Numerical precision radiative corrections to the Dalitz plot of baryon semileptonic decays including the spin-momentum correlation of the decaying and emitted baryons

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We calculate the radiative corrections to the angular correlation between the polarization of the decaying and the direction of the emitted spin one-half baryons in the semileptonic decay mode to order $(\alpha/\pi)(q/M_1)$, where q is the momentum transfer and M_1 is the mass of the decaying baryon. The final results are presented, first, with the triple integration of the bremsstrahlung photon ready to be performed numerically and, second, in an analytical form. A third presentation of our results in the form of numerical arrays of coefficients to be multiplied by the quadratic products of form factors is discussed. This latter may be the most practical one to use in Monte Carlo simulations. A series of cross-checks is performed. This paper is organized to make it accessible and reliable in the analysis of the Dalitz plot of precision experiments involving heavy quarks and is not compromised to fixing the form factors at predetermined values. It is assumed that the real photons are kinematically discriminated. Otherwise, our results have a general model-independent applicability.

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I. INTRODUCTION

The radiative corrections (RC) to spin one-half baryon semileptonic decays face three levels of complications. Despite the important progress achieved in the understanding of the fundamental interactions with the standard model [1], no first principle calculation of these corrections is yet possible. RC become then committed to model dependence and, what is worse, experimental analyses which use these calculations become model dependent, too. The second level comes from the fact that RC depend on the process characteristics, such as charge assignment of the baryons, type of the emitted charged lepton, size of the momentum transfer q involved, and whether real photons can be experimentally discriminated or not. RC also depend on the observable which is to be measured. All this requires RC to be recalculated every time the process characteristics and the observables are changed. At the third level one finds complications of a practical nature. It turns out that the final results of RC calculations are rather very inefficient to use or are long and tedious to the point that their use becomes unreliable. Fortunately, all the above complications can be solved rather satisfactorily.

Although the model dependence of the virtual RC cannot be eliminated, an extension to baryon semileptonic decays of an analysis of Sirlin [2] of these corrections in neutron beta decay shows that to orders $(\alpha/\pi)(q/M_1)^0$ and $(\alpha/\pi)(q/M_1)$, where M_1 is the mass of the decaying baryon, the corresponding model dependence amounts to several constants. These constants can all be absorbed into the already present form factors of the weak decay vertex. In addition, the theorem of Low [3] in its presentation by Chew [4] can be used to show that to these 2 orders of approximation the bremsstrahlung RC depend only on the nonradiative form factors and on the static electromagnetic multipoles of the particles involved. Accordingly, no model dependence is introduced in this other part of the RC. Within these orders of approximation it is then possible to obtain final expressions for the RC that can be used in model-independent experimental analyses. The price is that it will be the effective form factors (which may be indicated by putting a prime on them) that can be experimentally determined. The separation of the original form factors from the model dependence of RC is then a theoretical problem only. It is in this sense that the first level of complications is put under control.

To deal with the second level one must make an effort to calculate RC in a way as general as possible and to be able to obtain results which can be used directly to obtain the final results of other possible baryon semileptonic decays. In a recent publication [5] we showed that of the six allowed charge assignments to the baryons when heavy quarks are involved, namely, $A^- \to B^0 l^- \overline{\nu}_l$, $A^0 \to B^+ l^- \overline{\nu}$, $A^+ \to B^0 l^+ \nu_l$, $A^0 \to B^- l^+ \nu_l$, $A^{++} \to B^+ l^+ \nu_l$, and $A^+ \to B^{++} l^- \overline{\nu}_l$, it is necessary only to calculate the RC to the first two, to which we shall refer to as charged decaying baryon (CDB) and neutral decaying baryon (NDB), respectively. The RC to the other four cases are then obtained from these two. We also showed that this property is valid up to order $(\alpha/\pi)(q/M_1)$, when $l = e^\pm$, $\mu^\pm$, and $\tau^\pm$ and for any observable of baryon semileptonic decays. The problem of calculating RC is reduced considerably this way, although it will still be necessary to recalculate for different observables and whether real photons are discriminated or not in the first two cases.

The third level of complications has been dealt with by computing analytically the triple integrals over the real photon variables. A numerical calculation of these integrals makes the application of RC to a Monte Carlo simulation practically impossible, because every time the values of the kinematical variables are varied those integrals must be recalculated. The analytical form of RC solves this problem. However, the results are very long and tedious and the use of this latter form may become unreliable. To control this it is very important that the analytical result be well organized and that it be cross-checked with the triple numerical integration form. A successful cross-check allows the user to gain confidence on the analytical result and on its feeding into a Monte Carlo simulation. It may still be convenient to find a third presentation of RC, which would make their use more practical.

From the above discussion it is clear that the calculation of RC to baryon semileptonic decays must be done following a program (see Ref. [5], and references therein). In previous publications we obtained the RC to the Dalitz plot of unpolarized decaying baryons up to order $(\alpha/\pi)(q/M_1)$ [6,7]. In Ref. [8] we calculated to order $(\alpha/\pi)(q/M_1)^0$ the RC to the Dalitz plot with the angular correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ when the initial baryon is polarized along $\hat{\mathbf{s}}_1$ and the final baryon is emitted along $\hat{\mathbf{p}}_2$.

In the present paper we want to attain two goals. The first one is to continue with our program and to calculate to order $(\alpha/\pi)(q/M_1)$ the RC to the differential angular correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$. The second one is to present the RC in the form of numerical arrays which should be applied to the quadratic products of form factors that appear in the RC, up to order $(\alpha/\pi)(q/M_1)$. We shall cover both CDB and NDB cases.

The ordering of the paper is as follows. In Sec. II we present the results to order $(\alpha/\pi)(q/M_1)$ for the virtual RC. In Sec. III we give the results for the bremsstrahlung RC in the triple numerical integration form and combine them with the virtual RC results to obtain our first main result. In addition, we give the corresponding fully analytical results. In Sec. IV we perform several cross-checks and compare with other published results. In Sec. V we proceed towards our second goal. The last Sec. VI is dedicated to a summary and to concluding remarks.

In order not to obscure the physics we have moved the very many algebraic expressions that appear in the analytical results to Appendices A, B, and C. In this paper we exhibit only new expressions. However, previously published expressions are required in these results. We do not reproduce them here. Instead, we give all the necessary references so that the reader can identify them correctly. The text and these appendices are organized so as not to obscure the physics and to make accessible the use of our results. Performing the analytical integrals is long and tedious. In order to help the reader interested in checking our results we have introduced Appendix D, where the previous and new integrals can be identified. In Secs. IV and V we provide several numerical tables with the purposes of illustration and, more importantly, of helping the user to check his numerical results with ours.

II. VIRTUAL RADIATIVE CORRECTIONS

Our first purpose in this section is to review our notation and conventions. Next we shall discuss the virtual RC to the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ angular correlation over the Dalitz plot to order $(\alpha/\pi)(q/M_1)$. The uncorrected transition amplitude M_0 for the baryon semileptonic decays

$$A \to B l \overline{\nu}_l \tag{1}$$

is

$$\mathsf{M}_0 = \frac{G_V}{\sqrt{2}}[\overline{u}_B(p_2) W_\mu(p_1, p_2) u_A(p_1)][\overline{u}_l(l) O_\mu v_\nu(p_\nu)]. \tag{2}$$

G_V is the Fermi decay constant multiplied by the appropriate Cabibbo-Kobayashi-Maskawa factor [1]. A and B are spin one-half baryons, l is the charged lepton, and ν_l is the accompanying antineutrino or neutrino as the case may be. u_A, u_B, u_l or v_l, and v_ν or u_ν are their corresponding spinors. The weak interaction vertex is

$$\begin{aligned} W_\mu(p_1, p_2) = {} & f_1(q^2)\gamma_\mu + f_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_1} + f_3(q^2)\frac{q_\mu}{M_1} \\ & + \left[g_1(q^2)\gamma_\mu + g_2(q^2)\sigma_{\mu\nu}\frac{q_\nu}{M_1} \right. \\ & \left. + g_3(q^2)\frac{q_\mu}{M_1}\right]\gamma_5, \end{aligned} \tag{3}$$

where $O_\mu = \gamma_\mu(1 + \gamma_5)$, $\sigma_{\mu\nu} = (1/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, and γ_μ and γ_5 are Dirac matrices. $q \equiv p_1 - p_2$ is the four-momentum transfer, and $f_i(q^2)$ and $g_i(q^2)$ are the vector and axial-vector form factors, respectively. Each form factor is assumed to be real in this work. The four-momenta and masses of the particles involved in (1) are $p_1 = (E_1, \mathbf{p}_1)$ and M_1, $p_2 = (E_2, \mathbf{p}_2)$ and M_2, $l = (E, \mathbf{l})$ and m, and $p_\nu = (E_\nu^0, \mathbf{p}_\nu)$ and m_ν, respectively. Our calculations will be specialized to the center-of-mass frame of A. In this frame, p_1, p_2, l, and p_ν will also represent the magnitudes of the corresponding three-momenta; no confusion will arise from this. The directions of these momenta will be indicated by a caret, e.g., $\hat{\mathbf{p}}_2$.

Our approach to virtual RC follows the procedure of Ref. [2]. It has been discussed extensively in our previous works (see Ref. [6]), so only a few salient facts will be repeated here. The virtual RC can be separated into a model-independent part M_v and into a model-dependent one which amounts to six constants. These latter can be absorbed into the corresponding form factors of (3); this is indicated by a prime on M_0. The RC in M_v are finite in the ultraviolet, contain the infrared cutoff, and are gauge invariant. We shall limit ourselves here to exhibit explicitly only the new contributions of order $(\alpha/\pi)(q/M_1)$ to the Dalitz plot with the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ correlation. However, previous results are needed in the complete result. We shall give ample referencing to them.

The transition amplitude with virtual RC is

$$\mathsf{M}_V = \mathsf{M}_0' + \mathsf{M}_v. \tag{4}$$

The calculation of all the integrals over the virtual photon four-momentum that appear in M_V have been performed already to order $(\alpha/\pi)(q/M_1)$ in Ref. [6] for the CDB case and in Ref. [7] for the NDB case. The corresponding results are compactly expressed as

$$\mathsf{M}_{vi} = \frac{\alpha}{2\pi}[\mathsf{M}_0\Phi_i + \mathsf{M}_{\not{p}i}\Phi_i'], \tag{5}$$

where $i = C, N$ separates the CDB and NDB cases, respectively. The matrix element $\mathsf{M}_{\not{p}C}$ and the explicit forms of Φ_C and Φ_C' are found in Eqs. (8), (6), and (7) of Ref. [6], respectively. The corresponding ones of $\mathsf{M}_{\not{p}N}$, Φ_N, and Φ_N' are found in Eqs. (6)– (8) of Ref. [7], once the identifications $\Phi_N = 2\mathrm{Re}\phi$ and $\Phi_N' = 2\mathrm{Re}m\phi'$ are made.

The Dalitz plot with virtual radiative corrections is now obtained by leaving E and E_2 as the relevant variables in the differential decay rate for process (1) and specializing the result to exhibit explicitly the angular correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$. After making the replacement $u_A(p_1) \rightarrow \Sigma(s_1)u_A(p_1)$ in M_V [where the spin projector $\Sigma(s_1)$ is given by $\Sigma(s_1) = (1 - \gamma_5 \not{s}_1)/2$], squaring the resulting amplitude, and rearranging terms we obtain for the differential decay rate

$$d\Gamma_{iV} = d\Omega\bigg\{A_0' + \frac{\alpha}{\pi}(B_1'\Phi_i + B_{i1}''\Phi_i') - \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2\bigg[A_0'' + \frac{\alpha}{\pi}(B_2'\Phi_i + B_{i2}''\Phi_i')\bigg]\bigg\}. \tag{6}$$

In Eq. (6) the first two terms within curly brackets correspond to the unpolarized Dalitz plot. For $i = C$ they can be found in Ref. [6] where A_0', B_1', and B_{C1}'', correspond to Eqs. (10)–(12), respectively, of this reference. For $i = N$ the unpolarized Dalitz plot can be found in Ref. [7], where B_{N1}'' corresponds to Eq. (15). The spin-dependent part of Eq. (6) was obtained to order $(\alpha/\pi)(q/M_1)^0$ in Ref. [8]. There in Eq. (19) one can find the full expression for A_0''. To the next order of approximation, however, there appear the new contributions, namely,

$$B_2' = Ep_2\tilde{Q}_6 + Ely_0\tilde{Q}_7, \tag{7}$$

$$B_{C2}'' = Ep_2Q_8 + Ely_0Q_9, \tag{8}$$

and

$$B_{N2}'' = M_1p_2Q_{N8} + M_1ly_0Q_{N9}. \tag{9}$$

The phase space factor of Eq. (6) is $d\Omega = (G_V^2/2)\times [dE_2dEd\Omega_2d\varphi_l/(2\pi)^5]2M_1$, the cosine y_0 of the angle between the directions of the emitted baryon and the charged lepton is $y_0 = [(E_\nu^0)^2 - l^2 - p_2^2]/(2p_2l)$, and the neutrino energy is, by energy conservation, $E_\nu^0 = M_1 - E_2 - E$.

