CMB signals of neutrino mass generation

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We propose signals in the cosmic microwave background (CMB) to probe the type and spectrum of neutrino masses. In theories that have spontaneous breaking of approximate lepton flavor symmetries at or below the weak scale, light pseudo-Goldstone bosons recouple to the cosmic neutrinos after nucleosynthesis and affect the acoustic oscillations of the electron-photon fluid during the eV era. Deviations from the Standard Model are predicted for both the total energy density in radiation during this epoch, ΔN_ν, and for the multipole of the n'th CMB peak at large n, Δl_n. The latter signal is difficult to reproduce other than by scattering of the known neutrinos, and is therefore an ideal test of our class of theories. In many models, the large shift $\Delta l_n \approx 8 n_S$ depends on the number of neutrino species that scatter via the pseudo-Goldstone boson interaction. This interaction is proportional to the neutrino masses, so that the signal reflects the neutrino spectrum. The prediction for ΔN_ν is highly model dependent, but can be accurately computed within any given model. It is very sensitive to the number of pseudo-Goldstone bosons, and therefore to the underlying symmetries of the leptons, and is typically in the region of $0.03 < \Delta N_\nu < 1$. This signal is significantly larger for Majorana neutrinos than for Dirac neutrinos, and, like the scattering signal, varies as the spectrum of neutrinos is changed from hierarchical to inverse hierarchical to degenerate.

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I. INTRODUCTION

Over the last five years, a variety of experiments, involving neutrinos from the sun, atmospheric air showers, nuclear reactors and accelerators, have amassed compelling evidence that neutrinos have nonzero masses [1]. A remarkable feature of this data is that the two measured leptonic mixing angles are large. This was a surprise: theories which unify quarks and leptons had led to the expectation that the mixing amongst leptons, like that between quarks, would be small. Hence the data has sparked considerable activity directed toward understanding the origin of these large mixing angles. The more fundamental question of why the neutrino masses are so much smaller than the charged fermion masses has received less attention. There is a general belief that this problem was solved many years ago by the seesaw mechanism [2]. Indeed, one sometimes forgets that the data has not confirmed the seesaw mechanism, and it is worth stressing that there is no known experiment or observation which could test this plausible idea. Given our current theoretical understanding of effective field theories, the seesaw mechanism does indeed appear to give a very natural explanation for the lightness of the neutrinos. But this, like the belief in small mixing angles, is a theoretical prejudice, and in neutrino physics we have learnt to expect surprises.

In this paper we pursue an alternative idea: the neutrinos are light because they are protected from acquiring a mass by a global symmetry which is not broken until energies at or beneath the weak scale. The philosophy is precisely opposite to that of the seesaw mechanism—the underlying physics is not all at extremely high energies where it is hard to test, rather some is at very low energies, and we have missed it because it couples only to neutrinos. Instead of right-handed neutrinos acquiring very large masses at some high scale of lepton number breaking, neutrino masses arise from symmetry breaking at much lower energies. We explore the cosmological consequences of neutrino mass generation at a phase transition at or below the weak scale.

While we do not know of laboratory tests for this idea, signals in the cosmic microwave background (CMB) could not only answer whether the neutrinos are Majorana or Dirac, but also distinguish between the hierarchical, inverted and degenerate patterns of neutrino masses. This signal results because the acoustic oscillations at the eV era are sensitive to the total energy density in relativistic particles and to whether these relativistic particles scatter or free-stream. A measurement of the energy density of this radiation at the few percent level, and especially its scattering characteristics, will probe physical processes occurring in the neutrino fluid at and before the eV era.

II. NEUTRINO MASS GENERATION

A. Why are Neutrinos Light?

The mass scale of all the quarks and charged leptons is set by the scale v of electroweak symmetry breaking: $\langle h \rangle = v$, where h is the electroweak Higgs field. The interaction generating these masses is assumed to have the form $\overline{\psi}_L \psi_R h$, where ψ can be any of the quarks or charged leptons. If neutrino masses also originate from such an interaction, it is hard to understand why neutrinos would be so much lighter than the charged fermions. The beauty of the seesaw mechanism is that it explains why, of all the fermions, it is the neutrinos which are light. The only fermion that does not couple to the known gauge interactions is the right-handed neutrino, and hence it is not protected by gauge symmetry from acquiring a large Majorana mass, M_R. On integrating it out of the theory, the left-handed lepton doublet ℓ acquires an interaction which is bilinear in h

$$\frac{1}{M_R}\ell\ell hh, \tag{1}$$

leading to a neutrino mass which is quadratic in v, $m_\nu \approx v^2/M_R$, rather than the linear formula that applies to the charged fermions: $m \approx v$. Indeed, Eq. (1) provides a more primitive understanding of why the neutrinos are light. As long as the low energy effective theory does not contain right-handed neutrinos, there are no renormalizable operators that give a neutrino mass—the first neutrino mass operator appears at dimension 5.

We propose instead that the neutrinos are protected from acquiring a mass at the weak scale by a global symmetry, as in [3], [4]. The operator (1) does not possess such a symmetry, and hence is not the TeV description of neutrino masses we seek. The operators relevant for neutrino masses must involve a new scalar field ϕ which carries a charge under the global symmetry. Thus the TeV description of neutrino masses is given by operators of the form

$$\ell n\, h\left(\frac{\phi}{M}\right)^N, \qquad \ell\ell\frac{hh}{M}\left(\frac{\phi}{M}\right)^N, \tag{2}$$

where n represents the right-handed neutrino, M is a mass scale larger than v, and N is a positive integer. The first operator applies if lepton number is conserved, leading to Dirac neutrino masses, otherwise the second operator applies and the neutrinos are Majorana. Unlike the case of the seesaw mechanism, there is no preference for the neutrinos to be Majorana. Very stringent bounds would result if the Goldstone coupled to charged leptons and quarks; but these couplings are predicted to be absent because the charged fermion masses are not protected by the global symmetry.

At sufficiently high temperatures in the hot big bang, the ϕ and n particles will be in thermal equilibrium with the particles of the standard model. However, if ϕ and n only interact with standard model particles via (2), then they will drop out of thermal equilibrium as the universe cools so that there is an era of two separate sectors. During this era we assume that sufficient entropy is created in the standard model sector, from phase transitions or from heavy particle annihilations, so that the temperature rises significantly above that of the (ϕ, n) sector. Hence big bang nucleosynthesis is essentially unchanged by the extra sector.

Below we describe a minimal model for the flavor symmetry breaking sector. However, the details of any particular model are not as important as the model independent mechanism. Once symmetry breaking occurs in the ϕ sector, a set of Goldstone bosons, G, are produced. The CMB signals result from the interactions of G with neutrinos at very low energies, and will be discussed in section III.

B. A Minimal Model: $\mathrm{U}(1)_L$

We choose the global symmetry to be lepton number, $\mathrm{U}(1)_L$, defined with charge $+1$ on all neutrino fields (i.e., on both ν_i and n_i). Just below the scale of electroweak symmetry breaking the neutrino mass generation sector is described by

$$\mathcal{L}_\nu = \sqrt{2}g_i\left(\nu_i n_i\phi, \frac{1}{2}\nu_i\nu_i\phi\right) + \text{h. c.} \\ -\left[-\mu^2\phi^\dagger\phi + \frac{\lambda}{2}(\phi^\dagger\phi)^2\right] \tag{3}$$

together with kinetic energy terms for ν_i, ϕ (and for n_i if the neutrino is Dirac). For simplicity we have taken the neutrino interaction to be linear in ϕ by requiring the complex scalar ϕ to have lepton number -2. The index i runs over the three generations of neutrinos, and we have rotated the neutrino fields to a mass eigenstate basis. Small values for the dimensionless couplings g_i are perfectly natural, reflecting the hierarchy between v and M.

We have taken the sign of the scalar mass term negative to ensure that $\mathrm{U}(1)_L$ is spontaneously broken, $\langle\phi\rangle = f/\sqrt{2} = \mu/\sqrt{\lambda}$. This minimal model does not address the origin of the neutrino mass ratios, which follow from $g_{1,2}/g_3$. We assume the largest coupling, g_3, does not involve any small dimensionless parameter, so that g_3 is v/M for the Dirac case or $(v/M)^2$ for the Majorana case, giving a neutrino mass m_{ν_3} of $(v/M)f$ or $(v/M)^2 f$, respectively. The mass scale M is then $(f/m_{\nu_3})v$ or $\sqrt{f/m_{\nu_3}}v$, respectively. This should be compared with the seesaw result: $M_R \approx (v/m_{\nu_3})v \approx 10^{12}v$. For $f \ll v$, the scale of the underlying physics is reduced: $M \ll M_R$. At what scale, f, should the global symmetry be broken? If we take f all the way down to m_{ν_3}, then $M \approx v$ so that the physics generating the nonrenormalizable operators (2) becomes accessible to high energy colliders. However, in this case the dimensionless coupling g_3 is of order unity, so that in the early universe both ϕ and n become part of the thermal bath during the MeV era, conflicting with big bang nucleosynthesis [5]. If f were larger than the electroweak scale, then $g_3 \lesssim 10^{-12}$, which is too small to generate the CMB signals we have in mind. Hence we study an intermediate situation where

$$m_\nu \ll f \lesssim v. \tag{4}$$

We will construct theories in which such low symmetry breaking scales arise naturally in section VIII. Breaking lepton number below the weak scale also avoids the potential danger that the baryon asymmetry will be erased [6].

In the spontaneously broken phase, $\phi = (f + H + iG)/\sqrt{2}$, where G is a physical Goldstone boson, and H a Higgs boson. The coupling of the neutrino to G and H is given by

$$\mathcal{L}_\nu = g_i\left(\nu_i n_i, \frac{1}{2}\nu_i\nu_i\right)(H + iG). \tag{5}$$

The analysis of this paper is almost entirely based on these couplings, and the symmetry breaking sector that leads to them is of lesser importance. The masses of G and H play a very important role. If the self-interactions of ϕ are of order unity, then one expects $m_H \approx f$. In this case it is G which is of interest to us. We assume that G is actually a pseudo-Goldstone boson, with a nonzero mass m_G, as studied in [7]. If this mass arises from a dimension 5 interaction suppressed by the Planck scale, M_P, then we expect $m_G^2 \approx f^3/M_P$. We will not limit ourselves to this case, but take $m_G \ll f$ to be a free parameter. If ϕ is weakly coupled so that $m_H \ll f$, then the Higgs can also play an important role in generating CMB signals. However, Higgs particles lighter than f require further small parameters. In this paper we focus on signals from pseudo-Goldstone bosons (PGBs). They are naturally light, and they interact with neutrinos only via the couplings g_i. Furthermore, they generically have interactions among themselves with strength proportional to explicit symmetry breaking that gives PGBs mass. In more general models of spontaneously broken lepton flavor symmetry there are many PGBs, G_A.

