Large distance behavior of the light cone operator product in perturbative and nonperturbative QCD regimes

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We evaluate the coordinate space dependence of the matrix elements of the commutator of the electromagnetic and gluon currents in the vicinity of the light cone but at large distances. We found within the parton model, Doushitzer-Gribov-Lipatov-Altarelli, the resummation approaches to the small x behavior of deep-inelastic scattering (DIS) processes that an increase of the commutator with relative distance py as $\propto (py)f(py, y^2 = t^2 - r^2)$, where f is increasing with increase of py is the generic property of QCD at small but fixed space-time interval $y^2 = t^2 - r^2$ in perturbative and nonperturbative QCD regimes. We explain that the factor py follows within the dipole model (QCD factorization theorem) from the properties of Lorentz transformation. The increase of f disappears at central impact parameters if the cross section of DIS may achieve the unitarity limit. We argue that such long-range forces are hardly consistent with thermodynamic equilibrium while a unitarity limit may signal equilibration. Possible implications of this new long-range interaction are briefly discussed.

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I. INTRODUCTION

Commutators of the local currents in the coordinate space play an important role in the quantum field theory. In particular, they help to visualize the relationship between the quantum field theory and statistical mechanics of equilibrium and nonequilibrium systems. It has been understood that many properties of deep-inelastic processes follow from the operator product expansion. In particular, the dependence of the product of the local currents,

$$\langle N|j_\mu(y)j_\nu(o)|N\rangle = (1/y^2)^2\sum_n p_\mu p_\nu (py)^n\langle N|O_n(0)|N\rangle + \text{NLT terms} = p_\mu p_\nu[F(py, y^2)/(y^2)^2] + \text{NLT terms}, \quad (1.1)$$

on the space-time interval $y^2 = t^2 - r^2$ unambiguously follows from this expansion for the leading term (see review [1]) and from renormalization group. (For certainty we write formulas for the product of electromagnetic (e.m.) or gluon currents and neglect by longitudinal structure functions.) We will show in this paper that the dependence of operator product on relative distance (py) at fixed y^2 as

$$F(py, y^2) \propto (py)f(py, y^2), \quad (1.2)$$

with f increasing with (py), follows from the basic properties of QCD.

The actual behavior of the structure functions of the nucleon at small Bjorken $x = Q^2/2pq$ is still a challenging question now, as it was 30 years ago. (Here $-Q^2$ is mass^2 of an incoming photon.) So a variety of the new approaches to small x phenomena were developed such as the generalization of the QCD factorization theorem to the amplitudes of the hard diffractive processes which justifies the dipole approach [2,3]; the derivation within the Balitsky, Fadin, Kuraev, Lipatov (BFKL) approximation [4] of the dipole approach in the large N_c limit; the resummation of the perturbative QCD (pQCD) series within the Doushitzer-Gribov-Lipatov-Altarelli (DGLAP) approximation [5]; the next to leading order (NLO) BFKL approximation and resummation [6,7]; the McLerran-Venugopalan model [8]; the eikonal approximation where a "potential" is evaluated within the DGLAP or BFKL approximations [9–11], and the unitarity bound (blackbody limit) approach [12].

It is well known that the theoretical description of the high-energy processes is significantly simplified in the coordinate space even if actual calculations may appear rather cumbersome. The aim of this paper is to evaluate the amplitudes of the deep-inelastic scattering (DIS) processes in coordinate space and to visualize the dominant physics. The knowledge of the space-time evolution of DIS is especially important for the theoretical description of the RHIC program of heavy ion collisions, for the QCD part of the LHC program, and for the hunt for the new particles [13–15].

It has been demonstrated that DGLAP approximation describes well the increase of structure functions of a proton [16] observed by H1 and ZEUS. The experimental data can be fitted as

$$xG(x, Q^2), \qquad F_2(x, Q^2) \propto x^{-\lambda}, \quad (1.3)$$

with $\lambda \approx 0.25$ [16]. Basic features of hard diffractive processes observed by H1 and ZEUS [16] are well de-

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scribed by the QCD factorization theorem [3]. The success of DGLAP at the energy range covered by HERA is due to the energy-momentum conservation law restrictions on the possible number of gluons radiated in the multi-Regge kinematics. In the kinematics covered by HERA, this number is equal to 1–2. At even smaller x (this corresponds to the LHC kinematics and larger energies), the multiplicity of radiated gluons may achieve 5–6 and therefore pQCD approximations become unstable because of the necessity to account for large $\log(x_0/x)$ terms. So various resummation procedures were suggested [5,6].

Significant cross section of diffraction in DIS observed by H1 and ZEUS [16] is hardly consistent with the validity of the leading twist (LT) approximation at large Q^2 but sufficiently small x. The theoretical analysis shows that the LT approximation is probably violated in the kinematics which is not far from that investigated at HERA [16]. Account of the conservation of the probability leads to the unitarity bound which is the generalization to DIS of the Froissart limit familiar from the hadron-hadron collisions. The unitarity bound formula shows that the structure functions of a nucleon may increase with the energy as [12]

$$F_2, xG \propto \log^3(x_0/x). \tag{1.4}$$

Increase of the structure functions with the energy follows from the increase with the energy of the essential impact parameters factor $\log^2(x_0/x)$ and from the ultraviolet divergence of the renormalization constants in QCD factor $\log(x_0/x)$. The conservation of the probability permits a more moderate increase of the structure functions at the central impact parameters as

$$F_2, xG \propto \log(x_0/x). \tag{1.5}$$

It has been understood already in 60-s that the dependence of the amplitude of the deep-inelastic scattering on $\nu = 2pq$ and photon virtuality $-Q^2$ gives a unique possibility to probe the space-time behavior of the DIS processes [17]. (On the contrary, the amplitudes of the soft QCD processes are always on mass shell. So it is impossible to compare with the data the dependence of the commutator of currents on the space-time interval $y^2 = t^2 - r^2$ and the relative distance py.) An increase with the energy of the coherence length in the target rest frame has been suggested in Ref. [17], based on the analogy with the QED coherent phenomena in the high-energy electron interaction in the medium [18,19]. The formula for the coherence length, $l_c \approx 1/2m_N x$, follows from the properties of the Fourier transform in [17] since the amplitude of the DIS decreases with an increase of Q^2. Recently, the calculation of Ref. [20] found that account for the pQCD radiation leads to significantly smaller (but still increasing fast with the energy) coherent length as compared to that found within the parton model. The first theoretical analysis of the space-time evolution of high-energy processes in a quantum field theory was given by Gribov [21] and applied for the calculation of the nuclear shadowing in the electron-nucleus interactions [22].

The dependence of the DIS amplitude on the light cone interval y^2 was studied extensively in the parton model [1,23–25]. It was also studied in the framework of a Regge ansatz for small x_B structure function behavior [1,25] .