The Q_i in Eqs. (7)–(9) are functions of quadratic products of the form factors. They are new and are listed in Appendix A.

III. BREMSSTRAHLUNG RADIATIVE CORRECTIONS AND FINAL RESULTS

The radiative process that accompanies (1) is

$$A \rightarrow Bl\overline{\nu}_l\gamma, \tag{10}$$

where the real photon γ carries four-momentum $k = (\omega, \mathbf{k})$ and the neutrino energy is now $E_\nu = E_\nu^0 - \omega$.

The Dalitz plot for this four-body decay covers the three-body region of (1) and extends over it by a region where both E_ν and ω are always nonzero simultaneously. We shall refer to this extension as the four-body region. A detailed discussion of these two regions as well as explicit expressions of their boundaries in the (E, E_2) plane is given in Ref. [8]. Even if experiments have no provision to detect the real photons in (10), a precise measurement of E and E_2 still allows one to discriminate against photons belonging to the four-body region. We shall assume in this paper that this is the case and shall restrict our calculations to the three-body region of (10).

In order to establish our notation and conventions and to make the necessary connections with our previous work, we must briefly review the derivation of the bremsstrahlung differential decay rate. According to the Low theorem [3], the amplitude for process (10) with contributions of orders $1/k$ and k^0 depends only on the form factors of the nonradiative amplitude (2) and on the static electromagnetic multipoles of the particles involved. The model dependence included by the real photon appears in new form factors which vanish at least linearly with k.

These latter contribute to orders $(\alpha/\pi)(q/M_1)^2$ and higher to the differential decay rate. Thus, within the approximations of this paper this part of the RC is model independent. The transition amplitude consists of the sum of three terms, namely,

$$\mathsf{M}_{iB} = \mathsf{M}_{iB_1} + \mathsf{M}_{iB_2} + \mathsf{M}_{iB_3}. \tag{11}$$

As in Sec. II, the subindex $i = C, N$ is used to distinguish CDB and NDB cases, respectively. The detailed expressions of M_{CB_1}, M_{CB_2}, and M_{CB_3} are found in Eqs. (18)–(20), respectively, of Ref. [6] and of M_{NB_1}, M_{NB_2}, and M_{NB_3} are found in Eqs. (21)–(23) of Ref. [7], respectively. Using the $\Sigma(s_1)$ projector in Eq. (11), squaring the matrix element, performing the trace calculations, inserting the appropriate phase space factor, and indicating the integrations over the photon variables, the differential decay rate can be compactly given as

$$d\Gamma_{iB} = d\Gamma'_{iB} - d\Gamma^{(s)}_{iB}. \tag{12}$$

The analytical result to order $(\alpha/\pi)(q/M_1)$ of the unpolarized decay rate $d\Gamma'_{iB}$ was calculated in Ref. [6] for the CDB case and in Ref. [7] for the NDB case. They can be found in Eqs. (48) and (54) of such references, respectively. The polarized decay rate $d\Gamma^{(s)}_{iB}$ was calculated analytically to order $(\alpha/\pi)(q/M_1)^0$ in Ref. [8]. The final results are given in Eq. (101) of this reference, for both CDB and NDB cases.

The calculation to order $(\alpha/\pi)(q/M_1)$ of $d\Gamma^{(s)}_{iB}$ is new. Let us now proceed with it. This decay rate consists of the sum of three terms

$$d\Gamma^{(s)}_{iB} = d\Gamma^{(s)}_{iBI} + d\Gamma^{(s)}_{iBII} + d\Gamma^{(s)}_{iBIII}. \tag{13}$$

$d\Gamma^{(s)}_{iBI}$ comes from the product $\mathsf{M}^{(s)}_{iB1}\overline{\mathsf{M}}_{iB1}$ and it contains the infrared divergence and the finite terms that accompany it. To extract them we follow the procedure used in Ref. [8], which extended the formalism introduced in Ref. [9] for K_{l3} decays. The second and third summands in Eq. (13) come from the product $[\mathsf{M}^{(s)}_{iB1} + \mathsf{M}^{(s)}_{iB2}]\overline{\mathsf{M}}_{iBj}$ for $j = 2, 3$ respectively. They are infrared convergent and are computed with standard techniques. The product $\mathsf{M}^{(s)}_{iB3}\overline{\mathsf{M}}_{iB3}$ is left out because it contributes to orders $(\alpha/\pi)(q/M_1)^2$ and higher. The upper index s indicates where the $\gamma_5 \not{s}_1$ of $\Sigma(s_1)$ is contained in these amplitudes.

An important remark is in order here. It turns out that trying to compute the terms of order $(\alpha/\pi)(q/M_1)$ only and then adding them to the results of Ref. [8] is long and more cumbersome than doing from the start the full calculation containing both $(\alpha/\pi)(q/M_1)^0$ and $(\alpha/\pi)\times(q/M_1)$ contributions. Accordingly, our new expressions will contain the previous and the new contributions. It is then easy to verify that by eliminating the $(\alpha/\pi)(q/M_1)$ terms in the new expressions one obtains the ones of Ref. [8].

The procedure to calculate the CDB and NDB cases differs substantially. We shall deal with them successively in the next two subsections

A. Charged decaying baryon case

The polarized radiative differential decay rate can be cast into the form

$$d\Gamma^{(s)}_{CB} = \frac{\alpha}{\pi} d\Omega \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 (B'_2 I_{C0} + C^{(s)}_A). \tag{14}$$

I_{C0} contains the infrared divergence and the finite terms that accompany it. It was calculated already; its explicit form is found in Eq. (52) of Ref. [8]. B'_2 contains new (q/M_1) contributions. It coincides with Eq. (7) of the virtual RC. $C^{(s)}_A$ consists of the sum of three terms, namely,

$$C^{(s)}_A = \sum_{R=I}^{III} C_R, \tag{15}$$

where

$$C_R = \frac{p_2 l}{2\pi} \int_{-1}^{y_0} dy \int_{-1}^{1} dx \int_0^{2\pi} d\varphi_k \frac{|\mathsf{M}_R|^2}{D}. \tag{16}$$

The integrations over the photon three-momentum are to be performed through the variables $y = \hat{\mathbf{p}}_2 \cdot \hat{\mathbf{l}}$, $x = \hat{\mathbf{l}} \cdot \hat{\mathbf{k}}$, and the azimuthal angle φ_k of $\mathbf{k}$. The traces of the square of the matrix elements give

$$|\mathsf{M}_I|^2 = \frac{\beta^2(1-x^2)}{(1-\beta x)^2}\frac{E}{2}\left[-\frac{D}{p_2}\tilde{Q}_7 + \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2\tilde{Q}_9 + \frac{p_2(E+lx-D)}{M_1 E}Q_{10} + \frac{(1-\beta x)(p_2+2ly)}{M_1}Q_{11} + \frac{2ly(E^0_\nu + lx) + Dp_2}{M_1 E}Q_{12} + \frac{ly}{M_1}Q_{13} - \frac{Dp_2}{M_1 E}Q_{14}\right], \tag{17}$$

$$|\mathsf{M}_{II}|^2 = \frac{1}{1-\beta x}\Big\{\frac{p_2}{2}\tilde{Q}_6 + \frac{ly}{2}\tilde{Q}_7 + \frac{p_2}{2}R_1 Q_8 + \Big[\frac{\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2}{2}[(E_\nu-\omega)R_2+\beta\omega x] + \frac{ly}{2}R_1\Big]Q_9 + \frac{p_2}{2M_1}\Big[-(\hat{\mathbf{k}}\cdot\mathbf{p}_2+lx+2\omega)R_2 + \frac{\omega}{E}(2lx-D)\Big]Q_{10} + \frac{p_2}{2M_1}\Big[[-\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2(p_2+2ly+2\omega\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2)+lx]R_2 + \frac{2l\omega y}{p_2}(1-\beta x)\Big]Q_{11} + \frac{p_2}{M_1}\Big[\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2(p_2+ly+\omega\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2)R_2 + \frac{\omega}{2Ep_2}[Dp_2+2ly(D-\hat{\mathbf{k}}\cdot\mathbf{p}_2)]\Big]Q_{12} + \frac{l\omega y}{2M_1}Q_{13} - \frac{Dp_2\omega}{2M_1E}Q_{14} - \frac{E_\nu}{2}\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2R_2Q_{15}\Big\}, \tag{18}$$

and

$$|\mathsf{M}_{III}|^2 = \frac{2E_\nu l}{M_1}\frac{(x\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2-y)}{1-\beta x}Q_{16} - \frac{l}{M_1}\frac{(x\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2-y)}{1-\beta x}(E_\nu+\beta l+\beta p_2 y+\beta\omega x)Q_{17} + \frac{E}{M_1}\frac{\beta^2(1-x^2)}{1-\beta x}(p_2+ly+\omega\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2)Q_{18} + \frac{l}{M_1}\Big[\frac{\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2}{1-\beta x}(\beta E_\nu-p_2y-l-\omega x)+y\Big(E_\nu+\frac{D-2E_\nu}{1-\beta x}\Big)\Big]Q_{19} + \frac{l}{M_1}\Big[\frac{\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2}{1-\beta x}(\beta E_\nu+p_2y+l+\omega x)+y\Big(E_\nu-\frac{D}{1-\beta x}\Big)\Big]Q_{20} - \frac{l}{M_1}\frac{\beta y[x(E_\nu^0-D)+p_2y+l]}{1-\beta x}Q_{21} - \frac{\omega}{2M_1}\frac{\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2}{1-\beta x}(E_\nu-D+\beta l+\beta p_2y+\beta\omega x)Q_{22} + \frac{\omega}{2M_1}\frac{1}{1-\beta x}[\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2(E_\nu-\beta l-\beta p_2y-\beta\omega x)+\beta y(D-2E_\nu)]Q_{23} + \frac{E_\nu\omega}{2M_1}\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2Q_{24} - \frac{\omega}{2M_1}\times(p_2+ly+\omega\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2)Q_{25}. \tag{19}$$

Here $\beta = l/E$, $R_1 = -1 + \beta^2(1-x^2)/(1-\beta x) + \omega/E$, $R_2 = -1 + (1-\beta^2)/(1-\beta x) - \omega/E$, $D = E_\nu^0 + (\mathbf{l} + \mathbf{p}_2)\cdot\hat{\mathbf{k}}$, and $\omega = F/(2D)$, with $F = 2p_2l(y_0 - y)$.

The form factors of the vertex (3) are contained in the Q_i coefficients. These are collected in Appendix A.

The complete differential decay rate, containing the Dalitz plot with virtual and bremsstrahlung RC to order $(\alpha/\pi)(q/M_1)$, is compactly expressed as

$$d\Gamma_C = d\Gamma_{CV} + d\Gamma_{CB}, \tag{20}$$

where the detailed expressions of $d\Gamma_{CV}$ and $d\Gamma_{CB}$, containing $\hat{s}_1$, can be traced starting at Eqs. (6) and (14). One can check that the infrared cutoff λ contained in the virtual RC is canceled by its counterpart in the bremsstrahlung RC. Equation (20) is model independent to order $(\alpha/\pi)(q/M_1)$. The photon triple integrals of Eq. (16) remain to be performed numerically. This is our first main result, in the sense that it can already be used in a Monte Carlo simulation. It complies with all the requirements discussed in the Introduction to solve the difficulties of the first two levels. However, it still presents problems of the third level. The triple numerical integration form is still unpractical. This difficulty can be substantially solved because such triple integrations can be calculated analytically.