C. Origins of CMB Signals

The interactions of PGBs with neutrinos can alter the energy density in neutrinos and cause neutrinos to scatter rather than free-stream during the eV era. Both effects leave characteristic signals in the CMB. The PGBs, G_A, interact only via the small dimensionless couplings g_i to neutrinos, implying that in the early universe the rate for neutrinos to scatter into G_A has a recoupling form; that is, the rate increases relative to the expansion rate as the temperature, T, drops. At some recoupling temperature some subset of the n_i, G_A sector recouples to the left-handed neutrinos, ν_i, so that this subset gets reheated. Different subsets may get reheated at various recoupling temperatures. However, while this reheating creates entropy, it does not change the total radiation energy density, so recoupling itself does not lead to a change in the radiation energy density at the eV era.

As the ν, n, G fluid cools, the temperature will drop beneath the mass of one of the PGBs, G_H. Since G_H is in thermal equilibrium with ν, n and G_A, its number density becomes exponentially reduced via decays or annihilations and the neutrino fluid is adiabatically heated, which *does* lead to a change in the radiation energy density. The size of this signal depends on how many right-handed neutrinos and scalars are recoupled, which depends on whether the neutrinos are Dirac or Majorana and on the strength of the couplings g_i that each neutrino has with G_A. Provided G_H has recoupled and then disappeared before the eV era, the cosmic microwave background will have acoustic oscillations which reflect a radiation energy density that differs from standard cosmology and depends on the type and spectrum of the neutrinos.

If a light PGB recouples to neutrinos before the eV era, but has a mass less than 1 eV, it may prevent one or more of the neutrino species from free-streaming. In the standard model, neutrino free-streaming shifts the multipoles of the nth CMB peak by a large amount, $\Delta l \approx -23$, at large l, so the absence of free-streaming will produce a large signal in future CMB datasets.

III. THE CMB SIGNALS

In the previous section we have discussed an alternative origin for light neutrino masses, involving extra states at low energy, including Goldstone bosons, Higgs and possibly right-handed neutrino states. We have outlined how the interaction of these extra states could lead to signals in the CMB. In this section we study each of the CMB signals in some depth. We give general formulas for both signals in a model independent way, and discuss the range of new physics which could lead to each signal.

A. The Relativistic Energy Density Signal

Measurements of the precise form of the CMB acoustic oscillations provide a powerful constraint on the relativistic energy density of the universe during the eV era, $\rho_{\rm rel}$. In the standard model, $\rho_{\rm rel}$ is precisely predicted during this era, so these measurements are powerful probes of nonstandard physics. Increasing $\rho_{\rm rel}$ has several physical effects. For example, last scatter occurs at a fixed temperature, and hence as $\rho_{\rm rel}$ increases so the time at last scatter decreases. This decreases the horizon at last scatter and hence shifts the acoustic peaks to higher multipole l. An increase in $\rho_{\rm rel}$ leads to a lowering of the redshift of matter-radiation equality. This leads to larger amplitude acoustic peaks at low l [8], and a marked increase in the damping of the peaks at large l [9].

What mechanism would allow the total radiation energy density during the eV era to differ from that predicted by standard cosmology? We begin by discussing three important types of process: fragmentation, recoupling and disappearance by decay or annihilation.

In cosmology it is well-known that particles which interact with each other at very high temperature may no longer have thermal communication at lower temperature. In general one should expect that as the universe cools the fluid fragments into multiple components or sectors. This fragmentation occurs whenever there are no large renormalizable interactions between particles. Neutrinos provide the most familiar example: below the weak scale they only interact via the nonrenormalizable Fermi interaction and they fragment away from the electron/photon fluid at the MeV era. It appears quite likely that dark matter and dark energy are sectors that fragmented from the visible sector at some stage of cosmological evolution. We therefore write the relativistic energy density after fragmentation of the known neutrinos as

$$\rho_{\rm rel} = \frac{\pi^2}{30}\left(2T^4 + g_\nu T_\nu^4 + \sum_a g_a T_a^4\right), \qquad (6)$$

where T, T_ν, T_a are the temperatures of the photons, neutrinos and other sectors and the spin degeneracies g_ν, g_a for neutrinos and other radiation sectors include a factor of $7/8$ for fermions. It is not always necessary that each sector actually be in thermal equilibrium. For example, during this era in the standard cosmology the neutrinos are free-streaming. In the standard cosmology, $g_a = 0$, $g_\nu = 21/4$ and $T_\nu = (4/11)^{1/3}T$, so that

$$\rho_{\rm rel} = \frac{\pi^2}{30}(2 + 0.45N_\nu)T^4, \qquad (7)$$

where the number of neutrinos, N_ν, is 3. In nonstandard cosmologies we use (7) to define N_ν, so that N_ν may differ from three even if there are three species of neutrinos.

It is important to distinguish two very different ways in which CMB experiments could measure $N_\nu \neq 3$. If N_ν does not change between nucleosynthesis (BBN) and the eV era (CMB), then CMB could then discover $\delta N_\nu = \pm(0.5 - 1.0)$, depending on the uncertainties from BBN. Such a signal could be very significant given the small uncertainties possible in future CMB measurements. Since $N_\nu = (\rho_\nu + \sum_a \rho_a)/0.23\rho_\gamma$, this could probe the radiation density ρ_a in some sector that fragmented from the standard model sector before, perhaps much before, BBN.

Perhaps less well-known than the phenomenon of fragmentation is that of recoupling. If the renormalizable couplings between particles in different fragmented sectors are small rather than vanishing, then eventually, as the universe cools, the sectors will recouple back into a single thermal component. The smaller the renormalizable coupling the lower the recoupling temperature.

In this paper we will be concerned with recoupling contributing to a signal arising from a change in the ratio $(\rho_\nu + \sum_a \rho_a)/\rho_\gamma$ between BBN and CMB [10]. We find such a possibility particularly exciting because it indicates that new physics is affecting cosmological evolution after BBN. It could arise from a nonstandard evolution of ρ_γ or of $(\rho_\nu + \sum_a \rho_a)$. Particularly large effects result if the electron/photon fluid recouples to some previously fragmented sector of spin degeneracy g. If the temperature of the fragmented sector is small prior to recoupling, the photon cooling results in a large relative increase in the importance of the neutrinos, giving $\delta N_\nu = 3.7g$. This has already been excluded by the first year of the Wilkinson Microwave Anisotropy Probe (WMAP) data [11], except for the case of $g = 1$ [5][12]. Photon heating is also possible, for example, from the out-of-equilibrium decay of a nonrelativistic species.

A particularly important way of changing the radiation density of any sector is if some particle species in that sector becomes nonrelativistic. If the number density of the heavy particle maintains an exponentially suppressed equilibrium form, then the decay or annihilation of the particle occurs at constant entropy. This results in an increase in the temperature of the remaining radiation of that sector by a factor $(g_i/g_f)^{1/3}$, where g_i and g_f are spin degeneracies of the radiation of that sector before and after the disappearance of the heavy species. This mechanism is familiar from the annihilation of electron/positron pairs which heat the photons giving $T_\nu = (4/11)^{1/3}T_\gamma$. As an example of a nonstandard evolution of ρ_γ, suppose that the photon first recouples to a sector of spin degeneracy g, and then all the species of that sector become nonrelativistic before the eV era, the CMB signal is $\delta N_\nu = 3[2/(2+g)]^{1/3} - 3$. A wide range of g is allowed by the WMAP data and could be observed by future CMB experiments.

In this paper, we will be interested in the case that a CMB signal arises because of a nonstandard evolution of $\rho_\nu + \sum_a \rho_a$ after BBN, even though the signals tend to be smaller than those which arise from a nonstandard evolution of ρ_γ. Such a signal could arise whenever some particle of a fragmented sector becomes nonrelativistic and disappears. While the temperature of this sector would rise by a factor of $(g_i/g_f)^{1/3}$, this typically does not give an observable signal since the energy density in the sector is highly subdominant. We will explore theories of neutrino mass generation where the sector that generates the neutrino masses, including the right-handed neutrinos if they are Dirac, recouples to the sector of the left-handed neutrinos. This recoupling ensures that the physics of the new sector is not subdominant. While the recoupling of two such relativistic sectors does not by itself lead to a change in the total energy density[1], the

[1] When heat is exchanged between two relativistic sectors, the form of the energy-momentum tensor is not changed, hence the expansion rate of the universe is also unchanged. Since there is no change in the gravitational energy, the total fluid energy is unchanged.

effects of particle decay or annihilation in the new sector does give signals which depend on the type and spectrum of the neutrinos, and on the mass generation mechanism, and which can be computed from equilibrium thermodynamics.

Suppose that after BBN $n_{\rm rec}$ of the left-handed neutrino species recouple to some sector of G, n particles with spin degeneracy g. This recoupling may occur at a variety of temperatures, but we assume that at some stage the resulting ν, G, n fluid thermalizes at a single temperature. Subsequently we assume that some subset of the states of this recoupled sector, with spin degeneracy g_H, become heavy. As the universe expands further, but well before the eV era, the number density of the heavy states becomes exponentially suppressed so that their entropy is transferred to the lighter states. This may occur at several stages with different heavy species having different masses. If the whole process occurs without chemical potentials, then the prediction for the relativistic energy density during the eV era is then given by

$$N_\nu = 3 - n_{\rm rec} + n_{\rm rec}\left[\frac{n_{\rm rec} + \frac{4}{7}g}{n_{\rm rec} + \frac{4}{7}(g - g_H)}\right]^{1/3}. \qquad (8)$$

In the case where the recoupling reactions lead to chemical potentials this prediction is modified. It is no longer sufficient to calculate the neutrino temperature after the pseudo-Goldstone bosons disappear. Instead, both the temperature and all chemical potentials must be determined at each step. Typically all chemical potentials are related such that there exists only one additional degree of freedom.

At the first step, when the pseudo-Goldstone bosons equilibrate, the total energy density is conserved. In addition, the presence of a nonzero chemical potential implies that there exists an additional conserved charge. These two conservation laws provide us with two equations to solve for the two unknowns: temperature and chemical potential. When the Goldstone bosons disappear energy is not conserved. Instead comoving entropy density, together with the additional quantum number, are conserved. This again allows us to calculate the final temperature and chemical potential. From these two parameters we are able to calculate the final energy density in neutrinos and finally the effective number of neutrinos at matter-radiation equality

$$N_\nu = (3 - n_{\rm rec}) + n_{\rm rec}\frac{\rho_{\nu_R}(T, \mu)}{\rho_{\nu_R}(T_{SM}, 0)}, \qquad (9)$$

where $T_{SM} = (4/11)^{1/3}T_\gamma$ is the temperature the neutrinos would have had in the standard model and ρ_{ν_R} is the energy density in the $n_{\rm rec}$ neutrino species that recoupled.

B. The Neutrino Scattering Signal

Recently Bashinsky and Seljak have given an analytic understanding of the effects of neutrino free-streaming on the position and amplitudes of the CMB acoustic peaks in the standard model [9]. Here we briefly summarize some of their results, using their notation.

