Clothed particle representation in quantum field theory: Mass renormalization

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We consider the neutral pion and nucleon fields interacting via the pseudoscalar (PS) Yukawa-type coupling. The method of unitary clothing transformations is used to handle the so-called clothed particle representation, where the total field Hamiltonian and the three boost operators in the instant form of relativistic dynamics take on the same sparse structure in the Hilbert space of hadronic states. In this approach the mass counterterms are cancelled (at least, partly) by commutators of the generators of clothing transformations and the field interaction operator. This allows the pion and nucleon mass shifts to be expressed through the corresponding three-dimensional integrals whose integrands depend on certain covariant combinations of the relevant three-momenta. The property provides the momentum independence of mass renormalization. The present results prove to be equivalent to the results obtained by Feynman techniques.

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I. INTRODUCTION

Recently, the so-called unitary clothing transformation approach put forward in [1] and developed in [2] has been employed for an approximate treatment of the simplest eigenstates of the total field Hamiltonian H (see [3–5] and refs. therein). First of all, we mean the physical vacuum Ω (the lowest-energy H eigenstate) and the observable one-particle states $|\mathbf{p}\rangle$ with the momentum $\mathbf{p}$ since the procedure is aimed at reformulating quantum field theory (QFT) in terms of clothed or dressed particles. For a moment, the particle spin index is omitted, if any. By definition, the vector $|\mathbf{p}\rangle$ belongs to the H eigenvalue $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$, where m is the mass of a free particle (e.g., fermion). We call it the physical mass. Thus, m appears here in a natural way via the relativistic dispersion law. It is well known that the contemporary covariant approach uses the other prescription to find the physical mass, viz., it is determined as a pole of the full particle propagator (see, e.g., [6] (Chapter 7.1) and [7] (Chapters 10, 11)). The mass shift is expressed through the particle self-energy function evaluated in nontrivial field theories as an expansion in the coupling constants. The miscellaneous self-energy contributions give rise to undesirable divergences inherent in the existing applications of every local field model (at least, when employing the perturbative methods). Their removal requires considerable intellectual efforts associated with a consequent regularization of the divergent integrals involved. In the S-matrix calculations they are encountered as early as in the first nonvanishing approximation in the coupling constants (in particular, when evaluating the forward-scattering amplitudes, where the one-loop contributions must cancel the occurring mass counterterms). A few instructive examples of such a situation for the pion-nucleon and nucleon-nucleon scattering amplitudes can be found in the excellent exposition [8] both in the framework of the old-fashioned perturbation theory and the Dyson-Feynman approach.

In this context, we would like to note the paper [9] in which the one-particle energies have been calculated for the field model of interacting charged and neutral mesons. The author [9] has used the customary stationary perturbation theory, introduced a nonlocal extension of the interaction between the meson fields, and suggested a fresh look at the regularization problem within the noncovariant approach.

As shown in Refs. [10,11], the method of unitary transformations (UT's) can be helpful in this area. Thus, in [10,11] the Hamiltonian for interacting fields was block-diagonalized using Okubo's idea [12]. Note that while in [10] the π, ρ, ω, and σ mesons were coupled with nucleons via the Yukawa-type interactions, the authors of [11] dealt with scalar "nucleons" and mesons with a simpler coupling. This enabled them not only to derive the effective (Hermitian and energy independent) interactions ("quasipotentials") between nucleons, as done in [10], but also to separate the one-nucleon contribution to the Hamiltonian with the renormalized nucleon mass (cf., Eq. (23) of Ref. [10] and Eq. (36) of Ref. [11]).

The authors of [11] have shown that their expression for the second-order nucleon mass shift coincides with the corresponding expression found by Feynman technique. In particular, this shift is independent of the nucleon momentum. That has led to an expected momentum dependence (after Weisskopf [13]) of the nucleon energy shift. Besides, one should point out a direct (apparently, too sophisticated) way proposed in [11] (Appendix) to prove the reduction of the energy shift to the mass one.

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Note that the trick employed in [11] has common features with the so-called w-change of variables in the divergent integrals of interest (where w is a certain four-dimensional scalar in the energy-momentum space), which has been used many years ago in paper [14] devoted to the self-energy of a Dirac particle and its relativistic invariance.

Here we continue a study [5] of the mass renormalization problem in the clothed particle representation for the operator H. In this representation the primary mass counterterms must be compensated by the proper renormalization (''radiative'') parts of the effective interactions arising in the course of the clothing procedure (see Sect. II). It is achieved via normal ordering of the creation (destruction) operators for ''clothed'' particles (e.g., mesons, nucleons, and antinucleons). Details can be found in [4,5,15,16]. However, unlike [5], where priority has been given to deriving an analytic expression for the radiative correction to the ''bare'' pion mass and finding its covariance in the second order in the coupling constant, we will focus upon the elimination of the nucleon mass counterterm (see Sect. III).

In Sect. IV our results are compared with the simplest disconnected contributions to the pion-nucleon forward-scattering amplitude. They are evidently covariant and determined by the pion-nucleon one-loop diagram.

II. UNDERLYING FORMALISM: CLOTHED PARTICLES IN QUANTUM FIELD THEORY

The notion of clothed particles will be considered using the following model: a spinor (fermion) field ψ interacts with a neutral pseudoscalar meson field ϕ by means of the Yukawa coupling. The model Hamiltonian is $H = H_0 + V$ where

$$H_0 = \int d\mathbf{x}\bar{\psi}(\mathbf{x})[-i\gamma\nabla + m_0]\psi(\mathbf{x}) + \frac{1}{2}\int d\mathbf{x}[\pi^2(\mathbf{x}) + \nabla\phi(\mathbf{x})^2 + \mu_0^2\phi^2(\mathbf{x})], \quad (1)$$

$$V = \int d\mathbf{x}V(\mathbf{x}) = ig\int d\mathbf{x}\bar{\psi}(\mathbf{x})\gamma_5\psi(\mathbf{x})\phi(\mathbf{x}). \quad (2)$$

The Hamiltonian can be expressed through bare destruction (creation) operators $a(\mathbf{k})$ ($a^\dagger(\mathbf{k})$), $b(\mathbf{p}, r)$ ($b^\dagger(\mathbf{p}, r)$), and $d(\mathbf{p}, r)$ ($d^\dagger(\mathbf{p}, r)$) of the meson, the fermion, and the antifermion, respectively, (see Eqs. (8) and (16) below). Here $\mathbf{k}$ and $\mathbf{p}$ denote the momenta, r is the spin index. In what follows, the set of all these operators is denoted by a symbol a, while a_p is used for one of them. The state without bare particles Ω_0 and the one-bare-particle states $a^\dagger(\mathbf{k})\Omega_0$, $b^\dagger(\mathbf{p}, r)\Omega_0$, and $d^\dagger(\mathbf{p}, r)\Omega_0$ are not H eigenvectors.

