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Classification of supersymmetric and nonsupersymmetric chiral models from Abelian orbifolds AdS/CFT

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We classify compactifications of the type IIB superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5/\Gamma$, where Γ is an abelian group of order $n \leq 12$. Appropriate embedding of Γ in the isometry of S^5 yields both SUSY and non–SUSY chiral models that can contain the minimal SUSY standard model or the standard model. New non-SUSY three-family models with $\Gamma = Z_8$ are introduced, which lead to the right Weinberg angle for TeV trinification. These models form a small but interesting subclass of the set of $\mathrm{SU}^n(N)$ gauge theories with bifundamental matter.

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I. INTRODUCTION

When one bases models on conformal field theory gotten from the large N expansion of the anti-de Sitter/ conformal field theory (AdS/CFT) correspondence [1], stringy effects can arise at an energy scale as low as a few TeV. These models can potentially test string theory and examples with low energy scales are known in orbifolded $\mathrm{AdS}_5 \times \mathrm{S}^5$. The first three-family model of this type had $\mathcal{N} = 1$ SUSY and was based on a Z_3 orbifold [2], see also [3]. However, since then some of the most studied examples have been models without supersymmetry based on both abelian [4], [5], [6] and nonabelian [7], [8] orbifolds of $\mathrm{AdS}_5 \times \mathrm{S}^5$. Recently both SUSY and non–SUSY three-family Z_{12} orbifold models [9,10] have been shown to unify at a low scale ($\sim$ 4 TeV) and to have promise of testability. One motivation for studying the non–SUSY case is that the need for supersymmetry is less clear as: (1) the hierarchy problem is absent or ameliorated[1], (2) the difficulties involved in breaking the remaining $\mathcal{N} = 1$ SUSY can be avoided if the orbifolding already results in $\mathcal{N} = 0$ SUSY, and (3) many of the effects of SUSY are still present in the theory, just hidden. For example, the bose-fermi state count matches, renormalization group (RG) equations preserve vanishing β functions to some number of loops, etc., Here we concentrate on abelian orbifolds with and without supersymmetry, where the orbifolding group Γ has order $n = o(\Gamma) \leq 12$. We systematically study those cases with chiral matter (i.e., in the SUSY case, those with an imbalance between chiral supermultiplets and antichiral supermultiplets, and in the non–SUSY case with a net imbalance between left and right handed fermions). We find all chiral models for $n \leq 12$. Several of these contain the standard model (SM) or the minimal supersymmetric standard model (MSSM) with three or four families.

Before giving the details of model construction, let us pause to put our overall motivation into context. Many of the models we will construct can be considered from a purely quantum field theory perspective, and are the zero slope limit of the string theory construction. All the models will have gauge group $\mathrm{SU}^n(N)$, and all matter representations will be bifundamental. However, the string compactification procedure puts constraints on the allowed models, for instance when $\Gamma = Z_n$ the embedding into the isometry of S^5 is only consistent if it involves a partition of n, as described below. Other models, e.g., the nonpartition models defined below, that do not satisfy this criteria, are perfectly good quantum field theories (QFTs) and can have some of the desirable features of the orbifolded $\mathrm{AdS}_5 \times \mathrm{S}^5$ models. However, one would expect the models that do not satisfy all the string theory model criteria to have a lower probability of incorporating all the good behavior, e.g., they could lack radiative stability. Note the nonpartition models will fail one of the tests of proper string theory embedding; other QFT models with $\mathrm{SU}^n(N)$ gauge groups and bifundamental matter can easily be generated, but they will in general fail more of the tests of proper embedding with concomitant deleterious effects.

We begin with a summary of how orbifolded $\mathrm{AdS}_5 \times \mathrm{S}^5$ models are constructed (for more details see [8]). First we select a discrete subgroup Γ of the SO(6) $\sim$ SU(4) isometry of S^5 with which to form the orbifold $\mathrm{AdS}_5 \times \mathrm{S}^5/\Gamma$. The replacement of S^5 by S^5/Γ reduces the supersymmetry to $\mathcal{N} = 0$, 1 or 2 from the initial $\mathcal{N} = 4$, depending on how Γ is embedded in the isometry of S^5. The cases of interest here are $\mathcal{N} = 0$ and $\mathcal{N} = 1$ SUSY where Γ embeds irreducibly in the SU(4) isometry or in an SU(3) subgroup of the SU(4) isometry, respectively. I.e., to achieve $\mathcal{N} = 0$ we embed rep. $(\Gamma) \rightarrow \mathbf{4}$ of SU(4) as

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[1]Compare however the discussion in [11].

$\mathbf{4} = (\mathbf{r})$ where $\mathbf{r}$ is a nontrivial four dimensional representation of Γ; for $\mathcal{N} = 1$ we embed rep. $(\Gamma) \rightarrow \mathbf{4}$ of SU(4) as $\mathbf{4} = (\mathbf{1}, \mathbf{r})$ where $\mathbf{1}$ is the trivial irreducible representation (irrep.) of Γ and $\mathbf{r}$ is a nontrivial three dimensional representation of Γ.

