PHYSICAL REVIEW D, VOLUME 70, 084045

Gyrating strings: A new instability of black strings?

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(Received 18 May 2004; published 27 October 2004)

A thermodynamic argument is presented suggesting that near-extremal spinning D1-D5-P black strings become unstable when their angular momentum exceeds $J_{\rm crit} = 3Q_1Q_5/2\sqrt{2}$. In contrast, the dimensionally reduced black holes are thermodynamically stable. The proposed instability involves a phase in which the spin angular momentum above $J_{\rm crit}$ is transferred to gyration of the string in space, i.e., to orbital angular momentum of parts of the string about the mean location in space. Thus the string becomes a rotating helical coil. We note that an instability of this form would yield a counterexample to the Gubser-Mitra conjecture, which proposes a particular link between dynamic black string instabilities and the thermodynamics of black strings. There may also be other instabilities associated with radiation modes of various fields. Our arguments also apply to the D-brane bound states associated with these black strings in weakly coupled string theory.

DOI: 10.1103/PhysRevD.70.084045 PACS numbers: 04.50.+h, 04.65.+e, 11.27.+d

I. INTRODUCTION

Black holes remain one of the most intriguing objects in general relativity and are known for their simplicity and stability. Above $3 + 1$ dimensions, gravity can produce not only black holes, but also extended black objects such as strings and p-branes. In many ways, these extended black objects behave like their black hole cousins. Indeed, the most familiar examples of black strings and branes have translational symmetries, and dimensional reduction along these symmetries yields black hole solutions to lower dimensional theories.

However, properties of black branes can sometimes differ significantly from those of the associated black holes. The Gregory-Laflamme instability [1–3] is a classic example of such behavior. In [1] it was shown that certain black strings are dynamically unstable to a breaking of translational (but not rotational) symmetry along the string. The ultimate fate of this instability remains a matter of investigation, but conjectures [4] as to this fate have led to the discovery of inhomogeneous black strings [5] (though these solutions cannot be the endpoint of the Gregory-Laflamme instability, at least in $d \leq 13$ dimensions [6]; see also [7]) and related work on dynamics [8].

The general theory of such instabilities remains to be understood. An oft-discussed conjecture in this context was stated by Gubser and Mitra [9], who proposed that black branes might have dynamical instabilities precisely when they have thermodynamic instabilities, in the sense that the Hessian of second derivatives of the energy with respect to the conserved charges has negative eigenvalues. This conjecture has been proven in certain contexts [10].

Here we argue for an instability which places a new twist on such discussions. We examine the three-charge (D1-D5-P) spinning black brane of type IIB supergravity that saturates the Bogomol'nyi-Prasad-Sommerfield (BPS) bound, which becomes a $5 + 1$ black string when compactified on T^4. When compactified along the remaining translation symmetry and dimensionally reduced to $4 + 1$ dimensions, the resulting black hole is dual to that studied in [11] by Breckenridge, Myers, Peet, and Vafa and is a rotating version of the black hole whose entropy was counted [12] by Strominger and Vafa using D-brane techniques. As in the works above, we take the direction along the string to be compactified in a circle of length L in order to yield finite charges. The near-extremal solution was studied in [13–15]. In particular, from the results of [14] one can show that it has no thermodynamic instabilities in the sense of Gubser and Mitra.[1] Thus their conjecture predicts dynamical stability.[2]

However, this theory contains other BPS black strings carrying the same charges. In particular, strings were described in [17] in which all or part of the angular momentum is carried by *gyrations* of the string as opposed to spin. In such solutions the black string at any instant of time can be thought of as being helical in space,

[1]In addition, the microscopic entropy of such objects in string theory was computed in [16] using methods from the correspondence between string theory on anti-de sitter space and the associated conformal field theory.

[2]However, as will be discussed later, there is one direction which the system is thermodynamically only marginally stable.

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with the helical profile traveling along the string at the speed of light. Such solutions are easily generated by applying the technique of Garfinkle and Vachaspati [18] to the spinning strings of [11]. In fact, the shape of such gyrations need not be a helix but can be much more general. As a result, the space of such BPS black branes is highly degenerate; for fixed charges, one must still specify several functions on a circle in order to determine the solution uniquely. Oscillating versions of similar strings had previously been constructed in [19–22].