Now, we introduce new destruction (creation) operators

$$a_c(\mathbf{k})(a_c^\dagger(\mathbf{k})),\ b_c(\mathbf{p}, r)(b_c^\dagger(\mathbf{p}, r)),\ d_c(\mathbf{p}, r)(d_c^\dagger(\mathbf{p}, r)),\ \forall \mathbf{k}, \mathbf{p}, r \quad (3)$$

with the following properties:

i The physical vacuum (the lowest-energy H eigenstate) must coincide with a new no-particle state Ω, i.e., the state that obeys the equations

$$a_c(\mathbf{k})|\Omega\rangle = b_c(\mathbf{p}, r)|\Omega\rangle = d_c(\mathbf{p}, r)|\Omega\rangle = 0,\ \forall \mathbf{k}, \mathbf{p}, r \quad (4)$$

$$\langle\Omega|\Omega\rangle = 1.$$

ii New one-particle states $a_c^\dagger(\mathbf{k})\Omega$ etc. are H eigenstates as well.

iii The spectrum of indices that enumerate the new operators must be the same as that for the bare ones (this requirement has been used when writing Eq. (3)).

iv The new operators satisfy the same commutation rules as do their bare counterparts. For instance,

$$\begin{aligned}[][a_c(\mathbf{k}), a_c^\dagger(\mathbf{k}')] &= \delta(\mathbf{k} - \mathbf{k}'),\\ \{b_c(\mathbf{p}, r), b_c^\dagger(\mathbf{p}', r')\} &= \{d_c(\mathbf{p}, r), d_c^\dagger(\mathbf{p}', r')\}\\ &= \delta_{rr'}\delta(\mathbf{p} - \mathbf{p}').\end{aligned} \quad (5)$$

Following [1], [8] (Chapter XII) we shall call the new operators and states clothed. Note that the name is sometimes used in a sense which differs from that defined by the points (i)–(iv).

As one can see, the problem of clothing is equivalent to determination of some H eigenvectors. In fact, the property (iii) means that we do not pretend to find all H eigenstates which are one-particlelike. For example, H may have a deuteronlike eigenstate with a mass $<2m$, where m is the nucleon mass. The finding of similar states is considered in [3,5]. Now we intend to deal only with those one-particlelike eigenstates of H which have bare counterparts.

By definition, the bare one-fermion eigenstate $|\mathbf{p}, r\rangle_0$ of the operator H_0, being simultaneously the eigenstate of total momentum $\mathbf{P}$, belongs to the H_0 eigenvalue $E_{\mathbf{p}}^0 = \sqrt{\mathbf{p}^2 + m_0^2}$. Let us consider an H eigenstate $|\mathbf{p}, r\rangle$ for which $|\mathbf{p}, r\rangle_0$ is a zeroth approximation (ZA). Perturbation theory shows that the corresponding H eigenvalue $E_{\mathbf{p}}$ differs from $E_{\mathbf{p}}^0$. In the relativistic case the function $E_{\mathbf{p}}$ must be of the form $\sqrt{\mathbf{p}^2 + m^2}$ where m is the mass of an observed free fermion. Analogously, one can argue the appearance of the meson physical mass μ which differs from the trial mass μ_0.

Such an introduction of the masses m and μ can be used to divide the total Hamiltonian into the new free part H_F and the new interaction H_I. Namely, let us rewrite $H = H_0 + V$ as $H = H_F + H_I$ where

$$H_F = \int d\mathbf{x}\bar{\psi}(\mathbf{x})[-i\gamma\nabla + m]\psi(\mathbf{x}) + \frac{1}{2}\int d\mathbf{x}[\pi^2(\mathbf{x}) + \nabla\phi(\mathbf{x})^2 + \mu^2\phi^2(\mathbf{x})], \tag{6}$$

$$H_I = V + \delta m\int d\mathbf{x}\bar{\psi}(\mathbf{x})\psi(\mathbf{x}) + \frac{1}{2}\delta\mu^2\int d\mathbf{x}\phi^2(\mathbf{x}) \equiv V + M_{\text{ren}}, \tag{7}$$

with $\delta m = m_0 - m$ and $\delta\mu^2 = \mu_0^2 - \mu^2$.

The operator H_F can be brought to the "diagonal" form

$$H_F = \int d\mathbf{k}\,\omega_{\mathbf{k}} a^\dagger(\mathbf{k})a(\mathbf{k}) + \int d\mathbf{p}E_{\mathbf{p}}\sum_r[b^\dagger(\mathbf{p}, r)b(\mathbf{p}, r) + d^\dagger(\mathbf{p}, r)d(\mathbf{p}, r)], \tag{8}$$

by means of the standard expansions

$$\phi(\mathbf{x}) = (2\pi)^{-3/2}\int d\mathbf{k}(2\omega_{\mathbf{k}})^{-1/2}[a(\mathbf{k}) + a^\dagger(-\mathbf{k})] \times \exp(i\mathbf{k}\mathbf{x}), \tag{9}$$

$$\pi(\mathbf{x}) = -i(2\pi)^{-3/2}\int d\mathbf{k}(\omega_{\mathbf{k}}/2)^{1/2}[a(\mathbf{k}) - a^\dagger(-\mathbf{k})]\exp(i\mathbf{k}\mathbf{x}), \tag{10}$$

$$\psi(\mathbf{x}) = (2\pi)^{-3/2}\int d\mathbf{p}(m/E_{\mathbf{p}})^{1/2}\sum_r[u(\mathbf{p}, r)b(\mathbf{p}, r) + v(-\mathbf{p}, r)d^\dagger(-\mathbf{p}, r)]\exp(i\mathbf{p}\mathbf{x}), \tag{11}$$

where $u(\mathbf{p}, r)$ and $v(\mathbf{p}, r)$ are the Dirac spinors, which satisfy the conventional equations $(\hat{p} - m)u(\mathbf{p}, r) = 0$ and $(\hat{p} + m)v(\mathbf{p}, r) = 0$ with $\hat{p} = E_{\mathbf{p}}\gamma^0 - \mathbf{p}\gamma$, in the formulae $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ and $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + \mu^2}$.