In the conformal Newtonian gauge, the Robertson-Walker metric with scalar metric perturbations takes the form

$$ds^2 = a^2(\tau)[-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)d\mathbf{r}^2], \qquad (10)$$

where τ is conformal time and a is the cosmological scale factor. In this gauge, the density perturbation in the relative photon number density $d_\gamma = \delta n_\gamma(\mathbf{r})/n_\gamma(\mathbf{r})$ satisfies the equation

$$\frac{d^2 d_\gamma}{d\tau^2} - \frac{1}{3}\nabla^2 d_\gamma = \nabla^2(\Psi + \Phi). \qquad (11)$$

In the absence of any particle species which free-streams, the energy-momentum tensor takes the form for a locally isotropic fluid, resulting in the equality of the scalar metric perturbations: $\Psi = \Phi$. However, neutrino free-streaming introduces a direction at each locality, so that the energy-momentum tensor becomes anisotropic, with off-diagonal entries in the spatial subspace proportional to a free-streaming potential π_ν. This anisotropy induces a difference between the scalar metric perturbations:

$$\Psi - \Phi = 6\frac{R_\nu \pi_\nu}{\tau^2}, \qquad (12)$$

where R_ν is the fraction of radiation in neutrinos:

$$R_\nu = \frac{\rho_\nu}{\rho_\nu + \rho_\gamma} \simeq \frac{0.23N_\nu}{1 + 0.23N_\nu}. \qquad (13)$$

If the initial density perturbations are adiabatic, then the perturbation in the relative number density of all species, before they enter the horizon, are given by $d(\mathbf{r}) = -3\xi(\mathbf{r})$, where $\xi(\mathbf{r})$ describes the primordial perturbation. Solving first for π_ν, next for the metric perturbations, and finally for the photon perturbation at comoving wavenumber k, Bashinsky and Seljak find acoustic oscillations in the radiation dominated era for high k of the form

$$d_\gamma(\tau, k) = 3\xi(k)(1 + \Delta_\gamma)\cos\left(k\tau/\sqrt{3} + \delta\varphi\right), \qquad (14)$$

where, to leading order in R_ν, $\Delta_\gamma = -0.27R_\nu$ and

$$\delta\varphi = 0.19\pi R_\nu. \qquad (15)$$

The amplitude shift of the primordial spectrum, Δ_γ, can only be probed by observations which compare the photon and cold dark matter perturbations, and we do not consider them further. The nth peak of the CMB acoustic oscillations occurs at a wavenumber k_n such that the mode has n half cycles of oscillation between horizon crossing and last scatter (LS):

$$\frac{k_n \tau_{LS}}{\sqrt{3}} = n\pi - \delta\varphi, \tag{16}$$

where τ_{LS} is the time of last scatter. Since the multipole of the peak $l_n \propto k_n$, $\delta\varphi$ causes a shift in the position of the nth peak. This analytic solution accurately reproduces the numerical results obtained by the CMBFAST code, and it is apparent that this shift of the position of the peaks is purely due to the free-streaming of the neutrinos, since, in the absence of free-streaming, both π_ν and $\delta\varphi$ vanish. As pointed out in [9], this effect stems from the fact that neutrino perturbations are able to propagate faster than the speed of sound in the baryon-photon fluid. Thus, the free-streaming of N_ν species of neutrinos shifts the positions of the peaks, at large n, by

$$\Delta l_n \simeq -57\left(\frac{0.23N_\nu}{1+0.23N_\nu}\right)\left(\frac{\Delta l_{\rm peak}}{300}\right), \tag{17}$$

giving $\Delta l_n \simeq -23.3$ for $N_\nu = 3$. Here $\Delta l_{\rm peak}$ is the difference in multipole between successive peaks at large n. As the number of free-streaming neutrinos is increased above the standard model value of 3, so the peaks shift to lower l. The beauty of this signal is that, for adiabatic perturbations, it is not mimicked by changing other parameters of the theory. While other parameters do cause a shift in the positions of the peaks, only by changing the free-streaming behavior can one obtain a nonisotropic energy-momentum tensor which leads to a shift in the nth peak which is *independent* of n.

With these results in hand, we immediately see that there is an important signal in the position of the CMB peaks for theories where one or more neutrinos are scattering during the eV era rather than free-streaming. In general one must replace (13) by

$$R_\nu = \frac{\rho_{FS}}{\rho_{\rm rel}} = \frac{0.23N_{\nu FS}}{1+0.23N_\nu} \tag{18}$$

where ρ_{FS} is the energy density of the relativistic components that free-stream, while $\rho_{\rm rel}$ is the total relativistic energy density, including both free-streaming and scattering components. $N_{\nu FS}$ is the energy density of the relativistic free-streaming particles, expressed as an equivalent number of neutrino species in analogy to Eq. (7).

Consider a simple limit where the energy density of the free-streaming neutrinos and the total radiation is standard. If we indicate the number of free-streaming neutrino species by n_{FS}, then in this limit $N_{\nu FS} = n_{FS}$ and thus

$$\Delta l_n \simeq -7.8 n_{FS}\left(\frac{\Delta l_{\rm peak}}{300}\right). \tag{19}$$

Relative to the position of the peaks expected for the standard model with three free-streaming neutrinos, there is a uniform shift in the position of the peaks to *larger* l. This can only result if some of the known neutrinos are not free-streaming, and in this simple example the shift in l is 7.8 for each scattering neutrino species.

The result (19) applies if the energy densities are standard. However, since nonstandard physics is required to prevent free-streaming, it could well be that the neutrino energy densities are also nonstandard, or there could be energy density from light PGBs. How does this effect the change in the position of the peaks induced by the phase shift $\delta\phi$? From (18) we need expressions for $N_{\nu FS}$ and N_ν. The value of N_ν is predicted in our theories by (8), or (9) for a nonzero chemical potential, and a similar result can be derived for $N_{\nu FS}$. In terms of these quantities

$$\Delta l_n \simeq -57\left(\frac{0.23N_{\nu FS}}{1+0.23N_\nu}\right)\left(\frac{\Delta l_{\rm peak}}{300}\right). \tag{20}$$

Clearly, in general the position of the peaks could be determined by a combination of extra neutrinos (17), scattering of the known neutrinos (19), and nonstandard energy densities of the known neutrinos (20). In this paper we limit our consideration to the case of theories with three neutrinos. We will find that the mass generation mechanism gives large regions where the prediction (20) differs significantly from the shift of -23.3 expected in the standard model. The largest effect comes from $n_{FS} \neq 3$, but a significant deviation from (19) may result because the energy density in each free-streaming neutrino is nonstandard.

In this section we have discussed two different CMB signals that occur in theories with three neutrino species if their interactions are nonstandard. The first probes nonstandard neutrino energy densities which lead to effects such as delayed matter-radiation equality and enhanced damping of acoustic oscillations at higher l. However, we have shown that such signals can result from a variety of underlying physics origins which lead to a change of N_ν. In contrast, the second signal—a uniform shift of the CMB peaks to larger l, (19)—is a unique signal for the absence of free-streaming of one or more of the known neutrino species. If the physics leading to neutrino scattering also substantially affects the neutrino energy densities, then a measurement of Δl_n would not only reveal neutrino scattering, but would also confirm nonstandard energy densities.

IV. SIGNAL REGIONS FOR ONE NEUTRINO

A complete analysis of the CMB energy density and scattering signals from the interactions of PGBs with neutrinos is complicated. Several stages of spontaneous breaking of lepton flavor symmetries can each lead to several PGBs, and the combination of neutrino and PGB mass matrices leads to many parameters. In this section we study the simplest situation that leads to our CMB signals: a single PGB coupled to a single neutrino. There are three independent parameters, the PGB and neutrino masses, m_G and m_ν, and the coupling strength g of the interaction between them. This coupling parameter g can be traded for the symmetry breaking scale at which the PGB is produced $f = m_\nu/g$. We aim to discover whether, for typical values of the neutrino mass suggested by data, there are large regions in the (f, m_G) parameter space that give observable CMB signals. We will first consider the case that the neutrinos are Majorana, and later discuss the minor changes that must be made in the case of Dirac neutrinos.

A. Majorana Neutrinos

Since G is a pseudo-Goldstone boson, at low energies it does not possess significant self-interactions, so that the only interaction of interest is given by the Lagrangian term

$$\mathcal{L} \supset \frac{g}{2}\nu\nu(H + iG). \tag{21}$$

This interaction results in three possibly interesting processes: $\nu\nu \leftrightarrow G$, $\nu\bar{\nu} \leftrightarrow GG$ and $\nu\nu \leftrightarrow \nu\nu$. We will show that the first process has a rate that increases relative to the Hubble expansion rate as the temperature of the universe decreases. Therefore, this rate may lead to the recoupling required to produce the signals we seek. The second process may also couple neutrinos to the pseudo-Goldstone boson. However, for $T < \text{MeV} < m_H$ the rate has a decoupling form and is therefore unable to produce either signal. Instead, demanding that neutrinos and Goldstone bosons be decoupled prior to big bang nucleosynthesis will lead to an upper bound on the coupling constant g, and therefore a lower bound on f. Finally, the rate for $\nu\nu \leftrightarrow \nu\nu$ will be shown to be too slow to produce a signal.

The first process, $\nu\nu \leftrightarrow G$, is able to generate either a ΔN_ν and Δl signal. The regions of parameter space in which each signal is possible are clearly separated by simple kinematics. For a change in the effective number of neutrinos N_ν to be possible, G must go out of the bath prior to $T_{\rm eq} \sim 1$ eV. Therefore, the scalar mass must satisfy $m_G > 1$ eV. On the other hand, for the neutrinos to not free-stream during the era probed by the acoustic oscillations they must be scattering during the eV era. For this to be possible the scalar mass must satisfy $m_G < 1$ eV.

To see more precisely in what regions of parameter space a signal results, we will consider each process separately, beginning with the two-to-one process $\nu\nu \leftrightarrow G$. This process occurs at a rapid rate in the presence of a thermal bath of particles despite the severe kinematic restrictions. For $T \gg m_G > 2m_\nu$ the rate is approximately given by[2]

$$\Gamma(\nu\nu \leftrightarrow G) \approx \frac{m_\nu^2 m_G^2}{16\pi f^2 T}. \tag{22}$$

Defining the recoupling temperature as the temperature at which $\Gamma(T_{\rm rec}) = H(T_{\rm rec})$, we find that

$$T_{\rm rec}(\nu\nu \leftrightarrow G) \approx \left(\frac{m_\nu^2 m_G^2 M_{pl}}{16\pi f^2}\right)^{1/3}. \tag{23}$$

For temperatures below $T_{\rm rec}$, G will be in equilibrium with the neutrinos. If this is the only process that brings the G into thermal contact with the neutrinos, then there is a conserved quantity, $n_\nu + 2n_G$, that is left unchanged as the Goldstone boson comes into equilibrium. This conservation law implies the presence of a chemical potential satisfying $2\mu_\nu = 2\mu_{\bar{\nu}} = \mu_G$.