Below we study the general gyrating Breckenridge, Myers, Peet, and Vafa string with antiself dual angular momentum and find that, for large enough angular momentum ($J > J_{\rm crit} = 3Q_1Q_5/2\sqrt{2}$) it is entropically favorable for the excess angular momentum $J - J_{\rm crit}$ to be carried by gyrations. In particular, maximizing the entropy S over the class of solutions carrying antiself dual spin angular momentum J_{spin} and antiself dual gyrational angular momentum $J_{\rm gyro}$, we find that for $J > J_{\rm crit}$ the entropy S is an increasing function of $J_{\rm gyro}$ on the interval $[0, J - J_{\rm crit}]$, with positive slope for $0 \leq J_{\rm gyro} < J_{\rm crit}$. This indicates a first order phase transition and suggests that a small perturbation of a nongyrating black string could grow to become large, i.e., it suggests an instability of the black string[3] in the same way that supercooled water is unstable to the formation of ice and superheated water is unstable to boiling.

Some useful details of the solutions are described in Sec. II below, as they were suppressed in [17]. Section III then assembles the argument and comments on the importance of small deviations from extremality. We also discuss two possible scenarios for the final fate of this instability. Section IV contrasts our situation with that associated with super-radiance, observes that an analogous argument holds for the D-brane bound states associated with our black strings in weakly coupled string theory, and makes further final comments.

II. THE GYRATING BLACK STRING

Consider a D1-D5-P black brane solution which is asymptotically $R^5 \times S^1 \times T^4$. We will think of the T^4 as being small so that the solution is effectively a black string in 5 + 1 dimensions. For a translationally invariant brane, the ten-dimensional type IIB supergravity solution is

$$ds^2 = \left(1 + \frac{r_o^2}{r^2}\right)^{-1}\left[-dudv + \frac{p}{r^2}du^2 + \frac{2\gamma}{r^2}(\sin^2\theta d\varphi - \cos^2\theta d\psi)du\right] + \left(1 + \frac{r_o^2}{r^2}\right)dx_i dx^i + dy_a dy^a. \quad (2.1)$$

The index a runs over the four directions of the T^4 and i runs over the four space directions transverse to the branes. These latter four directions are associated with an S^3 labeled by constant values of $r^2 = \sum_i x^i x^i$. The angles θ, φ, ψ label this S^3, while the coordinates z, t label the worldvolume directions of the string. Here we have chosen the special case where D1- and D5-brane charges are tuned to achieve constant dilaton and four-torus volume, so that (2.1) is the metric in either the string or Einstein frame. In particular, the D1- and D5-charges are

$$Q_1 = \frac{Vr_0^2}{(2\pi)^4 g}, \qquad Q_5 = \frac{r_0^2}{g}, \quad (2.2)$$

where g is the string coupling and V is the volume of the four-torus. This solution has a null Killing vector field $\partial/\partial v$, so one may attempt to add travelling waves via the method of [18]. It turns out that there are many interesting such waves for this system, which were studied extensively in [17,22,23] following similar work of [19–21] for other systems.

We are interested here in the class of waves briefly discussed in [17] which can be viewed as *gyrations* of the brane itself in the x^i directions. Such solutions differ from (2.1) only by the addition of a term proportional to $\ddot{h}_i(u)x^i du^2$, where h_i are arbitrary functions of u:

$$ds^2 = \left(1 + \frac{r_o^2}{r^2}\right)^{-1}\left[-dudv + \frac{p}{r^2}du^2 + \frac{2\gamma}{r^4} \times (x^1dx^2 - x^2dx^1 - x^3dx^4 + x^4dx^3)du - 2\ddot{h}_i(u)x^i du^2\right] + \left(1 + \frac{r_o^2}{r^2}\right)dx_i dx^i + dy_i dy^i. \quad (2.3)$$

After the change of coordinates

$$v' = v + 2\dot{h}_i x^i + \int^u \dot{h}^2 du, \quad (2.4)$$

$$x'^i = x^i + h^i, \quad (2.5)$$

asymptotic flatness of the metric becomes manifest:

[3]The discussion in [17] explicitly focussed on the case where the gyrations are small enough that the object could be well described via dimensional reduction to a black *hole*. In such cases, it was shown that the effect of gyrations on the black string entropy is negligible. However, the gyrating string has significantly larger entropy only when $J - J_{\rm crit}$ is of order J, and in this case the oscillations are necessarily large.