We shall now proceed to obtain the analytical counterpart of the $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2$ correlation contained in Eq. (20). Within our approximations all the form factors are constant and can be factored out of the very many triple integrals. A convenient rearrangement of the C_R of Eq. (15) is

$$C_I = \sum_{i=1}^{8} Q_{i+6}\Lambda_i, \tag{21}$$

$$C_{II} = \sum_{i=6}^{15} Q_i\Lambda_{i+3}, \tag{22}$$

and

$$C_{III} = \sum_{i=16}^{25} Q_i\Lambda_{i+3}. \tag{23}$$

The Q_i are the quadratic functions of the form factors listed in Appendix A. The triple integrals are contained in the Λ_i. We shall not detail here their explicit form in terms of such integrals. We only give their final analytical expressions and collect them in Appendix B. Many of these integrals have been performed already in our previous work, although some are new. To help the reader interested in following our calculations in more detail, we have given in Appendix D the general form of the triple integrals and a guide to identify their analytical counterparts in our previous work. Only the results for the new integrals are explicitly given in this Appendix. In organizing Eq. (21) with one running index i it was necessary to introduce $\Lambda_2 = 0$, because Q_8 does not appear in this equation.

The completely analytical result for the Dalitz plot in the differential decay rate of CDB, Eq. (20), can be compactly written as

$$d\Gamma_C = d\Omega\left[A'_0 - A''_0\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2 + \frac{\alpha}{\pi}(\Theta_{CI} - \Theta_{CII}\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2)\right], \tag{24}$$

where

$$\Theta_{CI} = B'_1(\Phi_C + I_{C0}) + B''_{C1}\Phi'_C + C'_A, \tag{25}$$

and

$$\Theta_{CII} = B'_2(\Phi_C + I_{C0}) + B''_{C2}\Phi'_C + C_A^{(s)}. \tag{26}$$

In this last equation Φ_C, Φ'_C, B'_2, and B''_{C2} are the same as Eq. (6), I_{C0} is the one of Eq. (14), and $C_A^{(s)}$ is given by the sum of Eqs. (21)–(23). In Eq. (24) A'_0 and A''_0 are the ones of Eq. (6) and the analytic form of Θ_{CI} is found in Eq. (48) of Ref. [6]. One can check that when (q/M_1) contributions in Θ_{CII} and Θ_{CI} are neglected, one obtains the order $(q/M_1)^0$ result of Ref. [8]. In particular B'_2, B''_{C2}, and $C_A^{(s)}$ become A'_2, A''_2, and $D_3(\rho_1+\rho_3)+D_4(\rho_2+\rho_4)$ of Eqs. (20), (21), and (96), respectively, of this reference. Let us now proceed with the second case.

B. Neutral decaying baryon case

The calculation of $d\Gamma_{NB}^{(s)}$ proceeds in two ways. One possibility is to perform a straightforward calculation using the tools described in the previous section. Another possibility is to use the approach introduced in Ref. [7] to deal with the convergent pieces of $d\Gamma_{NB}$. All the $(\alpha/\pi)(q/M_1)$ terms can be obtained using the approximation

$$\frac{1}{p_2\cdot k} \simeq \frac{1}{p_1\cdot k} + \frac{q\cdot k}{(p_1\cdot k)^2}, \tag{27}$$

and this will allow us to incorporate all the terms of order $(\alpha/\pi)(q/M_1)$ that arise from this ratio. The advantage of this second possibility is that all the convergent terms of the NDB case are then obtained from their counterparts for the CDB case up to a few additional terms. This approximation, however, cannot be used in the divergent terms and hence we need standard techniques to calculate them. In the first term of Eq. (13) the infrared divergence is handled as in Ref. [7] and afterwards the approximation (27) is used. One gets

$$d\Gamma_{NBI}^{(s)} = \frac{\alpha}{\pi}d\Omega\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2(B'_2 I_{N0} + C_I + \tilde{C}_I^{(s)}). \tag{28}$$

The next term in Eq. (13) can be arranged using (27) into

$$d\Gamma_{NBII}^{(s)} = \frac{\alpha}{\pi}d\Omega\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2(C_{II} + \tilde{C}_{II}^{(s)}). \tag{29}$$

To calculate the third term in (13) we can use the approximations $W_\lambda \simeq \gamma_\lambda(f_1 + g_2\gamma_5)$ and $p_2 \simeq p_1$. The traces that arise are practically the same as in $d\Gamma_{CBIII}^{(s)}$. However, there is a difference to order $(\alpha/\pi)(q/M_1)$ which gives rise to a $\tilde{C}_{III}^{(s)}$ summand. Thus, one gets

$$d\Gamma_{NBIII}^{(s)} = \frac{\alpha}{\pi}d\Omega\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2(C_{III} + \tilde{C}_{III}^{(s)}). \tag{30}$$

The polarized decay rate becomes

$$d\Gamma_{NB}^{(s)} = \frac{\alpha}{\pi}d\Omega\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2(B'_2 I_{N0} + C_A^{(s)} + C_{NA}^{(s)}). \tag{31}$$

I_{N0} contains the infrared divergence and the finite terms that accompany it. It was calculated already; its explicit form is found in Eq. (40) of Ref. [7]. As before in Eq. (7), B'_2 contains the $(\alpha/\pi)(q/M_1)$ contributions. $C_A^{(s)}$ is the same as Eq. (14) of the CDB case and $C_{NA}^{(s)}$ is defined as

$$C_{NA}^{(s)} = \tilde{C}_I^{(s)} + \tilde{C}_{II}^{(s)} + \tilde{C}_{III}^{(s)}. \tag{32}$$

After some tedious but straightforward trace calculations their explicit forms are obtained. $\tilde{C}_i^{(s)}$ becomes

$$\tilde{C}_i^{(s)} = D_3\rho_i + D_4\rho'_i, \tag{33}$$

with $i = I, II, III$. Here $D_3 = 2(-g_1^2 + f_1 g_1)$ and $D_4 = 2(g_1^2 + f_1 g_1)$. ρ_i and ρ'_i are

$$\rho_I = \frac{p_2 l}{2\pi M_1}\int_{-1}^{y_0}dy\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\frac{\beta[-y + x\hat{\mathbf{p}}_2\cdot\hat{\mathbf{k}}]}{D(1-\beta x)} \times (DE_\nu^0 + p_2 l y), \tag{34}$$

$$\rho'_I = \frac{p_2 l}{2\pi M_1}\int_{-1}^{y_0}dy\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\frac{l[-y + x\hat{\mathbf{p}}_2\cdot\hat{\mathbf{k}}]}{D(1-\beta x)} \times [-D + \hat{\mathbf{k}}\cdot\mathbf{p}_2], \tag{35}$$

$$\rho_{II} = \frac{l p_2^2}{8\pi M_1}\int_{-1}^{y_0}dy\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\frac{E_\nu}{D}\left[1 + \frac{\beta y - \hat{\mathbf{p}}_2\cdot\hat{\mathbf{k}}}{1-\beta x}\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2\right], \tag{36}$$

$$\rho'_{II} = \frac{l p_2^2}{8\pi M_1}\int_{-1}^{y_0}dy\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\frac{E_\nu}{D}\left[\hat{\mathbf{p}}_2\cdot\hat{\mathbf{k}} + \frac{\beta y - \hat{\mathbf{p}}_2\cdot\hat{\mathbf{k}}}{1-\beta x}\right]\hat{\mathbf{p}}_\nu\cdot\hat{\mathbf{p}}_2, \tag{37}$$

$$\rho_{III} = \frac{p_2 l}{4\pi M_1}\int_{-1}^{y_0}dy\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\frac{E_\nu E}{D}\left\{\left[\frac{1-\beta^2}{1-\beta x} - \frac{2\omega}{E} - 1\right]\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2 - \beta y\left[1 - \beta\hat{\mathbf{p}}_\nu\cdot\frac{x\hat{\mathbf{k}} - \hat{\mathbf{l}}}{1-\beta x}\right]\right\}, \tag{38}$$

and

$$\rho'_{III} = \frac{p_2 l}{4\pi M_1}\int_{-1}^{y_0} dy \int_{-1}^{1} dx \int_0^{2\pi} d\varphi_k \frac{E E_\nu}{D}\left\{\left[\frac{1-\beta^2}{1-\beta x} - (1+\beta x) - \frac{2\omega}{E}\right]\hat{\mathbf{p}}_\nu \cdot \hat{\mathbf{p}}_2 - \beta y\left[\frac{1-\hat{\mathbf{p}}_\nu \cdot \hat{\mathbf{k}}}{1-\beta x}\right] + \left[\frac{1-\beta \hat{\mathbf{l}}\cdot \hat{\mathbf{p}}_\nu}{1-\beta x} - 1\right]\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}_2\right\}. \quad (39)$$

The complete differential decay rate that contains the Dalitz plot of the NDB case including the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ correlation can be expressed compactly as

$$d\Gamma_N = d\Gamma_{NV} + d\Gamma_{NB}, \quad (40)$$

where the detailed expressions of $d\Gamma_{NV}$ and $d\Gamma_{NB}$ containing $\hat{s}_1$ are traced starting at Eqs. (6) and (31). This is our second main result and the discussion of the previous subsection applies to it: its triple numerical integration form is still unpractical. This difficulty is solved by performing analytically the triple integrals contained in Eq. (31). We have to concentrate only on $C_{NA}^{(s)}$ of this equation; all other terms in it have been given analytically already. Then, Eq. (32) can be cast into the compact form

$$C_{NA}^{(s)} = D_3 \rho_{N3} + D_4 \rho_{N4}, \quad (41)$$

where

$$\rho_{N3} = \rho_I + \rho_{II} + \rho_{III}, \quad (42)$$

and

$$\rho_{N4} = \rho'_I + \rho'_{II} + \rho'_{III}. \quad (43)$$

The explicit analytical expressions of the ρ_i and ρ'_i are collected in Appendix C.

The completely analytical result for the Dalitz plot in Eq. (40) can be put in parallel with Eq. (24), namely,

$$d\Gamma_N = d\Omega\left[A'_0 - A''_0 \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 + \frac{\alpha}{\pi}(\Theta_{NI} - \Theta_{NII}\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2)\right], \quad (44)$$

where

$$\Theta_{NI} = B'_1(\Phi_N + I_{N0}) + B''_{N1}\Phi'_N + C'_A + C'_{NA}, \quad (45)$$

and

$$\Theta_{NII} = B'_2(\Phi_N + I_{N0}) + B''_{N2}\Phi'_N + C_A^{(s)} + C_{NA}^{(s)}. \quad (46)$$

In this last equation Φ_N, Φ'_N, B'_2, and B''_{N2} are the same as Eq. (6), $C_A^{(s)}$ and I_{N0} are the ones of Eq. (31), and $C_{NA}^{(s)}$ is given in Eq. (41). In Eq. (44) A'_0 and A''_0 are the ones of Eq. (6) and the analytical form of Θ_{NI} is found in Eq. (54) of Ref. [7]. One can check that when $(\alpha/\pi)(q/M_1)$ contributions in Eq. (44) are neglected one obtains the $(\alpha/\pi)(q/M_1)^0$ result of Ref. [8]. In particular B''_{N2} becomes A''_2 of this reference and $C_{NA}^{(s)}$ becomes zero.

IV. CROSS-CHECKS

There are several points we want to make in this section. One is that the analytical results are so long that it is important to check them. Another one is that there are some results already available in the literature [10] and we should compare with them. An even more important point is to provide the reader interested in using our result with numbers to be reproduced.

To cross-check the analytical results we use the triple numerical integration form of the RC. We make numerical comparisons of both forms by fixing the values of several form factors and of the Dalitz plot variables E and E_2. A complete cross-check requires the use of several choices of nonzero values for all six form factors and a range of values of the pair (E, E_2) over the Dalitz plot. Also, the comparison with the numerical results of Ref. [10] should be made in the several cases covered there. All these cross-checks and comparisons were satisfactory and it is not necessary to display all the details here. Accordingly, we shall present a minimum of numerical tables and limit our discussion to them.