For $\mathcal{N} = 0$ the fermions are given by $\sum_i \mathbf{4} \otimes R_i$ and the scalars by $\sum_i \mathbf{6} \otimes R_i$ where the set $\{R_i\}$ runs over all the irreps. of Γ. For abelian Γ, the irreps. are all one dimensional and as a consequence of the choice of N in the $1/N$ expansion, the gauge group [12] is $SU^n(N)$. In the $\mathcal{N} = 1$ SUSY case, chiral supermultiplets generated by this embedding are given by $\sum_i \mathbf{4} \otimes R_i$ where again $\{R_i\}$ runs over all the (irreps.) of Γ. Again for abelian Γ, the irreps. are all one dimensional and the gauge group is again $SU^n(N)$. Chiral models require the $\mathbf{4}$ to be complex ($\mathbf{4} \neq \mathbf{4}^*$) while a proper embedding requires $\mathbf{6} = \mathbf{6}^*$ where $\mathbf{6} = (\mathbf{4} \otimes \mathbf{4})_{\text{antisym}}$. (Even though the $\mathbf{6}$ does not enter the model in the $\mathcal{N} = 1$ SUSY case, mathematical consistency requires $\mathbf{6} = \mathbf{6}^*$, see [13].)

We now have the required background to begin building chiral models. We choose $N = 3$ throughout. If $SU_L(2)$ and $U_Y(1)$ are embedded in diagonal subgroups $SU^p(3)$ and $SU^q(3)$ respectively, of the initial $SU^n(3)$, the ratio $\frac{\alpha_2}{\alpha_Y}$ is $\frac{p}{q}$, leading to a calculable initial value of θ_W with, $\sin^2\theta_W = 3/[3 + 5(\frac{p}{q})]$. The more standard approach is to break the initial $SU^n(3)$ to $SU_C(3) \otimes SU_L(3) \otimes SU_R(3)$ where $SU_L(3)$ and $SU_R(3)$ are embedded in diagonal subgroups $SU^p(3)$ and $SU^q(3)$ of the initial $SU^n(3)$. We then embed all of $SU_L(2)$ in $SU_L(3)$ but $\frac{1}{3}$ of $U_Y(1)$ in $SU_L(3)$ and the other $\frac{2}{3}$ in $SU_R(3)$. This modifies the $\sin^2\theta_W$ formula to: $\sin^2\theta_W = 3/[3 + 5(\frac{\alpha_2}{\alpha_Y})] = 3/[3 + 5(\frac{3p}{p+2q})]$, which coincides with the previous result when $p = q$. One should use the second (standard) embedding when calculating $\sin^2\theta_W$ for any of the models obtained below. A similar relation holds for Pati-Salam type models [14] and their generalizations [15], but this would require investigation of models with $N \geq 4$ which are not included in this study. Note, if $\Gamma = Z_n$ the initial $\mathcal{N} = 0$ orbifold model (before any symmetry breaking) is completely fixed (recall we always are taking $N = 3$) by the choice of n and the embedding $\mathbf{4} = (\alpha^i, \alpha^j, \alpha^k, \alpha^l)$, so we define these models by M^n_{ijkl}. The conjugate models $M^n_{n-i,n-j,n-k,n-l}$ contain the same information, so we need not study them separately.

As we have previously studied chiral $\Gamma = Z_n$ models with $\mathcal{N} = 1$ SUSY, we first summarize those results before concentrating on $\mathcal{N} = 0$. At the end we consider both $\mathcal{N} = 1$ and $\mathcal{N} = 0$ models where Γ is abelian but not a single Z_n. For instance $\Gamma = Z_3 \times Z_3 \neq Z_9$.

II. SUMMARY OF $\mathcal{N} = 1$ CHIRAL Z_N MODELS

To tabulate the possible models for each value of n, we first show that a proper embedding (i.e., $\mathbf{6} = \mathbf{6}^*$) for $\mathbf{4} = (\mathbf{1}, \alpha^i, \alpha^j, \alpha^k)$ results when $i + j + k = n$. To do this we use the fact that the conjugate model has $i \rightarrow i' = n - i$, $j \rightarrow j' = n - j$ and $k \rightarrow k' = n - k$. Summing we find $i' + j' + k' = 3n - (i + j + k) = 2n$. But from $\mathbf{6} = (\mathbf{4} \otimes \mathbf{4})_{\text{antisym}}$ we find $\mathbf{6} = (\alpha^i, \alpha^j, \alpha^k, \alpha^{j+k}, \alpha^{i+k}, \alpha^{i+j})$, but $i + j = n - k = k'$. Likewise $i + k = j'$ and $j + k = i'$ so $\mathbf{6} = (\alpha^i, \alpha^j, \alpha^k, \alpha^{i'}, \alpha^{j'}, \alpha^{k'})$ and this is $\mathbf{6}^*$ up to an automorphism which is sufficient to provide a proper embedding (or to provide real scalars in the non-SUSY models). Models with $i + j + k = n$ (we will call these partition models) are always chiral, with total chirality (number of chiral states) $\chi = 3N^2n$ except in the case where n is even and one of i, j, or k is $n/2$ where $\chi = 2N^2n$. (No more than one of i, j, and k can be $n/2$ since they sum to n and are all positive.) This immediately gives us a lower bound on the number of chiral models at fixed n. It is the number of partitions of n into three non-negative integers. There is another class of models with $i' = k$ and $j' = 2j$, and total chirality $\chi = N^2n$; for example a Z_9 orbifold with $\mathbf{4} = (\mathbf{1}, \alpha^3, \alpha^3, \alpha^6)$. And there are a few other sporadically occurring cases like M^6_{124}, which typically have reduced total chirality, $\chi < 3N^2n$. Such "nonpartition"—i.e. neither partition nor double partition—models can fail other more subtle constraints on consistent embedding [13], but we list them here because they have vanishing anomaly coefficients and vanishing one loop β functions, and so are still of phenomenological interest from the gauge theory model building perspective.