$$ds^2 = \left(1 + \frac{r_o^2}{|\vec{x} - \vec{h}|^2}\right)^{-1}\Big[-dudv + \frac{p}{|\vec{x} - \vec{h}|^2}du^2 + \frac{2\gamma}{|\vec{x} - \vec{h}|^4}(\Delta^1 d\Delta^2 - \Delta^2 d\Delta^1 - \Delta^3 d\Delta^4 + \Delta^4 d\Delta^3)du + \left(2 + \frac{r_o^2}{|\vec{x} - \vec{h}|^2}\right)\frac{r_o^2}{|\vec{x} - \vec{h}|^2}(\dot{h}^2 du^2 - 2\dot{h}_i dx^i du)\Big] + \left(1 + \frac{r_o^2}{|\vec{x} - \vec{h}|^2}\right)dx_i dx^i + dy_i dy^i, \tag{2.6}$$

where $\Delta^i = x^i - h^i$. Note that the black string horizon is located at $x^i = h^i(u)$ in the new coordinates, suggesting that the string is indeed oscillating in the x^i directions. Following [17], we have in mind circular oscillations of the type associated with a net angular momentum and we refer to *gyrations* of the string in the x^i directions.

The conserved linear and angular momenta of the gyrating black string can be read off from the asymptotic form of (2.6). The results are as follows:

$$P = \frac{Lp}{\kappa^2} + P_{\rm gyro}, \qquad P_{\rm gyro} = \frac{2r_0^2}{\kappa^2}\int_0^L \dot{h}^2 du, \tag{2.7}$$

$$J_\phi \equiv J_{12} = J_{12}^{\rm gyro} + \frac{L\gamma}{\kappa^2}, \tag{2.8}$$

$$J_\psi \equiv J_{34} = J_{34}^{\rm gyro} - \frac{L\gamma}{\kappa^2}, \tag{2.9}$$

$$J_{ij}^{\rm gyro} = \frac{2r_0^2}{\kappa^2}\int_0^L (h_i\dot{h}_j - h_j\dot{h}_i)du. \tag{2.10}$$

Here $\kappa^2 = (2\pi)^5 g^2/V$. Note that the expressions for $P_{\rm gyro}$ and $J_{ij}^{\rm gyro}$ are identical to those of a material string with tension $2r_0^2/\kappa^2$.

Now, for any string the gyrational angular momentum is bounded by a linear function of gyrational momentum. This result may be derived in several ways. For example, one might note that the expressions for $P_{\rm gyro}$ and $J_{\rm gyro}$ have the same form as those of [24] for the angular momentum and charge of a supertube. Applying their argument here yields $|J_{12}| \leq \frac{P_{\rm gyro}L}{2\pi}$. Another argument notes that quantizing the string will yield vector particles, each carrying spin ± 1. Thus, the angular momentum must be bounded by the number $N_{\rm gyro} = P_{\rm gyro}L/2\pi$ of associated momentum quanta.

However, requiring our angular momentum to be antiself dual is not compatible with saturating this bound. To understand the additional constraint from antiself duality, consider expression (2.10) for $J_{\rm gyro}$ for the case where only a single wave number $k = 2\pi n/L$ is excited so that h_i takes the form

$$h_i = A_i\cos(2\pi n/L) + B_i\sin(2\pi n/L). \tag{2.11}$$

The resulting gyrational momentum and angular momentum are

$$J_{ij}^{\rm gyro} = \frac{4\pi n r_0^2}{\kappa^2}(A_iB_j - B_iA_j), \tag{2.12}$$

$$P_{\rm gyro} = \frac{4\pi^2 n^2 r_0^2}{L\kappa^2}\sum_i(A_iA_i + B_iB_i). \tag{2.13}$$

In particular, the angular momentum lies in the plane defined by the vectors A_i and B_j. Thus, to obtain an antiself dual J_{ij} requires excitations in at least two momentum modes and, if achieved with only two modes, requires the associated planes to be orthogonal.