If the recoupling temperature is below m_G, then the number of Goldstone bosons produced will be exponentially suppressed and they will not be able to generate a signal. Thus we must demand that $T_{\rm rec} > m_G$. This leads to a bound on the symmetry breaking scale f

$$f < f_1 = m_\nu\left(\frac{M_{pl}}{16\pi m_G}\right)^{1/2}. \tag{24}$$

If this limit is satisfied, and $m_G > 1$ eV, then the Goldstone boson will decay prior to $T_{\rm eq}$. This will alter the energy density in neutrinos as discussed in Sec. III A, changing the effective number of neutrinos, N_ν, that will be measured by a CMB experiment. The presence of a nonzero chemical potential does not alter this key result but will affect the magnitude of this shift. The line $f = f_1$ is displayed in Fig. 1.

On the other hand, if $m_G < 1$ eV, then $\nu\nu \leftrightarrow G$ will be kinematically allowed during acoustic oscillation and may therefore prevent the neutrinos from free-streaming during this period. However, the scattering angle is kinematically restricted to be quite small, $\theta \sim m_G/T$. For this

[2]The approximations we made here include: (a) neglecting the difference between T^n and the average of E^n, (b) using simply T^2/M_{pl} and T^3 for the expansion rate H and the number densities, respectively, neglecting prefactors such as $g\pi^2/30$ and subdominant corrections due to possible nonzero chemical potentials.

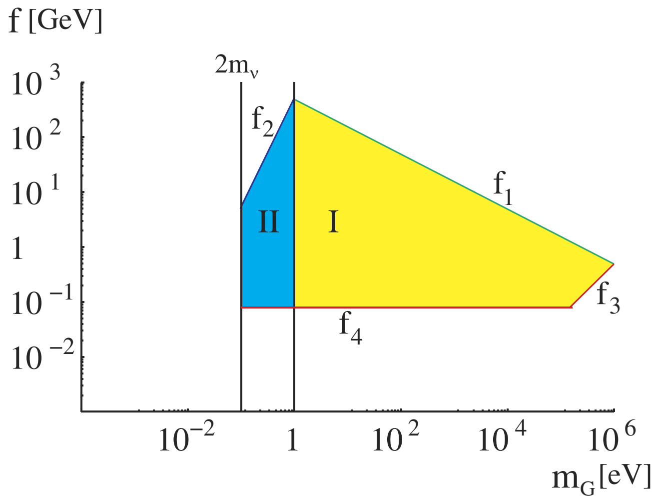

FIG. 1 (color online). Signal regions and cosmological bounds for a single Majorana neutrino coupled to a single pseudo-Goldstone boson. The lines and regions are labeled as in the text. CMB signals occur throughout the two shaded regions. The area below f_3 and f_4 is excluded by BBN, and in the region above f_1 and f_2 the PGB is too weakly coupled to give any signal. There is an energy density signal in region I and a scattering signal in region II. We have assumed $m_\nu = 0.05$ eV, $\lambda = 1$.

process to isotropize the neutrino momentum they must participate in N scatterings such that $\theta^2_{\rm tot} \approx (m_G/T)^2 N \approx 1$. In this case, we must demand that $\Gamma > HN$ resulting in the limit

$$f < f_2 = m_\nu \left(\frac{M_{pl} m_G^4}{16\pi T_*^5} \right)^{1/2}, \tag{25}$$

where $T_* \sim 1$ eV is the temperature at which the perturbation enters the horizon.

We must also require that G does not come into equilibrium with the neutrinos prior to the decoupling of the weak interactions. If this were to occur they would increase the energy density in radiation during big bang nucleosynthesis conflicting with observations of primordial elemental abundances. Therefore we must demand that $T_{\rm rec} < T_W \sim 1$ MeV. This places a bound on f of

$$f > f_3 = m_\nu \left(\frac{m_G^2 M_{pl}}{16\pi T_W^3} \right)^{1/2}. \tag{26}$$

We now turn to the two-to-two process $\nu\bar{\nu} \leftrightarrow GG$. In the nonderivatively coupled basis in which we work, this process is dominated by the exchange of a virtual Higgs boson for $T > m_\nu$. The rate is given approximately by

$$\Gamma(\nu\bar{\nu} \leftrightarrow GG) \approx \frac{m_\nu^2 T^3 m_H^4}{32\pi f^4 (T^2 - m_H^2)^2}. \tag{27}$$

For temperatures below the Higgs mass, m_H, this rate goes like T^3 and has a decoupling form. Therefore, this process cannot be used to recouple the Goldstone boson to the neutrinos. Instead, we must demand that this process not keep neutrinos and Goldstone bosons in equilibrium prior to BBN. To insure that the neutrino and Goldstone boson sectors are decoupled it suffices to demand that they not be in thermal contact at $T \sim m_H$ when the rate relative to the Hubble expansion rate is maximal. The requirement that $\Gamma(T = m_H) < H(T = m_H)$ leads to a lower bound on f given by

$$f > \left(\frac{m_H m_\nu^2 M_{pl}}{32\pi} \right)^{1/4}. \tag{28}$$

Setting $m_H = \sqrt{\lambda} f$ gives

$$f > f_4 = \left(\frac{\sqrt{\lambda} m_\nu^2 M_{pl}}{32\pi} \right)^{1/3}, \tag{29}$$

where λ is a dimensionless parameter describing the Higgs self coupling.

Finally, we consider $\nu\nu \leftrightarrow \nu\nu$ which is mediated by the exchange of a virtual Goldstone boson. The rate for this process is given approximately by

$$\Gamma(\nu\nu \leftrightarrow \nu\nu) \approx \frac{m_\nu^4 T^5}{16\pi f^4 (T^2 - m_G^2)^2}. \tag{30}$$

For our purposes, this process is only interesting if it is able to prevent the neutrino from free-streaming during the eV era. This will be the case if $\Gamma(T \sim 1~{\rm eV}) > H(T \sim 1~{\rm eV})$. For $m_G < 1$ eV this implies

$$f < m_\nu \left(\frac{M_{pl}}{16\pi~{\rm eV}} \right)^{1/4}. \tag{31}$$

For $m_\nu \lesssim 1$ eV, as implied by the recent WMAP data combined with the measurements of large scale structure [13] as well as tritium beta decay [14] and double beta decay experiments [15], this bound is smaller than the lower bound coming from $\nu\nu \leftrightarrow GG$. If $m_G > 1$ eV then the rate will be further suppressed by ${\rm eV}^4/m_G^4$. Therefore, as stated at the beginning of this section, the $\nu\nu \leftrightarrow \nu\nu$ process is unable to produce an interesting signal.

All of the above signal regions, together with the cosmological bounds arising from big bang nucleosynthesis, are displayed in Fig. 1. Note that the line for f_4 has been drawn for a self coupling parameter $\lambda = 1$. For smaller values of λ the allowed region grows to include lower values of f. There we see that there are two distinct signal regions. In region *I*, ΔN_ν, as measured from the relativistic energy density, is nonzero. Further, the neutrinos (and Goldstone bosons prior to decay) have a nonzero chemical potential. In region *II*, $\rho_{\rm rel}$ is unchanged, but the neutrino is no longer able to free-stream at $T \sim 1$ eV. As a result, there will be an overall phase shift in the angular power spectrum relative to the standard model prediction.

It is important to also understand if there are any bounds on our scenario from astrophysical processes. Because the pseudo-Goldstones couple to the quarks and leptons that make up astrophysical objects only through neutrinos, their effects only show up in highly dense regions like the cores of supernovae [16], [17]; for earlier work see, for example, [18]. The presence of pseudo-Goldstone bosons can affect a supernova in two different ways. The decay of electron neutrinos into Goldstone bosons can deleptonize the core prior to the "bounce" preventing the bounce from taking place. This puts a bound on the electron neutrino coupling to Goldstone bosons. Further, after the bounce, the supernova can lose energy too rapidly through neutrino or antineutrino decays into Goldstone bosons, which then free-stream out. This also puts bounds on the Goldstone boson couplings to the various neutrino species. All these constraints tend to depend in detail on the supernova dynamics but are typically at the $g \lesssim 10^{-6}$ level or so. This is a much weaker bound than that arising from considerations of big bang nucleosynthesis and is therefore not included in Fig. 1.

B. Dirac Neutrinos

For Dirac neutrinos the relevant interaction vertex involves both a left- and right-handed neutrino

$$\mathcal{L} \supset g\nu n(H + iG). \tag{32}$$

The two-to-two process $\nu\bar{\nu} \leftrightarrow GG$ is changed because now the Higgs boson couples to one left-handed neutrino and one right-handed neutrino. Consequently, the diagram with the virtual Higgs boson, which was the dominant one in the Majorana case, now needs a chirality flip for one of the initial left-handed neutrinos. The flip suppresses this diagram by a factor of m_ν/T in the amplitude. The amplitude now equals that from diagrams a virtual right-handed neutrino and no chirality flip. Thus the rate for both is suppressed by m_ν^2/T^2 relative to Eq. (27). Alternatively, the process could involve an initial state right-handed neutrino, in which case the rate will be suppressed by $r = n_n/n_\nu$. The value for r is expected to be smaller than 0.1 but still nonzero. Therefore, $\nu n \leftrightarrow GG$ may dominate over the process with the chirality flip. As a result the bound on f is lowered to

$$f > f_4 = \left(\frac{r\sqrt{\lambda} m_\nu^2 M_{pl}}{32\pi}\right)^{1/3}. \tag{33}$$

The rate with the chirality flip goes like $g^4 T$ and has a recoupling form. However, the region where this recoupling would lead to signals is already excluded by the above bound, $f > f_4$.

The two-to-one process $\nu\nu \leftrightarrow G$ is also changed. We must again introduce a mass insertion to convert one of the initial neutrinos into a right-handed neutrino. The rate will therefore be suppressed by a factor of m_ν^2/T^2 relative to the rate for the Majorana neutrino. Therefore, the recoupling temperature is given by

$$T_{\rm rec} \approx \left(\frac{m_G^2 m_\nu^4 M_{pl}}{16\pi f^2}\right)^{1/5}. \tag{34}$$

Demanding that $T_{\rm rec} > m_G$ then results in a limit

$$f < f_1 = \left(\frac{m_\nu^4 M_{pl}}{16\pi m_G^3}\right)^{1/2}. \tag{35}$$

Notice that this limit is a factor of m_ν/m_G lower than the corresponding limit in the Majorana neutrino case Eq. (24).