We now list all the $\mathcal{N} = 1$, Z_n orbifold models up to $n = 12$ along with the total chirality of each model, (see Table I).

A systematic search through $n \leq 7$ yields four models that can result a in three-family MSSM. They are M^3_{111}, M^5_{122}, M^6_{123}, and M^7_{133}. There may be many more models with sensible phenomenology at larger n, and we have given one example M^9_{333}, with particularly simple spontaneous symmetry breaking, that is also a member of an infinite series of models $M^n_{(n/3)(n/3)(n/3)}$, which all can lead to three-family MSSMs. The value of $\sin^2\theta_W$ at $SU^n(3)$ unification was calculated for all these three-family models in [3]. This completes the summary of $\mathcal{N} = 1$ chiral Z_n models, so we now proceed to investigate chiral Z_n models with no remaining supersymmetry.

III. $\mathcal{N} = 0$ CHIRAL Z_n MODELS

We begin this section by studying the first few $\mathcal{N} = 0$ chiral Z_n models. Insights gained here will allow us to generalize and give results to arbitrary n. First, the allowed $\Gamma = Z_2$ and Z_3, $\mathcal{N} = 0$ orbifolds have only real representations and therefore will not yield chiral models. Next, for $\Gamma = Z_4$ the choice $\mathbf{4} = (\alpha, \alpha, \alpha, \alpha)$ with $N = 3$ where $\alpha = e^{\pi i/2}$ (in what follows we will write $\alpha = e^{2\pi i/n}$ for the roots of unity that generate Z_n), yields an

TABLE I. All $\mathcal{N} = 1$ chiral Z_n orbifold models with $n \leq 12$. Three of the $n = 8$ models have $\chi/N^2 = 24$; the other two have $\chi/N^2 = 16$. Of the 12 models with $i + j + k = 12$, three have models $\chi/N^2 = 24$ and the other nine have $\chi/N^2 = 36$. Of the 60 models 53 are partition models, while the remaining seven models that do not satisfy $i + j + k = n$, are marked with an asterisk (*).

n	4	χ/N^2	comment
3	$(1, \alpha, \alpha, \alpha)$	9	$i + j + k = 3$; one model ($i = j = k = 1$)
3	$(1, \alpha, \alpha, \alpha^2)^*$	3	
4	$(1, \alpha, \alpha, \alpha^2)$	8	$i + j + k = 4$; one model
5	$(1, \alpha^i, \alpha^j, \alpha^k)$	15	$i + j + k = 5$; 2 models
6	$(1, \alpha^i, \alpha^j, \alpha^k)$	12	$i + j + k = 6$; 3 models
6	$(1, \alpha, \alpha^2, \alpha^4)^*$	6	
6	$(1, \alpha^2, \alpha^2, \alpha^4)^*$	6	
7	$(1, \alpha^i, \alpha^j, \alpha^k)$	21	$i + j + k = 7$; 4 models
8	$(1, \alpha^i, \alpha^j, \alpha^k)$	≤ 24	$i + j + k = 8$; 5 models
9	$(1, \alpha^i, \alpha^j, \alpha^k)$	27	$i + j + k = 9$; 7 models
9	$(1, \alpha, \alpha^4, \alpha^7)^*$	27	
9	$(1, \alpha^3, \alpha^3, \alpha^6)^*$	9	
10	$(1, \alpha^i, \alpha^j, \alpha^k)$	30	$i + j + k = 10$; 8 models
11	$(1, \alpha^i, \alpha^j, \alpha^k)$	33	$i + j + k = 11$; 10 models
12	$(1, \alpha^i, \alpha^j, \alpha^k)$	≤ 36	$i + j + k = 12$; 12 models
12	$(1, \alpha^2, \alpha^4, \alpha^8)^*$	12	
12	$(1, \alpha^4, \alpha^4, \alpha^8)^*$	12	

$SU^4(3)$ chiral model with the fermion content shown in Table II.

The scalar content of this model is given in Table III and a VEV for say a $(3, 1, \bar{3}, 1)$ breaks the symmetry to $SU_D(3) \times SU_2(3) \times SU_4(3)$ but renders the model vector-like, and hence uninteresting, so we consider it no further. The only other choice of embedding is a nonpartition model with $\Gamma = Z_4$ is $\mathbf{4} = (\alpha, \alpha, \alpha, \alpha^3)$ but it leads to the same scalars with half the chiral fermions so we move on to Z_5.

There is one chiral model for $\Gamma = Z_5$ and it is fixed by choosing $\mathbf{4} = (\alpha, \alpha, \alpha, \alpha^2)$, leading to $\mathbf{6} = (\alpha^2, \alpha^2, \alpha^2, \alpha^3, \alpha^3, \alpha^3)$ with real scalars. It is straightforward to write down the particle content of this M^5_{1112} model. The best one can do toward the construction of the standard model is to give a VEV to a $(3, 1, \bar{3}, 1, 1)$ to break the $SU^5(3)$ symmetry to $SU_D(3) \times SU_2(3) \times SU_4(3) \times SU_5(3)$. Now a VEV for $(1, 3, \bar{3}, 1)$ completes the breaking to $SU^3(3)$, but the only remaining chiral fermions are $2[(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3)]$ which contains only two families.