For definiteness, we shall work with the decays $\Sigma^- \to ne\overline{\nu}$ and $\Lambda \to pe\overline{\nu}$ as examples of CDB and NDB cases. The reason for this is that numerical RC for these two decays were produced in Ref. [10]. We shall accordingly fix the form factors at the values used in this reference, namely, $g_1/f_1 = -0.34$, $f_2/f_1 = -0.97$ for $\Sigma^- \to ne\overline{\nu}$ decay and $g_1/f_1 = 0.72$, $f_2/f_1 = 0.97$ for $\Lambda \to pe\overline{\nu}$. In addition, we use $f_1 = 1$ in $\Sigma^- \to ne\overline{\nu}$ and $f_1 = 1.2366$ in $\Lambda \to pe\overline{\nu}$ and to compare with Ref. [10] in both these decays we put $g_2 = 0$ and neglect f_3 and g_3 contributions. The values of the masses come from Ref. [1]. The anomalous magnetic moments of the baryons appear in our expressions of the RC. We use $\kappa(\Sigma^-) = 0.3764 M_N$, $\kappa(\Lambda) = 0.6130 M_N$, $\kappa(n) = 1.9130 M_N$, and $\kappa(p) = -1.7928 M_N$, where M_N is the nuclear magneton. These values are extracted from the corresponding total magnetic moments reported in [1] using Eq. (22) of Ref. [6]. We neglected the anomalous magnetic moment of the electron, due to its smallness.

As an example of the numerical cross-check we display Table I for $\Sigma^- \to ne\overline{\nu}$, where for generality we allowed g_2, g_3, and $f_3 \neq 0$. In the upper entries (a) we use the triple numerical integration form to obtain the RC for $C_A^{(s)}$ of the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ correlation covering a lattice of points over the Dalitz plot. The energies E and E_2 enter through $\delta = E/E_m$ and $\sigma = E_2/M_1$. E_m, $\sigma^{\max}$, and $\sigma^{\min}$ are determined using the boundaries of the three-body region given in Ref. [8]. The lower entries (b) contain the RC for the same $C_A^{(s)}$ calculated with the analytical form. An inspection of this table shows an agreement to two decimal places and the third one being close.

TABLE I. Values of $C_A^{(s)}$ in $\Sigma^- \to ne\overline{\nu}$ decay by (a) integrating it numerically and (b) evaluating it analytically. $C_A^{(s)}$ is given in units of GeV^2. The form factors have been given the arbitrary values $f_1 = 1.0$, $f_2 = -0.97$, $f_3 = -0.778$, $g_1 = -0.34$, $g_2 = 0.987$, and $g_3 = -1.563$.

σ					(a)					
0.8077	−0.0744	−0.0811	−0.0587	−0.0256	0.0101	0.0420	0.0650	0.0741	0.0648	0.0309
0.8056	−0.1350	−0.1435	−0.1048	−0.0507	0.0049	0.0528	0.0852	0.0955	0.0778	0.0253
0.8035		−0.1673	−0.1294	−0.0708	−0.0090	0.0443	0.0802	0.0910	0.0700	0.0102
0.8014		−0.1875	−0.1534	−0.0921	−0.0254	0.0327	0.0718	0.0831	0.0590	
0.7993		−0.2065	−0.1784	−0.1152	−0.0438	0.0193	0.0617	0.0734	0.0459	
0.7972			−0.2054	−0.1407	−0.0643	0.0042	0.0503	0.0623	0.0306	
0.7951			−0.2358	−0.1697	−0.0873	−0.0124	0.0379	0.0499	0.0128	
0.7930			−0.2720	−0.2042	−0.1141	−0.0309	0.0246	0.0363		
0.7909				−0.2480	−0.1469	−0.0520	0.0106	0.0211		
0.7888				−0.3115	−0.1913	−0.0772	−0.0035	0.0041		
0.7867					−0.2684	−0.1110	−0.0135			
					(b)					
0.8077	−0.0744	−0.0810	−0.0587	−0.0256	0.0098	0.0412	0.0639	0.0732	0.0643	0.0309
0.8056	−0.1350	−0.1435	−0.1048	−0.0507	0.0047	0.0521	0.0842	0.0947	0.0774	0.0253
0.8035		−0.1673	−0.1293	−0.0706	−0.0092	0.0437	0.0793	0.0902	0.0697	0.0102
0.8014		−0.1875	−0.1533	−0.0919	−0.0255	0.0322	0.0710	0.0823	0.0587	
0.7993		−0.2065	−0.1783	−0.1149	−0.0437	0.0189	0.0609	0.0727	0.0457	
0.7972			−0.2053	−0.1404	−0.0640	0.0040	0.0497	0.0617	0.0305	
0.7951			−0.2357	−0.1693	−0.0869	−0.0124	0.0374	0.0495	0.0127	
0.7930			−0.2719	−0.2036	−0.1135	−0.0308	0.0243	0.0360		
0.7909				−0.2473	−0.1460	−0.0516	0.0104	0.0210		
0.7888				−0.3106	−0.1900	−0.0765	−0.0034	0.0041		
0.7867					−0.2666	−0.1100	−0.0134			
δ	0.0500	0.1500	0.2500	0.3500	0.4500	0.5500	0.6500	0.7500	0.8500	0.9500
$\sigma^{\max}$	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078
$\sigma^{\min}$	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

To proceed with the comparison with Ref. [10] we must use the difference defined there, namely,

$$\delta\alpha_B(E, E_2) = \alpha_B(E, E_2) - \alpha_0(E, E_2), \tag{47}$$

where

$$\alpha_{Bi}(E, E_2) = -\frac{A_0''(E, E_2) + (\alpha/\pi)\Theta_{iII}(E, E_2)}{A_0'(E, E_2) + (\alpha/\pi)\Theta_{iI}(E, E_2)}, \tag{48}$$

and as before $i = C, N$ and $\alpha_0(E, E_2) = -A_0''(E, E_2)/A_0'(E, E_2)$ for both values of i.

One may interpret $\alpha_{Bi}(E, E_2)$ as the asymmetry parameter of the emitted baryon at (E, E_2) points of the Dalitz plot. Here we must choose the same (E, E_2) points as in Ref. [10] and, as already mentioned, use the same values of the form factors. However, it should be stressed that our final results are not compromised to fixing the values of the form factors.

Before proceeding with a detailed comparison with the numbers of this reference, there is a point that must be kept in mind. The approximations used in our work and in Ref. [10] are not quite the same. We used the Low theorem to calculate the bremsstrahlung RC and in this reference it was assumed that both baryons involved were pointlike and higher $(\alpha/\pi)(q/M_1)^n$ contributions $(n \geq 2)$ were included in this part. Another interesting thing is to compare our order $(\alpha/\pi)(q/M_1)$ results with our previous order $(\alpha/\pi)(q/M_1)^0$ results. As explained earlier these latter are reproduced here when the $(\alpha/\pi)(q/M_1)$ contributions are neglected.

We performed many comparisons and, as before, there is no need to present all the details. One example, the $\Lambda \to pe\overline{\nu}$ case, is enough for this discussion. The results are displayed in Table II. In the upper (a) part only the order $(\alpha/\pi)(q/M_1)^0$ is given. Both this order and the order $(\alpha/\pi)(q/M_1)$ contributions are added in the middle part (b). The numerical results of Ref. [10] are reproduced in the lower part (c). A numerical cross-check was also performed in producing parts (a) and (b). We do not reproduce it here; the agreement was as good as in Table I.

An inspection of Table II shows that the order $(\alpha/\pi) \times (q/M_1)$ is systematically perceptible at the second significant digit and even at the first one. In comparing with Ref. [10], one can see a better agreement with the middle table (b). The agreement at the first significant digit improves as the RC grow in size and also the variations in the second digit become smaller. There are differences, however. They may be explained as due to the different approximations used. Also, comparing entries (a) and (b)

TABLE II. $100\delta\alpha_B(E, E_2)$ with RC over the three-body region in $\Lambda \to pe\overline{\nu}$ decay. (a) gives the RC to order $(\alpha/\pi)(q/M_1)^0$; (b) gives the RC to order $(\alpha/\pi)(q/M_1)$; (c) corresponds to the RC computed in Ref. [10].

σ										
					(a)					
0.8530	0.1	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.4	1.4
0.8518	1.2	0.2	0.1	0.1	0.2	0.2	0.3	0.5	1.0	1.9
0.8505		0.5	0.3	0.3	0.3	0.4	0.6	0.9	1.5	
0.8492		1.0	0.5	0.5	0.5	0.6	0.8	1.2	1.6	
0.8480		1.7	0.9	0.7	0.7	0.8	1.1	1.4	1.4	
0.8467			1.3	1.0	1.0	1.1	1.3	1.5	0.5	
0.8454			1.9	1.4	1.3	1.4	1.5	1.5		
0.8442				1.9	1.7	1.7	1.7	1.0		
0.8429				2.7	2.3	2.1	1.6			
0.8416					3.4	2.4	0.1			
					(b)					
0.8530	0.5	0.2	0.2	0.2	0.1	0.1	0.1	0.2	0.3	0.9
0.8518	2.0	0.5	0.4	0.4	0.4	0.4	0.5	0.6	0.9	1.1
0.8505		0.9	0.7	0.6	0.6	0.7	0.8	1.0	1.3	
0.8492		1.5	1.0	0.9	0.9	0.9	1.1	1.3	1.3	
0.8480		2.3	1.3	1.1	1.1	1.2	1.3	1.4	1.0	
0.8467			1.8	1.5	1.4	1.5	1.5	1.5	0.2	
0.8454			2.3	1.8	1.7	1.7	1.7	1.3		
0.8442				2.2	2.0	1.9	1.7	0.7		
0.8429				2.8	2.4	2.1	1.5			
0.8416					3.2	2.2	0.1			
					(c)					
0.8530	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.2	0.3	0.9
0.8518	1.9	0.4	0.3	0.3	0.3	0.4	0.5	0.6	1.0	1.1
0.8505		0.8	0.6	0.5	0.6	0.6	0.8	1.0	1.3	
0.8493		1.4	0.9	0.8	0.8	0.9	1.1	1.3	1.4	
0.8480		2.4	1.3	1.1	1.1	1.2	1.3	1.5	1.1	
0.8467			1.7	1.4	1.4	1.5	1.6	1.5	0.3	
0.8455			2.3	1.8	1.7	1.7	1.7	1.3		
0.8442				2.2	2.0	2.0	1.7	0.8		
0.8429				2.8	2.5	2.2	1.5			
0.8417					3.2	2.3	0.1			
δ	0.0500	0.1500	0.2500	0.3500	0.4500	0.5500	0.6500	0.7500	0.8500	0.9500
$\sigma^{\max}$	0.8536	0.8536	0.8536	0.8536	0.8536	0.8536	0.8536	0.8536	0.8536	0.8536
$\sigma^{\min}$	0.8516	0.8479	0.8450	0.8428	0.8414	0.8410	0.8416	0.8433	0.8464	0.8508

one may conclude that for light quark hyperon semileptonic decays the order $(\alpha/\pi)(q/M_1)$ is perceptible enough and that when heavy quarks are involved contributions of this order become relevant in precision experiments.

Let us now turn to a different form to use our results. A form which may provide a more efficient use of them in a Monte Carlo simulation and which still is not compromised to fixing values of the form factors, as was the case in Table II.

V. NUMERICAL FORM OF THE RADIATIVE CORRECTIONS

We now come to our second goal in this paper. In the previous sections we have obtained the RC to CDB and NDB in two forms. The first one has triple integrals over the real photon variables ready to be performed numerically. The second one is fully analytical. Although this latter one is already practical it is still long and tedious. It still requires that the RC be calculated within the Monte Carlo simulation every time E and E_2 are varied. This is much faster than performing the triple integrals, but, it still represents a non-negligible computer effort. We shall now discuss a third form of the RC that may be more practical to use.

