

Probing strong-field scalar-tensor gravity with gravitational wave asteroseismology

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We present an alternative way of tracing the existence of a scalar field based on the analysis of the gravitational wave spectrum of a vibrating neutron star. Scalar-tensor theories in strong-field gravity can potentially introduce much greater differences in the parameters of a neutron star than the uncertainties introduced by the various equations of state. The detection of gravitational waves from neutron stars can set constraints on the existence and the strength of scalar fields. We show that the oscillation spectrum is dramatically affected by the presence of a scalar field, and can provide unique confirmation of its existence.

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I. INTRODUCTION

A natural alternative/generalization to general relativity is the scalar-tensor theory in which gravity is mediated by long-range scalar fields in addition to the usual tensor field present in Einstein's theory [1–6]. Scalar-tensor theories of gravity can be obtained from the low-energy limit of string theory or/and other gauge theories. The existence of scalar fields is crucial in explaining the accelerated expansion phases of the universe, e.g., inflation and quintessence. They are viable theories of gravity for a specific range of the function that couples the scalar field to gravity. Still, it is not clear how the scalar fields couple to gravity [7,8]. A basic assumption is that the scalar and gravitational fields φ and $g_{\mu\nu}$ are coupled to matter via an “effective metric” $\tilde{g}_{\mu\nu} = A^2(\varphi)g_{\mu\nu}$. The existence of the scalar field has not yet been verified while a number of experiments in the weak-field limit of general relativity disproved or set severe limits in the existence and the strength of the scalar fields [6,9].

The Fierz-Jordan-Brans-Dicke [1–3] theory assumes that the “coupling function” has the form $A(\varphi) = \alpha_0\varphi$, i.e., it is characterized by a unique free parameter $\alpha_0^2 = (2\omega_{\text{BD}} + 3)^{-1}$ and all its predictions differ from those of general relativity by quantities of order α_0^2 [10]. Solar system experiments set strict limits in the value of the Brans-Dicke parameter ω_{BD} , i.e., $\omega_{\text{BD}} \geq 40000$ which suggests a very small $\alpha_0^2 < 10^{-5}$ (see [9,11–13]). Damour and Esposito-Farese [10,14] showed that the predictions of scalar-tensor theories in the strong-field might be drastically different from those of general relativity. By studying neutron star models in a simplified

version of scalar-tensor theory where $A(\varphi) = \alpha_0\varphi + \beta\varphi^2/2$ they found that for certain values of the coupling parameter β , the stellar models develop some strong-field effects which induce significant deviations from general relativity. Their claim was based on the difference that one can observe in the properties of neutron stars by the introduction of a scalar field (even if the coupling constant α_0 is very small). Damour and Esposito-Farese described the sudden deviation from general relativity for specific values of the coupling constants as “spontaneous scalarization.” Harada [15] studied in more detail the stability of nonrotating neutron stars in the framework of the scalar-tensor theory and reported that “spontaneous scalarization” is possible for $\beta \lesssim -4.35$. In other words the “spontaneous scalarization” of Damour and Esposito-Farese suggests that weak-field experiments cannot constrain the effect of the scalar fields for the strong-field regime and prompts for alternative measurements. Such measurements can be based, for example, on accurate estimation of the orbital decay of binary systems [14,16], in the accurate monitoring of gravitational waveforms from neutron stars spiralling into massive black holes [17], or by direct observation of monopolar gravitational radiation during the collapse of compact objects. Collapse simulations have shown that indeed a scalar field can be observed by LIGO/EGO for the specific range of values of β if the event takes place in our Galaxy [18–23], while the space gravitational wave detector LISA can also provide constraints in the existence of scalar field [17,24].

Recently, DeDeo and Psaltis [25] suggested that the effects of the scalar fields might be apparent in the observed redshifted lines of the X-rays and γ -rays observed by Chandra and XMM-Newton. Testing strong gravity via electromagnetic observations is a really novel

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idea, which has been recently extended by the same authors [26] in suggesting tests via the quasiperiodic oscillations (QPOs).

In this paper we examine whether gravitational wave observations of the oscillation spectra of neutron stars can provide an alternative way of testing scalar-tensor theories. The aforementioned results [10,15,25] show that for a specific range of values of the coupling constant β , which are not constrained by the current experimental limits in the weak-field regime, neutron stars can have significantly larger masses and radii. This immediately suggests that the natural oscillation frequencies of the neutron stars will be altered accordingly and a possible detection of gravitational waves from such oscillations will not only probe the existence of the scalar field but might provide a way of estimating its strength.

The estimation of the stellar parameters (mass, radius and equation of state) via their oscillation properties is not a new idea. Helioseismology and asteroseismology are established fields in astronomy, and there is already a wealth of information about the interior of our sun and the stars via this approach. In the late '90s it was suggested [27,28] that the oscillation spectra of neutron stars can reveal in a unique way their properties. The radius, the mass and the equation of state can be easily deduced by an analysis of the oscillation spectrum of the f , p and w -modes [29]. Moreover, the rotation period of a neutron star can be revealed by the r -mode oscillations since their frequency is proportional to the rotation rate ($\omega_{r\text{-mode}} \sim 4\Omega/3$). Features such as superfluidity [30] or magnetic fields [31] can reveal their presence via detailed analysis of the gravitational and electromagnetic spectra. Recently, it has been suggested that compact stars with exotic equations of state, such as strange stars, have a spectrum which carries in a unique way their signature [32,33].

The oscillations of a neutron star in the scalar-tensor theory will produce not only gravitational but also scalar waves [5]. Detectable scalar waves will be a unique probe for the theory, but this might not be the case if the radiated energy is small. Still, we show that there is no need for direct observation of scalar waves—the presence of the scalar field will be apparent in the gravitational wave spectrum, since the spectra will be shifted according to the strength of the scalar field.

The paper is organized as follows. In the next section we present the basic equations for the construction of the unperturbed spherically symmetric stellar models. We also show the effect of the scalar field on the stellar structure. In Sec. III we derive the perturbation equations which will be used for the numerical estimation of the oscillation frequencies. Finally, in Sec. IV we discuss the results in connection to gravitational wave asteroseismology. Note that the present study will be based on two equations of state, which have been used earlier [10,15].