The aim of the paper is to show that for sufficiently small but fixed y^2 the product of the currents is the increasing function of py—distance (time), within both the leading log (LL) or resummation approaches to pQCD $\propto (py)f(py, y^2)$, where f is an increasing function of py at fixed y^2. Moreover, we shall see that the increase of the product of the color-neutral currents with (py) is valid for the phenomenological structure functions, describing HERA data. An increase of f disappears in the unitarity bound approximation but for the fixed impact parameters only. In particular, in perturbative QCD

$$F(py, y^2) \propto \theta(y^2)/(y^2)^2(\alpha_s N_c/\pi)^{1/4}\frac{\log(Q_0^2y^2)^{1/4}}{\log(py)^{3/4}} \times \exp[2\sqrt{(\alpha_s N_c/\pi)\log(py)\log(Q_0^2y^2)}], \tag{1.6}$$

and in the black limit

$$F(py, y^2) \propto \theta(y^2)\log^3(py)/(y^2)^2 + \text{peripheral terms.} \tag{1.7}$$

Note that a similar increase of correlators with relative distance is characteristic for turbulence. It is well known that the velocity-velocity correlator increases with distance in such a system for the case of homogeneous turbulence, i.e., for the scales much smaller than the scale of the entire system [26]. In turbulence such a behavior arises because the same piece of matter reveals itself in different points. Similarly in the deep-inelastic scattering in the target rest frame the same dipole reveals itself in different space-time points as a consequence of the large coherence length. This explains factor r in the matrix element of the commutator. An increase of $f(r, y^2)$ with r ($r \sim t$) indicates that the produced perturbative system is far from the thermodynamic equilibrium. Some caution is to the point: Our interest is in the distances less or comparable with coherence length. At larger distances deduced formulas are hardly applicable at the distances $\gg l_c$ where nonperturbative phenomena, such as confinement of color and phenomenon of spontaneously broken chiral symmetry, should be important. Discussion of this important question is beyond the scope of this publication. Note, however, that the regime leading to the unitarity bound corresponds to f not increasing with distance, which is a hint for the possibility of the equilibrium.

To visualize physics relevant for the shadowing effects, we investigate also the Fourier transform of the ratio of the distribution function and the invariant energy s. (Within the region of validity of QCD factorization theorem this ratio has a meaning of the cross section for the scattering of a dipole off a target [2].) The increase with the distance of a Fourier transform of this quantity shows the existence of the long-range pQCD interaction between the two colorless dipoles. (Remember that the long-range interaction related to the zero mass of the gluon is canceled out in the amplitudes of the collisions of color-neutral objects as the consequence of the gauge invariance).

Let us note that a similar increase with distance can be derived from the calculations of structure function evaluated within the Regge pole approximation [1,25] if the intercept of the Regge pole is $\alpha(0) > 1$ [Only the $\alpha(0) \leq 1$ case was considered in Refs. [1,25].]

Let us recall conventional definitions concerning the relationship between the products of the currents and the structure functions. The structure functions are defined through the current product as

$$\frac{1}{\pi} W_{\mu\lambda}(q, p) = -(\delta_{\mu\lambda} - q_\mu q_\lambda/q^2) W_1(x, q^2) + (1/m^2) \times [p_\mu + q_\mu/(2x)][p_\lambda + q_\lambda/(2x)] W_2, \quad (1.8)$$

$$W_{\mu\lambda} = \langle p | \int d^4 y \exp(iqy) J_\mu(y) J_\lambda(0) | p \rangle. \quad (1.9)$$

Here J_μ is the operator of the electromagnetic current. These structure functions are usually redefined into the dimensionless ones:

$$F_1 \equiv W_1, F_2 \equiv (Q^2/2m^2 x) W_2. \quad (1.10)$$

Within the DGLAP approximation these structure functions can be approximated at small x as

$$F_2(x, Q^2) = \int_x^1 ds G_2(x/s, Q^2) g_{\text{gluon}}(s, Q_0^2), \quad (1.11)$$

and

$$x F_1(x, Q^2) = \int_x^1 (ds/s) G_1(x/s, Q^2) g_{\text{gluon}}(s, Q_o^2). \quad (1.12)$$

The function $g_{\text{gluon}}(s, Q_0^2)$ is the nonperturbative gluon distribution that parametrizes long distance contributions while the functions G_i describe the distribution of gluons (sea quarks and antiquarks) within the gluon. For the gluon distribution similar convolution formulas are valid. (See, e.g., Ref. [27] for more detailed definitions.) Within the parton model the quark-gluon distribution functions are

$$G_1 = G_2 = \delta(x - s). \quad (1.13)$$

For the analysis of the light cone behavior it is convenient also to use functions

$$V_2 = W_2/(m_N^2 Q^2), \qquad V_L = [(Q^2/x^2) W_2 - W_1]/Q^2. \quad (1.14)$$

These functions are free from the kinematic singularities.

For the theoretical description of high-energy processes in the nucleon rest frame, it is useful to analyze the cross section

$$\sigma = F_2/Q^2, \quad (1.15)$$

instead of the infinite momentum frame parton distribution F_2.

For the gluon-gluon distribution function G this cross section has the sense of the dipole-target cross section [2]:

$$\sigma_d = 4\pi \alpha_s x_B G/Q^2. \quad (1.16)$$

In the framework of the Feynman parton model [1,23–25,28]

$$\begin{aligned} V_L(x^2, px) &= -2\pi i \epsilon(x_0) \delta(x^2) f_L(px), \\ V_2(x^2, px) &= 2\pi i \epsilon(x_0) \theta(x^2) f_2(px), \end{aligned} \quad (1.17)$$

while the calculation based on the Regge models [29] gives

$$f_L \sim (px)^{\alpha(0)} + \text{const}, \qquad f_2 \sim (px)^{\alpha(0)-2}. \quad (1.18)$$

Our main result is the current-current correlator and the cross section in the coordinate space at fixed and sufficiently small space-time interval y^2 but large relative distances py evaluated in QCD using both leading log and resummation models.

The paper is organized in the following way. In the second chapter we review the results of the parton model for the structure functions in coordinate space and show in detail how to properly account for the space-time structure of the commutators including causality. In the third chapter, we evaluate the light cone correlators of the currents within the DGLAP approximation, and find that at a fixed space-time interval they increase with the distance near the light cone. We found it difficult to calculate the Fourier transform of amplitude directly and to keep causality because of necessity to make approximations. Instead we generalized a method of calculations developed within the parton model in [23,24]. For this aim we found it convenient to use a method of moments including analytic continuation in the vicinity of $n \to 1$. In the fourth chapter, we evaluate Fourier transform into coordinate space of the phenomenological and theoretical gluon distributions in the small x limit, including both the experimental data and the recent resummation models. We also consider the space-time behavior of the structure functions if the unitarity bound is achieved at high energies. In general, we find that the rise of the distributions in the limit $x \to 0$ leads to the corresponding rise of the light cone product of the local currents. The fifth chapter is the conclusion.