The operators (3) are the milestones of the clothing procedure. Its urgent task is to find clothed operators which should satisfy the requirements (i)–(iv). Now, the symbol α will be used for set (3) with α_p being one operator of the set (cf. a and a_p). In order to meet the properties (iii) and (iv), we suppose that the clothed operators α are related to the bare ones a via a unitary transformation

$$\alpha_p = W^\dagger a_p W, \qquad W^\dagger W = WW^\dagger = 1, \tag{12}$$

where W is a function of all the bare operators a. Therefore, Eq. (12) represents α_p as a function (functional) of a.

Note that W is the same function of either clothed or bare operators (see [1]). Indeed, if $f(x)$ is a polynomial or a series of x, the relation $f(\alpha) = W^\dagger(a)f(a)W(a)$ follows from Eq. (12). Replacing $f(\alpha)$ by W leads to

$$W(\alpha) = W^\dagger(a)W(a)W(a) = W(a), \tag{13}$$

i.e., in the above statement. Hence, the operator a_p, when expressed in terms of α, is given by

$$a_p = W(\alpha)\alpha_p W^\dagger(\alpha). \tag{14}$$

Unitarity of W is automatically ensured if W is represented as the exponential of an antihermitian operator R: $W = \exp(R)$. For a given R, the r.h.s. of Eq. (14) can be evaluated with the help of

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2}[A, [A, B]] + \frac{1}{3!}[A, [A, [A, B]]] + \ldots, \tag{15}$$

that holds for any operators A and B, and the commutation rules (5).

In the context, the total Hamiltonian can be written as $H = H(a) = H_F + H_I$ where $H_F(a)$ is determined by Eq. (8) and $H_I = V(a) + M_{\text{ren}}(a)$ with

$$\begin{aligned} V(a) = \int d\mathbf{p}'d\mathbf{p}d\mathbf{k}\sum_{rr'}\{&b^\dagger(\mathbf{p}', r')V_{11}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r)b(\mathbf{p}, r) + b^\dagger(\mathbf{p}', r')V_{12}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r)d^\dagger(-\mathbf{p}, r) \\ &+ d(-\mathbf{p}', r')V_{21}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r)b(\mathbf{p}, r) + d(-\mathbf{p}', r')V_{22}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r)d^\dagger(-\mathbf{p}, r)\}[a(\mathbf{k}) + a^\dagger(-\mathbf{k})] \\ = \int d\mathbf{p}'d\mathbf{p}d\mathbf{k}\sum_{rr'}&F^\dagger(\mathbf{p}', r')V^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r)F(\mathbf{p}, r)a(\mathbf{k}) + \text{H.c.} \equiv \int d\mathbf{k}F^\dagger V^{\mathbf{k}}Fa(\mathbf{k}) + \text{H.c.}, \end{aligned} \tag{16}$$

where the operator column F and row $F^\dagger$ are composed of the bare nucleon and antinucleon operators (e.g., $F^\dagger(\mathbf{p}, r) \equiv [b^\dagger(\mathbf{p}, r), d(-\mathbf{p}, r)]$), and we have introduced the c-number matrices (cf., Appendix A of [5]),

$$\begin{aligned} V^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) &= \begin{bmatrix} V_{11}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) & V_{12}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) \\ V_{21}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) & V_{22}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) \end{bmatrix} \\ &= \frac{ig}{(2\pi)^{3/2}}\frac{m}{\sqrt{2\omega_{\mathbf{k}}E_{\mathbf{p}'}E_{\mathbf{p}}}}\delta(\mathbf{p} + \mathbf{k} - \mathbf{p}')\begin{bmatrix} \bar{u}(\mathbf{p}', r')\gamma_5 u(\mathbf{p}, r) & \bar{u}(\mathbf{p}', r')\gamma_5 v(-\mathbf{p}, r) \\ \bar{v}(-\mathbf{p}', r')\gamma_5 u(\mathbf{p}, r) & \bar{v}(-\mathbf{p}', r')\gamma_5 v(-\mathbf{p}, r) \end{bmatrix}. \end{aligned} \tag{17}$$

By using Eq. (14), one can replace the bare operators by the clothed ones

$$H(a) = H[W(\alpha)\alpha W^{\dagger}(\alpha)] \equiv K(\alpha). \tag{18}$$

The operator $K(\alpha)$ represents the same Hamiltonian, but it has another dependence on its argument α compared to $H(a)$. $K(\alpha)$ can be found as follows. First, Eq. (18) can be written as

$$K(\alpha) = W(\alpha)H(\alpha)W^{\dagger}(\alpha). \tag{19}$$

Second, putting $W(\alpha) = \exp(R(\alpha))$ and using Eq. (15) we have

$$\begin{aligned} H = K(\alpha) &= e^{R}[H_F + H_I]e^{-R} = \\ &= H_F(\alpha) + H_I(\alpha) + [R, H_F] + [R, H_I] \\ &\quad + \frac{1}{2}[R,[R,H_F]] + \frac{1}{2}[R,[R,H_I]] + \ldots \end{aligned} \tag{20}$$

III. THE CLOTHING PROCEDURE IN ACTION: ELIMINATION OF BAD TERMS INCLUDING MASS COUNTERTERMS

Equation (20) gives a practical recipe for the $K(\alpha)$ calculation: at the beginning one replaces a by α in the initial expression $H(\alpha)$ and then calculates $W(\alpha)H(\alpha)W^{\dagger}(\alpha)$ using Eqs. (15) and (5). The above transition $H(a) \rightarrow H(\alpha)$ generates a new operator $H(\alpha)$ as compared to $H(a)$, but Eqs. (18) and (19) show that $W(\alpha)H(\alpha)W^{\dagger}(\alpha)$ turns out to be equal to the original total Hamiltonian.