Our aim is to demonstrate the effect of the scalar field in the neutron star oscillation spectra, while more detailed analysis for a wide range of EOS is underway.

II. STELLAR MODELS IN SCALAR-TENSOR THEORIES OF GRAVITY

In this section we will study neutron star models in scalar-tensor theory of gravity with one scalar field. This is a natural extension of Einstein's theory, in which gravity is mediated not only by a second rank tensor (the metric tensor $g_{\mu\nu}$), but also by a massless long-range scalar field φ . The action is given by [4]

$$S = \frac{1}{16\pi G_*} \int \sqrt{-g_*} (R_* - 2g_*^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu}) d^4x + S_m[\Psi_m, A^2(\varphi)g_{*\mu\nu}], \quad (1)$$

where all quantities with asterisks are related to the ‘‘Einstein metric’’ $g_{*\mu\nu}$, then R_* is the curvature scalar for this metric and G_* is the bare gravitational coupling constant. Ψ_m represents collectively all matter fields, and S_m denotes the action of the matter represented by Ψ_m , which is coupled to the ‘‘Jordan-Fierz metric tensor’’ $\tilde{g}_{\mu\nu}$. The field equations formulated better in the ‘‘Einstein metric,’’ but all nongravitational experiments measure the ‘‘Jordan-Fierz’’ or ‘‘physical metric.’’ The ‘‘Jordan-Fierz metric’’ is related to the ‘‘Einstein metric’’ via the conformal transformation,

$$\tilde{g}_{\mu\nu} = A^2(\varphi)g_{*\mu\nu}. \quad (2)$$

Hereafter, we relate all tilded quantities with ‘‘physical frame’’ and those with asterisk with the ‘‘Einstein frame.’’ From the variation of the action S , we get the field equations in the Einstein frame

$$G_{*\mu\nu} = 8\pi G_* T_{*\mu\nu} + 2\left(\varphi_{,\mu} \varphi_{,\nu} - \frac{1}{2}g_{*\mu\nu}g_*^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta}\right), \quad (3)$$

$$\square_* \varphi = -4\pi G_* \alpha(\varphi) T_*, \quad (4)$$

where $T_*^{\mu\nu}$ is the energy-momentum tensor in the Einstein frame which is related to the physical energy-momentum tensor $\tilde{T}_{\mu\nu}$ as follows:

$$T_*^{\mu\nu} \equiv \frac{2}{\sqrt{-g_*}} \frac{\delta S_m}{\delta g_{*\mu\nu}} = A^6(\varphi) \tilde{T}^{\mu\nu}. \quad (5)$$

The scalar quantities T_* and $\alpha(\varphi)$ are defined as

$$T_* \equiv T_{*\mu}^{\mu} = T_*^{\mu\nu} g_{*\mu\nu}, \quad (6)$$

$$\alpha(\varphi) \equiv \frac{d \ln A(\varphi)}{d\varphi}. \quad (7)$$

It is apparent that $\alpha(\varphi)$ is the only field-dependent func-

tion which couples the scalar field with matter, for $\alpha(\varphi) = 0$ the theory reduces to general relativity.

Finally, the law for energy-momentum conservation $\tilde{\nabla}_\nu \tilde{T}^\nu_\mu = 0$ is transformed into

$$\nabla_{*\nu} T^\nu_{*\mu} = \alpha(\varphi) T_{*\mu} \nabla_{*\mu} \varphi, \quad (8)$$

and we set φ_0 as the cosmological value of the scalar field at infinity. In this paper, for simplicity, we adopt the same form of conformal factor $A(\varphi)$ as in Damour and Esposito-Farèse [10], which is

$$A(\varphi) = e^{\frac{1}{2}\beta\varphi^2}, \quad (9)$$

i.e., $\alpha(\varphi) = \beta\varphi$ where β is a real number. In the case for $\beta = 0$, the scalar-tensor theory reduces to general relativity, while the ‘‘spontaneous scalarization’’ occurs for $\beta \leq -4.35$ [15].

We will model the neutron stars as self-gravitating perfect fluids, made out of degenerate matter at equilibrium and admitting cold equation of state. Then the metric describing an unperturbed, nonrotating, spherically symmetric neutron star can be written as

$$ds_*^2 = g_{*\mu\nu} dx^\mu dx^\nu = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (10)$$

where

$$e^{-2\Lambda} = 1 - \frac{2\mu(r)}{r}, \quad (11)$$

while the ‘‘potential’’ function $\Phi(r)$ will be calculated later. The stellar matter is assumed to be a perfect fluid

$$\tilde{T}_{\mu\nu} = (\tilde{\rho} + \tilde{P})\tilde{U}_\mu\tilde{U}_\nu + \tilde{P}\tilde{g}_{\mu\nu}, \quad (12)$$

where \tilde{U}_μ is the four-velocity of the fluid, $\tilde{\rho}$ is the total energy density in the fluid frame, and \tilde{p} is the pressure.

Spherical symmetry simplifies significantly the procedure of constructing stellar models. Using Eqs. (3), (4) and (8), we can obtain the following set of equations for the background configuration [10,15]:

$$\frac{d\mu}{dr} = 4\pi G_* r^2 A^4 \tilde{\rho} + \frac{1}{2} r(r - 2\mu) \Psi^2, \quad (13)$$

$$\frac{d\Phi}{dr} = 4\pi G_* \frac{r^2 A^4 \tilde{P}}{r - 2\mu} + \frac{1}{2} r \Psi^2 + \frac{\mu}{r(r - 2\mu)}, \quad (14)$$

$$\frac{d\varphi}{dr} = \Psi, \quad (15)$$

$$\begin{aligned} \frac{d\Psi}{dr} = & 4\pi G_* \frac{r A^4}{r - 2\mu} [\alpha(\tilde{\rho} - 3\tilde{P}) + r(\tilde{\rho} - \tilde{P})\Psi] \\ & - \frac{2(r - \mu)}{r(r - 2\mu)} \Psi, \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{d\tilde{P}}{dr} = & -(\tilde{\rho} + \tilde{P}) \left[\frac{d\Phi}{dr} + \alpha\Psi \right] \\ = & -(\tilde{\rho} + \tilde{P}) \left[4\pi G_* \frac{r^2 A^4 \tilde{P}}{r - 2\mu} + \frac{1}{2} r \Psi^2 + \frac{\mu}{r(r - 2\mu)} \right. \\ & \left. + \alpha\Psi \right]. \end{aligned} \quad (17)$$