Next, we note that $P_{\rm gyro}$ in (2.13) is proportional to n^2 while $J_{ij}^{\rm gyro}$ is proportional only to n. Thus, if we wish to maximize $J^{\rm gyro}$ for a given $P_{\rm gyro}$, it is clear that we wish to use the lowest modes possible. Thus, we do best if we use only the $n = 1$ and $n = 2$ modes, say, with the fundamental mode carrying angular momentum J_{12} in the 12 plane while the $n = 2$ mode carries angular momentum J_{34} in the 34 plane. For antiself duality we require $J_{12} = -J_{34}$. It is also clear from (2.12) that we wish to choose the vectors A_i and B_i for any given mode to be orthogonal but of the same magnitude. But then the extra power of n in $P_{\rm gyro}$ implies that the $n = 2$ mode carries twice the gyrational momentum as the $n = 1$ mode. Thus, the largest antiself dual $J_{\rm gyro}$ is obtained by placing $N_{\rm gyro}/3$ excitations in the fundamental ($k = 1$) mode of the string and using them to carry $J_{12} = N_{\rm gyro}/3$, while also placing $N_{\rm gyro}/3$ excitations in the $k = 2$ mode and using it to carry $J_{34} = N_{gyro}/3$ and $2N_{\rm gyro}/3$ momentum quanta. This configuration has $J_{\rm gyro} = \frac{\sqrt{2}}{3}N_{\rm gyro}$ units of angular momentum. Other configurations with the same $J_{\rm gyro}$, $P_{\rm gyro}$ may be obtained through rotations that preserve $J_{ij}^{\rm gyro}$, but there are no configurations with greater antiself dual angular momentum.

III. ENTROPY AND INSTABILITY

In [17] it was shown that adding such gyrations does not affect the induced metric on the horizon, and, in particular, that the area of the horizon does not change.[4] Therefore, the entropy of the gyrating string has the same form as that of the spinning D1-D5-P string [11], but with the total momentum replaced by $P - P_{\rm gyro}$ and the total angular momentum replaced by $J - J_{\rm gyro}$. In terms of the number of momentum quanta $N = PL/2\pi$ and $N_{\rm gyro} =$

[4]However, a mild null shock-wave singularity does form along the horizon. The metric is C^0 at the horizon (so that, in particular, its area is well-defined) but it is not C^1. One expects that any amount of excitation above the BPS bound will smooth out this singularity, though the resulting solution is unlikely to be stationary. See the appendix of [23] for details of the extreme solutions.

$P_{\rm gyro}L/2\pi$, the result is

$$S = 2\pi\sqrt{Q_1Q_5(N - N_{\rm gyro}) - (J - J_{\rm gyro})^2}. \qquad (3.1)$$

Given a gyrating black string with $|J_{\rm gyro}| < \frac{\sqrt{2}}{3}N_{\rm gyro}$, we can always decrease $N_{\rm gyro}$ to obtain a solution with larger entropy. Thus maximally entropic strings saturate this bound to yield

$$S = 2\pi\sqrt{Q_1Q_5\Big(N - \frac{3}{\sqrt{2}}|J_{\rm gyro}|\Big) - (J - J_{\rm gyro})^2}. \qquad (3.2)$$

When the entropy is now maximized over $J_{\rm gyro}$, the absolute value in (3.2) leads to two distinct behaviors. For $J < J_{\rm crit}: = \frac{3Q_1Q_5}{2\sqrt{2}}$, the maximum is at $J_{\rm gyro} = 0$. However, for $J > J_{\rm crit}$, entropy is maximized for $J_{\rm gyro} = J - J_{\rm crit}$ and thus $J_{\rm spin} = J_{\rm crit}$.

We now use the above observations to argue for a new type of black string instability. Namely, we suggest that certain nongyrating spinning black strings are unstable to the development of gyration for $J > J_{\rm crit}$. Now, one does not expect BPS solutions to have a linear instability.[5] Indeed, we have seen that they are marginally stable to developing gyration, as any amount of gyration leads to a stationary solution. That is, the parameter $J_{\rm gyro}$ effectively labels a moduli space of BPS solutions.

However, a generic perturbation of the spinning string will result in motion through this moduli space (as well as some amount of excitation of the string off of the moduli space). The important observation is that near $J_{\rm gyro} = 0$ motion in the direction of increasing $J_{\rm gyro}$ is entropically favored over motion in the direction of decreasing $J_{\rm gyro}$. In fact, we have seen that the entropy is maximized at $J_{\rm gyro} = J - J_{\rm crit}$, so that this value is entropically stable. As one may expect[6] this entropic stability to be enforced dynamically, we conjecture that the gyrating string with $J_{\rm gyro} = J - J_{\rm crit}$ is *dynamically* stable, perhaps due to higher order dynamical effects beyond the linear level. Similarly, we conjecture that the spinning nongyrating string with $J > J_{\rm crit}$ is dynamically *unstable*, perhaps due to higher order effects.