In order to meet the requirements (i) and (ii), the r.h.s. of Eq. (20) must not contain some undesirable terms that prevent the no-clothed-particle state Ω and one-clothed-particle states to be H eigenvectors. Such terms which we call bad as in [5] enter in the operator $V(\alpha)$ that is derived from $V(a)$ by means of the replacement $a \rightarrow \alpha$.

Let us eliminate from $K(\alpha)$ the bad terms of the g^1-order. For this purpose we choose such R that

$$V + [R, H_F] = 0. \tag{21}$$

According to [5], the corresponding operator $R(\alpha) = \mathcal{R} - \mathcal{R}^{\dagger}$ with

$$\mathcal{R} = \int d\mathbf{k} F_c^{\dagger} R^{\mathbf{k}} F_c a_c(\mathbf{k}). \tag{22}$$

In cumbersome formulae summations over the dummy spin indices are sometimes omitted. Here, unlike the fermion operators F and $F^{\dagger}$ in Eq. (16) the operator column F_c and row $F_c^{\dagger}$ are composed of the clothed nucleon and antinucleon operators. The explicit expression for c-number matrices $R^{\mathbf{k}}$ can be written as

$$\begin{aligned} R_{i,j}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) = V_{i,j}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r)/[(-1)^{i-1}E_{\mathbf{p}'} \\ -(-1)^{j-1}E_{\mathbf{p}} - \omega_{\mathbf{k}}], \\ (i, j = 1, 2). \end{aligned} \tag{23}$$

Once $[R, H_F] = -V$, Eq. (20) can be rewritten as

$$\begin{aligned} K(\alpha) = H_F(\alpha) + M_{\text{ren}}(\alpha) + \frac{1}{2}[R, V] + [R, M_{\text{ren}}] \\ + \frac{1}{3}[R,[R,V]] + \ldots \end{aligned} \tag{24}$$

Thus, we have removed from $K(\alpha)$ all the bad terms of the g^1-order.

However, the r.h.s. of Eq. (24) embodies other bad terms of the g^2- and higher orders. In particular, we imply the terms bilinear in the meson and fermion operators, that arise from the commutator

$$\begin{aligned} [R, V] = &\int d\mathbf{k}_1 d\mathbf{k}_2 F_c^{\dagger}[R^{\mathbf{k}_1}, V^{\mathbf{k}_2}]F_c a_c(\mathbf{k}_1)a_c(\mathbf{k}_2) \\ &+ \int d\mathbf{k}_1 d\mathbf{k}_2 F_c^{\dagger}[R^{\mathbf{k}_1}, V^{-\mathbf{k}_2}]F_c a_c^{\dagger}(\mathbf{k}_2)a_c(\mathbf{k}_1) \\ &+ \int d\mathbf{k} F_c^{\dagger} R^{\mathbf{k}} F_c \cdot F_c^{\dagger} V^{-\mathbf{k}} F_c + \text{H.c.} \end{aligned} \tag{25}$$

after normal ordering. They may be cancelled by the respective counterparts from the operator $M_{\text{ren}}(\alpha) = M_{\text{mes}}(\alpha) + M_{\text{ferm}}(\alpha)$.

It has been shown in [5] that in the model under this consideration the meson mass shift of the g^2-order is determined by

$$\begin{aligned} \delta\mu^2 &= \frac{2g^2}{(2\pi)^3}\int \frac{d\mathbf{p}}{E_{\mathbf{p}}}\left\{\frac{p_- k}{\mu^2 + 2p_- k} - \frac{pk}{\mu^2 - 2pk}\right\} \\ &= \frac{2g^2}{(2\pi)^3}\int \frac{d\mathbf{p}}{E_{\mathbf{p}}}\left\{1 + \frac{\mu^4}{4(pk)^2 - \mu^4}\right\}, \end{aligned} \tag{26}$$

i.e., it is independent of the meson momentum $\mathbf{k}$. Here we have introduced the 4-vectors $p = (E_{\mathbf{p}}, \mathbf{p})$, $p_- = (E_{\mathbf{p}}, -\mathbf{p})$, and $k = (\omega_{\mathbf{k}}, \mathbf{k})$. Note that the similar formula (A.20) from [5] contains a misprint in signs.

Now, we will show how the second-order contributions to the fermion mass counterterm can be cancelled by certain terms of the commutator $[R, V]$. In this connection, we will look for all the terms bilinear in the fermion operators, which arise from $\frac{1}{2}[R, V]$ (more exactly, the third integral of Eq. (25)). Like Eq. (16) this fermionic two-operator contribution can be written as

$$\frac{1}{2}[R, V]_{2\mathrm{ferm}} = \int d\mathbf{k} F_c^\dagger X^{\mathbf{k}} F_c = \int d\mathbf{k} \Big\{ b_c^\dagger X_{11}^{\mathbf{k}} b_c + b_c^\dagger X_{12}^{\mathbf{k}} d_c^\dagger + d_c X_{21}^{\mathbf{k}} b_c + d_c X_{22}^{\mathbf{k}} d_c^\dagger \Big\}. \quad (27)$$