Near the center, the background quantities φ , Ψ , Φ , \tilde{P} , and μ can be expanded as:

$$\varphi(r) = \varphi_c + \frac{1}{2} \varphi_2 r^2 + O(r^4), \quad (18)$$

$$\Psi(r) = \varphi' = \varphi_2 r + O(r^3), \quad (19)$$

$$\Phi(r) = \Phi_c + \frac{1}{2} \Phi_2 r^2 + O(r^4), \quad (20)$$

$$\tilde{P}(r) = \tilde{P}_c + \frac{1}{2} \tilde{P}_2 r^2 + O(r^4), \quad (21)$$

$$\mu(r) = O(r^3), \quad (22)$$

where the expansion coefficients are given by

$$\Phi_2 = 4\pi G_* A_c^4 \tilde{P}_c, \quad (23)$$

$$\varphi_2 = \frac{4\pi}{3} G_* A_c^4 \alpha_c (\tilde{\rho}_c - 3\tilde{P}_c), \quad (24)$$

$$\tilde{P}_2 = -(\tilde{\rho}_c + \tilde{P}_c)(\Phi_2 + \alpha_c \varphi_2), \quad (25)$$

where $A_c \equiv A(\varphi_c)$ and $\alpha_c \equiv \alpha(\varphi_c)$.

Outside the star, the metric is assumed to be static and spherically symmetric, and since we deal with only one scalar field, it gets the form [4]

$$ds_*^2 = -e^\nu dt^2 + e^{-\nu} d\bar{r}^2 + e^{-\nu+\lambda} (d\theta^2 + \sin^2\theta d\phi^2), \quad (26)$$

$$e^\nu = \left(1 - \frac{a}{\bar{r}}\right)^{b/a}, \quad e^\lambda = \bar{r}(\bar{r} - a), \quad (27)$$

where \bar{r} is a radial coordinate, given by the following relation

$$r^2 = \bar{r}^2 \left(1 - \frac{a}{\bar{r}}\right)^{1-b/a}. \quad (28)$$

Moreover, in the above metric (27), a and b are some constants, which are connected with the total scalar charge ω_A and the total ADM mass M_{ADM} , i.e., $a^2 - b^2 = 4\omega_A^2$ and $b = 2M_{\text{ADM}}$. Finally, the asymptotic form, at spatial infinity, for the metric and scalar field will be given as functions of the total ADM mass and the total scalar charge

$$g^{*\mu\nu} = \eta_{\mu\nu} + \frac{2M_{\text{ADM}}}{r} \delta_{\mu\nu} + O\left(\frac{1}{r^2}\right), \quad (29)$$

$$\varphi = \varphi_0 + \frac{\omega_A}{r} + O\left(\frac{1}{r^2}\right), \quad (30)$$

where $\eta_{\mu\nu}$ is the Minkowskian metric.

By matching this exterior solution and the interior metric, one gets the following relations [10]:

$$M_{\text{ADM}} = \frac{R^2 \Phi'_s}{G_*} \left(1 - \frac{2\mu_s}{R}\right)^{1/2} \times \exp\left[-\frac{\Phi'_s}{\sqrt{(\Phi'_s)^2 + \Psi_s^2}} \operatorname{arctanh}\left(\frac{\sqrt{(\Phi'_s)^2 + \Psi_s^2}}{\Phi'_s + 1/R}\right)\right], \quad (31)$$

$$\varphi_0 = \varphi_s + \frac{\Psi_s}{\sqrt{(\Phi'_s)^2 + \Psi_s^2}} \operatorname{arctanh}\left[\frac{\sqrt{(\Phi'_s)^2 + \Psi_s^2}}{\Phi'_s + 1/R}\right], \quad (32)$$

$$\Phi'_s = \frac{1}{2} R \Psi_s^2 + \frac{\mu_s}{R(R - 2\mu_s)}, \quad (33)$$

where the functions with the subscript s correspond to their actual values at the stellar surface and the prime denotes the derivative with respect to r . A more general scheme for fixing the asymptotic value of ϖ_0 can be derived using the cosmological model of Damour and Nordtvedt [34], (see, for example, [35]). Still, this approach has been applied for $\beta > 0$, which is not the subject of our present study.

In order to determine the stellar properties, an additional equation is needed, i.e., the equation of state (EOS). Here we will use a polytropic one given by

$$\tilde{P} = K n_0 m_b \frac{\tilde{n}}{n_0}, \quad (34)$$

$$\tilde{\rho} = \tilde{n} m_b + \frac{\tilde{P}}{\Gamma - 1}, \quad (35)$$

$$m_b = 1.66 \times 10^{-24} \text{ g}, \quad (36)$$

$$n_0 = 0.1 \text{ fm}^{-3}. \quad (37)$$

We have selected $\Gamma = 2.46$ and $K = 0.00936$ in agreement with a fitting to tabulated data for EOS A [36], and $\Gamma = 2.34$ and $K = 0.0195$ to fit EOS II [37]. In other words the parameters Γ and K are adjusted to fit the tabulated data from these two realistic equations of state which have been used in earlier studies of the problem [10,15]. The present study will be constrained to these two EOS since our aim is to demonstrate the effect of the scalar field in the neutron star oscillation spectra while

more detailed analysis for a wide range of EOS is underway.

A. Neutron Star Models

Here we present some typical stellar models for the two equations of state under discussion, i.e., EOS A and EOS II. For the construction of the stellar models, there exist three freely specifiable parameters. These are: the constant β of the conformal factor, the value φ_0 of the scalar field φ at infinity, and the central density $\tilde{\rho}_c$. We will also consider positive values of φ_0 , because the basic Eqs. (13)–(17) are symmetric under the reflection $\varphi \rightarrow -\varphi$. Also, we will only consider stellar models with $\beta = -6$ and $\beta = -8$; for these range of values, the effect of the scalar field is more pronounced. Binary pulsar data already suggest larger values for β , i.e., $\beta \gtrsim -4.5$, but this work is based on a different framework and the results can be used as an alternative way of constraining the appearance of “spontaneous scalarization” in neutron stars.