Similarly, the bounds that must be satisfied to prevent the neutrino from free-streaming at $T = T_*$ and to agree with big bang nucleosynthesis are changed to

$$\begin{aligned} f < f_2 &= \left(\frac{m_G^4 m_\nu^4 M_{pl}}{16\pi T_*^7}\right)^{1/2}, \\ f > f_3 &= \left(\frac{m_G^2 m_\nu^4 M_{pl}}{16\pi T_W^5}\right)^{1/2}. \end{aligned} \tag{36}$$

An additional process is also possible in the case of a Dirac neutrino mass, $\nu n \leftrightarrow G$. Again, this rate is suppressed by r relative to $\nu\nu \leftrightarrow G$ in the case of Majorana neutrinos because of the initial state right-handed neutrino. More importantly, $n_\nu + n_n + 2n_G$ is conserved. As a result, the number of Goldstone bosons present will never exceed $n_n = rn_\nu$. Such a small number of Goldstone bosons will not produce a sizable change in the neutrino energy density. However, if the rate is large enough, it may still contribute to neutrino scattering thus producing a phase shift in the angular power spectrum. For a signal to result, the scale f must be lowered by a factor of $1/\sqrt{r}$ compared to Eq. (25). The rate and limit on f are given by

$$\Gamma \approx \frac{m_\nu^2 m_G^2}{8\pi f^2 T} r, \qquad f < f_2' = \left(\frac{r m_G^4 m_\nu^2 M_{pl}}{8\pi T_*^5}\right)^{1/2}. \tag{37}$$

The signal regions and big bang nucleosynthesis bounds are shown in Fig. 2 for $m_\nu = 0.05$ eV and $r = 0.0001$. For these values the relevant scattering process is $\nu\nu \leftrightarrow G$; identical to the process which equilibrates the Goldstone boson. The signal regions are the same as those for Majorana neutrinos.

V. SIGNAL REGIONS FOR THREE NEUTRINOS

We are now in a position to study the cosmological signals of three neutrinos interacting with a single pseudo-Goldstone boson. As in the single neutrino case, if recoupling occurs we expect an energy density signal for $m_G > 1$ eV and a scattering signal for $m_G < 1$ eV. The crucial difference, however, is that now the number

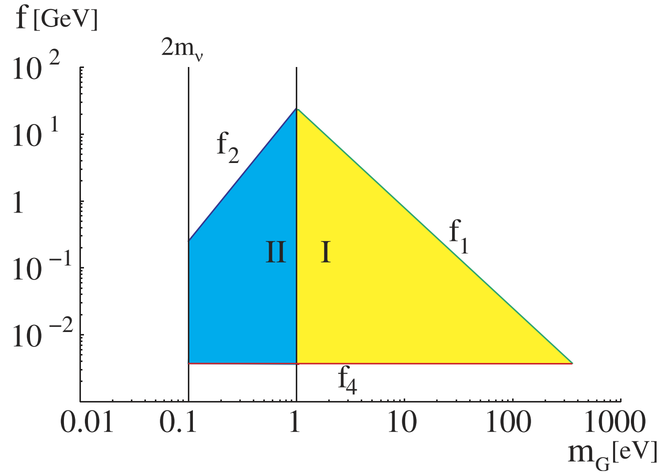

FIG. 2 (color online). Signal regions and cosmological bounds for a single Dirac neutrino coupled to a single pseudo-Goldstone boson. The region below line f_4 is excluded by BBN. In the region above lines f_1 and f_2 the Goldstone boson is too weakly coupled to give any signal. There is a signal in $\rho_{\rm rel}$ in region I, and in region II there is an overall phase shift. We have assumed $m_\nu = 0.05$ eV, $\lambda = 1$ and $r = 0.0001$.

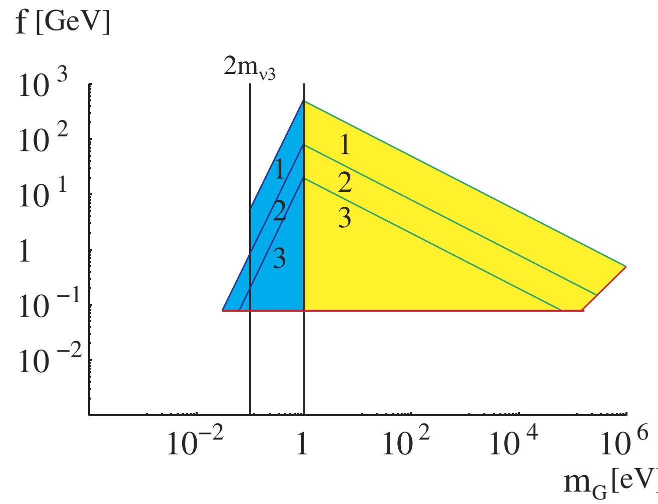

FIG. 3 (color online). The signal regions and bounds for three Majorana neutrinos with hierarchical masses $m_\nu = 0.05$, 0.008, 0.002 eV. The regions are labeled by the number of neutrinos species that recouple to the pseudo-Goldstone boson for $m_G > 1$ eV and by the number of neutrinos that scatter at $T \sim 1$ eV for $m_G < 1$ eV.

of neutrinos that recouple to the pseudo-Goldstone boson may be one, two or three. Since the pseudo-Goldstone boson couples to each neutrino with a strength proportional to its mass, even at the quantum level [19], the pattern of neutrino masses determines the number of neutrinos which recouple, and thereby the magnitudes of the energy density and scattering signals. This is very exciting, because it implies that a careful investigation of the cosmic microwave background may help determine the pattern of neutrino masses.

The neutrinos may be Majorana or Dirac, the pattern of their masses hierarchical, inverse hierarchical or degenerate. We consider each of these cases separately, starting with the case of hierarchical Majorana neutrinos. Oscillation data reveals that for such a pattern the heaviest neutrino has a mass of about 0.05 eV, while the intermediate neutrino has mass of about 0.008 eV. The mass of the lightest neutrino is significantly smaller, and for concreteness we take it to be 0.002 eV. The allowed signal region is plotted as a function of the symmetry breaking scale f and the pseudo-Goldstone boson mass m_G in Fig. 3. The various regions in this plot are generically labeled by an integer which lies between one and 3. If the region lies in the $m_G > 1$ eV portion of the plot this number indicates the number of neutrinos in equilibrium with the heavy pseudo-Goldstone boson when it disappeared, $n_{\rm rec}$. If it lies in the $m_G < 1$ eV region of the plot it indicates the number of neutrinos scattering at an eV, n_S.

The hierarchy in the masses of the three neutrinos implies that there are large, distinct parts of the plot where one, two or three neutrinos contribute to the signal. This is to be contrasted with the case where the spectrum of neutrino masses is inverse hierarchical or degenerate. If the neutrino masses exhibit an inverted hierarchy, the mass of both the two heavier neutrinos is close to 0.05 eV, with a splitting smaller than 0.001 eV. This implies that there is almost no region of parameter space in f where only one neutrino contributes to a signal, and in general we expect either two or three neutrinos to contribute. If the neutrino masses are degenerate, then the small mass splitting (less than 0.001 eV) between each pair of neutrinos implies that in the entire signal region all three neutrinos will contribute. It may therefore be possible to determine the pattern of neutrino masses from a careful measurement of the cosmic microwave background.

How well can these different patterns be distinguished? That depends on the magnitude of the signal for each case. The energy density signals for a single pseudo-Goldstone boson decaying or annihilating into

TABLE I. Table of effective number of neutrinos as determined by the relativistic energy density. Predictions are given for both Dirac and Majorana masses and for cases in which 1, 2 or 3 neutrinos recouple to the pseudo-Goldstone boson.

	Dirac		Majorana	
n_R	$\mu = 0$	$\mu \neq 0$	$\mu = 0$	$\mu \neq 0$
1	3.09	3.03	3.16	3.08
2	3.09	3.03	3.17	3.10
3	3.09	3.03	3.18	3.12

one, two or three Majorana neutrinos are shown in Table I. In the case that the chemical potential vanishes, N_ν can be easily found from Eq. (8). In the case of nonzero chemical potential we solve Eq. (9) numerically, imposing energy conservation when the pseudo-Goldstone bosons equilibrate and entropy conservation when they decay or annihilate as well as number conservation associated with the chemical potential. We see from the table that the differences in the energy density signal between the various patterns is rather small, which means that it is unlikely that we will be able to distinguish between them in upcoming experiments. If the PGB interacts only via $\nu\nu \leftrightarrow G$ then the relevant signal corresponds to the column of Table I labeled "$\mu \neq 0$", since the PGB and neutrinos possess a chemical potential. In theories with multiple PGBs, the reaction $G' \leftrightarrow GG$ will force the chemical potential to vanish, as we discuss in VII, giving larger signals, as shown in the "$\mu = 0$" column. The signal is enhanced due to the fact that the PGB number density while relativistic is not suppressed by a chemical potential so that their eventual decay or annihilation will have a larger impact on the final neutrino energy density.

However, the differences in the scattering signal, which can immediately be read off from Eq. (19), are large enough that in this region of parameter space there is indeed the distinct possibility of distinguishing between different patterns of neutrino masses.

We now move over to the case of three Dirac neutrinos. As before we first consider a hierarchical pattern of masses. The signal region is as shown in Fig. 4, where we have used the same values for the neutrino masses as

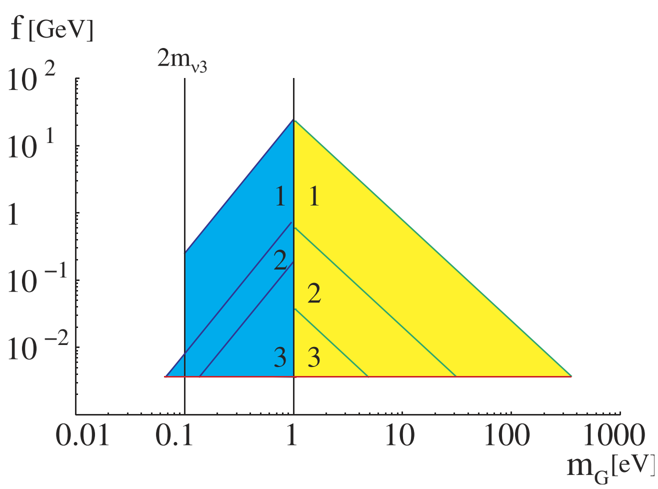

FIG. 4 (color online). The signal regions and bounds for three Dirac neutrinos with hierarchical masses $m_\nu = 0.05$, 0.008, 0.002 eV. The regions are labeled by the number of neutrinos species that recouple to the pseudo-Goldstone boson for $m_G >$ 1 eV and by the number of neutrinos that scatter at $T \sim 1$ eV for $m_G < 1$ eV and $r = 0.0001$.

in the Majorana case. As in the one neutrino model, the signal region differs from that of Majorana neutrinos. The reason is that, because there are a reduced number of pseudo-Goldstone bosons and right-handed neutrinos in the bath initially, the only $2 \leftrightarrow 1$ processes that can significantly alter the energy density in radiation necessarily involve a chirality flip on one of the neutrino legs, and are therefore suppressed except at very low temperatures. The reduced number of pseudo-Goldstones and right-handed neutrinos initially present also weaken the bounds from BBN. The scattering signal region is also altered because of the reduced number of right-handed neutrinos available for scattering. As in the Majorana case, we obtain either an energy density signal or a scattering signal, depending on whether or not the pseudo-Goldstone boson mass exceeds an eV.