Moving on to $\Gamma = Z_6$ we find two models where, as with the previous Z_5 model, the $\mathbf{4}$ is arranged so that $\mathbf{4} = (\alpha^i, \alpha^j, \alpha^k, \alpha^l)$ with $i + j + k + l = n$. These have $\mathbf{4} = (\alpha, \alpha, \alpha, \alpha^3)$ and $\mathbf{4} = (\alpha, \alpha, \alpha^2, \alpha^2)$ and were defined as partition models in [3] when i was equal to zero. Here we generalize and call all models satisfying $i + j + k + l = n$ partition models. We have now introduced most of the background and notation we need, so at this point (before completing the investigation of the $\Gamma = Z_6$ models) it is useful to give a summary (see Table IV) of all $\mathcal{N} = 0$ chiral Z_n models with real $\mathbf{6}$'s for $n \leq 12$. We note that the $n = 8$ partition model with $\mathbf{4} = (\alpha, \alpha, \alpha^2, \alpha^4)$ has $\chi/N^2 = 16$; the other four have $\chi/N^2 = 32$. Of the nine Z_{10} partition models, two have $\chi/N^2 = 30$ and the other seven have $\chi/N^2 = 40$. The Z_{12} partition models derived from $\mathbf{4} = (\alpha, \alpha, \alpha^4, \alpha^6)$, $\mathbf{4} = (\alpha, \alpha^2, \alpha^3, \alpha^6)$, and $\mathbf{4} = (\alpha^2, \alpha^2, \alpha^2, \alpha^6)$ have $\chi/N^2 = 36$; the others have $\chi/N^2 = 48$.

TABLE II. Fermion content for the model M^4_{1111}. The $\times^4$ entry at the $(1, \alpha)$ position means the model contains $4(3, \bar{3}, 1, 1)$ of $SU^4(3)$, etc. Hence, the fermions in this table are $4[(3, \bar{3}, 1, 1) + (1, 3, \bar{3}, 1) + (1, 1, 3, \bar{3}) + (\bar{3}, 1, 1, 3)]$. Diagonal entries do not occur in this model but, if they did, an $\times$ at say $((\alpha^2, \alpha^2)$ would correspond to $(1, 8 + 1, 1, 1)$, etc., see models below.

$M^4_{1111}(F)$	1	α	α^2	α^3
1		$\times^4$		
α			$\times^4$	
α^2				$\times^4$
α^3	$\times^4$			

TABLE III. Scalar content of the model M^4_{1111}.

$M^4_{1111}(F)$	1	α	α^2	α^3
1			$\times^6$	
α				$\times^6$
α^2	$\times^6$			
α^3		$\times^6$		

TABLE IV. All chiral $\mathcal{N} = 0$, Z_n orbifold models with $n \leq 12$. The 13 nonpartition models are marked with an asterisk (*). For further explanations see text.

n	4	χ/N^2	comment
4	$(\alpha, \alpha, \alpha, \alpha)$	16	$i + j + k + l = 3$; one model $(i = j = k = l)$
4	$(\alpha, \alpha, \alpha, \alpha^3)^*$	8	nonpartition model
5	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	20	$i + j + k + l = 5$; 1 model
6	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	≤ 24	$i + j + k + l = 6$; 2 models
6	$(\alpha, \alpha, \alpha^3, \alpha^5)^*$	6	nonpartition
6	$(\alpha, \alpha^2, \alpha^3, \alpha^5)^*$	6	nonpartition
6	$(\alpha, \alpha^3, \alpha^4, \alpha^4)$	24	double partition
7	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	28	$i + j + k + l = 7$; 3 models
8	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	≤ 32	$i + j + k + l = 8$; 5 models
8	$(\alpha, \alpha^2, \alpha^3, \alpha^6)^*$	16	nonpartition
8	$(\alpha^2, \alpha^2, \alpha^2, \alpha^6)^*$	16	analog of $Z_4(\alpha, \alpha, \alpha, \alpha^3)$ model
8	$(\alpha, \alpha^4, \alpha^5, \alpha^6)$	32	double partition
9	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	36	$i + j + k + l = 9$; 7 models
9	$(\alpha, \alpha^3, \alpha^4, \alpha^7)^*$	36	nonpartition
9	$(\alpha, \alpha^4, \alpha^6, \alpha^7)$	36	double partition
10	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	≤ 40	$i + j + k + l = 10$; 9 models
10	$(\alpha, \alpha^3, \alpha^8, \alpha^8)$	40	double partition
10	$(\alpha, \alpha^5, \alpha^6, \alpha^8)$	40	double partition
11	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	44	$i + j + k + l = 11$; 11 models
12	$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$	≤ 48	$i + j + k + l = 12$; 15 models
12	$(\alpha, \alpha^4, \alpha^9, \alpha^{10})$	48	double partition
12	$(\alpha, \alpha^5, \alpha^9, \alpha^9)$	48	double partition
12	$(\alpha, \alpha^6, \alpha^7, \alpha^{10})$	48	double partition
12	$(\alpha, \alpha^6, \alpha^8, \alpha^9)$	36	double partition
12	$(\alpha, \alpha^7, \alpha^8, \alpha^8)$	48	double partition
12	$(\alpha^2, \alpha^6, \alpha^8, \alpha^8)$	36	double partition
12	$(\alpha, \alpha, \alpha^5, \alpha^9)^*$	48	nonpartition
12	$(\alpha, \alpha^3, \alpha^5, \alpha^9)^*$	24	nonpartition
12	$(\alpha, \alpha^3, \alpha^7, \alpha^{11})^*$	24	nonpartition
12	$(\alpha, \alpha^5, \alpha^5, \alpha^9)^*$	48	nonpartition
12	$(\alpha^2, \alpha^2, \alpha^6, \alpha^{10})^*$	12	nonpartition
12	$(\alpha^2, \alpha^3, \alpha^4, \alpha^9)^*$	24	nonpartition
12	$(\alpha^2, \alpha^4, \alpha^6, \alpha^{10})^*$	24	nonpartition
12	$(\alpha^3, \alpha^3, \alpha^3, \alpha^9)^*$	24	nonpartition