In Fig. 1 we show stellar models with $\beta = -6$. In the left column we plot models for the EOS A, and in the right column, models for the EOS II. In the two upper panels we plot the ADM mass M_{ADM} as functions of the central density; in the two middle panels we plot the value of the scalar field in the center of the star, φ_c as functions of the central density. It is apparent that the effect of the scalar field is pronounced for central densities higher than $5 \times 10^{14} \text{ gr/cm}^3$; for lower densities there is no way that one can trace the existence of a scalar field. The lower panel is a “typical” mass-radius relation where one can observe the dramatic effect of the scalar field in the stellar structure. For comparison in every panel the general relativistic stellar models are shown by a solid line (GR)

In Fig. 2 we present stellar models for $\beta = -8$, the ordering of the graphs is as in Fig. 1. Now the effect of the scalar field is more pronounced than before. Its presence can be traced for even lower densities while for higher densities the stellar models divert considerably from those of GR. The effects on the mass and radius are even more dramatic in the mass-radius diagrams in the lower panels.

Finally, in order to stress the effect of the scalar field on the maximum mass of neutron stars, we show in two tables the relevant parameters for each EOS. The maximum masses are well beyond the observed values of neutron star. This is also true for the observed redshifts. It is worth mentioning that the redshift of the maximum mass models is quite unaffected by the changes in the central density and the scalar field parameter. Although for these maximum mass models the redshift is unusually large, for the typical models that we will use, the surface redshift z is below the maximum observed redshift, which is $z = 0.35$ [38]. This means that neutron stars of

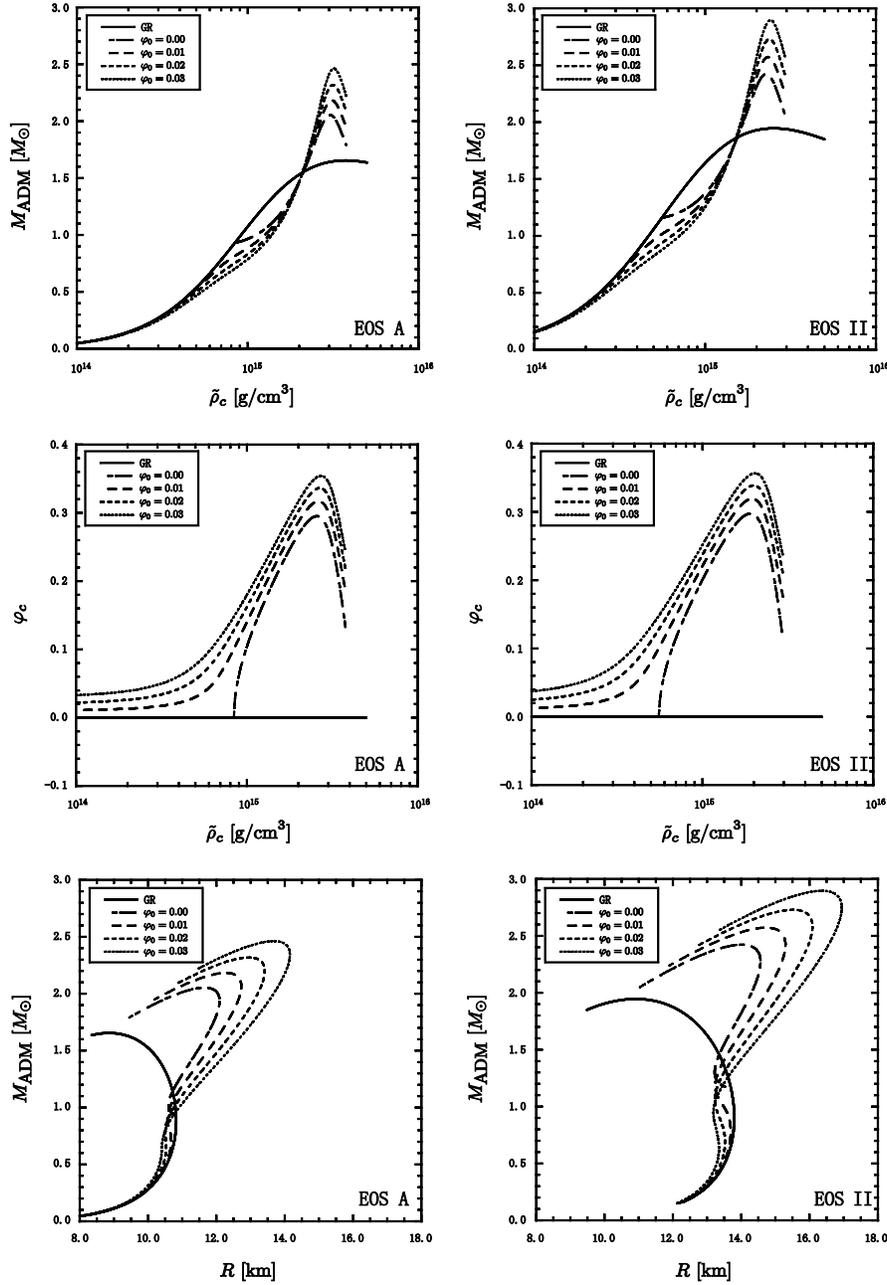


FIG. 1. Stellar models with $\beta = -6$ are shown. The left column corresponds to models for the EOS A while in the right to models for EOS II. In the upper panels the ADM mass M_{ADM} and the value of the scalar field in the center of the star, φ_c are plotted as functions of the central density. The effect of the scalar field is apparent for $\rho_c \geq 5 \times 10^{14} \text{ gr/cm}^3$. The lower panel is a mass-radius diagram where the dramatic effect of the scalar field can be observed. In every panel, the general relativistic stellar models are shown by a solid line (GR)

the scalar-tensor theories cannot be excluded by the present electromagnetic observations.