II. PARTON MODEL IN THE COORDINATE SPACE

Let us briefly review the calculations of the structure functions in coordinate space for $y^2 \to 0$ within the parton model [1,23,24,28]. Within the parton model approximation, the structure functions are the functions of only x. The calculations were carried through in the early seventies assuming dependence on x as given by Regge formulas, with $\alpha_P(0) \leq 1$. We need to calculate

$$F(y^2, py) = \int \frac{d^4q}{(2\pi)^3} F(q^2, pq) \exp(iqy). \tag{2.1}$$

The structure function in the parton model can be derived in the nonperturbative QCD as the discontinuity on the cut in the complex pq plane [24,30]:

$$F(q^2, pq) = \int_0^1 dx (2pq)\epsilon(pq)\{\delta[q^2 + 2(pq)x] + \delta[q^2 - 2(pq)x]\}F(x). \tag{2.2}$$

Here $F(x)$ is the nonperturbative parton distribution in the target. For simplicity we consider here spinless quarks. Generalization to spin of quark 1/2 is trivial and does not introduce new theoretical phenomena. Let us briefly review the standard way of the calculation of these Fourier transforms [24]. We shall start from the integral that is the particular case of the integral (2.1), the integral

$$R(y^2, py) = \int \frac{d^4q}{(2\pi)^3} \int_0^1 dx \epsilon(pq)\{\delta[q^2 + 2(pq)x] + \delta[q^2 - 2(pq)x]\}. \tag{2.3}$$

We first calculate the integral

$$I(y^2, py) = \int \frac{d^4q}{(2\pi)^3} \epsilon(pq)\{\delta[q^2 + 2(pq)x] + \delta[q^2 - 2(pq)x]\}. \tag{2.4}$$

This integral is equal to a sum of two integrals; the first of them corresponds to the contribution of the region $pq > 0$, and the second of the region $pq < 0$:

$$I(y^2, py) = I^+(y^2, py) + I^-(y^2, py). \tag{2.5}$$

Here

$$I^{\pm}(x^2, px) = \int d^4q\{\delta[q^2 + 2(pq)x] + \delta[q^2 - 2(pq)x]\} \times \exp(iqy)\theta(\pm qp). \tag{2.6}$$

Making substitution $q \to q + px$ in the first term and $q \to q - px$ in the second, we obtain

$$I^{\pm} = 2\cos[(py)x] \int d^4q/(2\pi)^3 \delta(q^2)\theta(\pm pq) \exp(iqy). \tag{2.7}$$

The latter integrals are well known (see, e.g., Ref. [31]):

$$I^{\pm}(y^2, py) = 2i\cos[x(py)]D^{\pm}[(y^2, py)]. \tag{2.8}$$

Taking the sum we obtain

$$I(y^2, py) = 2i\cos[x(py)]D[y^2, (py)]. \tag{2.9}$$

Integrating now over x we obtain the integral (2.3)

$$R(y^2, py) = 2i \int_0^1 dx \cos[x(py)]D(y^2, py). \tag{2.10}$$

Here the function

$$D(y^2, py) = [1/(2\pi)]\{\epsilon(py)[\delta(y^2)] - [(mx)/(2\sqrt{y^2})]\theta(y^2)J_1(mx\sqrt{x^2})\} \tag{2.11}$$

is the Pauli-Jordan commutator of the scalar particles [see, e.g., Ref. [31] for a detailed analysis of the singular functions in quantum field theory (QFT)]. We retained the full dependence on the nucleon mass in order to be sure that there are no singularities in the limit $m \to 0$. To account for the spin of the quarks, one should substitute function D by the Green function of spin 1/2 particle $S(y)$. In the LT approximation it is necessary to neglect masses of quarks. Taking now the $m \to 0$ limit in the latter equation, we obtain

$$R(y^2, py) = (i/\pi)\epsilon(py)\delta(y^2)\sin(py)/(py). \tag{2.12}$$

We can go now to other structure functions discussed in the introduction. The corresponding Fourier transforms differ from the integral (2.3) that we had taken by the powers $(q^2)^n(pq)^m$, where n and m are integers (but generally nonpositive numbers). If both n and m are positive, we can take the relevant integrals just by using the corresponding differential operators. In the parton model the scaling leads to the general form of the structure function $F(x)$ and one immediately obtains, acting on the Eq. (2.11) with the operator $pq = -ip\partial_y$, that near the light cone [24]

$$F(y^2, py) \to \epsilon(py)/(2\pi) \int_0^1 dx\{4i(py)\delta'(y^2)\cos[x(py)] - 2m^2x\sin(py)\delta(y^2) + 2(p\partial_y)[\theta(y^2)mx/\sqrt{y^2}] \times J_1(xm\sqrt{y^2})\}F(x). \tag{2.13}$$

Here we once again retain nonzero m to be sure there are no singularities. Taking the limit $m \to 0$, we immediately obtain the following for the parton model:

$$F(y^2, py) = (1/\pi)\int_0^1 dx\epsilon(py)\cos[x(py)]\delta'(y^2) \times[2(py)]F(x). \quad (2.14)$$

In particular for $F(x) = 1$ we obtain the Fourier transform

$$K(y^2, py) = (2/\pi)\sin(py)\delta'(y^2)\epsilon(py). \quad (2.15)$$

The integral (2.14) is well defined for $F(x) \sim x^\alpha$ if $\alpha > -1$. This is however not the general case. The most interesting structure functions are $F_1(x) = 1/x$, $V_2(q^2, pq) \sim 1/(q^2 pq) \sim x/q^4$, $V_L \sim pq/q^4 \sim 1/(xq^2)$. It is easy to see that for these structure functions the integral (2.14) formally diverges logarithmically and must be regularized. In order to define these integrals and satisfy the requirement of causality, we follow Ref. [24], namely, use the differential equations: If two functions A and B are connected as

$$A = B/q^2,$$

then $B(q^2, pq) = q^2 A(q^2, pq)$, and in coordinate space we obtain

$$\Box A(y^2, py) = B(y^2, py).$$

Let us use this method for the calculation of the structure functions defined above. Let us start from $F_1 = 1/x = -2(pq)/q^2$. Then one has

$$\Box F_1(x^2, px) = +2L(pq) = -2i(p\partial_x)K(x^2, px). \quad (2.16)$$

Here L means a Fourier image of the corresponding structure function. Since $K = 4\sin(py)\delta'(y^2)\epsilon(py)$, one obtains the equation

$$\Box F_1(y2, py) = -(4i/\pi)(py)\epsilon(py)\sin(py)\delta''(y^2). \quad (2.17)$$

We look for the solution in the form

$$F_1(y^2, py) = A(v)B(u),$$

where $v = y^2$, $u = py$. Since

$$\Box H(v, u) = 4(2F_v + vF_{vv} + uF_{uv}),$$

where we used $p^2 = 0$, we obtain

$$\Box[A(v)B(u)] = 4[2A_v B(u) + vA_{vv}B(u) + uB_u A_v].$$

Note now that

$$v\delta^n(v) = -n\delta^{n-1}(v), \quad (2.18)$$

as easily proven by direct calculation. Then if we take $A(v) = \delta'(v)$, one obtains

$$\Box H = 4[-\delta''(v)B(u) + uB_u(u)\delta''(v)].$$

Comparing this expression with the right-hand side of Eq. (2.17), we obtain

$$(-i/\pi)u\sin(u) = uB_u - B(u). \quad (2.19)$$

This differential equation can be easily solved with the boundary condition $B(0) \to 0$. The solution is

$$B(u) = -(i/\pi)u\int_0^u ds\sin(s)/s. \quad (2.20)$$

Then one obtains

$$F_1 \to -(i/\pi)\epsilon(py)\delta'(y^2)(py)\mathrm{Si}(py). \quad (2.21)$$

Here Si is the integral sinus function

$$\mathrm{S\,i}(u) = \int_0^u \frac{\sin(s)}{s}ds \quad (2.22)$$

(see, e.g., Ref. [32] for the detailed review of its properties). In the limit of large py in which we are interested in this paper, we obtain

$$F_1(y^2, py) \to -(i/2)\epsilon(py)(py)\delta'(y^2). \quad (2.23)$$

Note that the results do not contain any logs as it will follow naively from the corresponding diverging integral (2.14) and are causal.