A new class of models appears in Table IV; these are the double partition models. They have $i + j + k + l = 2n$ and none are equivalent to single partition models (if we require that i, j, k, and l are all positive integers) with $i + j + k + l = n$. The $\mathcal{N} = 1$ nonpartition models have been classified [13], and we find 11 $\mathcal{N} = 0$ examples in Table IV. While they have a self-conjugate **6**, this is only a necessary condition that may be insufficient to insure the construction of viable string theory based models [13]. However, as is the $\mathcal{N} = 1$ case, the $\mathcal{N} = 0$ nonpartition models may still be interesting phenomenologically and as a testing ground for models with the potential of broken conformal invariance.

For Z_n orbifold models with n a prime number, only partition models arise. The nonpartition and double partition models only occur when n is not a prime number, and only a few are independent. Consider $n = 12$; here we can write $Z_{12} = Z_4 \times Z_3$. If we write an element of this group as $\gamma \equiv (a, b)$, where a is a generator of Z_4 and b of Z_3, then $\gamma^2 \equiv (a^2, b^2)$, $\gamma^3 \equiv (a^3, 1)$, etc. The full group is generated by any one of the elements $\gamma = (a, b)$, $\gamma^5 = (a, b^2)$, $\gamma^7 = (a^3, b)$, or $\gamma^{11} = (a^3, b^2)$. The other choices do not faithfully represent the group. Letting $\alpha = \gamma^{11}$ give a conjugate model, e.g., it transforms $(\alpha, \alpha^6, \alpha^8, \alpha^9)$ into $(\gamma^{11}, \gamma^6, \gamma^4, \gamma^3)$, so this pair of double partition models are equivalent, while letting $\alpha = \gamma^5$ transforms $(\alpha, \alpha^6, \alpha^8, \alpha^9)$ into the equivalent model $(\gamma^5, \gamma^6, \gamma^4, \gamma^9)$, and $\alpha = \gamma^7$ transforms $(\alpha, \alpha^6, \alpha^8, \alpha^9)$ into the equivalent model $(\gamma^7, \gamma^6, \gamma^8, \gamma^3)$. Hence a systematic use of these operations on the nonpartition and double partition models can reduce them to the equivalence classes listed in the tables.

It is easy to prove we always have a proper embedding (i.e., **6**=**6***) for the **4**=$(\alpha^i, \alpha^j, \alpha^k, \alpha^l)$ when $i + j + k +$

$l = n$ (or $2n$). To show this note from $\mathbf{6} = (\mathbf{4} \otimes \mathbf{4})_{\text{antisym}}$ we find $\mathbf{6} = (\alpha^{i+j}, \alpha^{i+k}, \alpha^{i+l}, \alpha^{j+k}, \alpha^{j+l}, \alpha^{k+l})$ but $i+j = n-k-l = -(k+l)\text{mod}n$, $i+k = n-j-l = -(j+l)\text{mod}n$, and $i+l = n-j-k = -(j+k)\text{mod}n$, so this gives $\mathbf{6} = (\alpha^{-(k+l)}, \alpha^{-(j+l)}, \alpha^{-(j+k)}, \alpha^{j+k}, \alpha^{j+l}, \alpha^{k+l}) = \mathbf{6}^*$. A simple modification of this proof also applies to the double partition models.

Now let us return to $\Gamma = Z_6$ where the partition models of interest are: (1) $\mathbf{4} = (\alpha, \alpha, \alpha^2, \alpha^2)$ where one easily sees that VEVs for $(3, 1, \bar{3}, 1, 1, 1)$ and then $(1, 3, \bar{3}, 1, 1)$ lead to at most two families, while other spontaneous symmetry breaking (SSB) routes lead to equal or less chirality. (2) $\mathbf{4} = (\alpha, \alpha, \alpha, \alpha^3)$ where VEVs for $(3, 1, \bar{3}, 1, 1, 1)$ followed by a VEV for $(1, 3, \bar{3}, 1, 1)$ leads to an $\text{SU}^4(3)$ model containing fermions $2[(3, \bar{3}, 1, 1) + (1, 3, \bar{3}, 1) + (1, 1, 3, \bar{3}) + (\bar{3}, 1, 1, 3)]$. However, there are insufficient scalars to complete the symmetry breaking to the standard model. In fact, one cannot even achieve the trinification spectrum.