III. PERTURBATION EQUATIONS IN THE COWLING APPROXIMATION

In this section we present the perturbation equations for nonradial oscillations of spherically symmetric neu-

tron stars in scalar-tensor theory. We derive the equations, describe the numerical method for getting the spectra, and finally we present results for a few characteristic values of the scalar field β . The presentation in this section is quite extended; we have chosen to show all the details of the calculation since this is the first article dealing with the stellar perturbations in scalar-tensor

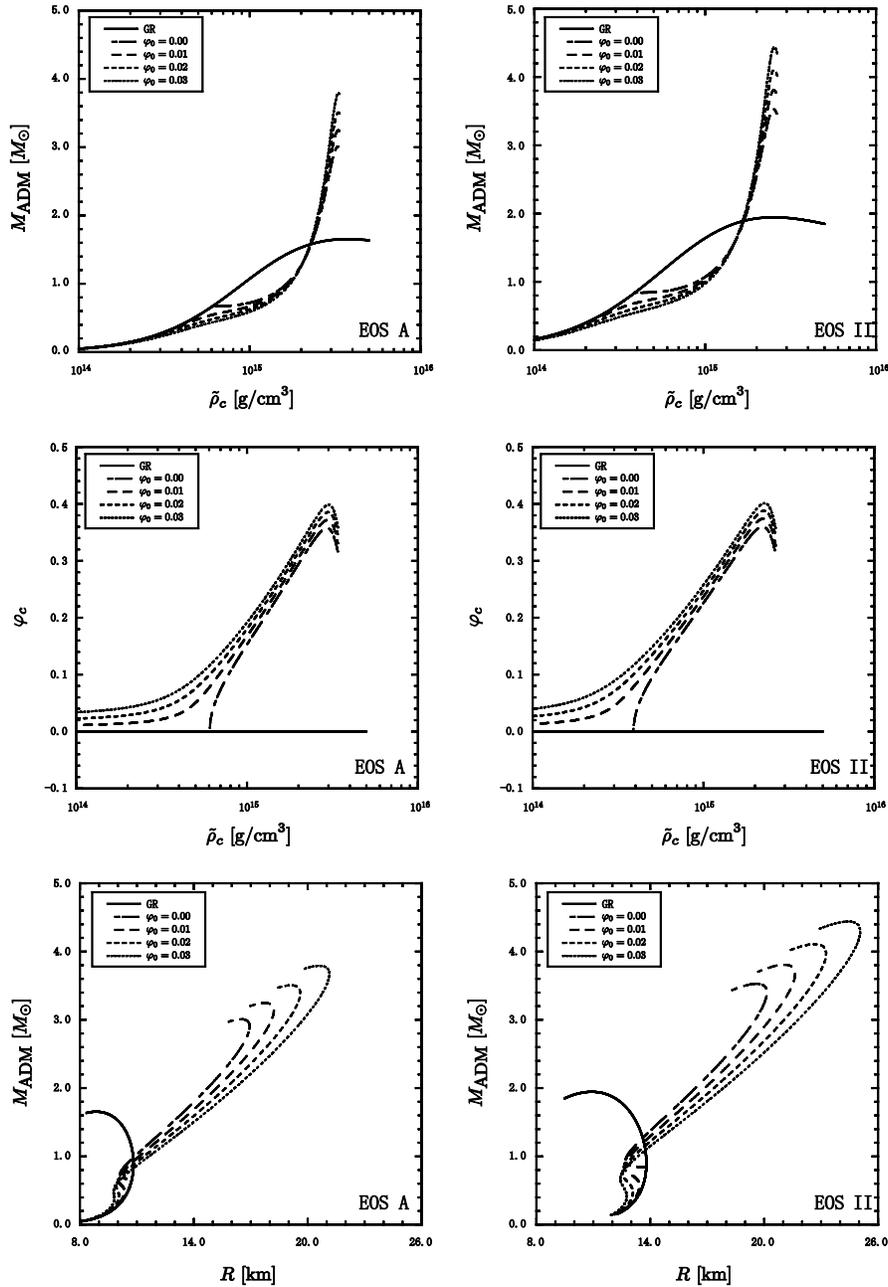


FIG. 2. Stellar models with $\beta = -8$ are shown. The left column corresponds to models for the EOS A and the right to models for EOS II. In the upper panels the ADM mass M_{ADM} and the value of the scalar field in the center of the star φ_c are plotted as functions of the central density. The effect of the scalar field is apparent for $\rho_c \geq 3 \times 10^{14} \text{ gr/cm}^3$. The lower panel is a mass-radius diagram where the dramatic effect of the scalar field can be observed. In every panel, the general relativistic stellar models are shown by a solid line (GR)

theory and there is a wealth of new functions and notations. We will also keep arbitrary the functional form of the scalar function $\alpha(\varphi)$ which will be fixed only later during the numerical calculations. In this first investigation we will not consider the full problem but instead will restrict it to the so called ‘‘Cowling approximation.’’ This means that the fluid is perturbed on a fixed background. In

this way we freeze the spacetime and scalar field perturbations, i.e., $\delta\tilde{g}_{\mu\nu} = 0$ and $\delta\varphi = 0$. The Cowling approximation limits our study to the modes which are directly related to the fluid perturbations, i.e., the f , p and g -modes, while we cannot study the emission of gravitational and scalar waves, neither the families of the spacetime [39] and scalar modes.