Instead of considering BPS objects, one might consider strings with energies slightly in excess of the BPS bound. For a nearly BPS object with J substantially greater than $J_{\rm crit}$, one expects a similar entropy formula and a similar instability. In particular, if some form of continuity holds then we have that:

(1) Near-BPS solutions can also be labeled by a parameter $J_{\rm gyro}$. Since one expects non-BPS gyrating strings to radiate, these solutions are unlikely to be stationary. Their gyrating phase will be transient. However, this means only that J_{gyro} will refer to the gyrational angular momentum at some particular moment of time.

(2) The derivative $\frac{\partial S}{\partial J_{\rm gyro}}$ will be positive at $J_{\rm gyro} = 0$, where the derivative is taken with all conserved charges held fixed.

Thus, one expects a near-BPS nongyrating string with $J > J_{\rm crit}$ to be unstable to transfer of angular momentum from spin to gyration.

It is interesting to speculate as to the final state into which this string decays. There are two natural possibilities. The first is that the string sheds its excess angular momentum through classical radiation and eventually becomes a *stable* nongyrating string with $J \leq J_{\rm crit}$. The second is that the dominant effect is shedding of excess *energy* and that the final state is a gyrating *BPS* string. One might expect that either final state can arise and that the outcome depends on the particular values of the parameters. Note, however, that if we were to place the unstable string in a small reflecting cavity, this would prevent the loss of significant amounts of either E or J, so that one would expect decay into a stable non-BPS gyrating black string. Because of the small domain and simple boundary conditions, this might be a particularly interesting arena for numerical simulations. It would also lead to a clear signal: an equilibrium state that is far from being rotationally invariant.

IV. DISCUSSION

We have argued for an instability of D1-D5-P near-BPS black strings with $J > J_{\rm crit} = \frac{3Q_1Q_5}{2\sqrt{2}}$. Note that such strings exist only when the number N of momentum quanta exceeds a certain bound: $N \geq \frac{9}{8}Q_1Q_5$. Thus, like the original Gregory-Laflamme instability, the instability arises only for sufficiently long strings.

Now, the thermodynamics noted above might also be taken to suggest an instability to simply radiating angular momentum (and momentum) to infinity. In fact, this latter sort of potential instability could in principle occur at a smaller value of J, since gravitational waves can carry more J for a given amount of P. It is natural to take guidance from the study of $3 + 1$ dimensional Kerr black holes, where one finds a similar thermodynamics: Kerr black holes have an entropy that decreases with increasing J. Thus, radiation of a small amount of energy is allowed if it carries a large angular momentum. In the case of Kerr one finds no instability for massless fields but merely super-radiance: a given incident wave undergoes a finite amount of amplification and then disperses to infinity. In contrast, a true linear instability does result [28,29] if the black hole is surrounded by a large mirror (or is placed in a large anti-de sitter space [30]) so that the wave is continually redirected toward the black hole.

[5]For static solutions, the BPS bound and the results of [25] work together to forbid linear instabilities. However, we know of no general theorems for the stationary case.

[6]Were the horizons completely smooth, this would be enforced by the area theorem [26,27].

However, an instability for Kerr can arise for massive fields, which can be bound to the black hole by the gravitational potential. Results are known for minimally coupled scalar fields [31–34]. In our context, one may expect radiation modes with momentum in the z-direction (i.e., Kaluza-Klein modes) to behave similarly, and our gyrational mode is much like such a bound state. Indeed, in the non-BPS case it is not clear to us to what extent it can be meaningfully distinguished from such bound states. But while a study of such bound states for non-BPS strings is difficult, it is clear the gyrational mode is the unique such bound state in the BPS limit. Since this limit will be important below, we focus on the gyrational mode.

We have seen that gyrations cannot be reabsorbed into the black string since $\frac{\partial S}{\partial J_{\rm gyro}} > 0$. Thus, such gyrations can decay only through radiation to infinity. Whether or not a linear instability occurs will then be determined by a competition between two effects: the amplification of the traveling wave and the tendency to radiate the gyrational traveling wave to infinity. Both are expected to vanish in the BPS limit. However, one expects the amplification to increase with $J - J_{\rm crit}$, while there is no reason for this parameter to affect the rate at which the gyrational traveling wave is radiated to infinity. Thus, one expects that, at least by tuning parameters so that $J - J_{\rm crit}$ is large while the string remains nearly BPS, one can indeed produce an instability.