For fixed values of E and E_2, Eqs. (25) and (26) for the CDB case and Eqs. (45) and (46) for the NDB case take the form

$$\Theta_m = \sum_{i\leq j=1}^{6} a_{ij}^m f_i f_j, \tag{49}$$

TABLE III. Numerical arrays of the coefficients a_{ij}^{NII} in GeV^2 of Eq. (49) evaluated at ten points (E, E_2) (headings of columns) over the polarized Dalitz plot of $\Lambda \to pe^- \bar{\nu}$ decay.

	(0.05,0.8518)	(0.35,0.8518)	(0.65,0.8518)	(0.95,0.8518)	(0.25,0.8480)	(0.55,0.8480)	(0.75,0.8480)	(0.45,0.8442)	(0.65,0.8442)	(0.55,0.8416)
f_1^2	6.812×10^{-4}	1.550×10^{-4}	-1.024×10^{-4}	6.505×10^{-4}	7.218×10^{-4}	-1.622×10^{-4}	-6.987×10^{-5}	7.198×10^{-5}	-1.227×10^{-4}	-4.887×10^{-5}
f_2^2	1.604×10^{-3}	-1.875×10^{-4}	-5.732×10^{-4}	2.204×10^{-3}	6.787×10^{-4}	-1.268×10^{-3}	-3.588×10^{-4}	-7.888×10^{-4}	-5.807×10^{-4}	-2.633×10^{-4}
f_3^2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
g_1^2	7.470×10^{-2}	-8.900×10^{-3}	-2.635×10^{-2}	1.026×10^{-1}	4.812×10^{-2}	-9.116×10^{-2}	-2.537×10^{-2}	-1.250×10^{-1}	-9.181×10^{-2}	-2.073×10^{-1}
g_2^2	1.882×10^{-3}	-2.199×10^{-4}	-6.722×10^{-4}	2.585×10^{-3}	1.227×10^{-3}	-2.293×10^{-3}	-6.488×10^{-4}	-3.131×10^{-3}	-2.305×10^{-3}	-5.219×10^{-3}
g_3^2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$f_1 f_2$	2.010×10^{-3}	-4.892×10^{-5}	-5.756×10^{-4}	2.395×10^{-3}	1.258×10^{-3}	-1.173×10^{-3}	-3.495×10^{-4}	-4.985×10^{-4}	-5.694×10^{-4}	-2.477×10^{-4}
$f_1 f_3$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$f_2 f_3$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$g_1 g_2$	-2.362×10^{-2}	2.889×10^{-3}	8.504×10^{-3}	-3.252×10^{-2}	-1.509×10^{-2}	2.906×10^{-2}	8.219×10^{-3}	3.975×10^{-2}	2.911×10^{-2}	6.574×10^{-2}
$g_1 g_3$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$g_2 g_3$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$f_1 g_1$	1.683×10^{-1}	3.604×10^{-1}	1.875×10^{-2}	-5.329×10^{-2}	2.878×10^{-1}	8.435×10^{-2}	-1.250×10^{-1}	1.258×10^{-1}	-8.629×10^{-2}	-3.368×10^{-2}
$f_1 g_2$	-1.763×10^{-3}	-6.857×10^{-3}	6.870×10^{-4}	-9.573×10^{-4}	-1.808×10^{-2}	-4.425×10^{-3}	9.176×10^{-3}	-1.410×10^{-2}	1.046×10^{-2}	5.085×10^{-3}
$f_1 g_3$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$f_2 g_1$	-2.482×10^{-2}	-8.236×10^{-3}	-6.172×10^{-3}	2.989×10^{-2}	-2.846×10^{-2}	-1.758×10^{-2}	-4.052×10^{-3}	-1.703×10^{-2}	-9.910×10^{-3}	-5.365×10^{-3}
$f_2 g_2$	3.970×10^{-3}	1.329×10^{-3}	9.192×10^{-4}	-4.778×10^{-3}	4.634×10^{-3}	2.719×10^{-3}	4.868×10^{-4}	2.659×10^{-3}	1.401×10^{-3}	7.390×10^{-4}
$f_2 g_3$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$f_3 g_1$	-5.014×10^{-5}	-9.348×10^{-5}	-6.324×10^{-5}	-5.962×10^{-7}	-1.323×10^{-4}	-1.003×10^{-4}	-2.914×10^{-5}	-9.289×10^{-5}	-2.412×10^{-5}	-9.643×10^{-6}
$f_3 g_2$	-4.271×10^{-5}	-8.147×10^{-5}	-5.522×10^{-5}	-5.513×10^{-7}	-1.136×10^{-4}	-8.644×10^{-5}	-2.517×10^{-5}	-7.876×10^{-5}	-2.061×10^{-5}	-8.140×10^{-6}
$f_3 g_3$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE IV. Numerical arrays of the coefficients a_{ij}^{CII} in GeV^2 of Eq. (49) evaluated at ten points (E, E_2) (headings of columns) over the polarized Dalitz plot of $\Lambda_c^+ \to \Lambda e^+ \nu$ decay.

	(0.15,0.5995)	(0.45,0.5995)	(0.75,0.5995)	(0.95,0.5995)	(0.25,0.5602)	(0.55,0.5602)	(0.85,0.5602)	(0.45,0.5210)	(0.75,0.5210)	(0.65,0.4948)
f_1^2	-1.532×10^{-2}	1.036×10^{-1}	6.859×10^{-3}	-3.894×10^{-1}	7.582×10^{-2}	2.298×10^{-1}	-3.036×10^{-1}	2.295×10^{-1}	-3.851×10^{-2}	6.446×10^{-2}
f_2^2	-4.818×10^{-3}	2.436×10^{-1}	1.849×10^{-2}	-8.624×10^{-1}	2.289×10^{-1}	5.389×10^{-1}	-6.710×10^{-1}	5.611×10^{-1}	-8.135×10^{-2}	1.481×10^{-1}
f_3^2	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
g_1^2	-1.466×10^{-2}	1.076	8.152×10^{-2}	-3.808	1.534	3.546	-4.416	7.830	-1.139	$1.00 \times 10^{+1}$
g_2^2	-5.571×10^{-3}	2.817×10^{-1}	2.138×10^{-2}	-9.972×10^{-1}	3.943×10^{-1}	9.282×10^{-1}	-1.156	2.047	-2.967×10^{-1}	2.632
g_3^2	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
f_1f_2	-3.445×10^{-2}	3.133×10^{-1}	2.160×10^{-2}	-1.159	2.497×10^{-1}	6.956×10^{-1}	-9.031×10^{-1}	7.047×10^{-1}	-1.130×10^{-1}	1.941×10^{-1}
f_1f_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
f_2f_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
g_1g_2	8.296×10^{-3}	-1.114	-8.738×10^{-2}	3.897	-1.571	-3.659	4.514	-8.037	1.150	$-1.030 \times 10^{+1}$
g_1g_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
g_2g_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
f_1g_1	9.235	9.436×10^{-1}	-9.728	-2.410	8.070	-2.512	-5.571	5.378	-6.121	-2.391
f_1g_2	-4.429×10^{-1}	8.400×10^{-2}	7.031×10^{-1}	-8.759×10^{-1}	-1.671	7.110×10^{-1}	5.766×10^{-1}	-2.146	2.184	1.134
f_1g_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
f_2g_1	-4.903×10^{-1}	-2.339×10^{-1}	1.770×10^{-2}	3.505	-2.921×10^{-2}	-6.449×10^{-1}	3.069	1.706	7.521×10^{-1}	2.504×10^{-1}
f_2g_2	3.501×10^{-1}	1.071×10^{-1}	-1.408×10^{-1}	-1.853	2.102×10^{-1}	2.404×10^{-1}	-1.777	-7.944×10^{-1}	-5.591×10^{-1}	-1.778×10^{-1}
f_2g_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
f_3g_1	-2.067×10^{-2}	-2.617×10^{-2}	-1.189×10^{-2}	-4.642×10^{-4}	-3.808×10^{-2}	-3.174×10^{-2}	-3.158×10^{-3}	-3.269×10^{-2}	-4.636×10^{-3}	-3.500×10^{-3}
f_3g_2	-1.261×10^{-2}	-1.597×10^{-2}	-7.253×10^{-3}	-2.833×10^{-4}	-2.147×10^{-2}	-1.790×10^{-2}	-1.781×10^{-3}	-1.704×10^{-2}	-2.417×10^{-3}	-1.731×10^{-3}
f_3g_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

because they are quadratic in the form factors. For the sake of simplicity, in Eq. (49) we have momentarily redefined $g_1 = f_4$, $g_2 = f_5$, and $g_3 = f_6$. Notice that the restriction $i \leq j$ reduces the sum in Eq. (49) to 21 terms. The index m takes the values $m = CI, CII, NI$, and NII. The third form of RC we propose consists of calculating arrays of the a_{ij}^m coefficients determined at fixed values of (E, E_2) and that these pairs of (E, E_2) cover a lattice of points on the Dalitz plot.

To calculate the coefficients a_{ij}^m it is not necessary to rearrange our final results, either analytical or to be integrated, so that they take the form (49). One can calculate them following a systematic procedure. One chooses fixed (E, E_2) points. Then one fixes $f_1 = 1$ and $f_i = 0$, $i \neq 1$ and obtains a_{11}^m; one repeats this calculation for $f_2 = 1$, $f_i = 0$, $i \neq 2$ to obtain a_{22}^m, and again until $f_6 = 1$, $f_i = 0$, $i \neq 6$, and a_{66}^m are obtained. Next, one repeats the calculation with $f_1 = 1$, $f_2 = 1$, $f_i = 0$, $i \neq 1, 2$ and from this results one subtracts a_{11}^m and a_{22}^m; this way one obtains the coefficient a_{12}^m. One repeats this last step changing i and j until all the interference coefficients a_{ij}^m, $i \neq j$, have been calculated.

To illustrate all this and to further discuss it we have produced arrays presented in two tables, selecting in each one ten points (E, E_2) over the Dalitz plot. We have chosen two examples, $\Lambda \to pe\overline{\nu}$ of a NDB case which is displayed in Table III and $\Lambda_c^+ \to \Lambda e^+ \nu$ of a CDB case which is displayed in Table IV. This latter also serves as an example of a heavy quark decay. As in the previous section, the more important purpose is to provide the user with numbers to compare with. The arrays of these two tables were obtained using the RC in the analytical form. In the Λ_c^+ case we used the formulas for the charge assignments A^-, B^0, l^- of the CDB case of the previous sections and then applied the rules of Ref. [5] to obtain the results for the charge assignment A^+, B^0, l^+ of this particular case. In these tables we have restored our standard notation for the axial-vector form factors g_1, g_2, and g_3. The masses used are those of Sec. IV, $M_1(\Lambda_c^+)$ comes from Ref. [1], and we assume an estimate for $\kappa(\Lambda_c^+) = 0.1106 M_N$.

The first fact that appears in these tables is that the RC do not depend on the form factor products f_3^2, g_3^2, $f_1 f_3$, $f_2 f_3$, $g_1 g_3$, $g_2 g_3$, $f_1 g_3$, $f_2 g_3$, and $f_3 g_3$ in Tables III and IV. The nonappearance of these products cannot be seen easily in our final results of Sec. III. The other fact is that the nonzero RC to each form factor product vary appreciably from one (E, E_2) point to another. This means that replacing the precision results of Sec. III with an array of only a few columns over the Dalitz plot is far from satisfactory. Therefore the lattice of (E, E_2) points must be much finer than only a few points.

The use of this third presentation of RC is very practical in the sense that such RC can be calculated separately and only the arrays should be fed into the Monte Carlo simulation. However, in a precision experiment possibly involving 150, 200, and even 300 bins over the Dalitz plot the number of columns in the RC arrays should be at least just as many. It may be required that several columns be produced in finer subdivisions within each bin, possibly four, eight, or even more. For example, one may require that the numerical changes of the a_{ij}^m coefficients between neighboring (E, E_2) points do not exceed two decimal places within rounding of the third decimal place.

To close this section let us stress that none of the three forms of our RC results is compromised to fixing from the outset values for the form factors when RC are applied in a Monte Carlo simulation. To fix them at prescribed values may be not too bad an assumption for hyperon semileptonic decays, but it is not acceptable at all for decays involving heavy quarks where the Cabibbo theory [1] is no longer reliable for fixing the form factors.