The fluid perturbations will be described by the Lagrangian displacement vector

$$\begin{aligned}\tilde{\xi}^i &= (\tilde{\xi}^r, \tilde{\xi}^\theta, \tilde{\xi}^\phi) \\ &= \left(e^{-\Lambda} W, -V \frac{\partial}{\partial \theta}, -\frac{1}{\sin^2 \theta} V \frac{\partial}{\partial \phi} \right) \frac{1}{r^2} Y_{lm},\end{aligned}\quad (38)$$

where W and V are functions with of t and r . The unperturbed 4-velocity $\tilde{U}^{(B)\mu}$ is

$$\tilde{U}^{(B)\mu} = \left(\frac{1}{A(\varphi)e^\Phi}, 0, 0, 0 \right); \quad (39)$$

hereafter, an index “(B)” will denote the unperturbed quantities. The perturbed 4-velocity $\delta\tilde{U}^\mu$ has the form

$$\begin{aligned}\delta\tilde{U}^\mu &= \left(0, e^{-\Lambda} \frac{\partial W}{\partial t}, -\frac{\partial V}{\partial t} \frac{\partial}{\partial \theta}, -\frac{1}{\sin^2 \theta} \frac{\partial V}{\partial t} \frac{\partial}{\partial \phi} \right) \\ &\times \frac{1}{A(\varphi)r^2} e^{-\Phi} Y_{lm}.\end{aligned}\quad (40)$$

The perturbed energy-momentum tensor $\delta\tilde{T}_\mu^\nu$ in the Cowling approximation gets the form

$$\begin{aligned}\delta\tilde{T}_\mu^\nu &= (\delta\tilde{\rho} + \delta\tilde{P})\tilde{g}_{\mu\beta}^{(B)}\tilde{U}^{(B)\beta}\tilde{U}^{(B)\nu} + (\tilde{\rho}^{(B)} \\ &+ \tilde{P}^{(B)})\tilde{g}_{\mu\beta}^{(B)}(\tilde{U}^{(B)\beta}\delta\tilde{U}^\nu + \delta\tilde{U}^\beta\tilde{U}^{(B)\nu}) + \delta\tilde{P}\delta_\mu^\nu.\end{aligned}\quad (41)$$

The Lagrangian variation of the baryon number density $\Delta\tilde{n}$ is

$$\frac{\Delta\tilde{n}}{\tilde{n}} = -\tilde{\nabla}_k^{(3)}\tilde{\xi}^k - \frac{\delta\tilde{g}}{2\tilde{g}^{(B)}}, \quad (42)$$

where $\tilde{\nabla}_k^{(3)}$ denotes the covariant derivative in a 3-dimensional with metric $\tilde{g}_{\mu\nu}$ and Δ the Lagrangian variation. In this equation, the first term of the right hand side expresses the 3-dimensional divergence of the fluid, and the second term is the amount of the volume change due to the metric perturbation, but because we use the Cowling approximation we neglect this second term. By employing the above perturbation variables, Eq. (42) for the Lagrangian variation of the baryon number density will be written as

$$\frac{\Delta\tilde{n}}{\tilde{n}} = -\left[\frac{1}{r^2} e^{-\Lambda} \frac{\partial W}{\partial r} + \frac{l(l+1)V}{r^2} + 3\alpha\Psi e^{-\Lambda} \frac{W}{r^2} \right] Y_{lm}.\quad (43)$$

For adiabatic perturbations, using the first law of thermo-

dynamics, we can get the following relation between the change of the baryon number density and the pressure

$$\Delta\tilde{\rho} = \frac{\tilde{\rho} + \tilde{P}}{\tilde{n}} \Delta\tilde{n}.\quad (44)$$

Therefore we can express the Eulerian density variation $\delta\tilde{\rho}$ as,

$$\delta\tilde{\rho} = (\tilde{\rho}^{(B)} + \tilde{P}^{(B)}) \frac{\Delta\tilde{n}}{\tilde{n}} - \frac{\partial\tilde{\rho}^{(B)}}{\partial r} e^{-\Lambda} \frac{W}{r^2} Y_{lm}.\quad (45)$$

Here we have used the relation between the Lagrangian perturbation Δ and Eulerian perturbation δ

$$\Delta\tilde{\rho}(t, r) = \tilde{\rho}(t, r + \tilde{\xi}^r) - \tilde{\rho}^{(B)}(t, r) \simeq \delta\tilde{\rho} + \frac{\partial\tilde{\rho}^{(B)}}{\partial r} \tilde{\xi}^r.\quad (46)$$

From the definition of the adiabatic constant

$$\gamma \equiv \left(\frac{\partial \ln \tilde{P}}{\partial \ln \tilde{n}} \right)_s = \frac{\tilde{n} \Delta \tilde{P}}{\tilde{P} \Delta \tilde{n}}, \quad (47)$$

we can derive the form of the Eulerian variation of the pressure

$$\delta\tilde{P} = \gamma \tilde{P}^{(B)} \frac{\Delta\tilde{n}}{\tilde{n}} - \frac{\partial\tilde{P}^{(B)}}{\partial r} e^{-\Lambda} \frac{W}{r^2} Y_{lm}.\quad (48)$$

Finally, the combination of Eqs. (44) and (47), provides the standard form of the adiabatic constant

$$\gamma = \frac{\tilde{\rho} + \tilde{P}}{\tilde{P}} \left(\frac{\partial\tilde{P}}{\partial\tilde{\rho}} \right)_s.\quad (49)$$

By taking a variation of the equation for the conservation of energy-momentum $\tilde{\nabla}_\nu \tilde{T}_\mu^\nu = 0$, we can get the equation describing the perturbations of the fluid in the Cowling approximation:

$$\delta\tilde{T}_{\mu,\nu}^\nu - \Gamma_{*\mu\nu}^\alpha \delta\tilde{T}_\alpha^\nu + \Gamma_{*\alpha\nu}^\nu \delta\tilde{T}_\mu^\alpha - \alpha\varphi_{,\mu} \delta\tilde{T}_\nu^\nu + 4\alpha\varphi_{,\nu} \delta\tilde{T}_\mu^\nu = 0.\quad (50)$$