The double partition Z_6 model $\mathbf{4} = (\alpha, \alpha^3, \alpha^4, \alpha^4)$ is relatively complicated, since there are 24 different scalar representations in the spectrum, and this makes the SSB analysis rather difficult. We have investigated a number of possible SSB pathways, but have found none that lead to the SM with at least three families. However, since our search was not exhaustive, we cannot make a definitive statement about this model. As stated elsewhere, the non-partition models are difficult to interpret, if not pathological, so we have not studied the SSB pathways for these Z_6 models.

We move on to Z_7, where there are three partition models: (1) for $\mathbf{4} = (\alpha, \alpha^2, \alpha^2, \alpha^2)$, we find no SSB pathway to the SM. There are paths to an SM with less than three families, e. g., VEVs for $(3, 1, 1, \bar{3}, 1, 1, 1)$, $(1, 3, 1, \bar{3}, 1, 1)$, $(3, \bar{3}, 1, 1, 1)$, and $(1, 3, \bar{3}, 1)$ lead to one family at the $\text{SU}^3(3)$ level; (2) for $\mathbf{4} = (\alpha, \alpha, \alpha, \alpha^4)$, again we find only paths to family-deficient standard models. An example is where we have VEVs for $(3, 1, \bar{3}, 1, 1, 1, 1)$, $(1, 3, \bar{3}, 1, 1, 1)$, $(3, 1, \bar{3}, 1, 1)$, and $(1, 3, \bar{3}, 1)$, which lead to a two-family $\text{SU}^3(3)$ model; (3) finally, $\mathbf{4} = (\alpha, \alpha, \alpha^2, \alpha^3)$ is the model discovered in [4], where VEVs to $(1, 3, 1, \bar{3}, 1, 1, 1)$, $(1, 1, 3, \bar{3}, 1, 1)$, $(1, 1, 3, \bar{3}, 1)$ and $(1, 1, 3, \bar{3})$ lead to a three-family model with the correct Weinberg angle at the Z-pole, $\sin^2\theta_W = 3/13$.

For Z_n with $n \geq 8$, the number of representations of matter multiplets has already grown to a degree where it makes a systematic analysis of the models prohibitively time-consuming. It is thus helpful to have further motivation to study particular examples or limited sets of these models with large n values. Thus we searched for examples which break $\text{SU}(3)^8$ down to diagonal subgroups $\text{SU}(3)^4 \times \text{SU}(3)^3 \times \text{SU}(3)$, since this implies the right Weinberg angle for TeV trinification [16], $\sin^2\theta_W = 3/13$, when embedding $\text{SU}(3)_L$ and $\text{SU}(3)_R$ into the diagonal subgroups of $\text{SU}(3)^4$ and $\text{SU}(3)$, respectively. There are actually 11 different possibilities to break $\text{SU}(3)^8$ down to $\text{SU}(3)^4 \times \text{SU}(3)^3 \times \text{SU}(3)$, assuming the necessary scalars exist. While none of these paths was successful for $\mathbf{4} = (\alpha, \alpha, \alpha, \alpha^5)$, the model $\mathbf{4} = (\alpha, \alpha, \alpha^2, \alpha^4)$ leads to the three family SM. Assigning VEVs to $(3, 1, \bar{3}, 1, 1, 1, 1, 1)$, $(3, 1, 1, \bar{3}, 1, 1, 1)$, $(3, \bar{3}, 1, 1, 1, 1)$, $(1, 3, \bar{3}, 1, 1)$ and $(1, 3, \bar{3}, 1)$ breaks $\text{SU}(3)^8$ down to $\text{SU}(3)_{1235} \times \text{SU}(3)_{467} \times \text{SU}(3)_8$.

Another option exists for $\mathbf{4} = (\alpha, \alpha^4, \alpha^5, \alpha^6)$, when assigning VEVs to $(3, \bar{3}, 1, 1, 1, 1, 1, 1)$, $(3, \bar{3}, 1, 1, 1, 1, 1)$, $(3, 1, 1, \bar{3}, 1, 1)$, $(1, 3, \bar{3}, 1, 1)$ and $(1, 3, 1, \bar{3})$ [2]. These models have not been discussed in the literature so far and have potential interesting phenomenology.

IV. $\mathcal{N} = 1$ AND $\mathcal{N} = 0$ CHIRAL MODELS FOR ABELIAN PRODUCT GROUP ORBIFOLDING

Now let us consider abelian orbifold groups of order $o(G) \leq 12$, that are not just Z_n. There are only four, but they will be sufficient to teach us how to deal with this type of orbifold. We will search for both $\mathcal{N} = 1$ and $\mathcal{N} = 0$ models since neither have been studied in general in the literature. Three groups, $Z_2 \times Z_4$, $Z_3 \times Z_3$, and $Z_2 \times Z_2 \times Z_3$ fit our requirements. We have dispensed with $Z_2 \times Z_2 \times Z_2$ since all their representations are real and it cannot produce chiral models.