After lengthy transformations (again, with the aid of normal ordering) one can obtain the following formulae for evaluation of the c-number matrix elements $X_{ij}^{\mathbf{k}}$

$$2X_{11}^{\mathbf{k}} = R_{11}^{\mathbf{k}} V_{11}^{-\mathbf{k}} - V_{12}^{-\mathbf{k}} R_{21}^{\mathbf{k}} + \mathrm{H.c.},$$

$$2X_{12}^{\mathbf{k}} = 2X_{21}^{\mathbf{k}\dagger} = R_{11}^{\mathbf{k}} V_{12}^{-\mathbf{k}} - V_{12}^{-\mathbf{k}} R_{22}^{\mathbf{k}} + (R_{21}^{\mathbf{k}} V_{11}^{-\mathbf{k}} - V_{22}^{-\mathbf{k}} R_{21}^{\mathbf{k}})^\dagger,$$

$$2X_{22}^{\mathbf{k}} = R_{21}^{\mathbf{k}} V_{12}^{-\mathbf{k}} - V_{22}^{-\mathbf{k}} R_{22}^{\mathbf{k}} + \mathrm{H.c.}.$$

The fermion mass counterterm has the form

$$M_{\mathrm{ferm}} = \delta m \int d\mathbf{x} \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}), \quad (28)$$

or

$$M_{\mathrm{ferm}}(\alpha) = m\delta m F_c^\dagger M F_c = m\delta m \{ b_c^\dagger M_{11} b_c + b_c^\dagger M_{12} d_c^\dagger + d_c M_{21} b_c + d_c M_{22} d_c^\dagger \}, \quad (29)$$

where the matrix M is given by

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \frac{\delta(\mathbf{p}' - \mathbf{p})}{E_{\mathbf{p}}} \begin{bmatrix} \delta_{r'r} & \bar{u}(\mathbf{p}', r') v(-\mathbf{p}, r) \\ \bar{v}(-\mathbf{p}', r') u(\mathbf{p}, r) & -\delta_{r'r} \end{bmatrix}. \quad (30)$$

Under certain conditions the separate terms in Eq. (29) will cancel the terms of the same operator structure in Eq. (27). First of all, we are interested in cancellation of the $b_c^\dagger b_c$ and $d_c d_c^\dagger$ terms responsible for the transitions one fermion → one fermion to get a prescription in determining the fermion (nucleon) mass renormalization (of course, in the g^2-order). Doing so, we assume that

$$m\delta m^{(2)} M_{11} + \int d\mathbf{k} X_{11}^{\mathbf{k}} = 0, \qquad m\delta m^{(2)} M_{22} + \int d\mathbf{k} X_{22}^{\mathbf{k}} = 0, \quad (31)$$

or in the spinor space,

$$m\delta m^{(2)} \frac{\delta(\mathbf{p}' - \mathbf{p})}{E_{\mathbf{p}}} \delta_{r'r} = -\int d\mathbf{k} X_{11}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r), \qquad m\delta m^{(2)} \frac{\delta(\mathbf{p}' - \mathbf{p})}{E_{\mathbf{p}}} \delta_{r'r} = \int d\mathbf{k} X_{22}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r). \quad (32)$$

It is evident that these equations impose the following constraints upon the integrals, viz., each of them must depend on the fermion momentum and spin as $C\delta(\mathbf{p}' - \mathbf{p})\delta_{r'r}/E_{\mathbf{p}}$ with a constant C. The further task is to check this property and find the constant, if any.

As shown in Appendix,

$$\int d\mathbf{k} X_{11}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) = -\frac{g^2}{4(2\pi)^3} \frac{\delta(\mathbf{p}' - \mathbf{p})}{E_{\mathbf{p}}} \delta_{r'r} I(p), \quad (33)$$

where

$$I(p) = I_1(p) + I_2(p),$$

with

$$I_1(p) = \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} pk \Big\{ \frac{1}{\mu^2 - 2pk} - \frac{1}{\mu^2 + 2pk} \Big\},$$

$$I_2(p) = \int \frac{d\mathbf{q}}{E_{\mathbf{q}}} \Big\{ \frac{m^2 - pq}{2[m^2 - pq] - \mu^2} + \frac{m^2 + pq}{2[m^2 + pq] - \mu^2} \Big\}.$$

Thus, the mass shift of interest is

$$\delta m^{(2)} = \frac{g^2}{4m(2\pi)^3} I(p) = \frac{g^2}{4m(2\pi)^3} [I_1(m, 0, 0, 0) + I_2(m, 0, 0, 0)]. \quad (34)$$

The second relation (32) leads to the same result since $X_{22}^{\mathbf{k}} = -X_{11}^{\mathbf{k}}$. The integrals involved in Eq. (34) can be reduced to the elementary ones.

The crossed $b_c^\dagger d_c^\dagger$ and $d_c b_c$ terms in Eq. (27) are bad having nonvanishing matrix elements between the vacuum Ω and two-fermion states. It turns out that they are not covariant (see Appendix) and, unlike the meson mass renormalization, are not cancelled with the respective terms of the operator $M_{\mathrm{ferm}}(\alpha)$.

IV. COMPARISON WITH AN EXPLICITLY COVARIANT CALCULATION: ELIMINATION OF DIVERGENCES IN THE *S*-MATRIX

We have seen that the considered procedure enables us to remove from the Hamiltonian in the clothed particle representation not only the ''bad'' terms (at least, up to any given order in the coupling constant g). Simultaneously, the ''good'' two-particle terms are eliminated too being compensated with the corresponding mass counterterms. Along the guideline some ultraviolet divergences inherent in the conventional form of H cannot appear in the S-matrix. In the context, let us recall the Dyson-Feynman expansion for the S operator,

$$S = 1 - i\int_{-\infty}^{\infty} dt_1 H_I(t_1) + (-i)^2 \frac{1}{2!} \times \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 P[H_I(t_1) H_I(t_2)] + \ldots, \quad (35)$$

where, as usual, $H_I(t) = \exp[iH_F(a)t]H_I(a) \times \exp[-iH_F(a)t]$ is an interaction in the Dirac picture. To be definite, we consider the interacting neutral pion and nucleon fields with the operator $H_I(a) = V(a) + M_{\rm ren}(a)$ (see Eqs. (16) and (29)) and the matrix elements $\langle f|S^{(2)}|i\rangle$ of the S operator in the g^2-order, sandwiched between the initial and final $\pi^0 N$ states,

$$|i\rangle = a^\dagger(\mathbf{k})b^\dagger(\mathbf{p}, r)\Omega_0, \qquad |f\rangle = a^\dagger(\mathbf{k}')b^\dagger(\mathbf{p}', r')\Omega_0. \tag{36}$$