The analytic form of the above equation for the values of the index $\mu = 1, 2$ is

$$\begin{aligned} & \tilde{\nabla}_\nu \delta \tilde{T}_1^\nu = 0 \\ \Leftrightarrow & (\tilde{\rho} + \tilde{P}) \frac{1}{r^2} e^{\Lambda-2\Phi} \frac{\partial^2 W}{\partial t^2} - \frac{\partial}{\partial r} \left[\gamma \tilde{P} \left\{ \frac{1}{r^2} e^{-\Lambda} \frac{\partial W}{\partial r} + \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right\} \right] - \frac{\partial}{\partial r} \left(e^{-\Lambda} \frac{d\tilde{P}}{dr} \frac{W}{r^2} \right) + \\ & \frac{d\tilde{P}}{dr} \left[\frac{1}{r^2} e^{-\Lambda} \frac{\partial W}{\partial r} + \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right] - \left(\frac{d\Phi}{dr} + \alpha \Psi \right) e^{-\Lambda} \frac{d\tilde{\rho}}{dr} \frac{W}{r^2} + \frac{d\tilde{P}}{dr} \frac{d\tilde{P}}{d\tilde{\rho}} \left[\frac{1}{r^2} e^{-\Lambda} \frac{\partial W}{\partial r} + \right. \\ & \left. \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right] - \left(\frac{d\Phi}{dr} + \alpha \Psi \right) e^{-\Lambda} \frac{d\tilde{P}}{dr} \frac{W}{r^2} = 0. \end{aligned} \quad (51)$$

$$\tilde{\nabla}_\nu \delta \tilde{T}_2^\nu = 0 \Leftrightarrow (\tilde{\rho} + \tilde{P}) e^{-2\Phi} \frac{\partial^2 V}{\partial t^2} + \gamma \tilde{P} \left[\frac{1}{r^2} e^{-\Lambda} \frac{\partial W}{\partial r} + \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right] + e^{-\Lambda} \frac{d\tilde{P}}{dr} \frac{W}{r^2} = 0. \quad (52)$$

By assuming a harmonic dependence on time, the perturbation functions will be written as $W(t, r) = W(r)e^{i\omega t}$ and $V = V(t, r) = V(r)e^{i\omega t}$, and the above system of equations gets the form

$$\begin{aligned} & \omega^2 (\tilde{\rho} + \tilde{P}) e^{\Lambda-2\Phi} \frac{W}{r^2} + \frac{d}{dr} \left[\gamma \tilde{P} \left\{ \frac{1}{r^2} e^{-\Lambda} \frac{dW}{dr} + \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right\} \right] + \frac{d}{dr} \left(e^{-\Lambda} \frac{d\tilde{P}}{dr} \frac{W}{r^2} \right) - \\ & \frac{d\tilde{P}}{dr} \left[\frac{1}{r^2} e^{-\Lambda} \frac{dW}{dr} + \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right] + \left(\frac{d\Phi}{dr} + \alpha \Psi \right) e^{-\Lambda} \frac{d\tilde{\rho}}{dr} \frac{W}{r^2} - \frac{d\tilde{P}}{dr} \frac{d\tilde{P}}{d\tilde{\rho}} \left[\frac{1}{r^2} e^{-\Lambda} \frac{dW}{dr} + \right. \\ & \left. \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right] + \left(\frac{d\Phi}{dr} + \alpha \Psi \right) e^{-\Lambda} \frac{d\tilde{P}}{dr} \frac{W}{r^2} = 0, \end{aligned} \quad (53)$$

$$-\omega^2 e^{-2\Phi} (\tilde{\rho} + \tilde{P}) V + \frac{\gamma \tilde{P}}{r^2} \left[e^{-\Lambda} \frac{dW}{dr} + l(l+1)V + 3\alpha \Psi e^{-\Lambda} W \right] + e^{-\Lambda} \frac{d\tilde{P}}{dr} \frac{W}{r^2} = 0. \quad (54)$$

A further simplification can be achieved for the first of the above equations by using an appropriate combination of the form $d(54)/dr - (53)$, this leads to the equation

$$\begin{aligned} & -\omega^2 \frac{d}{dr} \left[e^{-2\Phi} (\tilde{\rho} + \tilde{P}) V \right] - \omega^2 (\tilde{\rho} + \tilde{P}) e^{\Lambda-2\Phi} \frac{W}{r^2} + \frac{d\tilde{P}}{dr} \left[\frac{1}{r^2} e^{-\Lambda} \frac{\partial W}{\partial r} + \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right] - \\ & \left(\frac{d\Phi}{dr} + \alpha \Psi \right) e^{-\Lambda} \frac{d\tilde{\rho}}{dr} \frac{W}{r^2} + \frac{d\tilde{P}}{dr} \frac{d\tilde{P}}{d\tilde{\rho}} \left[\frac{1}{r^2} e^{-\Lambda} \frac{\partial W}{\partial r} + \frac{l(l+1)V}{r^2} + 3\alpha \Psi e^{-\Lambda} \frac{W}{r^2} \right] - \left(\frac{d\Phi}{dr} + \alpha \Psi \right) e^{-\Lambda} \frac{d\tilde{P}}{dr} \frac{W}{r^2} = 0, \end{aligned} \quad (55)$$

which can be further simplified by proper substitutions of $d\tilde{P}/dr$ and dW/dr from Eqs. (17) and (54):

$$\frac{dV}{dr} = 2 \frac{d\Phi}{dr} V - e^\Lambda \frac{W}{r^2}. \quad (56)$$

Thus from Eqs. (54) and (56), we get the following simple system of couple ODEs

$$\begin{aligned} \frac{dW}{dr} &= \frac{d\tilde{\rho}}{d\tilde{P}} \left[\omega^2 r^2 e^{\Lambda-2\Phi} V + \left(\frac{d\Phi}{dr} + \alpha \Psi \right) W \right] \\ &- l(l+1)e^\Lambda V - 3\alpha \Psi W, \end{aligned} \quad (57)$$

$$\frac{dV}{dr} = 2 \frac{d\Phi}{dr} V - e^\Lambda \frac{W}{r^2}, \quad (58)$$

which, together with the appropriate boundary conditions at the center and at the surface, constitute an eigenvalue problem for the parameter ω (the eigenfrequency).