Exactly in the same way, one can calculate V_2 and V_L. For V_2 one gets

$$\Box V_2(y^2, py) = -[i/(2\pi)]\epsilon(px)\delta(y^2)\sin(py)/(py). \quad (2.24)$$

This equation can be easily solved using the ansatz

$$V_2 = \theta(y^2)\epsilon(py)B(py). \quad (2.25)$$

Repeating the steps that lead to the solution of the previous equation, we obtain

$$uB_u + B = \sin(u)/u. \quad (2.26)$$

This equation has the solution

$$B(u) = \frac{1}{u}\mathrm{Si}(u).$$

We immediately obtain

$$V_2(y^2, py) = -(i)/(2\pi)(1/py)\mathrm{Si}(py)\theta(y^2)\epsilon(py), \quad (2.27)$$

Asymptotically one obtains

$$V_2 \sim (-i/4)/(py). \quad (2.28)$$

This is just the result of Ioffe [24] (obtained practically by the same method). Note that one does not obtain any large logs using such a method as one will obtain making naively Fourier transform (see next section). Finally, using the same approach, one can calculate the function $V_L \sim (pq)/q^4$:

$$\Box^2 V_L = 2(py)\delta''(y^2)\sin(py). \quad (2.29)$$

The general solution is

$$V_L = [-i/(2\pi)]\delta(y^2)\epsilon(py)\cos(py), \quad (2.30)$$

as in Ref. [24].

Summarizing, in this chapter we reviewed the method of differential equations due to Ref. [24] of obtaining the Fourier transform of the scaling functions, and stressed that this method permits one to calculate Fourier transforms without violating causality. We considered the Fourier transforms of F_1 in the parton model under the condition that $F_1 \sim 1/x$ near $x \to 0$, i.e., has Pomeranchuk behavior with $\alpha_P(0) = 1$.

III. COORDINATE SPACE REPRESENTATION OF THE SEA QUARK, GLUON DISTRIBUTION FUNCTIONS OF THE GLUON

A. Fourier transform of the current-current correlators

In the previous section we performed the Fourier transform of the structure functions in the parton model. Let us now go to the leading log QCD. Let us make the actual calculation for the simplest case—Fourier transform into coordinate space of the structure function of the gluons within the gluon. In the case of quark structure function of a quark or a gluon all calculations are practically identical. So there is no need to repeat them. All calculations will be made in the target rest frame because the space-time evolution is most straightforward in this frame. Our calculations will be legitimate in the limit of the fixed space-time interval y^2 but $py \to \infty$. We choose this limit because there exists a rather direct correspondence between the structure functions at small x and Fourier transform. We start from the expression for the gluon structure function which is the solution of the DGLAP equation in the double-logarithmic approximation [33,34]. To derive analytic formulas, we neglect the running of the coupling constant which should be slow because of the smallness of y^2. The DGLAP equation is

$$Q^2 \frac{d}{dQ^2} G(x, Q^2) = [\alpha_s/(2\pi)] \times \int_x^1 (dx'/x') \gamma_{GG}(x/x') G(x', Q^2). \tag{3.1}$$

Here γ_{GG} is the kernel in the QCD evolution equation,

$$\gamma_{GG} = 2N_c/x. \tag{3.2}$$

The solution of this equation is given by

$$G(x, Q^2) = \int dn/(2\pi i)(x_0)^{n-1}/x^n (Q^2/Q_0^2)^{\alpha_s N_c/[\pi(n-1)]}. \tag{3.3}$$

Here the contour integration over n runs along a straight line parallel to the imaginary axis to the right of all singularities of the integral. We use the notation $Q^2 = -q^2$ if $q^2 \leq 0$ and $Q^2 = q^2$ if $q^2 \geq 0$. The Bjorken scaling variable is defined in a usual way:

$$x = -q^2/(2pq). \tag{3.4}$$

The above solution corresponds to simple initial conditions

$$xG(x, Q^2) = \delta(x - x_0).$$

We shall need the Fourier transform of Eq. (3.3):

$$G(py, y^2) = \int d^4q G(x, Q^2) \exp(iqy). \tag{3.5}$$

Let us first determine the integration area. The structure function is symmetric between the u and s channels. So $\mathrm{Im}_s F_1 = \mathrm{Im} F_u$. This means the invariance on the substitution $x \to -x$, $pq \to -pq$. Thus, one can limit the area of integration by the region ($pq \geq 0$). In this area there are four subregions:

(i) $q^2 \geq 0$ and $0 \geq x_B \geq -1$ which corresponds to the e^+e^- fragmentation into hadrons in the field of target. Within the LL approximation this structure function is zero.

(ii) $q^2 \geq 0$ and $x \leq -1$. Amplitude in this kinematics can be related with the inclusive process: $e^+e^- \to N + X$. We will show that this region gives no significant contribution to the kinematics of interest in this paper. In the LL approximation this amplitude is connected to DIS amplitude by the Gribov-Lipatov relation (see below).

(iii) $q^2 \leq 0$, and $1 \geq x \geq 0$ that corresponds to the DIS.

(iv) $q^2 \leq 0$ and $x \geq 1$. In this area the structure function is 0 because of the energy-momentum conservation laws.

In the second region, one has an additional kinematical restriction:

$$x \geq 1, \qquad q^2 \geq 4m^2x^2, \tag{3.6}$$

which just expresses the condition $q_0^2 \geq \vec{q}^2$ in this kinematical area.

In the third region, it is worthwhile to use $Q^2 = -q^2$ instead of q^2 as an invariant variable, and it is easy to see that kinematically $x \leq 1$.

We shall start from the DIS region.