Notice that for illustrative purposes we have chosen a somewhat small value of r. As a result, for the largest neutrino mass the scattering process is mass suppressed and the associated signal region is bounded by the line f_2 given in Eq. (36). However, for the smallest neutrino mass the most rapid scattering process is $\nu n \leftrightarrow G$ which is r suppressed. The scattering signal region is therefore bounded by f_2' given by Eq. (37). (For the intermediate mass the two rates are comparable.) As a result, the region in which there exists a scattering signal shrinks less rapidly as m_ν is lowered from the intermediate mass to the smallest mass than the region in which $\Delta N_\nu \neq 0$. This is because the equilibration process $\nu\nu \leftrightarrow G$ is suppressed by m_ν^2/T^2. This leads to a discontinuity in the signal region at $m_G = 1$ eV for light neutrinos.

Since the neutrino masses are hierarchical, there are large, distinct regions where one, two or three neutrinos contribute to the signal. As in the Majorana case, for an inverted hierarchy we expect either two or three neutrinos to contribute, while for the degenerate case all three are expected to contribute. Once again, this opens the door to the possibility of determining the pattern of neutrino masses from precision measurements of the cosmic microwave background.

The energy density signals for a pseudo-Goldstone boson decaying or annihilating into one, two or three Dirac neutrinos are shown in Table I, and can be contrasted with the Majorana case. Once again the differences in the energy density signals are too small to distinguish between the various patterns of neutrino masses in upcoming experiments. However the differences in energy density signals between the Dirac and Majorana cases are large enough that it may be possible to distinguish between these two cases. As before, for a given number of scattering neutrinos, the scattering signal may be immediately obtained from Eq. (19). In this region of parameter space we expect that it will indeed be possible to distinguish between different patterns of neutrino masses.

VI. MULTIPLE PGBS FROM APPROXIMATE LEPTON FLAVOR SYMMETRIES

Until now we have assumed that the neutrinos couple to a single pseudo-Goldstone boson, G, resulting from the spontaneous breakdown of an Abelian, flavor-diagonal symmetry. In the Majorana case, G is the Majoron of broken lepton number symmetry. However, the flavor symmetries of the neutrino sector are much richer than this, offering the hope of larger CMB signals which could probe the mass generation mechanism at a deeper level.

The most general lepton flavor symmetries acting on three generations of left-handed fermions ℓ and right-handed e and n are $\mathrm{U}(3)_\ell \times \mathrm{U}(3)_e \times \mathrm{U}(3)_n$. Here and below we give the symmetries for the Dirac case, with the understanding that the Majorana case is trivially obtained by deleting any symmetries on n. The charged lepton masses arise from operators such as $\ell \Sigma e h$ when Σ acquires a vev. Part of the flavor symmetry is broken, and any Goldstones produced at this stage are constrained to be very weakly coupled. However, some flavor symmetries escape breaking at this stage, and remain as symmetries of the low energy neutrino interactions. They necessarily include $\mathrm{U}(3)_n$ and a flavor symmetric U(1) acting on the neutrinos, which we write as $e + \mu + \tau = U(1)_L$. They could also include two flavor asymmetric U(1)s, $e - \mu$ and $\mu - \tau$. The Goldstones which result from the spontaneous breaking of these symmetries we call G_N because the symmetries can only be broken by Neutral lepton masses not charged lepton masses. Some of the symmetries broken by the charged lepton masses may also be of interest for neutrino physics. It could be that these symmetries reappear as accidental symmetries of the neutrino mass sector, i.e., of the interactions of (2). If they are broken by vevs of ϕ, then Goldstones G_C appear, where the label indicates that explicit symmetry breaking arises from the Charged lepton masses.

We assume that all global symmetries are not exact but receive explicit breakings from some mass scale M_E, which might be as high as the Planck scale. All the Goldstones are therefore really pseudo-Goldstones bosons, PGBs, and acquire small masses. If these arise from dimension 5 operators, ϕ^5/M_E, then we expect

$$m_G \approx \sqrt{\frac{f}{M_E}} f \approx \mathrm{eV}\left(\frac{f}{\mathrm{GeV}}\right)^{3/2}\left(\frac{M_{Pl}}{M_E}\right)^{1/2}, \tag{38}$$

where f is the vev of the relevant ϕ field. This result is very interesting. All global symmetries are expected to be broken at the Planck scale, and the simplest possibility leads to a correlation between m_G and f that passes right through our signal regions—for both ΔN_ν and Δl signals.

This dimension 5 explicit symmetry breaking operator also induces a low energy self interaction for the PGB, $(f/M_E)G^4$. Depending on parameters, this may recouple the $GG \leftrightarrow GG$ reaction. However, such a process appears not to have any signals. For the case that there are several PGBs, a much more interesting question is whether the explicit symmetry breaking can lead to the reaction $G' \leftrightarrow GG$ recoupling. This would have the important consequence of forcing the chemical potential of the Goldstones and neutrinos to vanish, and, as we have seen, this gives a large increase to the size of the ΔN_ν signal. We find that the recoupling of this reaction is generic to theories of multi-PGBs. Suppose the PGBs result from fields $\phi, \phi' \ldots$ and that the explicit symmetry breaking at dimension 5 includes several operators $(1/M_E)(a\phi^5 + b\phi^4\phi' + c\phi^3\phi'^2 + \ldots)$. Substituting $\phi = (f + iG)/\sqrt{2}, \phi' = (f' + iG')/\sqrt{2}, \ldots,$ and performing field phase redefinitions to remove linear terms in $G, G' \ldots$, one discovers that the phases of $a, b, c, \ldots$ are not all removed, so that interactions of the form $G'GG$ appear in the low energy theory, even after rotating to the PGB mass basis. The recoupling temperature for $G' \leftrightarrow GG$ is

$$\begin{aligned} T_{\mathrm{rec}}(G' \leftrightarrow GG) &\approx \left(\frac{m_{G'}^4 M_{pl}}{f^2 16\pi}\right)^{1/3} \\ &\approx \mathrm{keV}\left(\frac{m_{G'}}{\mathrm{keV}}\right)^{4/3}\left(\frac{\mathrm{GeV}}{f}\right)^{2/3}, \end{aligned} \tag{39}$$

where f is the larger of f and f', and we have taken G' heavier than G. For the entire region of f and m_G of interest for the ΔN_ν signal, $T_{\mathrm{rec}} > m_{G'}$. Hence, for multi-PGB theories, the generic situation is that $G' \leftrightarrow GG$ recouples and we expect the larger ΔN_ν signals appropriate for vanishing chemical potential.

The difference between the G_N and G_C PGBs, is that the G_C also have contributions to their mass from explicit breakings of the accidental low energy neutrino symmetries by the interactions that generate the charged fermion masses. Although, these contributions are model dependent, for a wide range of theories we can estimate their size. Suppose that the explicit symmetry breaking appears in the low energy neutrino theory as powers of the insertion λ_E, the charged lepton Yukawa coupling matrix, which transforms under $\mathrm{SU}(3)_\ell \times \mathrm{SU}(3)_e$ as (3, 3). Assume also that, in the symmetric limit, a potential for ϕ is generated with a mass term of order $-f^2\phi^\dagger\phi$. Including the effects of explicit symmetry breaking, this term will be modified to $-f^2\phi^\dagger(1 + c\lambda_E\lambda_E^\dagger)\phi$, where c depends on coupling parameters and loop factors. Ignoring c, gives

$$m_{G_C} \approx \lambda'_E f \approx (\mathrm{keV} - 10\ \mathrm{MeV})\frac{f}{\mathrm{GeV}}, \tag{40}$$

where λ'_E are the eigenvalues of the charged lepton Yukawa coupling matrix. The G_C PGBs have hierarchical masses typically in the range to give a ΔN_ν CMB signals, while G_N are typically lighter and could give either a ΔN_ν or a Δl CMB signal.

TABLE II. The number of G_N and G_C PGBs in theories with three generations of Majorana or Dirac neutrinos and maximal flavor symmetry. For the Majorana (Dirac) case there are a total of 9 (18) PGB. The number of G_C is increased from 6 to 7 or 8 if the physics responsible for the charged lepton masses breaks $e-\mu$ and $\mu-\tau$ symmetries.

	Majorana	Dirac
G_N	1–3	10–12
G_C	8–6	8–6

In Table II we list the number of each type of PGB, for both Majorana and Dirac cases, when the flavor symmetry is maximal. It is immediately clear that if there are anywhere near these numbers of PGB, then the ΔN_ν signal is likely to be much larger than in the previous sections. How many Goldstones do we expect? Zero, 1 or of order 10? If the mass scale of the physics generating neutrino masses is substantially below the weak scale then it is plausible that this scale, f, results from the breaking of a global symmetry. There need only be one symmetry, leading to a single PGB as discussed in the previous sections. However, if the entire structure of the neutrino mass matrix follows from broken symmetries, in analogy to the Froggatt-Nielson symmetry breaking frequently studied in the charged sector, then many more Goldstones are to be expected. While the number need not necessarily be the maximal number listed in Table II, no non-Abelian subgroup can escape breaking, since we know that the three neutrinos have no exact degeneracies, and hence, if the underlying symmetries of the neutrino sector is $U(3)_\ell \times U(3)_n$, we expect the number of Goldstones to be near maximal.

As an example, consider a $U(3)_\ell$ theory of Majorana neutrinos based on operators of the form $g\nu_i\phi_{ij}\nu_j$ where the generation indices i, j run over 1,2,3. Suppose that the vevs of the ϕ multiplet are hierarchical, and we have chosen the basis where the largest vev lies in the 33 direction, breaking $U(3)_\ell$ to $U(2)_\ell$, creating 5 PGB. Since 4 of these PGBs are associated with off-diagonal generators, they are G_C. These 5 PGB all have the same coupling, g, to neutrinos and for a large range of parameters will be produced by the $2 \leftrightarrow 1$ process $\nu_i\nu_j \leftrightarrow G_{ij}$. Other ϕ components with smaller vevs will break the remaining $U(2)_\ell$ symmetry creating further PGBs. Since the majority of PGBs are G_C rather than G_N, they are likely to have hierarchical masses, for example, as in (40), that are heavy enough to give a CMB ΔN_ν signal. A similar analysis would occur for a $U(3)_\ell \times U(3)_n$ theory of Dirac neutrinos, with two important differences. First, many more G_N are expected from the $U(3)_n$ symmetry. However, countering this, for any Goldstone with dominant coupling to mass eigenstates $\nu_i n_j$ the rate for being recoupled by the $2 \leftrightarrow 1$ process is suppressed by a factor $(m_j/T)^2$ at temperature T. This would make the ΔN_ν signal sensitive to the spectrum of both the neutrinos and the PGBs.