VI. SUMMARY AND CONCLUSIONS

We have obtained in Secs. II and III the RC to the angular correlation $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2$ to order $(\alpha/\pi)(q/M_1)$. Our final results are given in two forms. The first one is the triple numerical integration form, in which the integrations over the real photon variables are explicitly exhibited and remain to be performed numerically. The second one is the analytical form where those integrations have all been calculated analytically. We covered two cases, the CDB and the NDB ones whose final results are given in Eqs. (24) and (44), respectively. The analytical results are very long and tedious. To make their use more accessible we have collected the numerous Q_i and Λ_i algebraic expressions which appear in the CDB case in Appendices A and B, respectively. The NDB case uses these expressions and also the ρ_i ones. These latter were collected in Appendix C.

Our analytical results were cross-checked and compared with other results available in the literature. This we have done in Sec. IV. We have limited ourselves to discuss the decay $\Sigma^- \to ne\overline{\nu}$ as an example of the cross-checks and $\Lambda \to pe\overline{\nu}$ as an example of the comparisons with Ref. [10]. In addition, in this latter decay we also included a comparison between our RC to orders $(\alpha/\pi) \times (q/M_1)^0$ and $(\alpha/\pi)(q/M_1)$.

We have discussed in Sec. V another possibility to use our results in an experimental analysis. One can calculate the numerical factors of the quadratic products of the form factors that appear in the RC at fixed values of (E, E_2). These factors can be organized in arrays to be multiplied upon such products, covering a lattice of (E, E_2) points over the Dalitz plot. We discussed two examples of this possibility, a CDB one and a NDB one.

Apart from illustration purposes, the tables in this paper provide numbers to compare with. Also, our calculations rely heavily on previous results. Apart from discussions in text, we have given in Appendix D details to allow the identification of our previous and new analytical results for the many integrals.

To close, let us recall that our results are general within our approximation. They can be applied in the other four charge assignments of baryons involving heavy quarks and whether the charged lepton is $e^\pm$, $\mu^\pm$, or $\tau^\pm$. They are model independent and are not compromised to fixing the form factors at prescribed values. The above calculations should be extended to cover precision RC in the $\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}}$ correlation [11] and in the four-body region [12]. We hope to return to these cases in the near future.

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APPENDIX A: COLLECTION OF THE Q_i COEFFICIENTS

The coefficients Q_i introduced in Secs. II and III are long quadratic functions of the form factors. The coefficients $Q_1, \ldots, Q_7$ have been computed in previous works [6,8] and can be found there. The new coefficients are listed below. They read

$$\tilde{Q}_6 = F_1^2\left[\frac{E_2 - M_2 - \beta p_2 y_0}{M_1}\right] + G_1^2\left[\frac{E_2 + M_2 - \beta p_2 y_0}{M_1}\right] + 2F_1G_1\left[\frac{E_2 - \beta p_2 y_0}{M_1}\right] - (F_1F_2 - G_1G_2)\left[\frac{\beta p_2 y_0}{M_1}\right] - F_1G_2\left[\frac{M_1 - M_2 + E_\nu^0 - E}{M_1} - \frac{q^2}{2M_1E}\right] + F_2G_1\left[\frac{M_1 + M_2 + E_\nu^0 - E}{M_1} - \frac{q^2}{2M_1E}\right] - F_2G_2\left[\frac{2E_\nu^0}{M_1} - \frac{q^2}{2M_1E}\right],$$

$$\tilde{Q}_7 = F_1^2\left[1 + \frac{M_2}{M_1}\right]\left[\frac{E_2 - M_2}{E}\right] + G_1^2\left[1 - \frac{M_2}{M_1}\right]\left[\frac{E_2 + M_2}{E}\right] - 2F_1G_1\left[\frac{E_\nu^0 - E}{E}\right] + F_1G_2\left[\frac{E_2 - M_2}{M_1}\right]\left[\frac{E_\nu^0 - E}{E}\right] - F_2G_1\left[\frac{E_2 + M_2}{M_1}\right]\left[\frac{E_\nu^0 - E}{E}\right] + (F_1F_2 - G_1G_2)\left[\frac{p_2^2}{M_1E}\right],$$

$$Q_8 = F_1^2\left[\frac{E_2 - M_2}{M_1}\right] + G_1^2\left[\frac{E_2 + M_2}{M_1}\right] + 2F_1G_1\left[\frac{E_2}{M_1}\right] + F_1G_2\left[\frac{E - M_1 + M_2}{M_1}\right] - F_2G_2\left[\frac{E_\nu^0}{M_1}\right] - F_2G_1\left[\frac{E - M_1 - M_2}{M_1}\right] + F_3G_1\left[\frac{E(E_2 + M_2)}{M_1^2}\right],$$

$$Q_9 = F_1^2\left[\frac{E_2 - M_2}{M_1}\right] + G_1^2\left[\frac{E_2 + M_2}{M_1}\right] + 2F_1G_1\left[\frac{E_2}{M_1}\right] - F_1G_2\left[\frac{E_2 - M_2}{M_1}\right] + F_2G_1\left[\frac{E_2 + M_2}{M_1}\right] - F_3G_1\left[\frac{E_2}{M_1} - 1\right] \times \left[\frac{E_2 + M_2}{M_1}\right],$$

$$Q_{N8} = F_1^2\left[\frac{(M_1 - E)(E_2 - M_2)}{M_1^2}\right] + G_1^2\left[\frac{(M_1 - E)(E_2 + M_2)}{M_1^2}\right] + 2F_1G_1\left[\frac{M_2^2}{M_1^2} - \frac{E_\nu^0}{M_1}\right] + F_1G_2\left[\frac{M_2(E_2 - M_2 - E_\nu^0)}{M_1^2}\right] + F_2G_1\left[\frac{M_2(E_2 + M_2 - E_\nu^0)}{M_1^2}\right] - F_2G_2\left[\frac{E_2E_\nu^0}{M_1^2}\right] + F_3G_1\left[\frac{E(E_2 + M_2)}{M_1^2}\right],$$

$$Q_{N9} = -F_1^2\left[\frac{M_2(E_2 - M_2)}{M_1^2}\right] + G_1^2\left[\frac{M_2(E_2 + M_2)}{M_1^2}\right] + 2F_1G_1\left[\frac{M_2^2}{M_1^2}\right] - F_1G_2\left[\frac{E_2(E_2 - M_2)}{M_1^2}\right] + F_2G_1\left[\frac{E_2(E_2 + M_2)}{M_1^2}\right] + F_3G_1\left[\frac{M_2 + E_2}{M_1}\right]\left[1 - \frac{E_2}{M_1}\right],$$

$$Q_{10} = -F_2G_1 + F_1G_2 + F_2G_2,$$

$$Q_{11} = \left[\frac{E_2 + M_2}{M_1}\right]G_1F_3,$$

$$Q_{12} = 2F_1G_1,$$

$$Q_{13} = -F_1^2\left[\frac{E_2 - M_2}{E}\right] - G_1^2\left[\frac{E_2 + M_2}{E}\right] + 2F_1G_1\left[\frac{E_2}{E}\right] + F_2G_1\left[\frac{E_2 + M_2}{E}\right] - F_1G_2\left[\frac{E_2 - M_2}{E}\right] - F_3G_1\left[1 - \frac{E_2}{M_1}\right] \times\left[\frac{M_2 + E_2}{E}\right],$$

$$Q_{14} = -F_1^2 - G_1^2 - F_1F_2 + G_1G_2,$$

$$Q_{15} = 2F_1^2\left[\frac{E_2 - M_2}{M_1}\right] + 2G_1^2\left[\frac{E_2 + M_2}{M_1}\right],$$

$$Q_{16} = f_1(g_2 - g_1) - f_2g_1,$$

$$Q_{17} = f_1g_2 + f_3g_1,$$

$$Q_{18} = \tfrac{1}{2}(f_1^2 - g_1^2) + f_2(f_1 + g_1) - g_1(f_3 - g_2),$$

$$Q_{19} = 2f_1g_1\left[\frac{1}{2M_1} + \frac{\kappa_1}{e}\right]M_1,$$

$$Q_{20} = -2g_1^2\left[\frac{1}{2M_1} + \frac{\kappa_1}{e}\right]M_1,$$

$$Q_{21} = \tfrac{1}{2}(f_1^2 - g_1^2) + f_2(f_1 - g_1) + g_1(f_3 + g_2),$$

$$Q_{22} = (f_1 - g_1)(f_2 - g_2) + M_1\frac{\kappa_2}{e}(f_1 - g_1)^2 - M_1\frac{\kappa_1}{e} \times (f_1^2 - g_1^2),$$

$$Q_{23} = -(f_1 + g_1)(f_2 - g_2) + M_1\frac{\kappa_1}{e}(f_1 + g_1)^2 - M_1\frac{\kappa_2}{e} \times (f_1^2 - g_1^2),$$

$$Q_{24} = -(f_1 - g_1)^2 + g_1(2f_1 + 3f_2 + 2f_3 + g_2 - 2g_1) - f_1(f_2 + g_2) - M_1\frac{\kappa_1}{e}(f_1 - g_1)^2 + M_1\frac{\kappa_2}{e}(f_1^2 - g_1^2) - 4M_1\frac{\kappa_1}{e}g_1^2, \quad (57)$$

and

$$Q_{25} = -(f_1^2 - g_1^2) - (f_1 + g_1)(f_2 + g_2) + 2g_1(f_3 - f_2) + M_1\frac{\kappa_2}{e}(5g_1^2 + f_1^2 + 2f_1g_1) - M_1\frac{\kappa_1}{e}(f_1^2 - g_1^2). \quad (58)$$

Here, κ_1 and κ_2 denote the anomalous magnetic moments of the decaying and emitted baryons, respectively.

The tildes on Q_6 and Q_7 indicate that contributions of order $(q/M_1)^2$ and higher have been subtracted. Also, $Q_8, \ldots, Q_{25}$ have contributions only up to order q/M_1.

Although we have not made it explicit, in the above expressions the primed form factors, containing the model dependence of virtual RC should be used. This is valid to order $(\alpha/\pi)^2$ rearrangements. In the coefficients $Q_6, \ldots, Q_{15}$ we have used the Harrington's form factors F_i, G_i. They are related to the Dirac's form factors f_i, g_i as $F_1 = f_1 + (1 + M_2/M_1)f_2$, $G_1 = g_1 - (1 - M_2/M_1)g_2$, $F_2 = -2f_2$, $G_2 = -2g_2$, $F_3 = f_2 + f_3$, and $G_3 = g_2 + g_3$.