Naively, in order to carry the Fourier transform in Eq. (3.5), one can use the Gribov-Ioffe-Pomeranchuk (GIP) approximation [17,24]. In this approximation, one takes into account that the integrand in the laboratory reference frame is dominated by

$$q_0^2 \geq (q^2)^2/(4m^2) \gg |q^2|. \tag{3.7}$$

Correspondingly, one can expand $\sqrt{q_0^2 - q^2} \sim q_0 - q^2/(2q_0)$. Using this approximation, one obtains the integrand directly as a function of y^2. The arising integrals can be easily calculated. However, they do not satisfy the causality condition: The commutator is nonzero for $y^2 \leq 0$, and this condition must be imposed by hand. It is easy to see, taking as a pattern the calculations from the previous chapter for the parton model and trying to do them explicitly calculating the integrals in the GIP approximation, that the problem is the limitation of the integration area by the condition (3.7). Then even when we obtain the convergent integrals it is not clear how to obtain casuality naturally. Instead, we shall adopt here a different approach. It is possible to prove that the Fourier transform of $(q^2)^n/x^m$ is the analytical function of n, uniquely defined by its values in integer n, where the latter function is understood as a generalized one. Let us start from the integral (2.15) and multiply the integrand by $(q^2)^n$. The integral is obtained by acting with the operator $\Box^n$ on the result of the integration. For the leading term in the asymptotics in py one obtains

$$(q^2)^n \to \Box^n (2/\pi)(-1)^n \int_0^1 dx \cos[x(py)]\delta'(y^2)(py). \tag{3.8}$$

After differentiating one gets

$$q^{2n}F(x) \to \int_0^1 dx 2^{2n}(py)^{n+1}\delta^{n+1}(y^2)x^n \times \cos[x(py) + n\pi/2]F(x). \tag{3.9}$$

There exists, however, the unique generalized function such that it is an analytic function of n and is equal to $\delta^n(u)$ for positive integer n [35]. This function is

$$J(s, u) = u_+^{s-1}/\Gamma(s). \tag{3.10}$$

For this function

$$\lim_{s\to -n} J(s, u) = \delta^n(u), \tag{3.11}$$

and we denote $u_+ \equiv \theta(u)u$ (the standard notation in the mathematical literature [35]). Then we can extend Eq. (3.9) to noninteger n as

$$q^{2s}F(x)/2^{-2s} \to \int_0^1 dx x^{-s}(py)^{-s+1}\cos[x(px) - s\pi/2] \times F(x)(y_+^2)^{-s-2}/\Gamma(-s-1). \tag{3.12}$$

Here $y_+^2 = \theta(y^2)y^2$.

Let us briefly discuss the result from the mathematical point of view. It is easy to check by using the inverse operator of Laplace as we did in the previous section that the asymptotics (3.12) is valid for negative n. Thus, we have the problem of restoring the function that is known for all integer n, analytical in n, and has power like asymptotics. Such a function is uniquely defined [36], as it is well known from the theory of complex variables (and Regge calculus, where the corresponding procedure is called Gribov-Froissart projection). Then Eq. (3.10) fully defines the function. It is straightforward to see that this function coincides with the one obtained in GIP approximation, except for an important difference: We automatically achieve causality. Thus, our approach, analytically continuing the result of the parton model, is the only possible approach to the Fourier transform. Once we know how to deal with the powers of q^2, we must put $s = \alpha_s N_c/[\pi(n-1)]$ and carry the remaining integration over x. We obtain the following using Eq. (3.11):

$$G(py, y^2) = [\pi/(2py)] \int dn/(2\pi i) \int_0^1 (dx/x^2) \cos(-\{\alpha_s N_c/[\pi(n-1)] + 2\}\pi/2 + x(py))\Gamma\{\alpha_s N_c/[\pi(n-1)] + 2\} \times (x)^{\alpha_s N_c/[\pi(n-1)]+2-n}/|[y^2/(2py)]|^{\alpha_s N_c/[\pi(n-1)]+2}(Q_0^2)^{-\alpha_s N_c/[\pi(n-1)]}x_0^{n-1}(2m)^{\alpha_s N_c/[\pi(n-1)]+2}. \tag{3.13}$$

The integration in the above formula can be easily performed [32,37,38]:

$$\int_0^1 \sin(a + mxr)x^{\mu-1}dx = \{\sin(a)[F(imr) + F(-imr)] - i\cos(a)[F(imr) - F(-imr)]\}/(2\mu). \tag{3.14}$$

Here the function F is the confluent hypergeometric function:

$$F(x) = {}_1F_1(\mu, \mu+1, x), F(0) = 1. \tag{3.15}$$

We are actually interested in the limit of large distances (times), $r \to \infty$. In this limit, one uses the asymptotics

$$F(imr) = \Gamma(\mu+1)\exp(i\pi\mu)/(imr)^\mu + \mu\exp(imr)/(imr), \tag{3.16}$$

$$F(-imr) = \Gamma(\mu+1)\exp(-i\pi\mu)/(-imr)^\mu + \mu\exp(-imr)/(-imr), \tag{3.17}$$

or, since

$$F(imr) + F(-imr) = 2[\Gamma(\mu+1)\cos(\pi\mu/2)/(mr)^\mu + \mu\sin(mr)/(mr)], \tag{3.18}$$

and

$$F(imr) - F(-imr) = 2i[\Gamma(\mu+1)\sin(\pi\mu/2)/(mr)^\mu - 2\mu\cos(mr)/(mr)], \tag{3.19}$$

we have

$$\int_0^1 dx x^{\mu-1}\sin(a + mxr) \sim 2[\Gamma(\mu)\sin(a + \pi\mu/2)/(mr)^\mu - \cos(mr + a)]. \tag{3.20}$$

Using the latter integral, we finally obtain the contribution from the deep-inelastic scattering region in the Fourier transform. In our case

$$a + \mu\pi/2 = 2\alpha_s N_c/[\pi(n-1)]\pi/2 - (n+1)\pi/2, \tag{3.21}$$

$$\mu = \alpha_s N_c/[\pi(n-1)] - n + 1,$$
$$a = -\{+\alpha_s N_c/[\pi(n-1)] + 1\}\pi/2. \tag{3.22}$$

Then we have

$$G(py, y^2) = [\pi/(py)]\int [dn/(2\pi i)]\big[\Gamma\{\alpha_s N_c/[\pi(n-1)] + 2\}\cos\big(py - \{\alpha_s N_c/[\pi(n-1)]\}\pi/2\big)/(py) + \Gamma\{\alpha_s N_c/[\pi(n-1)] - n + 1\}\sin(n\pi/2)\big]/\{(py)^{\alpha_s N_c/[\pi(n-1)]-n+1}|[y^2/(2py)]|^{\alpha_s N_c/[\pi(n-1)]+2}\} \times (Q_0^2)^{-\alpha_s N_c/[\pi(n-1)]} x_0^{n-1}. \tag{3.23}$$

We have found the contribution due to the DIS into the integral over n. The contribution due to the fragmentation of e^+e^- in the nucleon color field into the integral over n is zero in the leading log approximation (see above and Ref. [39]).