First for $Z_2 \times Z_4$, we can write elements as $(\alpha^i, \beta^{i'})$ where $\alpha^2 = 1$, and $\beta^4 = 1$. The supersymmetry after orbifolding is determined by the embeddings. These are of the form:

$$\mathbf{4} = [(\alpha^i, \beta^{i'}), (\alpha^j, \beta^{j'}), (\alpha^k, \beta^{k'}), (\alpha^l, \beta^{l'})]. \quad (1)$$

If all four entries are nontrivial, then $\mathcal{N} = 0$ SUSY results. If one is trivial, then we have $\mathcal{N} = 1$. We can think of the SUSY breaking as a two step process, where we first embed the α's in the $\mathbf{4}$ and then the β's. Let us proceed this way and include only the partition, and possibly double partition models. (As we noted above, the nonpartition models have potential pathologies.) Thus for the α's we must have either $\mathbf{4}_{\alpha_1} = (-1, -1, -1, -1)$ or $\mathbf{4}_{\alpha_2} = (1, 1, -1, -1)$. The $\mathbf{4}_{\alpha_1}$ results in $\mathcal{N} = 0$ SUSY, while $\mathbf{4}_{\alpha_2}$ gives $\mathcal{N} = 2$. We do not include trivial Z_n factors $\mathbf{4}$=(1,1,1,1) in the discussion, since these models contain very little new information. [Note, for any product groups $Z_n \times Z_m$, the α's of Z_n must be self conjugate in the $\mathbf{6}$, as are the β's of Z_m. Hence, the full $\mathbf{6}$ is self conjugate since the subgroups Z_n and Z_m are orthogonal. This generalizes to more complicated products $Z_n \times Z_m \times Z_p \times \ldots$].

Now for the β's. These are to be combined with the α's so we must consider the $\mathbf{4}_{\alpha_1}$ and $\mathbf{4}_{\alpha_2}$ separately. For $\mathbf{4}_{\alpha_1}$, the inequivalent $\mathbf{4}_\beta$'s are $\mathbf{4}_{\beta_1} = (\beta, \beta, \beta, \beta)$ and

[2]This SSB pathway has first been derived by Yasmin Anstruther.

$\mathbf{4}_{\beta_2} = (1, \beta, \beta, \beta^2)$. (Models with $\mathbf{4} = (1, 1, \beta^2, \beta^2)$ are uninteresting since they all are nonchiral.) Both cases have $\mathcal{N} = 0$ SUSY since we were already at $\mathcal{N} = 0$ after the $\mathbf{4}_{\alpha_1}$ embedding. For $\mathbf{4}_{\alpha_2}$ we find five possible inequivalent embeddings, again we can have $\mathbf{4}_{\beta_1} = (\beta, \beta, \beta, \beta)$ or $\mathbf{4}_{\beta_2} = (1, \beta, \beta, \beta^2)$, but now we can also have $\mathbf{4}_{\beta_3} = (1, \beta^2, \beta, \beta)$, $\mathbf{4}_{\beta_4} = (\beta, \beta, 1, \beta^2)$ and $\mathbf{4}_{\beta_5} = (\beta^2, \beta, 1, \beta)$. The embeddings $\mathbf{4}_{\beta_1}$, $\mathbf{4}_{\beta_4}$ and $\mathbf{4}_{\beta_5}$ lead to $\mathcal{N} = 0$ SUSY while $\mathbf{4}_{\beta_2}$ and $\mathbf{4}_{\beta_3}$ leave $\mathcal{N} = 1$ SUSY unbroken. A similar analysis can be carried out for $Z_3 \times Z_3$, and $Z_2 \times Z_2 \times Z_3$, with the obvious generalization to a triple embedding for $Z_2 \times Z_2 \times Z_3$.

For $Z_3 \times Z_3$ there are five models. We can choose $\mathbf{4}_{\alpha} = (1, \alpha, \alpha, \alpha)$ as the embedding of the first Z_3. Then the embedding of the second Z_3 can be $\mathbf{4}_{\beta_1} = (1, \beta, \beta, \beta)$, $\mathbf{4}_{\beta_2} = (\beta, 1, \beta, \beta)$, $\mathbf{4}_{\beta_3} = (1, 1, \beta, \beta^2)$, $\mathbf{4}_{\beta_4} = (\beta, 1, 1, \beta^2)$, or $\mathbf{4}_{\beta_5} = (\beta^2, 1, 1, \beta)$. The first and third result in $\mathcal{N} = 1$ SUSY models while the other three are $\mathcal{N} = 0$.

For $Z_2 \times Z_2 \times Z_3$ we find nine chiral models. Rather than belabor the details, we summarize all our results for $Z_2 \times Z_4$, $Z_3 \times Z_3$, and $Z_2 \times Z_2 \times Z_3$ in Table V.

V. CONCLUSIONS

We have now completed our task of summarizing all $\mathcal{N} = 0$ and $\mathcal{N} = 1$ SUSY chiral models of phenomenological interest derivable from orbifolding $AdS_5 \times S^5$ with abelian orbifold group Γ of order $o(\Gamma) \leq 12$. The models fall into three classes: partition models, double partition models, and nonpartition models as determined by how the equation $i + j + k + l = sn$ is satisfied by the embedding where $s = 1$ for partition models, $s = 2$ for double partition models and s is non-integer for nonpartition models. For Z_n orbifolds with $\mathcal{N} = 1$ SUSY, there are 53 partition models, and seven nonpartition models, and for $\mathcal{N} = 0$ SUSY, we find 54 partition, eleven double partition, and 13 nonpartition models. The nonpartition models have potential pathologies if they are to be interpreted as coming from string theory, but they still may be of phenomenological and technical interest, so they have been included in our classification of Z_n models. See also the related discussions in [17,18].