APPENDIX B: COLLECTION OF THE Λ_i FUNCTIONS

Here we give the analytical expressions of the Λ_i functions that appear in Sec. III in the analytical form of the RC to the polarized decay rate:

$$\Lambda_1 = -El\theta_0,$$

$$\Lambda_3 = \frac{E}{2}[(\beta^2 - 1)\chi_{12} + 2\chi_{11} - \chi_{10}],$$

$$\Lambda_4 = \frac{Elp_2^2}{2M_1}\left[2Y_2 - Y_3 - \frac{2\theta_0}{E}\right],$$

$$\Lambda_5 = \frac{El}{2M_1}\left[p_2^2Y_3 + 2Z_2 + \frac{2p_2l^2}{E}Y_1\right],$$

$$\Lambda_6 = \frac{l}{M_1}[p_2^2\theta_0 + (E + E_\nu^0)Z_1 - EZ_2 - l^2p_2Y_1],$$

$$\Lambda_7 = \frac{El}{2M_1}Z_1,$$

$$\Lambda_8 = -\frac{p_2^2l}{M_1}\theta_0,$$

$$\Lambda_9 = \tfrac{1}{2}lp_2^2\theta_3,$$

$$\Lambda_{10} = \tfrac{1}{2}l\zeta_{11},$$

$$\Lambda_{11} = \tfrac{1}{2}\beta p_2^2(\gamma_0 - E\theta_3),$$

$$\Lambda_{12} = \frac{1}{2}\left[-l\zeta_{10} - \beta Z_3 - \frac{X_3}{E} + \frac{1}{2}(\chi_{21} - \chi_{20}) + \frac{E_\nu^0}{E}X_2\right],$$

$$\Lambda_{13} = \frac{lp_2^2}{2M_1}\left\{Y_4 - \frac{X_2}{El} - 2\eta_0 - \frac{\chi_{21}}{2El} - \frac{E + E_\nu^0}{E}\left[\frac{1}{2}\theta_7 + E(\theta_4 - \theta_3)\right]\right\},$$

$$\Lambda_{14} = -\frac{lp_2^2}{2M_1}\left[\gamma_0 - \beta l\theta_3 + \frac{X_2}{El} - \eta_0\right] + \frac{1}{2}\frac{E+E_\nu^0}{M_1 E}X_3 - \frac{X_4}{2M_1} + \frac{p_2 l}{M_1}\left[-\frac{y_0}{E}X_2 + \frac{\eta_0}{4}(l(y_0-1) - 2p_2)\right],$$

$$\Lambda_{15} = \frac{\beta(E+E_\nu^0)}{4M_1}\left[p_2^2\theta_7 + 2p_2^2E(\theta_4-\theta_3) + 2\zeta_{21} - \frac{2}{l}X_3\right] + \frac{X_4}{2M_1} - \frac{p_2 l^2}{4M_1}(y_0^2-1) + \frac{p_2}{4M_1E}[4(ly_0 + p_2)X_2 + p_2\chi_{21}],$$

$$\Lambda_{16} = \frac{l}{4M_1}\zeta_{21},$$

$$\Lambda_{17} = -\frac{\beta p_2^2}{4M_1}\left[\frac{\chi_{21}}{l} - 2E\eta_0 + (E+E_\nu^0)[\theta_7 + 2E(\theta_4-\theta_3)]\right],$$

$$\Lambda_{18} = \frac{1}{4E}[X_3 - 2E_\nu^0 X_2],$$

$$\Lambda_{19} = \frac{E}{M_1}[\chi_{20} - \chi_{21} + 2E_\nu^0(\chi_{11}-\chi_{10}) + \beta(\zeta_{21} - 2E_\nu^0\zeta_{11})],$$

$$\Lambda_{20} = \frac{E}{M_1}\left[\beta l^2 p_2(Y_5 - Y_1) - (E_\nu^0 + l\beta)(\chi_{11} - \chi_{10} - \beta\zeta_{11}) + \beta(E+E_\nu^0)(\zeta_{11}-\zeta_{10}) + \frac{1}{2}l\beta p_2(1 - y_0)(\theta_0+\eta_0) + \beta^2 l p_2^2 I\right],$$

$$\Lambda_{21} = \frac{E}{M_1}\left[\beta l^2 p_2(Y_5+Y_1) - \frac{1}{2E}X_4 + lp_2^2Y_3 + lZ_2\right],$$

$$\Lambda_{22} = \frac{E}{M_1}\left[\frac{1}{2}\chi_{20} + \beta(\beta E_\nu^0 - l)\chi_{11} - \frac{1}{2}(1+\beta^2)\chi_{21} + \beta(E - E_\nu^0)(\zeta_{11}-\zeta_{10}) + \beta\zeta_{21} + \frac{1}{2}l\beta p_2(1-y_0^2)\right],$$

$$\Lambda_{23} = \frac{l}{M_1}\left[\frac{X_4}{2l} + (\beta E_\nu^0 + l)\chi_{11} - (E+E_\nu^0)(\zeta_{11}-\zeta_{10}) + \frac{1}{2}p_2 l(1-y_0^2)\right],$$

$$\Lambda_{24} = \frac{l}{M_1}\left[\frac{1}{2}p_2 l(y_0-1)\theta_0 + E_\nu^0\zeta_{10} - (E_\nu^0 + l\beta)\zeta_{11} - \beta l p_2^2 I\right],$$

$$\Lambda_{25} = \frac{p_2 l}{4M_1}\left[2y_0\chi_{11} + l(\theta_0 + 2\eta_0)(1-y_0) + \frac{2}{p_2}(E + E_\nu^0 - \beta p_2 y_0)\zeta_{11} - \frac{2E}{p_2}\zeta_{10} - (E_\nu^0 + \beta l)\frac{\chi_{21}}{p_2 l} + \beta(E+E_\nu^0)\frac{\zeta_{21}}{p_2 l} + N_1 + 2\beta l p_2 I\right],$$

$$\Lambda_{26} = \frac{1}{4M_1}[(E_\nu^0 - l\beta)\chi_{21} - \beta(E_\nu^0 - E)\zeta_{21} + p_2 l N_1 - \chi_{31} + \beta\zeta_{31}],$$

$$\Lambda_{27} = \frac{1}{4M_1}[E_\nu^0\chi_{20} + p_2 l N_2],$$

and

$$\Lambda_{28} = -\frac{p_2 l}{4M_1}N_2.$$

In these Λ_i we introduced the definitions

$$X_2 = \frac{m^2}{E}\chi_{12} - E\chi_{11} - \frac{1}{2}\chi_{21},$$

$$X_3 = \frac{m^2}{E}\chi_{22} - E\chi_{21} - \frac{1}{2}\chi_{31},$$

$$X_4 = \frac{m^2}{E}\chi_{21} - E\chi_{20},$$

$$Y_1 = \theta_{19} - \frac{l}{p_2}\theta_{20} - \frac{E_\nu^0}{p_2}\theta_{10},$$

$$Y_2 = 2\theta_3 + (\beta^2-1)\theta_2 - \theta_4,$$

$$Y_3 = (\beta^2-1)\theta_3 + \theta_4 + \beta\theta_5,$$

$$Y_4 = \gamma_0 - 3E\theta_3 + E\theta_4 + 2\theta_7 - 3l\theta_5 + (1-\beta^2)(2E\theta_2 - \theta_6) + \frac{1}{2E}\theta_9,$$

$$Y_5 = \theta_{19} - \frac{1}{2p_2}\left[\theta_{21} + \frac{E_\nu^0}{l}\theta_{14}\right] - 2Y_1 + y_0\theta_5,$$

$$Z_1 = (\beta^2-1)\zeta_{12} + 2\zeta_{11} - \zeta_{10},$$

$$Z_2 = (\beta^2-1)\zeta_{11} + \zeta_{10},$$

$$Z_3 = \frac{m^2}{E}\zeta_{12} - E\zeta_{11} - \frac{1}{2}\zeta_{21},$$

$$N_1 = l\eta_0\left[\frac{3}{2}(1-y_0) - \frac{p_2}{l}\right],$$

$$N_2 = l\eta_0\left[\frac{1}{2}(y_0-1) + \frac{p_2}{l}\right],$$

and

$$\gamma_0 = -\frac{m^2}{E}\theta_2 + E\theta_3 + \frac{1}{2}\theta_7.$$

η_0 is defined as $\eta_0 = 1 + y_0$. All of the quantities

ζ_{pq}, χ_{mn} except ζ_{31} as functions of the $\theta_1, \ldots, \theta_{18}$ come from previous work [8]. The $\theta_0, \ldots, \theta_{18}$ are found in Refs. [7,8]. ζ_{31}, I, $\theta_{19}, \ldots, \theta_{22}$ are all new functions and they are given by

$$I = \frac{3}{2\beta p_2}(E + E_\nu^0)(\theta_{13} - \theta_{12}) + \frac{1}{2}y_0\theta_{12} + \frac{\beta E_\nu^0 + l - p_2}{2\beta p_2^2}\theta_0 + \frac{\eta_0 E_\nu^0}{p_2^2} + \frac{1}{2p_2^2\beta^2}[3(E_\nu^0)^2 - l^2 + 3E(E + 2E_\nu^0)](\theta_3 - \theta_4 - \beta\theta_5) + \frac{EE_\nu^0}{p_2^2}(\theta_4 - \theta_3) - \frac{(E_\nu^0)^2}{2p_2^2}\theta_3 + \frac{3E}{2p_2}Y_1 - \frac{3E(E + E_\nu^0)}{2p_2^2}\theta_{10} + \frac{1}{2\beta^2}Y_3,$$

$$\zeta_{31} = p_2 l y_0[2(3E^2 - l^2)\theta_3 - 6E^2(\theta_4 + \beta\theta_5) + \theta_9] - 30lE^2 p_2\theta_{13} - 30l^2 E p_2\theta_{19} - \frac{6l^3}{\beta^4}[5(l + \beta E_\nu^0) + 3\beta^2(p_2 y_0 - l)] \times (\theta_3 - \theta_4 - \beta\theta_5) - 18l^2 E E_\nu^0(\theta_4 - \theta_3) + 6p_2 l^3 y_0\theta_3 + 30lE^2(l + \beta E_\nu^0)\theta_{10} + 30El^3\theta_{20} - \frac{1}{2}\theta_{22} - 6p_2\left[l^2E(\beta^2 - 5) - \frac{2lp_2^2 + 2\beta p_2 l y_0(E + E_\nu^0)}{b^+ b^-}\right]\theta_{12}.$$

The functions $\theta_{19}, \ldots, \theta_{22}$ are

$$\theta_{19} = \frac{1}{p_2}(T_{19}^+ + T_{19}^-), \qquad \theta_{20} = \frac{1}{p_2}(T_{20}^+ + T_{20}^-), \qquad \theta_{21} = \frac{1}{p_2}(T_{21}^+ + T_{21}^-),$$

and

$$\theta_{22} = \frac{1}{p_2}(T_{22}^+ + T_{22}^-),$$

with

$$T_{19}^\pm = \frac{1}{3p_2}\left[p_2 - l + \frac{1}{2}E_\nu^0(x_0^2 - 3)x_0\right],$$

$$T_{20}^\pm = \frac{1}{4}\left[x_0^4 \ln\left|\frac{1 \pm x_0}{\pm x_0 \pm a^\pm}\right| + (a^\pm)^4 \ln\left|\frac{\pm x_0 \pm a^\pm}{1 \pm a^\pm}\right| + \ln\left|\frac{1 \pm a^\pm}{1 \pm x_0}\right| - \frac{1}{3}(1 \mp x_0^3)(1 \mp a^\pm) + \frac{1}{2}(1 - x_0^2)[1 - (a^\pm)^2] - (1 \mp x_0)[1 \mp (a^\pm)^3]\right],$$

$$T_{21}^\pm = \tfrac{2}{3}[p_2 - l - E_\nu^0 x_0^3] \mp p_2 y_0^\pm a^\pm[a^\pm I_2^\pm - 2] - E_\nu^0 x_0^\pm[2a^\pm x_0 - x_0^2 + 1 + (a^\pm)^2 J_2^\pm],$$

and

$$\frac{T_{22}^\pm}{2l} = \frac{2}{\beta}[(E_\nu^0)^3 x_0 + (l - p_2)^3] - 6p_2 E\eta_0(p_2 + l) + 12\eta_0 p_2 l a^\pm\left[\pm E + p_2\frac{y_0^\pm}{b^\pm}\right] + \frac{[E_\nu^0(1 - \beta x_0)]^3}{\beta(b^\pm)^2}J_1 + \frac{1}{b^\pm}[\mp 3(a^\pm \mp 1)\eta_0 p_2 l(\beta\eta_0 p_2 + 2E + 2E_\nu^0)]I_1 + \frac{1}{(b^\pm)^2}\left[\mp(p_2\eta_0)^3\beta^2 - \frac{1}{\beta}(E_\nu^0\beta + l - p_2)^3 - 3p_2\eta_0(E_\nu^0\beta + l - p_2) \times (p_2\beta y_0 + E_\nu^0 + E)\right]I_1 + \left[\frac{3\eta_0 p_2}{\beta}(E(l - p_2) + p_2(E_\nu^0 \mp 2l \pm 2p_2)) - 3\frac{(a^\pm \mp 1)}{b^\pm}p_2^2 l(\pm\eta_0^2 + 2a^\pm(\eta_0 + y_0^\pm))\right]I_2^\pm + \frac{1}{(b^\pm)^2}[\mp\beta(p_2\eta_0)^3 - p_2^2(a^\pm \mp 1)^2(3(E_\nu^0\beta + l - p_2) - 2p_2\beta(a^\pm \mp 1))]I_2^\pm + \frac{1}{(b^\pm)^2}\left[-\frac{3p_2\eta_0}{\beta} \times (E_\nu^0\beta + l - p_2)(p_2\beta y_0 + E_\nu^0 + E)\right]I_2^\pm - \frac{(E_\nu^0 x_0^\pm)^3}{b^\pm}(J_3^\pm \pm I_3^\pm) + \left[\frac{(E_\nu^0)^3(x_0^\pm)^2}{(b^\pm)^2}(3 - \beta x_0 + 2\beta a^\pm) - 6p_2 l E_\nu^0\left(\frac{a^\pm y_0^\pm x_0^\pm}{b^\pm}\right)\right]J_2^\pm,$$

where $y_0^\pm = y_0 \pm a^\pm$, $b^\pm = 1 + \beta a^\pm$, and $x_0^\pm = x_0 + a^\pm$. The functions $a^\pm$, x_0, I_1, $I_2^\pm$, $I_3^\pm$, J_1, $J_2^\pm$, and $J_3^\pm$ are found in Ref. [8].