Let us now consider the contribution due to the annihilation. The current commutator in the annihilation region can be expressed through the correlation function in DIS via the Gribov-Lipatov relation [39]

$$G^a(x, q^2) = xG(1/x, Q^2). \tag{3.24}$$

Here x is the Bjorken variable defined in the same way as above, only for the different kinematic region. It is straightforward to obtain the annihilation contribution to the commutator:

$$G(py, y^2) = [\pi/(2py)]\int dn/(2\pi i)\int_1^\infty \times dx\cos\big(+\{\alpha_s N_c/[\pi(n-1)] + 2\} + (py)x\big) \times \Gamma\{\alpha_s N_c/[\pi(n-1)] + 2\}(x)^{\alpha_s N_c/[\pi(n-1)]-n}/| \times [y^2/2py)]|^{\alpha_s N_c/[\pi(n-1)]+2} \times (Q_0^2)^{-\alpha_s N_c/[\pi(n-1)]} x_0^{n-1}. \tag{3.25}$$

The integral over x can be easily taken using the integral

$$\int_1^\infty \sin(a + mrx)x^{\mu-1}dx = \int_0^\infty \sin(a + mrx)x^{\mu-1}dx - \int_0^1 \sin(a + mrx)x^{\mu-1}dx. \tag{3.26}$$

Then the first integral can be easily taken explicitly, while the second was taken above:

$$\int_1^\infty \sin(a + mrx)x^{\mu-1}dx = \Gamma(\mu)\sin(\mu\pi/2 + a) - \int_0^1 \sin(a + mrx)x^{\mu-1}dx. \tag{3.27}$$

The asymptotic expansion of the latter integral is known, and it is straightforward to see that for large r

$$\int_1^\infty \sin(a + mrx)x^{\mu-1}dx \sim +\cos(mr + a)/mr. \tag{3.28}$$

The reason that the latter integral does not depend on μ is that the asymptotics is dominated by the area $x \sim 1$, where

$$x^{\mu-1} = \exp[(\mu - 1)\log(x)] \sim 1.$$

It is clear that the similar term in Eq. (3.13) also comes from the region $x \sim 1$, and these terms correspond to the contribution of the parton model. The two contributions are very similar with the only difference that we must use in Eq. (3.28)

$$a = -\{\alpha_s N_c/[\pi(n-1)] + 2\}\pi/2. \tag{3.29}$$

Now we can write the expression for the structure funcion that includes both the DIS and annihilation regions:

$$G(py, y^2) = [\pi/(py)]\int dn/(2i\pi)\big[\Gamma\{\alpha_s N_c/[\pi(n-1)] + 2\}\sin(py)\cos\big(\{\alpha_s N_c/[\pi(n-1)]\}\pi/2\big) + \Gamma\{\alpha_s N_c/[\pi(n-1)] + 2\}\Gamma\{\alpha_s N_c/[\pi(n-1)] - n + 1\} \times \sin(n\pi/2)\big]/(py)^{\alpha_s N_c/[\pi(n-1)]-n+1})/|[y^2/(2py)]|^{\alpha_s N_c/[\pi(n-1)]+2}(Q_0^2)^{-\alpha_s N_c/[\pi(n-1)]} x_0^{n-1}. \tag{3.30}$$

We see that the current-current correlator contains two distinct contributions. The first is due to $x \sim 1$. The second is solely due to perturbative gluon effects. This part is dominated by moderately small x.

We can now take an integral over n. Let us start from the $x \sim 1$ contribution:

$$G_1(py, y^2) = -[1/(2py)] \int [dn/(2\pi i)] \cos(\{\alpha_s N_c/[\pi \times(n-1)]\}\pi/2) \sin(py)/(py) \Gamma\{\alpha_s N_c/[\pi \times(n-1)] + 2\}/|[y^2/(2py)]|^{\alpha_s N_c/[\pi(n-1)]+2} \times(Q_0^2)^{-\alpha_s N_c/[\pi(n-1)]} x_0^{n-1} (2m)^{\alpha_s N_c/[\pi(n-1)]+2}. \tag{3.31}$$

The second contribution is due to the moderately small x. It is equal to

$$G_2 = \Gamma\{\alpha_s N_c/[\pi(n-1)] - n + 1\} \Gamma\{\alpha_s N_c/[\pi \times(n-1)] + 2\} \sin[(n)\pi/2]/[(py)^{\alpha_s N_c/[\pi(n-1)]-n+1}| \times[y^2/(2py)]|^{\alpha_s N_c/[\pi(n-1)]+2}](Q_0^2)^{-\alpha_s N_c/[\pi(n-1)]} x_0^{n-1} \times(2m)^{\alpha_s N_c/[\pi(n-1)]+2}. \tag{3.32}$$

These integrals can be taken using the saddle point method. Consider first the contribution where $xG \approx$ const for small x to honor the distinctive property of the soft QCD amplitudes to significantly more slowly increase with energy as compared to the amplitudes of hard processes. This contribution can be studied using the saddle point approximation. Indeed, we have

$$G_1(py, y^2) = \int dn/(2\pi i)(1/y^2) \cos(py) \cos\{\alpha_s N_c/[\pi(n - 1)]\pi/2\}[2py/(y^2 Q_0^2)] \cos\{\alpha_s N_c/[\pi(n - 1)]\pi/2\} \Gamma\{\alpha_s N_c/[\pi(n-1)] + 2\}(x_0)^{n-1}. \tag{3.33}$$

The saddle point is at

$$n - 1 = \sqrt{(\alpha_s N/\pi) \log(2py/y^2 Q_0^2)/\log(x_0)}. \tag{3.34}$$

We obtain

$$G_1 = [\cos(py)/(y^2)^2] \cos \times[\sqrt{(\alpha_s N_c/\pi) \log(x_0)/\log(2py/y^2 Q_0^2)} \times\Gamma[2 + \sqrt{(\alpha_s N_c/\pi) \log(x_0)/\log(2py/y^2 Q_0^2)}] \times \exp\sqrt{\alpha_s N_c/\pi \log(py) \log(x_0) \log(y^2 Q_0^2)} \times(\alpha_s N_c/\pi)^{1/4} \log(2py/y^2 Q_0^2)^{1/4}/\log(x_0)^{3/4}. \tag{3.35}$$

The last line corresponds to the preexponential.

Consider now the second integral (3.32). The integral can be rewritten in dimensionless variables as

$$G_2 = \Gamma\{\alpha_s N_c/[\pi(n-1)] - n + 1\} \Gamma\{\alpha_s N_c/[\pi \times(n-1)] + 2\} \sin[(n+1)\pi/2] \times(py)^{n+1}/(Q_0^2 y^2)^{\alpha_s N_c/[\pi(n-1)]+2} x_0^{n-1} \times(2)^{2\{\alpha_s N_c/[\pi(n-1)]+2\}}. \tag{3.36}$$

We immediately see that the saddle point is determined from the equation

$$\log(pyx_0) = -\alpha_s N_c/[\pi(n-1)^2] \log(Q_0^2 y^2). \tag{3.37}$$

We obtain

$$n - 1 = \sqrt{(\alpha_s N_c/\pi)[\log(1/Q_0^2 y^2)]/\log(pyx_0)}. \tag{3.38}$$

Then substituting the latter expression into the integral over n, one immediately obtains the asymptotics

$$G_2 = \Gamma\left\{\sqrt{\frac{\alpha_s N_c}{\pi}}[\sqrt{\log(pyx_0)/\log(y^2 Q_0^2)} - \sqrt{\log(pyx_0)/\log(y^2 Q_0^2)}]\right\} \Gamma\left[\sqrt{\left(\frac{\alpha_s N_c}{\pi}\right) \frac{\log(py)}{\log y^2 Q_0^2}} + 2\right] \times \sin\left[\sqrt{\frac{\alpha_s N_c}{\pi} \frac{\log(Q_0^2 y^2)}{[\log(pyx_0)/2]}}\right] \left(\frac{\alpha_s N_c}{\pi}\right)^{1/4} \frac{\log(Q_0^2 y^2)^{1/4}}{\log(py)^{3/4}} \left(\frac{1}{y^2}\right)^2 \exp\left[2\sqrt{\frac{\alpha_s N_c}{\pi} \log(py) \log(Q_0^2 y^2)}\right]. \tag{3.39}$$

Let us now check the applicability of the saddle point method. It is easy to see that the condition is

$$\alpha_s \log(py) \log(y^2 Q_0^2) \gg 1. \tag{3.40}$$

Thus Eqs. (3.37), (3.38), and (3.39) are not valid in the limit $\alpha_s \to 0$ that corresponds to parton model.