TABLE III. Table of effective number of neutrinos as determined by the relativistic energy density. Predictions are given for both Dirac and Majorana masses and for cases in which n_G Goldstone bosons recouple to $n_{\rm rec} = 3$ neutrinos and then decay. We have assumed that all Goldstone bosons equilibrate prior to any decaying. We assume that all heavy Goldstone bosons have decayed or annihilated prior to the equilibration of any light PGBs.

	Dirac		Majorana	
n_G	$\mu = 0$	$\mu \neq 0$	$\mu = 0$	$\mu \neq 0$
2	3.18	3.06	3.34	3.19
8	3.62	3.18	4.08	3.42
16	4.08	3.27	X	X

Rather than compute the spectrum for any specific model, in Table III we present the energy density signal for generic situations. A particular model is likely to have a signal which can be computed and differs from those in the Table. However, the Table does give an impression of the size of the signals which should be expected. For illustration we have taken Majorana models with 2 or 8 PGB that have recoupled to 1,2 or 3 neutrinos and are heavier than 1 eV. This could be the situation for models based on $U(1)_e \times U(1)_\mu \times U(1)_\tau$ or U(3), where only the PGB for overall lepton number is lighter than 1 eV. For Dirac neutrinos we have chosen 2, 8 or 16 heavy PGB, corresponding to models based on $U(1)_e \times U(1)_\mu \times U(1)_\tau$, $U(3)_{\ell+n}$ or $U(3)_\ell \times U(3)_n$ symmetries, where again one or two flavor-diagonal PGB are taken lighter than 1 eV. We note that in both Majorana and Dirac cases, planned CMB experiments are now able to distinguish between cases where the PGB recouples to 1,2 and 3 neutrinos, giving sensitivity to differing neutrino spectra. The energy density signal clearly grows substantially when more PGB contribute.

In the Dirac case one might question whether it is really plausible that so many PGB are recoupled—as mentioned earlier, a PGB G_A that couples predominantly to neutrino mass eigenstates via $\nu_i T^A_{ij} n_j$ has a recoupling rate suppressed by $(m_j/T)^2$. However, the issue here is whether the mass eigenstate bases for the PGB and the neutrinos line up. Since the explicit symmetry breaking from very high scales, or from the charged lepton mass sector, is unrelated to the neutrino mass generation, it seems that the two bases will be very different. Indeed, for the symmetry breaking coming from the charged lepton sector, neutrino oscillation data already tell us that there are large mixing angles between the two bases. Hence, if we consider the coupling to the heaviest mass eigenstate neutrino $\nu_3 n_3 G_{33}$, we expect that G_{33} will be a linear combination of all the PGB mass eigenstates without any

TABLE IV. Table of effective number of neutrinos as determined by the relativistic energy density. Predictions are given for both Dirac and Majorana masses and for cases in which $n_G = 8$ Goldstone bosons recouple to $n_{\rm rec}$ neutrinos and then decay. We have assumed that all Goldstone bosons equilibrate prior to any decaying. We assume that all heavy Goldstone bosons have decayed or annihilated prior to the equilibration of any light PGBs.

	Dirac		Majorana	
$n_{\rm rec}$	$\mu = 0$	$\mu \neq 0$	$\mu = 0$	$\mu \neq 0$
3	3.62	3.18	4.08	3.42
2	3.58	3.15	3.97	3.35
1	3.49	3.12	3.77	3.24

TABLE V. Illustration of how the scattering signal, Δl_n, depends on ΔN_ν and $\Delta N_{\nu FS}$. These predictions are for the case that one Majorana neutrino scatters due to interactions with one light pseudo-Goldstone boson. In this illustration, ΔN_ν and $\Delta N_{\nu FS}$ are calculated by assuming that $n_{\rm rec}$ neutrinos, including the one that scatters, are heated by the disappearance of 8 heavy PGBs. These Δl_n predictions should be compared to the value -15.6 that results for $N_\nu = 3$ and $N_{\nu FS} = 2$.

$n_{\rm rec}$	$N_{\nu FS}$	Δl_n
1	2.00	-14.04
2	2.16	-14.77
3	2.28	-15.45

small mixing angles. Hence the suppression factor for all of them is only $(m_3/T)^2$. If one of the PGB recouples we typically expect that they all do.

This does not mean that the signal is insensitive to the neutrino spectrum, since the signal does depend on the number of neutrinos that get recoupled. From Table IV we see that the dependence on the number of recoupled neutrinos is large and future CMB experiments will be able to probe the nature of the neutrino spectrum, unlike the case of a single PGB discussed in the previous section.

The change in the neutrino energy density is now so large that this has repercussions for the size of the scattering signal. This results from large deviations of N_ν from three in Eq. (20). Additionally, if some neutrinos that were coupled to the heavy pseudo-Goldstone bosons are not coupled to the light PGB and hence free-stream, it is possible that $N_{\nu FS} \neq (3 - n_S)$. Remember that N_ν and $N_{\nu FS}$ refer to energy densities, expressed as equivalent numbers of neutrino species, while n_S is the actual number of neutrino species that do not free-stream. Therefore, in general it is also true that $N_{\nu FS} \neq N_\nu - n_S$. Assuming that all scattering neutrinos and light Goldstone bosons were heated by the disappearance of the heavy PGBs, $N_{\nu FS}$ is given by

$$N_{\nu FS} = (3 - n_{\rm rec}) + \frac{n_{\rm rec} - n_S}{n_{\rm rec} + g_G/g_\nu}[N_\nu - (3 - n_{\rm rec})], \tag{41}$$

where $g_\nu = 7/4$ or $7/2$ if the neutrinos are Majorana or Dirac and g_G is the spin degeneracy of the light PGBs. This, combined with Eq. (20) allows us to calculate Δl_n.

Notice that Δl_n is no longer proportional to n_S and in fact has a dependence on $n_{\rm rec}$ and n_G through $N_{\nu FS}$ and N_ν. As a result, the phase of the acoustic oscillations may give us information about the number of neutrino species that were coupled to the heavy pseudo-Goldstone bosons and the number of heavy PGBs as well as the number of neutrinos that are scattering at an eV. An example of this is shown in Table V, which shows the case of Majorana neutrinos coupled to 8 heavy and 1 light PGB. We assume that one neutrino species is scattering and that this species was also heated by the decay or annihilation of the heavy PGBs. We see that in this case the Δl_n shift has a noticeable dependence on the number of recoupled neutrino species, in contrast with Eq. (19), and could be discovered with an observational resolution of $\Delta l_n \leq \pm 1$.

VII. PROBING THEORIES OF NEUTRINO MASS

One aspect of the PGB couplings is very general, and is independent of the underlying symmetry structure, the ϕ multiplet structure and the interactions coupling ϕ to neutrinos. If a PGB is produced at a symmetry breaking of strength f that produces a mass term m_ν for some neutrinos, then the PGB coupling strength to these neutrinos is $g = m_\nu/f$. This means that the analysis of section IV applies to any individual PGB, providing m_ν is interpreted as the neutrino mass arising from this particular PGB coupling. For the Majorana case, the lines of Fig. 1 do not depend on m_ν, hence the only modification necessary is to rescale the f axis by a factor $m_\nu/0.05$ eV. From this we see that CMB signals may arise in *any* theory of Majorana neutrinos where neutrino flavor symmetries are broken in the mass range

$$f = (50\ {\rm MeV} - 500\ {\rm GeV})\frac{m_\nu}{0.05\ {\rm eV}}. \tag{42}$$

For the Dirac case, the slopes of some of the lines do depend on m_ν, and some limits depend on r, so that the relevant regions are shifted to somewhat lower values of f. We seek theories where lepton flavor symmetries are broken at the weak scale, or up to four orders of magnitude below the weak scale. Whatever breaks the weak interaction might also break lepton flavor symmetries, either directly or at one or two loop order.

This large range in f allows a wide variety of models, not just those of Eq. (2), but does not include the popular seesaw models where lepton number is broken quite close to the scale of grand unification. In general two factors contribute to explaining why the neutrinos are much lighter than the weak scale: f/v and v/M. The relative importance of these two factors is model dependent.

However, for all models it is clear that a crucial ingredient is that an approximate lepton flavor symmetry is broken at some scale f much less than the energy scale, M, responsible for the physics of neutrino masses. Below we choose two models to illustrate the rich range of possibilities.

A. A low energy seesaw

It may be that $M \approx v$ and the suppression of neutrino masses is entirely due to a small value for f/v. As an example of such a theory, consider the seesaw model:

$$L = nn\phi + \ell nh\left(\frac{\phi'}{v}\right)^2, \tag{43}$$

where order unity couplings are understood. Suppose that lepton number is spontaneously broken at the weak scale $\phi \approx ve^{iG/v}$, while other lepton flavor symmetries are broken at some lower scale f: $\phi' \approx fe^{iG'/f}$. The light neutrinos are Majorana with a mass seesawed down to f^4/v^3, so that f should be of order 1 GeV. There are two very different types of PGB: G and G' with symmetry breaking parameters v and f. From Fig. 1 we find that either could give scattering or energy density CMB signals. The mass of G would need to be close to 1 eV, but G' could give a signal for a wide range of masses.

B. A SU(3) $\times$ U(1)$_L$ theory

A discussion of realistic three neutrino theories with a single Goldstone was given in section V. With more than one Goldstone an important new ingredient appears: multiple symmetry breaking scales f. Neutrino mass ratios may now arise from a hierarchy of g parameters or from a hierarchy of f parameters. Of course, given the observed pattern of neutrino oscillations the hierarchies need not be large. In the case of hierarchical flavor symmetry breaking, for a fixed pattern of neutrino masses, smaller values of f translate into larger values for g. Hence the couplings of the PGBs to the lighter neutrinos are larger than for the case of a single symmetry breaking scale, discussed in section V, and the signal regions are correspondingly enhanced for the lighter neutrinos compared to the regions shown in Figs. 3 and 4.

A particularly interesting model with a hierarchy of global symmetry breaking scales is a $SU(3) \times U(1)_L$ theory of Majorana neutrinos described by

$$L = \frac{1}{M^3}\ell_i(\phi_{A_{ij}}\phi_L)\ell_j hh, \tag{44}$$

where ϕ_L carries overall lepton number, while the multiplet ϕ_A is a representation of SU(3). This theory not only has multiple symmetry breaking scales with hierarchical vevs of ϕ_A, but neutrino masses occur at second order in symmetry breaking, rather than at linear order.