We are interested in competition between the fermion mass renormalization contribution to $S^{(2)}$ and the so-called fermion self-energy diagram contribution. The latter can be written as

$$\langle f|S^{(2)}_{SE}|i\rangle = -\frac{g^2}{(2\pi)^3} m^2 I_F(p)\frac{\delta(\mathbf{p}'-\mathbf{p})}{E_{\mathbf{p}}} \times \delta(\mathbf{k}'-\mathbf{k})\delta(E_{\mathbf{p}'} + \omega_{\mathbf{k}'} - E_{\mathbf{p}} - \omega_{\mathbf{k}})\delta_{r'r}, \tag{37}$$

$$I_F(p) = \int \frac{d^4q}{q^2 - \mu^2 + i0}\left\{1 - \frac{p(p-q)}{m^2}\right\} \times \frac{1}{(p-q)^2 - m^2 + i0},$$

or

$$I_F(p) = \int d\mathbf{q} \int_{-\infty}^{\infty} dq_0 \frac{1}{q_0^2 - \omega_q^2 + i0} \times \left\{1 - \frac{p_0(p_0 - q_0) - \mathbf{p}(\mathbf{p}-\mathbf{q})}{m^2}\right\} \times \frac{1}{(p_0 - q_0)^2 - E^2_{\mathbf{p}-\mathbf{q}} + i0}.$$

It is pertinent to stress that, unlike the notation $p - k = (E_{\mathbf{p}-\mathbf{k}}, \mathbf{p} - \mathbf{k})$ adopted in the Appendix, the vector $p - q$ here is the difference of the two 4-vectors p and q, i.e., $p - q = (p_0 - q_0, \mathbf{p} - \mathbf{q})$.

The "forward-scattering" process associated with this diagram would be responsible for the appearance of certain infinity in the πN scattering amplitude $\langle f|\mathcal{T}|i\rangle$. Following a common practice, the divergence should be compensated by the $\langle f|M^{(2)}_{\rm ferm}(a)|i\rangle$ piece, viz., it is required that

$$2\pi i\langle f|M^{(2)}_{\rm ferm}(a)|i\rangle\delta(E_f - E_i) = \langle f|S^{(2)}_{SE}|i\rangle. \tag{38}$$

At this point, one should emphasize that similar well-known steps become unnecessary if from the beginning we operate with the clothed particle representation $K(\alpha)$ of the Hamiltonian $H(a)$. This new form of H does not contain ultraviolet divergences and, being constructed via the sequential unitary transformations, gives new unitarily equivalent forms of the S operator [17]. It is important that the approach enables us to evaluate one and the same S matrix with nonperturbative methods (cf. an akin approach elaborated in [16]).

In addition, we would like to note that the crossed (nondiagonal) terms in (29) do not contribute to $\langle f|M^{(2)}_{\rm ferm}(a)|i\rangle$. In fact,

$$\begin{aligned}\langle f|M^{(2)}_{\rm ferm}(a)|i\rangle &= \langle\Omega_0|a(\mathbf{k}')b(\mathbf{p}', r')M^{(2)}_{\rm ferm}a^\dagger(\mathbf{k})b^\dagger(\mathbf{p}, r)|\Omega_0\rangle \\ &= \langle\Omega_0|b(\mathbf{p}', r')M^{(2)}_{\rm ferm}b^\dagger(\mathbf{p}, r)|\Omega_0\rangle\delta(\mathbf{k}'-\mathbf{k}) \\ &= m\delta m^{(2)}\langle\Omega_0|b(\mathbf{p}', r') \times b^\dagger M_{11}bb^\dagger(\mathbf{p}, r)|\Omega_0\rangle\delta(\mathbf{k}'-\mathbf{k}) \\ &= m\delta m^{(2)}\frac{\delta(\mathbf{p}'-\mathbf{p})}{E_{\mathbf{p}}}\delta_{r'r}\delta(\mathbf{k}'-\mathbf{k}).\end{aligned}$$

Obviously, this observation is related to all g^{2n}-orders $(n = 1, 2, \ldots)$.

Now, by taking into account the pole disposition for the propagators involved and carrying out the q_0-integration, one can get

$$\begin{aligned}\langle f|S^{(2)}_{SE}|i\rangle = &\frac{\pi i}{2}\frac{g^2}{(2\pi)^3}\frac{\delta(\mathbf{p}'-\mathbf{p})}{E_{\mathbf{p}}} \times \delta(\mathbf{k}'-\mathbf{k})\delta(E_{\mathbf{p}'} + \omega_{\mathbf{k}'} - E_{\mathbf{p}} - \omega_{\mathbf{k}}) \\ &\times \delta_{r'r}\int \frac{d\mathbf{q}}{E_{\mathbf{p}-\mathbf{q}}\omega_{\mathbf{q}}}\left\{\frac{m^2 - E_{\mathbf{p}}E_{\mathbf{p}-\mathbf{q}} + \mathbf{p}(\mathbf{p}-\mathbf{q})}{E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{q}} - \omega_{\mathbf{q}}} - \frac{m^2 + E_{\mathbf{p}}E_{\mathbf{p}-\mathbf{q}} + \mathbf{p}(\mathbf{p}-\mathbf{q})}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{q}} + \omega_{\mathbf{q}}}\right\}.\end{aligned} \tag{39}$$

The three-dimensional integral in (39) coincides with the integral $I(p)$ defined by Eq. (44). Hence, one can write

$$\langle f|S^{(2)}_{SE}|i\rangle = \frac{\pi i}{2}\frac{g^2}{(2\pi)^3}I(p)\frac{\delta(\mathbf{p}'-\mathbf{p})}{E_{\mathbf{p}}} \times \delta(\mathbf{k}'-\mathbf{k})\delta(E_{\mathbf{p}'} + \omega_{\mathbf{k}'} - E_{\mathbf{p}} - \omega_{\mathbf{k}})\delta_{r'r}. \tag{40}$$

It follows from (37) and (40) that

$$I_F(p) = -\frac{\pi i}{2m^2}I(p), \tag{41}$$

i.e., we have found another proof of the p-independence of $I(p)$ since $I_F(p)$ is the explicitly covariant quantity. Besides, we have expressed the Feynman one-loop integral $I_F(p)$ through other covariant quantities (see formula (47)).