We have found the asymptotics of the current-current correlator in the double-logarithmic limit of QCD. In this limit the saddle point method is applicable and the correlator increases with distances. The applicability condition of this method is evidently the existence of two large logarithms: the parameter

$$(\alpha_s N_c/\pi) \log(Q_0^2 y^2) \log(py) \gg 1.$$

Note that, generally speaking, it is beyond of the accuracy of the method to keep single logs in the arguments of the exponents in the above expressions, and the legitimate answer for asymptotics:

$$G^{\rm DGLAP}(py, y^2) = \theta(y^2)\frac{py}{(y^2)^2}(\alpha_s N_c/\pi)^{1/4}\frac{\log(Q_0^2y^2)^{1/4}}{\log(py)^{3/4}} \times \exp[2\sqrt{(\alpha_s N_c/\pi)\log(py)\log(Q_0^2y^2)}]. \quad (3.41)$$

Here we put all terms with a single log in the arguments of exponents to 1. Note that the leading asymptotics is given by the integral (3.39).

Thus, we obtained asymptotics for perturbative QCD. Note that delta-function singularities on the light cone for F_2 and F_1 were translated into $1/(y_+^2)^2$ behavior in the perturbative QCD.

B. Coordinate space physics relevant for cross sections

In the previous section we discussed the space-time asymptotics of the current-current commutator in the LL approximation of pQCD. This commutator has a well-defined probabilistic interpretation in the infinite momentum frame . However, in the target rest frame significantly more direct interpretation has a cross section for the dipole scattering of a target and related shadowing effects . The cross section is equal to the correlator divided by an invariant energy, i.e., by s, which means the commutator must be multiplied by $2x/Q^2$. Thus, in the notations of the introduction we have to calculate

$$D(py, y^2) = \int d^4q \exp(iqy)G(x, Q^2)/(s) \equiv \int d^4q \exp(iqy)(x/Q^2)G(x, Q^2). \quad (3.42)$$

Here $s = Q^2/x$ is the invariant energy squared.

This quantity is sometimes considered a potential for the interaction between color-neutral dipoles. For this quantity we may repeat the analysis of the previous section. It is straightforward to see that, for both parton model and large Bjorken x, large Q^2 regime of DGLAP equations, the effect is the loss of one power of y^2 in the denominator and the loss of one power of (py) in the nominator, i.e., the correlator increases logarithmically in the parton model (see the previous subsection). For the perturbative QCD asymptotics we obtain

$$D(py, y^2) = \theta(y^2)\frac{1}{y^2}(\alpha_s N_c/\pi)^{1/4}\frac{\log(Q_0^2y^2)^{1/4}}{\log(py)^{3/4}} \times \exp[2\sqrt{\alpha_s N_c/\pi\log(py)\log(Q_0^2y^2)}]. \quad (3.43)$$

The function D thus increases in the perturbative QCD.

C. More about parton model

The modern definition of the parton model ordinary refers to nonperturbative distributions without taking into account perturbative QCD evolution, i.e., for initial conditions for evolution equations, such as DGLAP. The most popular form of the initial conditions (that also gives a best agreement with the experimental data) is

$$F_2(x) = C/x^\alpha, \quad (3.44)$$

where $\alpha > 0$. This case was not considered in the early seventies since at that time it was assumed that $\alpha \leq 0$ for the physical cases. The Fourier transform of this function, except the special case of the integer α, is clearly given by the analytic continuation of the asymptotics obtained in the previous section. If we continue analytically the equations from the last section and use the results of Sec. II, we immediately obtain

$$F_2 \to \int_0^1 dx \cos[x(py)]\delta'(y^2)(py)x^{(1-\alpha)-1} \sim 2\Gamma(1-\alpha)\sin(\pi\alpha)(py)^\alpha. \quad (3.45)$$

Note that for $\alpha = 0$ this term becomes zero and the asymptotics will be given by the oscillating term $\sim\sin(py)/(py)$ (times the same type of light cone singularity).

For the structure function F_1, the behavior is $\sim x_B^{-(\alpha+1)}$, and one continues analytically Eq. (3.45) obtaining the increase of the commutator as

$$F_1 \to 2\Gamma(-\alpha)\cos(\pi\alpha/2)(py)^{\alpha+1}\delta'(y^2). \quad (3.46)$$

This expression has a pole singularity for $\alpha = 0$, when we return to the function already considered in the framework of the parton model.

IV. PHENOMENOLOGICAL DISTRIBUTIONS

A. Small structure functions

In the previous section, we analyzed the space-time structure of the correlators due to a parton model and within the area of applicability of leading order DGLAP equations. Including NLO will not change our conclusions. However, at extremely small x (kinematics of LHC?) where the energy conservation law does not preclude a large number of gluon radiations in the multi-Regge kinematics, a small x limit of the DGLAP equation is, literally speaking, not available and instead one needs to use for G either by resummation approaches [5,6], or phenomenological ones or phenomenological one from HERA [16]:

$$G_p(x, Q^2) \sim (1/x)^{\alpha+1}(Q^2)^\beta, \quad (4.1)$$

where

$$\alpha \sim 0.25, \qquad \beta \sim 0.25. \quad (4.2)$$

The theoretical distributions expected within the resummation approaches of [5,6] differ from a phenomenological one by logarithmic terms:

$$G_T(x, Q^2) \sim (1/x)^{1.25}(Q^2)^{0.25}/\sqrt{\log(x)^3}. \quad (4.3)$$

It is straightforward to carry Fourier transform of this distribution. Once again we carry the analytical continuation of the parton model formula. The singularities $\delta(y^2)$ and $\delta'(y^2)$ are smoothened into $1/(y_+^2)$ and $1/(y_+^2)^2$, respectively. We obtain

$$G(py, y^2) = \pi\Gamma(\beta+2)(py)^{\beta+1}/(y^2)^{2+\beta}\int_0^1 dx x^{\beta-\alpha-1} \times \sin[x(py) - \beta\pi/2]/(py),$$
$$D(py, y^2) = \pi\Gamma(\beta+1)(py)^{\beta}/(y^2)^{1+\beta}\int_0^1 dx x^{\beta-\alpha-1} \times \cos[x(py) - \beta\pi/2]/(py). \tag{4.4}$$

The relevant asymptotics is obtained from the asymptotics of the confluent hypergeometric function as in the previous section. For $\beta - \alpha \leq 0$, the integrals must be considered in the analytic continuation sense; for $\beta = \alpha$ one obtains the logarithmic divergence that must be dealt with as in the parton model in the previous subsection.