The non-Z_n abelian product groups of interest (we consider partition models here) with $o(\Gamma) \leq 12$ are $Z_2 \times Z_4$ with five $\mathcal{N} = 0$ and two $\mathcal{N} = 1$ chiral models; $Z_3 \times Z_3$ with three $\mathcal{N} = 0$ and two $\mathcal{N} = 1$ chiral models, and $Z_2 \times Z_2 \times Z_3$ with seven $\mathcal{N} = 0$ and two $\mathcal{N} = 1$ chiral models.

We have explored the relation to the SM and MSSM in some detail only for Z_n models with $o(\Gamma) \leq 7$, but have only given a few examples with $o(\Gamma) > 7$, and have indicated how to build abelian orbifold models for any $o(\Gamma)$. Two Z_8 models have been introduced, which can lead to the right Weinberg angle, when broken down to the

TABLE V. All chiral $\mathcal{N} = 0$ and $\mathcal{N} = 1$ SUSY partition models for product orbifolding groups $Z_2 \times Z_4$, $Z_3 \times Z_3$, and $Z_2 \times Z_2 \times Z_3$, where the embedding is nontrivial in all factors. Our notation is: $\mathbf{4} = [(\alpha^i), (\alpha^j), (\alpha^k), (\alpha^l)] \times [(\beta^{i'}), (\beta^{j'}), (\beta^{k'}), (\beta^{l'})] = [(\alpha^i, \beta^{i'}), (\alpha^j, \beta^{j'}), (\alpha^k, \beta^{k'}), (\alpha^l, \beta^{l'})]$, etc.

Group	4	χ/N^2	$\mathcal{N}$
$Z_2 \times Z_4$	$(-1, -1, -1, -1) \times (\beta, \beta, \beta, \beta)$	32	0
$Z_2 \times Z_4$	$(-1, -1, -1, -1) \times (\mathbf{1}, \beta, \beta, \beta^2)$	16	0
$Z_2 \times Z_4$	$(1, 1, -1, -1) \times (\beta, \beta, \beta, \beta)$	32	0
$Z_2 \times Z_4$	$(1, 1, -1, -1) \times (\mathbf{1}, \beta, \beta, \beta^2)$	16	1
$Z_2 \times Z_4$	$(1, 1, -1, -1) \times (\mathbf{1}, \beta^2, \beta, \beta)$	16	1
$Z_2 \times Z_4$	$(1, 1, -1, -1) \times (\beta, \beta, 1, \beta^2)$	16	0
$Z_2 \times Z_4$	$(1, 1, -1, -1) \times (\beta, \beta^2, 1, \beta)$	16	0
$Z_3 \times Z_3$	$(\mathbf{1}, \alpha, \alpha, \alpha) \times (1, \beta, \beta, \beta)$	27	1
$Z_3 \times Z_3$	$(\mathbf{1}, \alpha, \alpha, \alpha) \times (\beta, 1, \beta, \beta)$	36	0
$Z_3 \times Z_3$	$(\mathbf{1}, \alpha, \alpha, \alpha) \times (1, 1, \beta, \beta^2)$	18	1
$Z_3 \times Z_3$	$(\mathbf{1}, \alpha, \alpha, \alpha) \times (\beta, 1, 1, \beta^2)$	36	0
$Z_3 \times Z_3$	$(\mathbf{1}, \alpha, \alpha, \alpha) \times (\beta^2, 1, 1, \beta)$	36	0
$Z_2 \times Z_2 \times Z_3$	$(1, 1, -1, -1) \times (1, 1, -1, -1) \times (1, \gamma, \gamma, \gamma)$	48	1
$Z_2 \times Z_2 \times Z_3$	$(1, 1, -1, -1) \times (-1, 1, 1, -1) \times (1, \gamma, \gamma, \gamma)$	48	0
$Z_2 \times Z_2 \times Z_3$	$(1, 1, -1, -1) \times (-1, -1, -1, -1) \times (1, \gamma, \gamma, \gamma)$	48	0
$Z_2 \times Z_2 \times Z_3$	$(-1, -1, 1, 1) \times (-1, -1, 1, 1) \times (1, \gamma, \gamma, \gamma)$	48	0
$Z_2 \times Z_2 \times Z_3$	$(-1, -1, 1, 1) \times (-1, -1, -1, -1) \times (1, \gamma, \gamma, \gamma)$	48	0
$Z_2 \times Z_2 \times Z_3$	$(1, 1, -1, -1) \times (-1, -1, 1, 1) \times (1, \gamma, \gamma, \gamma)$	48	0
$Z_2 \times Z_2 \times Z_3$	$(1, 1, -1, -1) \times (1, -1, -1, 1) \times (1, \gamma, \gamma, \gamma)$	48	1
$Z_2 \times Z_2 \times Z_3$	$(-1, 1, 1, -1) \times (-1, 1, -1, 1) \times (1, \gamma, \gamma, \gamma)$	48	0
$Z_2 \times Z_2 \times Z_3$	$(-1, -1, -1, -1) \times (-1, -1, -1, -1) \times (1, \gamma, \gamma, \gamma)$	48	0

SM. We hope our results will be useful to model builders and phenomenologists alike.

Interesting examples of field theory models of the type sketched here (models with $SU^n(N)$ gauge groups and bifundamental matter) have been studied recently [19–21] and show promise. While these have yet to be fit into the subclass of $AdS_5 \times S^5$ models, it would be interesting to do so.

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