Finally, Eqs. (37) and (38) yield the result (34) found alternatively.

V. SUMMARY

We have demonstrated here how the mass shifts in the system of interacting pion and nucleon fields can be calculated by the use of the clothed particle representation, where the total Hamiltonian and other generators of the Poincaré group take on a certain sparse structure in the Hilbert space. This application of the method of unitary clothing transformations refers to the Yukawa-type πN interaction with the PS coupling. The respective mass counterterms are compensated and determined due to normal ordering of the clothed creation (destruction) operators involved in the commutator $[R, V]$ of the model interaction V and the generator R of the corresponding clothing transformation.

The procedure described above has an important feature, viz., the mass renormalization is made simultaneously with the construction of a new family of quasipotentials (Hermitian and energy independent) between the physical particles (the quasiparticles of the method). Explicit expressions for the quasipotentials can be found in [5].

Being three-dimensional, the approach demands some efforts to prove the momentum independence of the expressions for the particle mass shifts. By using a comparatively simple analytical means, we could show that the three-dimensional integrals, which determine the pion and nucleon renormalizations in the second order in the coupling constant g, can be written in terms of the Lorentz invariants composed of the particle three-momenta. In other words, these integrals are independent of the particle momentum. An essential result of our work is the observation (see Sect. III and the Appendix) that unlike those operations with the spinless particles (see [11] and Appendix A in [5]) only the particle-conserving part of the nucleon mass counterterm (responsible for the one fermion → one fermion transition) may be cancelled via one and the same clothing transformation. The rest consists of bad terms which have nonvanishing matrix elements between the vacuum Ω and the fermion-antifermion states. They should be removed via a subsequent UT linear in them.

The experience acquired has allowed us, on the one hand, to reproduce the manifestly covariant result obtained by Feynman techniques and, on the other hand, to derive a new representation (see Eqs. (41) and (47)) for the Feynman integral that corresponds to the fermion self-energy diagram. Of course, here we are dealing with the coincidence of the two divergent quantities: one of them is determined by the nucleon mass renormalization one-loop integral, while the other stems from the commutator $[R, V]$. Trying to overcome this drawback, we face the problem of ultraviolet divergences in the quantum theory of fields. Within the clothing procedure they are removed step by step directly in the Hamiltonian H. As noted in Sect. IV, the form $K(\alpha)$ for $H(a)$ in the clothed particle representation does not contain ultraviolet divergences.

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APPENDIX: EVALUATION OF THE RENORMALIZATION INTEGRALS

By using Eq. (23) we find

$$2\int d\mathbf{k} X_{11}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) = \int d\mathbf{k} \int d\mathbf{q} \sum_{r} \left\{ \frac{V_{11}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{q}, s) V_{11}^{-\mathbf{k}}(\mathbf{q}, s; \mathbf{p}, r)}{E_{\mathbf{p}} - E_{\mathbf{q}} - \omega_{\mathbf{k}}} + \frac{V_{12}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{q}, s) V_{21}^{-\mathbf{k}}(\mathbf{q}, s; \mathbf{p}, r)}{E_{\mathbf{p}} + E_{\mathbf{q}} + \omega_{\mathbf{k}}} + \text{H.c.} \right\}, \quad (42)$$

whence it follows (cf. the transition from Eq. (A.17) to Eq. (A.20a) in [5]) that

$$2\int d\mathbf{k} X_{11}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r) = -\frac{g^2}{4(2\pi)^3} \frac{\delta(\mathbf{p}' - \mathbf{p})}{E_{\mathbf{p}}} \delta_{r'r} I(p), \quad (43)$$

$$I(p) = \int \frac{d\mathbf{k}}{E_{\mathbf{p}-\mathbf{k}} \omega_{\mathbf{k}}} \left\{ \frac{m^2 - E_{\mathbf{p}} E_{\mathbf{p}-\mathbf{k}} + \mathbf{p}(\mathbf{p} - \mathbf{k})}{E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}} - \frac{m^2 + E_{\mathbf{p}} E_{\mathbf{p}-\mathbf{k}} + \mathbf{p}(\mathbf{p} - \mathbf{k})}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}} \right\}. \quad (44)$$

In order to convert the integrand of (44) into a covariant form it is convenient to write down the two equivalent forms:

$$\begin{aligned} \frac{m^2 - E_{\mathbf{p}} E_{\mathbf{p}-\mathbf{k}} + \mathbf{p}(\mathbf{p} - \mathbf{k})}{E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}} - \frac{m^2 + E_{\mathbf{p}} E_{\mathbf{p}-\mathbf{k}} + \mathbf{p}(\mathbf{p} - \mathbf{k})}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}} &\equiv C(-\omega_{\mathbf{k}}) - D(\omega_{\mathbf{k}}) \\ &= D(-\omega_{\mathbf{k}}) - D(\omega_{\mathbf{k}}) + C(-\omega_{\mathbf{k}}) - D(-\omega_{\mathbf{k}}) \\ &= C(-\omega_{\mathbf{k}}) - C(\omega_{\mathbf{k}}) - D(\omega_{\mathbf{k}}) + C(\omega_{\mathbf{k}}). \end{aligned} \quad (45)$$