Altogether one obtains, if $\beta \neq \alpha$,

$$G(py, y^2) = \theta(y^2)\pi\Gamma(\beta+2)\Gamma(\beta-\alpha)\sin(\pi\alpha/2) \times (py)^{\alpha+1}/(y^2)^{\beta+2}, \tag{4.5}$$

and

$$D(py, y^2) = \theta(y^2)\pi\Gamma(\beta+2)\Gamma(\beta-\alpha)(1/2)\cos[\pi(\alpha)/2] \times (py)^{\alpha}/(y^2)^{\beta+1}. \tag{4.6}$$

If $\beta = \alpha \neq 0$, one obtains logarithmic asymptotics:

$$G(py, y^2) = \pi\Gamma(\beta+2)\sin(\beta\pi/2)(py)^{1+\beta}\log \times (py)/(y^2)^{\beta+2}, \tag{4.7}$$

and

$$D(py, y^2) = \pi\Gamma(\beta+2)\cos(\beta\pi/2)(py)^{\beta}\log(py)/(y^2)^{\beta+1}. \tag{4.8}$$

For the HERA phenomenological case, one has $\beta \sim \alpha$; the same is true for recent phenomenological asymptotics due to Refs. [5,6]. Thus, for them we obtain the logarithmic times power increase of D and G functions on the light cone.

It is interesting to note that recently Ciafaloni *et al.* [6] suggested a resummation model where the structure function may have a dip in the energy dependence, postponing an increase to smaller x than in the kinematics of HERA. This will postpone an increase of D to larger x. An additional factor in the asymptotics $\sim 1/\log^{3/2}(1/x)$ claimed in Ref. [6] may change the behavior of D, making it slowly increasing with distance as $(py)^{0.2}/\sqrt{\log(py)}$ and having a dip for some interval of py.

In the case of the leading order BFKL approximation, we have $\beta = 1/2$, $\alpha \sim 0.8$, and we have asymptotics

$$G(px, x^2) \sim (py)^{1.8}/(y^2)^{5/2},$$
$$D(py, y^2) \sim (py)^{0.8}/(y^2)^{3/2}. \tag{4.9}$$

B. Black limit

It is worth analyzing the coordinate space dependence for the unitarity limit for structure functions for the small x behavior of structure functions [12] unitarity bound = black body approximation

$$G(x, Q^2) \propto (1/x)(Q^2/Q_0^2)\log^3(x_0/x) + \text{peripheral} = \text{DGLAP terms}, \tag{4.10}$$

and

$$\sigma(s) \sim \log^3(x/x_0), \tag{4.11}$$

where x_0 is a weak function of Q^2. Doing Fourier transform, we obtain

$$G(py, y^2) \propto (py)\theta(y^2)\log^3(py)/(y^2)^3 + \text{peripheral terms}, \tag{4.12}$$

$$D(py, y^2) \propto \theta(y^2)\log^3(py)/(y^2)^2 + \text{peripheral terms}. \tag{4.13}$$

Thus, in the black limit both G and D at given impact parameter contain a trivial increase with distance: Factor r is because the same dipole is probed at different space-time points; one $\ln(1/x)$ is due to ultraviolet divergence of renormalization of e.m. charge. $\ln^2(x_0/x)$ is due to an increase with energy of impact parameters in the scattering process. Excluding the above factors, we find that an increase of commutator with distance is stopped within this limit. The structure function continues to increase with energy for the configuration in the photon wave function with $4k_t^2 \geq Q^2$ as the consequence of renormalizability of QCD, and because of an increase with energy of essential impact parameters.

C. Hard diffractive processes

Another interesting question is a question of space-time evolution of hard diffractive processes. These are the processes $\gamma^* + p \rightarrow X + p$, where X is vector meson, dijet, etc. For this process the relevant amplitude behaves as

$$A \sim (s/Q^2)1/Q^n, \tag{4.14}$$

where $n = 1/2$ or 1. Repeating calculations as above, we obtain that coordinate space amplitudes increase with distances as amplitudes of LT processes. However, dependence on y^2 will be weaker by the factor $(y^2)^n$.

V. CONCLUSION

We have studied the dependence of the current-current correlators (gluon distributions) in coordinate space on py at fixed y^2 close to light cone. Quite surprisingly, we found that all theoretical approaches (DGLAP, BFKL, recent resummation models of small x behavior [5,6], unitarity bound [12], and phenomenological description of data) all lead to the increasing with the distance current-current correlators,

$$G(py, y^2) \sim py/(y^2)^2(py)^\alpha/(y^2)^\beta. \tag{5.1}$$

Here indexes α and β are model dependent at present but positive. The DGLAP equations in double-logarithmic approximation lead to an increase:

$$\sim \theta(y^2)(py)/(y^2)^2(\alpha_s N_c/\pi)^{1/4}\frac{\log(Q_0^2y^2)^{1/4}}{\log(py)^{3/4}} \times \exp[2\sqrt{\alpha_s N_c/\pi \log(py)\log(Q_0^2y^2)}]. \tag{5.2}$$

Thus, increasing the current-current correlators with a distance near the light cone is the general feature of high-energy scattering processes. Moreover, we see that, apart from the kinematical multiplier py, this feature appears due to the interaction with the gluons and is absent in the parton model, where only fixed (except the kinematical multiplier py) amplitude oscillations occur.

The increase of the commutator with the distance at the light cone is a relativistic effect, present in the Minkowsky space only, and it is absent in the Euclidean space.

This feature is closely connected with the known increase of the correlation length at high energies.

Such an increase seems to be a characteristic feature of a Pomeron, i.e., of a contribution into amplitudes of a t exchange with vacuum quantum numbers. Fourier transform of amplitude with nonvacuum quantum numbers in a t channel (contribution into cross section) decreases with distance as $(1/r)^{\alpha(0)-1}$. Here α is the intercept of trajectory of dominant Regge pole contribution.

Let us stress that we consider the asymptotics near the light cone, i.e., $r \sim t$. On the other hand, for equal time commutator, i.e., small time t but $r \to \infty$, dominant contribution into Fourier transform arises due to the region of large $q_0 \sim 1/t$ and small space momenta, i.e., the region around $x = 1$ but $q^2 \to \infty$. In this region Fourier transform oscillates.

We can also evaluate a product of four currents in the same way and obtain similar results as above $j(y)j(z)j(0)j(0)$ in the kinematics

$$y^2 \to 0, \qquad z^2 \to 0, \qquad r_y, r_z \to \infty.$$

This correlator appears in heavy ion collisions as a correlation function between two hard processes which occur at different space-time points.

Finally, we found that the function $D(y^2, py)$ that can be interpreted as a dipole-target potential increases with py, in all above cases. This increase disappears for the Fourier transform of the unitarity bound formula after excluding effects beyond long-range dynamics. Thus, we have found another instability of the description of the physical state in terms of quarks and gluons. The physical consequences of this fact will be discussed in more detail in future publications.

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