{ GeV}^2$  to  $s_0 = 2.2 \text{ GeV}^2$ .

Concerning the sources of errors in the form factors, we would comment as follows. In the present work, leading  $\mathcal{O}(\alpha_s)$  QCD corrections to the correlator are not calculated, and, as has already been noted, they can bring  $\sim$ 25% uncertainty to the final result. When this uncertainty is taken into account together with the uncertainties coming from the variation of  $s_0$ ,  $M^2$ , and higher order twists, one can conclude that the form factors are determined to within 35% uncertainty. The  $\rho$  meson form factors are also calculated in the framework of other approaches, such as three-point QCD sum rules [5], Dyson-Schwinger based models [15,16], covariant lightfront approaches with constituent quark model [17], lightfront formalism [18], light-front quark model [19], and lattice calculation [20]. In Table I, we present the values of the charge, magnetic, and quadrupole form factors pre-



FIG. 6. The same as in Fig. 4, but for the quadrupole form factor  $G_{Q}$ .

TABLE I. Comparison of our predictions on  $\rho$  meson form factors with other approaches at three different values of  $Q^2$  (Note that in [15], only the magnitudes of the form factors are given).

	$Q^2 = 1 \text{ GeV}^2$			$Q^2 = 2 \text{ GeV}^2$			$Q^2 = 3 \text{ GeV}^2$		
	$G_C$	$G_M$	$G_Q$	$G_C$	$G_M$	$G_Q$	$G_C$	$G_M$	$G_Q$
[19]	0.38	0.93	-0.23	0.18	0.59	-0.15	0.08	0.41	-0.10
[5]	0.10	0.46	016	0.016	0.27	-0.11	-0.03	0.18	-0.10
[15]	0.17	0.85	0.51	0.04	0.45	0.32	0.11	0.25	0.23
ours	0.25	0.58	-0.49	0.13	0.28	-0.24	0.08	0.17	-0.15

dicted by different approaches at different values of  $Q^2$ , when all form factors are reexpressed in terms of our parametrization and when leading  $\mathcal{O}(\alpha_s)$  QCD corrections are neglected.

It follows from Table I that, at three selected values of  $Q^2$ , the form factors predicted by different approaches are quite different. For example, our results for  $G_M(Q^2)$ , at  $Q^2 = 1, 2, 3 \text{ GeV}^2$ , practically coincide with the predictions of [5], while  $G_C(Q^2)$  and  $G_Q(Q^2)$  are 2 times larger compared to that of [5]. Our prediction of  $G_Q(Q^2)$  at the above-mentioned values of  $Q^2$  are almost 2 times larger compared to that given in [19]. As far as  $G_C(Q^2)$  is concerned, there is 50% difference between our results and that of [19] at  $Q^2 = 1, 2 \text{ GeV}^2$ , but they coincide at  $Q^2 = 3 \text{ GeV}^2$ . As it comes to a comparison between our results of the form factors), there seems to be an overall 50% difference [except for  $G_C(Q^2)$  at  $Q^2 = 2 \text{ GeV}^2$ , where the difference is almost 3 times]. Therefore, more data on  $\rho$ 

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meson is needed in order to choose the "right" model in calculating the form factors.

Unfortunately, the sum rules fail working at small  $Q^2$ , and hence do not allow determination of the magnetic moment of the  $\rho$  meson with better accuracy compared to the results in the literature, such as the results predicted by the sum rules [6,7,15–20]. Our analysis predicts that, starting from  $Q^2 = 0.5 \text{ GeV}^2$ , the ratio  $G_M(Q^2)/G_C(Q^2)$ is around 2.3 at all values of  $Q^2$ . If we assume that this behavior holds at smaller values of  $Q^2$ , we can conclude that  $\mu \simeq 2.3$ , which is quite close to the prediction on the magnetic moment of the  $\rho$  meson given in [7,20].

In conclusion, we have presented the results for the  $\rho$  meson form factors in the frame work of QLCSR. We have obtained that, in the region of applicability of the method, at all values of  $Q^2$ , the ratio  $G_M(Q^2)/G_C(Q^2)$  is larger than 2; more precisely, this ratio varies around 2.3. This result can be checked when more precise data on  $\rho$  meson form factors is available.

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