One can see that the differences $D(-\omega_{\mathbf{k}}) - D(\omega_{\mathbf{k}})$, $D(\pm\omega_{\mathbf{k}}) - C(\pm\omega_{\mathbf{k}})$, and $C(-\omega_{\mathbf{k}}) - C(\omega_{\mathbf{k}})$ are in a simple way expressed through the scalar products $p_{\pm}k$ and $p(p-k)_{\pm}$ where $p - k \equiv (E_{\mathbf{p}-\mathbf{k}}, \mathbf{p} - \mathbf{k}) \neq (E_{\mathbf{p}}, \mathbf{p}) - (E_{\mathbf{k}}, \mathbf{k})$, viz.,

$$D(-\omega_{\mathbf{k}}) - D(\omega_{\mathbf{k}}) = 2\omega_{\mathbf{k}} \frac{m^2 + p(p-k)_-}{2[m^2 + p(p-k)_-] - \mu^2},$$

$$D(\pm\omega_{\mathbf{k}}) - C(\pm\omega_{\mathbf{k}}) = \pm 2E_{\mathbf{p}-\mathbf{k}} \frac{p_\mp k}{\mu^2 \pm 2p_\mp k},$$

$$C(-\omega_{\mathbf{k}}) - C(\omega_{\mathbf{k}}) = 2\omega_{\mathbf{k}} \frac{m^2 - p(p-k)}{2[m^2 - p(p-k)] - \mu^2}.$$

Now, one can see that

$$I(p) = \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} \frac{pk}{\mu^2 - 2pk} - \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} \frac{p_- k}{\mu^2 + 2p_- k} + \int \frac{d\mathbf{k}}{E_{\mathbf{p}-\mathbf{k}}} \frac{m^2 - p(p-k)}{2[m^2 - p(p-k)] - \mu^2} + \int \frac{d\mathbf{k}}{E_{\mathbf{p}-\mathbf{k}}} \frac{m^2 + p(p-k)_-}{2[m^2 + p(p-k)_-] - \mu^2}. \quad (46)$$

Then with the help of the changes: $\mathbf{k} \to -\mathbf{k}$ in the second integral of (46), $\mathbf{p} - \mathbf{k} \to \mathbf{q}$ in the third integral of (46), and $\mathbf{p} - \mathbf{k} \to -\mathbf{q}$ in the fourth integral of (46), these integrals can be transformed to the explicitly covariant quantities,

$$I(p) = \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} pk \left\{ \frac{1}{\mu^2 - 2pk} - \frac{1}{\mu^2 + 2pk} \right\} + \int \frac{d\mathbf{q}}{E_{\mathbf{q}}} \times \left\{ \frac{m^2 - pq}{2[m^2 - pq] - \mu^2} + \frac{m^2 + pq}{2[m^2 + pq] - \mu^2} \right\}. \quad (47)$$

Apparently, such a proof of the p-independence of the renormalization integral is simpler than those based on the different changes of variables that have been mentioned in Section I.

The previous experience makes our evaluation of the crossed (bad) $b^\dagger d^\dagger$ terms a little shorter. Indeed, we have

$$2 \int d\mathbf{k} X_{12}^{\mathbf{k}} = \int d\mathbf{k} [R_{11}^{\mathbf{k}} V_{12}^{-\mathbf{k}} - V_{12}^{-\mathbf{k}} R_{22}^{\mathbf{k}}] + \int d\mathbf{k} [R_{21}^{\mathbf{k}} V_{11}^{-\mathbf{k}} - V_{22}^{-\mathbf{k}} R_{21}^{\mathbf{k}}]^\dagger.$$

For brevity, we have replaced $X_{ij}^{\mathbf{k}}(\mathbf{p}', r'; \mathbf{p}, r)$ with $X_{ij}^{\mathbf{k}}$.

After a simple calculation we arrive at

$$2 \int d\mathbf{k} X_{12}^{\mathbf{k}} = -\frac{g^2}{(2\pi)^3} \frac{m^2}{2} \frac{\delta(\mathbf{p}' - \mathbf{p})}{E_{\mathbf{p}}} I_{r'r}(p), \quad (48)$$

$$I_{r'r}(p) = \int \frac{d\mathbf{k}}{E_{\mathbf{p}-\mathbf{k}} \omega_{\mathbf{k}}} \left\{ \frac{1}{E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}} - \frac{1}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}} \right\} \bar{u}(\mathbf{p}, r') \frac{\hat{k}}{m} v(-\mathbf{p}, r). \quad (49)$$

Transformation properties of $I_{r'r}(p)$ are convenient to study via the decomposition

$$\begin{aligned} \frac{1}{E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}} - \frac{1}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}} &= \frac{1}{E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}} - \frac{1}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}} \\ &\quad + \frac{1}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}} - \frac{1}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}} \\ &= \frac{2E_{\mathbf{p}-\mathbf{k}}}{\mu^2 - 2pk} + \frac{2\omega_{\mathbf{k}}}{2[m^2 + p(p-k)_-] - \mu^2}, \end{aligned} \quad (50)$$

that yields

$$I_{r'r}(p) = A_{r'r}(p) + B_{r'r}(p) + C_{r'r}(p), \quad (51)$$

where

$$A_{r'r}(p) = 2 \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} \frac{\bar{u}(p, r') \frac{\hat{k}}{m} v(-p, r)}{\mu^2 - 2pk},$$

$$B_{r'r}(p) = 2 \int \frac{d\mathbf{q}}{E_{\mathbf{q}}} \frac{\bar{u}(p, r') \frac{\hat{q}}{m} v(-p, r)}{2[m^2 + pq] - \mu^2},$$

and

$$C_{r'r}(p) = 2\bar{u}(\mathbf{p}, r') \frac{\hat{p}}{m} v(-\mathbf{p}, r) \int \frac{d\mathbf{q}}{E_{\mathbf{q}}} \frac{1}{2[m^2 + pq] - \mu^2} = 8\pi \bar{u}(\mathbf{p}, r') \frac{\hat{p}}{m} v(-\mathbf{p}, r) \times \int \frac{q^2 dq}{E_q} \frac{1}{2m[m + E_q] - \mu^2}.$$

These quantities are zero at the point $p = (m, 0, 0, 0)$ together with the coefficients $\bar{u}(\mathbf{p}, r') v(-\mathbf{p}, r)$ in the decomposition (29) of M_{ferm}. However, being p-dependent they do not nullify the g^2-order nondiagonal (crossed) contributions to H in the clothed particle representation (at least, at the first stage of our procedure). These contributions can be eliminated via the correspondent g^2-order clothing transformation.

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