APPENDIX C: COLLECTION OF THE ρ_i FUNCTIONS

Here we give the analytical results for the ρ_i functions after performing the integrals displayed in Eqs. (34)–(39).

$$\rho_I = \frac{l}{M_1}\left\{E_\nu^0 p_2(y_0-1)\left[\frac{1-\beta^2}{2\beta}\theta_0 - \beta\eta_0\right] + (E+E_\nu^0)(\zeta_{10}-\zeta_{11}) + \frac{p_2 l}{2}(y_0-1)\theta_0 + l^2 p_2 Y_1 - \beta l p_2^2 I\right\},$$

$$\rho_I' = \frac{El}{M_1}[(E_\nu^0+E)(\beta p_2\theta_{12}-\theta_0) - p_2 l\theta_{13}],$$

$$\rho_{II} = \frac{p_2 l}{2M_1}\left\{\frac{p_2 E_\nu^0}{2}\theta_4 + \frac{\eta_0}{4}[(y_0-1)(2E_\nu^0\beta - 3l) - 2p_2] - \frac{E_\nu^0}{2\beta p_2}\chi_{10} + \frac{1}{4\beta p_2}\chi_{20} + \frac{1}{2}\left[y_0 + \frac{E_\nu^0(E_\nu^0+E)}{p_2 l}\right]\chi_{11} - \frac{E_\nu^0+E}{4p_2 l}\chi_{21} + \frac{y_0-1}{4\beta}[(\beta^2-1)E_\nu^0 - \beta l]\theta_0 + \frac{E_\nu^0-E}{2p_2}\zeta_{10} + \frac{1}{2}\left[\frac{E^2-(E_\nu^0)^2}{Ep_2} - \beta y_0\right]\zeta_{11} + \frac{E_\nu^0+E}{4Ep_2}\zeta_{21} + \frac{\beta p_2 l}{2}I\right\},$$

$$\rho_{II}' = \frac{p_2 l}{8M_1}\left[2(p_2+ly_0)(\theta_0-\beta p_2\theta_{12}) + l(1-y_0^2) + 2l^2Y_5 + \frac{E+E_\nu^0}{p_2 l}(\beta\zeta_{21}+\chi_{20}-\chi_{21})\right],$$

$$\rho_{III} = \frac{p_2 l}{2M_1}\left[\frac{E}{2p_2 l}[2E_\nu^0(1-\beta^2)\chi_{11} - 2E_\nu^0\chi_{10} - (1-\beta^2)\chi_{21} + \chi_{20}] - \frac{2E_\nu^0}{p_2}\zeta_{10} + \frac{l(y_0^2-1)}{2} + \frac{E_\nu^0+\beta l}{p_2}\zeta_{11} + \beta p_2 lI - \frac{l(y_0-1)}{2}\theta_0\right] - \frac{p_2 l}{4M_1}\eta_0[l(y_0-1)+2p_2] - \frac{E_\nu^0}{2M_1}\chi_{20},$$

and

$$\rho_{III}' = \frac{p_2 l}{2M_1}\left\{E(p_2-ly_0)\theta_4 + \frac{EE_\nu^0}{p_2}\eta_0 + l(p_2+ly_0)\theta_5 - \frac{p_2 m^2}{E}\theta_3 - \frac{E}{p_2}\left[2-\beta^2+\frac{E_\nu^0}{E}\right]\zeta_{11} + \frac{3E}{p_2}\zeta_{10} + \frac{E}{p_2 l}(E_\nu^0 + \beta l)\chi_{11} - \frac{1-\beta^2}{2\beta p_2}\chi_{21} + \frac{E}{p_2 l}\chi_{20} - \frac{EE_\nu^0}{p_2 l}\chi_{10} - \frac{l^2}{2p_2}(\theta_{21}-2l\theta_{20}) - \frac{l}{2p_2}(E_\nu^0-E)(\theta_{14}-2l\theta_{10}) + \frac{\eta_0}{2}[l(y_0-1) + 2p_2]\right\}.$$

All the algebraic expressions that appear in these ρ_i and ρ_i' are defined in Appendix B.

APPENDIX D: NEW AND PREVIOUS ANALYTICAL INTEGRALS

In this Appendix, we give a brief discussion that allows the identification of previous and new analytical integrals over the photon variables that emerge in the present calculation. Such integrals can all be put into the general form

$$\int_{-1}^{y_0} dy F^p \int_{-1}^{1} dx \int_0^{2\pi} d\varphi_k \frac{x^r \hat{\mathbf{s}}_1\cdot\hat{\mathbf{a}}}{D^m(1-\beta x)^n},$$

with $\hat{\mathbf{a}} = \hat{\mathbf{p}}_2$, $\hat{\mathbf{l}}$, $\hat{\mathbf{k}}$ and $x = \hat{\mathbf{l}}\cdot\hat{\mathbf{k}}$. F and D were defined after Eq. (19). The powers of x, F, D, and $(1-\beta x)$ are denoted by $\{rpmn\}$. Since each one of these quantities is a rotational scalar, one may change the orientation of the space axes. For $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{a}}$ we use the rule [8]

$$\hat{\mathbf{s}}_1\cdot\hat{\mathbf{a}} \to (\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2)(\hat{\mathbf{a}}\cdot\hat{\mathbf{p}}_2),$$

where $\hat{\mathbf{a}} = \hat{\mathbf{p}}_2$, $\hat{\mathbf{l}}$, or $\hat{\mathbf{k}}$. This considerably simplifies the calculation of such integrals. One can classify these integrals into three groups. In the first one we can directly identify integrals previously performed. This occurs for $\hat{\mathbf{a}} = \hat{\mathbf{p}}_2$ and $\{rpmn\} = \{0012, 0011, 0010\}$, then one identifies θ_2, θ_3 and θ_4 of Ref. [8]. For $\hat{\mathbf{a}} = \hat{\mathbf{l}}$ and $\{rpmn\} = \{0010, 0011, 0012, 0121\}$ one identifies the functions ζ_{pq}. For $\hat{\mathbf{a}} = \hat{\mathbf{k}}$ and $\{rpmn\} = \{0010, 0120, 0011, 0012, 0121, 0122, 0231\}$ one identifies the functions χ_{pq}.

In the second group are new integrals that can be expressed as combinations of previous results in terms of $\eta_0 = 1 + y_0$ and $\theta_0, \theta_2, \ldots, \theta_{18}$ also of Ref. [8]. This occurs for $\hat{\mathbf{a}} = \hat{\mathbf{p}}_2$ and $\{rpmn\} = \{0000, 0001, 00(-1)1, 0111, 0121, 0122, 0231, 1010\}$, for $\hat{\mathbf{a}} = \hat{\mathbf{l}}$ and $\{rpmn\} = \{0000, 0001, 0002, 0120, 0101\}$, and for $\hat{\mathbf{a}} = \hat{\mathbf{k}}$ and $\{rpmn\} = \{0001, 1010, 0111, 0112, 0230\}$. Omitting details, such combinations are accommodated into the Λ_i of Appendix B.

The third group contains, after applying the above rule, only four new integrals with $\hat{\mathbf{a}} = \hat{\mathbf{p}}_2$ and the powers are $\{rpmn\} = \{0211, 2120, 1110, 0331\}$. The first three of them are straightforward. Explicitly they are

$$\int_{-1}^{1} dx \frac{1}{1-\beta x} \int_{-1}^{y_0} dy F^2 \int_0^{2\pi} d\varphi_k \frac{1}{D} = 2\pi(2p_2 l)^2\left[I + y_0^2\theta_3 - \frac{2y_0}{p_2 l}\zeta_{11}\right],$$

where

$$I = \int_{-1}^{1} dx \frac{1}{1-\beta x} \int_{-1}^{y_0} dy \frac{y^2}{\sqrt{R}},$$

$$\int_{-1}^{1} dx x^2 \int_{-1}^{y_0} dy F \int_0^{2\pi} d\varphi_k \frac{1}{D^2} = 2\pi(\theta_{21} - 2l\theta_{20}),$$

and

$$\int_{-1}^{1} dx x \int_{-1}^{y_0} dy F \int_0^{2\pi} d\varphi_k \frac{1}{D} = 2\pi(2p_2 l)(y_0\theta_5 - Y_1).$$

θ_{20} and θ_{21} are new and they are listed below. θ_5 is found in [8] and Y_1 is found in Appendix B. The computation of the fourth integral,

$$J = \int_{-1}^{1} dx \frac{1}{1-\beta x} \int_{-1}^{y_0} dy F^3 \int_0^{2\pi} d\varphi_k \frac{1}{D^3},$$

although long and tedious, can be performed by using standard techniques. The final result can be organized as

$$\frac{J}{(2\pi)(12 l^3)} = \frac{1}{\beta^4}[5(l + \beta E_\nu^0) + 3\beta^2(p_2 y_0 - l)](\theta_3 - \theta_4 - \beta\theta_5) + \frac{3E_\nu^0}{\beta}(\theta_4 - \theta_3) - p_2 y_0\theta_3 - \frac{5}{\beta^2}(l + \beta E_\nu^0)\theta_{10} - 5E\theta_{20} + \frac{1}{12 l^3}\theta_{22} + \frac{5p_2}{\beta^2}\theta_{13} + \frac{5p_2}{\beta}\theta_{19} + p_2\left[\frac{\beta^2 - 5}{\beta} - \frac{2p_2^2 + 2\beta p_2 y_0(E + E_\nu^0)}{l^2 b^+ b^-}\right]\theta_{12}.$$

The functions $\theta_{19}, \ldots, \theta_{22}$ in the four new integrals are

$$\theta_{19} = \int_{-1}^{1} x\xi_4(x)dx, \qquad \theta_{20} = \int_{-1}^{1} x^3\xi_1(x)dx,$$

$$\theta_{21} = \int_{-1}^{1} x^2\xi_2(x)dx,$$

and

$$\theta_{22} = \int_{-1}^{1} \frac{\xi_6(x)}{1-\beta x}dx.$$

The other $\xi_1(x)$, $\xi_2(x)$, and $\xi_4(x)$ are used in Ref. [6]; the function $\xi_6(x)$ is new and it reads

$$\frac{\xi_6(x)}{2l} = p_2^2\eta_0^3\left[\frac{1}{(x+a^-)^2} - \frac{1}{(x+a^+)^2}\right] + 3l\eta_0[p_2\eta_0 + 2x(E_\nu^0 + lx)]\left[\frac{a^- + 1}{x + a^-} - \frac{a^+ - 1}{x + a^+}\right] - 3\eta_0[p_2 y_0 + x(E_\nu^0 + lx)] \times [E_\nu^0 + (l - p_2)x]\left[\frac{1}{(x+a^-)^2} + \frac{1}{(x+a^+)^2}\right] + \frac{(E_\nu^0)^3}{p_2}[|x - x_0|^3 - (1 + xd)^3]\left[\frac{1}{(x+a^-)^2} + \frac{1}{(x+a^+)^2}\right] - 6lp_2^2\left[-\frac{a^+ y_0^+}{b^+(x+a^+)} - \frac{a^- y_0^-}{b^-(x+a^-)}\right](1 - \beta x)\xi_4(x),$$

where $d = (l - p_2)/E_\nu^0$.

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