The neutrino mass is a product of symmetry breaking terms

$$m_\nu = \frac{f_A f_L}{M}\frac{v^2}{M^2}, \tag{45}$$

that could lead, for example, to a hierarchical spectrum. The coupling of the Goldstone of overall lepton number, G_L, is proportional to the SU(3) symmetry breaking:

$$g_L = \frac{f_A}{M}\frac{v^2}{M^2}, \tag{46}$$

while the couplings of the Goldstones of broken SU(3) are proportional to the breaking of lepton number:

$$g_A = \frac{f_L}{M}\frac{v^2}{M^2}. \tag{47}$$

The CMB signals for this model vary drastically as (f_A, f_L) are varied. If $f_{A,L} > v$ there are no CMB signals. If only $f_L < v$ then there is a single flavor-diagonal PGB, G_L, with signals as given in section V. On the other hand if $f_A < v$ while $f_L > v$, then there are several PGB, G_A, which contribute to CMB signals and which may have a hierarchy of f_A. If $f_{A,L} < v$ we have a model of Majorana neutrinos where the maximal number of PGBs can contribute to CMB signals. Since G_L is a member of G_N, it may have a small mass and give rise to a scattering signal. Even though it derives from overall lepton number symmetry, it has a different coupling to each neutrino species, so that the scattering signal may correspond to 1,2 or 3 neutrinos depending on the neutrino spectrum. The 8 flavor PGB, G_A, may all be G_C type with larger masses, giving a very large ΔN_ν signal as shown in Table III.

VIII. $f < v$ FROM SUPERSYMMETRY

CMB signals are possible even if the lepton flavor symmetries are broken at scales as high as the electroweak scale, v. However, observable signals result from a much wider range of PGB masses if $f < v$. There are a variety of scenarios for naturally inducing symmetry breaking scales well beneath the weak scale; in this section we study a supersymmetric theory.

Consider the superpotential below, where in addition to a Dirac mass for the neutrino of the form of Eq. (2), we have added a mass M_N for the right-handed neutrino. As we will argue later, the natural size of M_N is of order the weak scale, and therefore the neutrino masses in this theory are Majorana. The theory has an R symmetry corresponding to lepton number under which L, N and Φ are all charged. In the absence of the mass term M_N for the right-handed neutrino the theory has an additional right-handed lepton number symmetry under only N and Φ are charged; this symmetry is broken at the weak scale generating a right-handed neutrino mass.

$$\mathrm{W} = \frac{\lambda}{M}LNH_u\Phi + M_N N^2 + \frac{\alpha}{3!}\Phi^3. \tag{48}$$

After supersymmetry and electroweak symmetry breaking, we obtain the coupling $\sqrt{2}g_{ij}\nu_i n_i \phi$ which leads to a Dirac mass term, and the scalar potential

$$V = \tilde{m}^2(|\tilde{\nu}_i|^2 + |\tilde{n}_i|^2) + |g_{ij}\tilde{\nu}_j\phi + 2(M_N)_{ij}\tilde{n}_j|^2 + |g_{ij}\tilde{n}_j|^2|\phi|^2 + \left|\frac{\alpha}{2}\phi^2 + g_{ij}\tilde{\nu}_i\tilde{n}_j\right|^2, \quad (49)$$

where we have taken a common soft mass, $\tilde{m}$, for $\tilde{\nu}_i$ and $\tilde{n}_i$ for simplicity. Note that we have made a crucial assumption that Φ does not feel supersymmetry breaking directly; namely, there is no soft mass for ϕ. This would occur, for example, in certain theories of gauge-mediated supersymmetry breaking if Φ does not have any gauge interactions. Since Φ couples through g_i to particles which do feel supersymmetry breaking, radiative corrections must induce a supersymmetry breaking mass at least as big as

$$\delta m_\phi^2 = -\frac{g^2}{16\pi^2}\tilde{m}^2, \quad (50)$$

where g is the largest coupling of Φ to neutrinos. Note that the sign is negative, and will induce symmetry breaking, in a very similar fashion to radiative electroweak symmetry breaking in the standard supersymmetric model. Therefore, the VEV of ϕ, namely f, is given by

$$f \simeq \sqrt{\frac{-2\delta m_\phi^2}{\alpha^2}} \approx \frac{g\tilde{m}}{4\pi\alpha}. \quad (51)$$

For α of order unity, we see that supersymmetry protects f down to the scale $f_{\rm susy} \approx g\tilde{m}/4\pi$. If there are several Φ multiplets with a hierarchy of couplings to neutrinos, then their symmetry breaking scales will reflect that hierarchy. The coupling g is naturally small, $g \approx v/M$, and is related to the observed neutrino mass as

$$m_\nu \approx g^2\frac{f^2}{M_N}. \quad (52)$$

Eliminating g between Eqs. (51) and (52) we obtain an approximate expression for the scale f given below

$$f_{\rm susy}^2 \approx \frac{\tilde{m}}{4\pi}\sqrt{m_\nu M_N} \quad (53)$$

A natural value for the scale M_N at which right-handed lepton number is broken is the weak scale. This scale could, for example, be generated from a term of the form $\lambda_N\Phi_n NN$ in the superpotential. Here ϕ_n acquires a VEV and thereby breaks right-handed lepton number, and λ_N is a coupling constant. If λ_N is of order one and the $\tilde{n}$'s have a soft mass of order the weak scale then ϕ_n can acquire a VEV of order the weak scale radiatively, exactly the way the Higgs does in the minimal supersymmetric model.

For $m_\nu = 10^{-2}$ eV and $\tilde{m} = 100$ GeV we find that supersymmetry is able to protect f down to 30 MeV. This is the lowest f for which our CMB signals are compatible with BBN constraints. Larger f could be obtained in this simple model of radiative lepton flavor symmetry breaking in several ways, for example, by reducing the coupling α or by introducing additional couplings of Φ to the supersymmetry breaking sector. Hence we conclude that no fine tuning is necessary anywhere in the wide range of (f, m_G) parameter space that leads to CMB signals.

IX. CONCLUSIONS

We have proposed that future measurements of the CMB will provide a powerful probe of theories of neutrino mass that have lepton flavor symmetries spontaneously broken at or below the weak scale. Such theories lead to light pseudo-Goldstone bosons that interact with neutrinos with couplings proportional to the neutrino masses. Such interactions can modify the acoustic oscillations of the electron-photon fluid during the eV era. In particular there is a change in the relativistic energy density, parameterized by an effective change in the number of neutrino species, ΔN_ν, and a change in the multipole of the nth CMB peak, Δl_n, for large n. While other new physics could lead to an energy density signal, a uniform shift in the high n peaks to larger l can only result from scattering of the known neutrinos, and is therefore an ideal test of our class of theories.

The present experimental limit on deviations from the relativistic energy density predicted by the standard model is roughly $-2 < \Delta N_\nu < 4$ [5] [12], although the precise numbers depend what other data are included in the fit. Our predictions are well within these bounds, typically in the range $0 < \Delta N_\nu < 1$. The Planck experiment is expected to reach a sensitivity of ± 0.20 at 1 standard deviation, and the proposed satellite experiment CMBPOL could probe to ± 0.05 [9]. These projections assume that the neutrinos are free-streaming; in the presence of our scattering signal we do not know how well the energy density can be measured. The inherent limit on ΔN_ν from cosmic variance is at the level of ± 0.04 for measurements up to l of 2000, if the CMB is to determine all the cosmological parameters [20]. If the CMB is used only to determine N_ν, then the cosmic variance limit drops to ± 0.003. All numbers assume polarization correlations are measured as well as the temperature correlations. While the positions of the low n CMB peaks are now accurately determined, the higher n peaks will have to await future measurements at higher l. To first approximation, our signal is $\Delta l_n \approx +8n_S$, where $n_S = 1, 2, 3$ is the number of neutrino species that scatter by PGB exchange during the eV era. This is a clear order of magnitude larger than the expected reach of Planck, $\Delta l \approx \pm 2$, and CMBPOL, $\Delta l \approx \pm 1$, always at 1 standard deviation [9].

The signals can be precisely calculated in any particular theory of neutrino masses, and reflect both the spectrum of the neutrinos and whether the neutrinos are Majorana or Dirac. Signals are expected for a wide range of lepton symmetry breaking parameters, 10 MeV $<$ $f <$ TeV, and for a wide range of the PGB masses, $m_G <$ MeV. The signal regions for a Goldstone boson produced at scale f with mass m_G are shown in Fig. 1 for Majorana neutrinos and in Fig. 2 for Dirac neutrinos. In the case that the same symmetry breaking scale f produces all three neutrino masses, the signal regions are shown in Figs. 3 and 4, for a hierarchical pattern of neutrino masses. The size of the signals differs in the regions labeled 1,2,3, and the sizes of these regions change as the pattern of neutrino masses changes.

In many cases the scattering signal depends only on the number of scattering neutrino species and is given by $\Delta l \approx +7.8 n_S(\Delta l_{\rm peak}/300)$, relative to the prediction of the standard model, which will be easily seen in upcoming experiments. On the other hand, the size of ΔN_ν is highly dependent on the number and spectrum of PGBs and the neutrino spectrum. For a single PGB the signal is small; for example, with a nonzero chemical potential $\Delta N_\nu = 0.03$, 0.10 for the Dirac, Majorana cases, for the PGB recoupling to $n_{\rm rec} = 2$ neutrino species. The dependence on $n_{\rm rec}$ is mild, so that in this case the signal does not allow a discrimination between hierarchical, inverted or degenerate spectra.

The size of the ΔN_ν signal increases dramatically in theories with multiple PGB. This is partly due to the larger number of degrees of freedom, and partly because the interactions between different PGB generically sets the chemical potential to zero. For example, for 8 PGB, corresponding to breaking a SU(3) lepton flavor symmetry, if all three neutrinos are recoupled, $n_{\rm rec} = 3$, then $\Delta N_\nu = 0.62$, 1.08 for the Dirac and Majorana cases. The signals for $n_{\rm rec} = 1, 2, 3$ are 0.49, 0.58, 0.62 for the Dirac case and 0.77, 0.97, 1.08 for the Majorana case. Hence a precise measurement of ΔN_ν has the ability to distinguish both the type and spectrum of the neutrinos. In cases such as this, where there is a large ΔN_ν signal, there are deviations from the simple prediction for the scattering signal, Δl_n, which now also depends on $n_{\rm rec}$ and the number of PGB.

For a theory of neutrino masses to give a CMB signal, the crucial ingredient is the spontaneous breaking of lepton flavor symmetry at a scale f of the weak scale or below. This does not occur in the conventional seesaw models. We have shown that the well-known radiative symmetry breaking mechanism of supersymmetry can yield such values for f, and we have demonstrated that the explicit breaking of the global lepton symmetries expected from the Planck scale leads to PGB masses precisely in the range that gives CMB signals. There is a rich variety of models with CMB signals. We have given two illustrations where the interactions of the neutrinos are bilinear in the field ϕ that breaks the lepton symmetries. In the first model the right-handed neutrinos are at the weak scale, v, and the lightness of the neutrinos is due to powers of f/v. In the second model there are two types of PGB, the flavor-diagonal Majoron, and the flavor-changing PGB associated with SU(3). One possibility is that the latter contribute to a large ΔN_ν signal, while the Majoron is lighter and gives a scattering signal. It is remarkable that the CMB offers a powerful probe of this physics.

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