Our argument is highly suggestive, but clearly falls short of a proof. We have also described two possible final states. The system clearly calls for more detailed investigation and may yield a variety of interesting phenomena. In addition to those mentioned above, it may also be fruitful to investigate relations between gyrating black strings and black tubes or black rings [35–37].

Supposing now that an instability (of any type discussed above) does occur, let us briefly reflect on the broader implications. An interesting attempt to understand the general nature of black string instabilities is encoded in the Gubser-Mitra conjecture of [9]. Quoting from [9], this is the conjecture that "...for a black brane with translational symmetry, a Gregory-Laflamme instability exists precisely when the brane is thermodynamically unstable. Here, by Gregory-Laflamme instability we mean a tachyonic mode in small perturbations of the horizon; and by thermodynamically unstable we mean that the Hessian matrix of second derivatives of the mass with respect to the entropy and the conserved charges or angular momenta has a negative eigenvalue."

This conjecture has been proven for a certain class of black strings [10], but our system appears to be a counter-example to the conjecture holding in complete generality. Let us consider a slightly non-BPS spinning string with $J > J_{\rm crit}$. We choose the non-BPS case as, with asymptotically flat boundary conditions, we expect gyrating strings to radiate so that non-BPS nongyrating strings will form an isolated family in the space of stationary solutions. Thus we may cleanly talk about "the entropy of the black string with fixed conserved charges and angular momenta." Nearly BPS objects are generally thermodynamically stable. The details of the nonextremal solutions can be found in [14], and show that no instabilities are present near extremality. Our discussion above suggests a dynamical instability and thus a counter-example to the above conjecture. However, it should be noted that, even at a finite distance from extremality, one finds an interesting conspiracy that forces the Hessian to have a single zero eigenvalue. Thus, it is possible that the conjecture may be preserved if nonextremal strings have a marginally stable mode leading to gyration but no linear instabilities.

Shortly after the initial appearance of this work, another argument for a counter-example to Gubser-Mitra was given in [38] by exhibiting a marginally stable mode in a thermodynamically stable but near-BPS black string. It is interesting to rephrase our results in the same terms: on general grounds, one may expect that some Kaluza-Klein modes around rotating black strings are unstable as they should act like massive fields around Kerr black holes. However, in general we expect such black strings to be thermodynamically unstable. It is only in the BPS limit that one expects thermodynamic stability. The gyrational mode of the BPS black string studied here allows us to see that a small number of marginal bound states persist for this string in the extremal limit, suggesting the presence of stable, marginal, and unstable modes within any neighborhood of the BPS limit, and, in particular, in the thermodynamically stable regime.

As a final comment, the reader may wish to return to the discussion of [11] and ask how the entropy of gyrating BPS black strings is to be understood from string theory. The answer is that gyration of a D-brane bound state is described by the U(1) "center-of-mass" degrees of freedom, since the D-branes gyrate collectively in a way that does not excite the relative motion degrees of freedom. The counting of states for the spinning black string given in [11] does not include the effect of this degree of freedom as it is known to carry little entropy (see, e.g., [39]). Thus, momentum that goes into exciting gyration of the bound state produces no entropy. But this is just what was observed in the black string entropy in Eq. (3.1). The point is that the U(1) degrees of freedom *can* carry angular momentum, and can do so more "cheaply" than can the collective modes. Thus, allowing the U(1) degrees of freedom carry linear momentum P_{gyro} and angular momentum $J_{\rm gyro}$ leads directly to the entropy (3.1) for the gyrating D-brane bound state. In particular, this means that for certain values of the global charges D-brane bound states are also unstable to gyration in the presence of interactions (e.g., via closed-string exchange) between the center-of-mass U(1) and other degrees of freedom.

ACKNOWLEDGMENTS

This work was inspired by questions that arose from discussion with Iosif Bena. We also thank Henriette Elvang, Eric Gimon, Andrés Gomberoff, Gary Horowitz, Harvey Reall, and Rob Myers for other useful discussions. This work was supported in part by NSF Grant Nos. PHY00-98747, PHY99-07949, and PHY03-42749 by funds from Syracuse University and by funds from the University of California. B. C. P. received support from Syracuse University. In addition, B. C. P. thanks the University of California at Santa Barbara for its hospitality during this work.

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