The above system of perturbation Eqs. (57) and (58) near the stellar center gets the following simple form:

$$\frac{dW}{dr} + l(l+1)V \approx 0, \quad (59)$$

$$\frac{dV}{dr} + \frac{W}{r^2} \approx 0, \quad (60)$$

with the following set of approximate normal solutions

$$W(r) = B r^{l+1} + \dots, \quad (61)$$

$$V(r) = -\frac{B}{l} r^l + \dots, \quad (62)$$

where B is an arbitrary constant. The two approximate solutions near the center, Eq. (61) and (62), suggest the introduction of two new perturbation functions, $W(r) \equiv \bar{W}(r)r^{l+1}$ and $V(r) \equiv \bar{V}(r)r^l$. After this change, the approximate solutions near the center become

$$\bar{W}(r) = \bar{W}_c + \frac{1}{2} \bar{W}_2 r^2 + \dots, \quad (63)$$

$$\bar{V}(r) = \bar{V}_c + \frac{1}{2}\bar{V}_2 r^2 + \dots, \quad (64)$$

where $\bar{W}_c = B$, $\bar{V}_c = -B/l$, and the coefficients \bar{W}_2 and \bar{V}_2 of the second order terms are

$$\bar{V}_2 = \frac{1}{2l+3} \left[3\alpha_c \varphi_2 \bar{W}_c + 2(l+3)\Phi_2 \bar{V}_c - \frac{\tilde{\rho}_2}{\tilde{P}_2} \{ \omega^2 e^{-2\Phi_c} \bar{V}_c + (\Phi_2 + \alpha_c \varphi_2) \bar{W}_c \} \right], \quad (65)$$

$$\bar{W}_2 = 4\Phi_2 \bar{V}_c - (l+2)\bar{V}_2. \quad (66)$$

At the surface, the boundary condition is the vanishing of the Lagrangian perturbation of the pressure $\Delta\tilde{P} = 0$. The Lagrangian perturbation of the pressure is described by $\Delta\tilde{P} = \gamma\tilde{P}\Delta\tilde{n}/\tilde{n}$, and thus making use of Eq. (47), we get the following boundary condition at $r = R$

$$(\tilde{\rho} + \tilde{P}) \left[\omega^2 e^{-2\Phi} V + e^{-\Lambda} \left(\frac{d\Phi}{dr} + \alpha\Psi \right) \frac{W}{r^2} \right] = 0. \quad (67)$$

For this new function the system of perturbation equations becomes

$$\frac{d\bar{W}}{dr} = \frac{d\tilde{\rho}}{d\tilde{P}} \left[\omega^2 e^{\Lambda-2\Phi} r \bar{V} + \left(\frac{d\Phi}{dr} + \alpha\Psi \right) \bar{W} \right] - \frac{l(l+1)}{r} e^{\Lambda} \bar{V} - \left(\frac{l+1}{r} + 3\alpha\Psi \right) \bar{W}, \quad (68)$$

$$\frac{d\bar{V}}{dr} = \left(2\frac{d\Phi}{dr} - \frac{l}{r} \right) \bar{V} - e^{\Lambda} \frac{\bar{W}}{r}, \quad (69)$$

with following boundary conditions at the center and the surface

$$\bar{W} = -lV \quad (\text{at } r = 0), \quad (70)$$

$$\omega^2 e^{-2\Phi} r \bar{V} + e^{-\Lambda} \left(\frac{d\Phi}{dr} + \alpha\Psi \right) \bar{W} = 0 \quad (\text{at } r = R). \quad (71)$$

Equations (68), (69), (70), and (71) form a well posed eigenvalue problem for the real eigenfrequency ω^2 . Using shooting method we can get in a quite simple way the eigenvalues, i.e., the characteristic frequencies of the oscillations. It should be noticed that the above system of perturbation equations and boundary conditions has been derived for an arbitrary form of the scalar function $\alpha(\varphi)$; later in the numerical calculations we will fix its functional form to the one of equation $\alpha(\varphi)$.

A. The Oscillation Spectra

The spectrum of an oscillating neutron star is directly related to its parameters, mass, radius, and EOS [28]. As we have seen in Sec. II, the presence of the scalar field on the background star is influencing both the mass and the

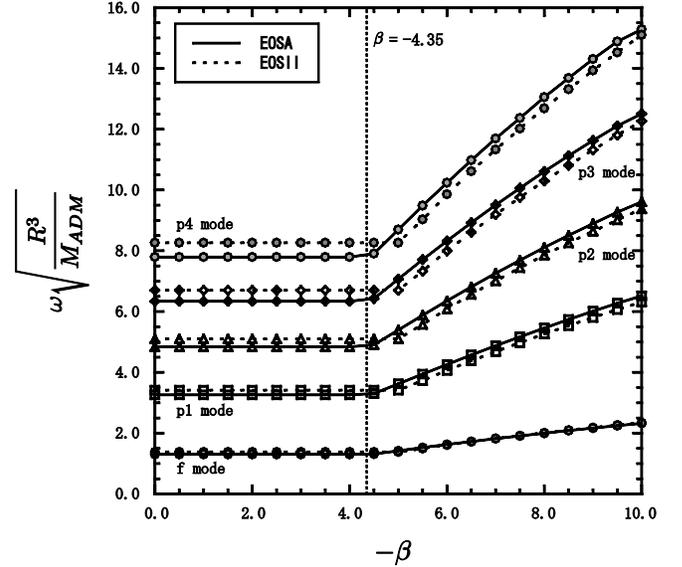


FIG. 3. The normalized eigenvalues ω for the first few modes (f , p_1 , p_2 , p_3 , and p_4) are shown as functions the parameter β for the equations of state EOS A (solid line) and EOS II (dashed line). The effect of the “spontaneous scalarization” is more pronounced for the higher modes. The asymptotic values of the scalar field and of ADM mass are fixed to the values $\varphi_0 = 0.0$ and $M_{\text{ADM}} = 1.4M_{\odot}$.

radius, while in the way that it enters in the equilibrium equations it has a role of an extra pressure term, i.e., it seems to alter the actual EOS. As before, we will restrict our study only two equations of state, i.e., EOS A and EOS II, and we will use models from those described in Sec. IIA.

First, we will examine the effect of the scalar factor β on the frequency and especially whether the “spontaneous scalarization” can be traced in the spectrum. A discontinuous change in a system, as one varies its parameters, signals a catastrophic behavior and the so called “spontaneous scalarization” [10] is related to it. Harada [15] showed that in the scalar-tensor theory the value $\beta = -4.35$ is the critical one for the “spontaneous scalarization”. DeDeo and Psaltis [25] verified that the effects of the scalar field on the line redshifts of neutron stars become pronounced for values $\beta \lesssim -4.35$. We show here that the spontaneous scalarization is also present in the oscillation spectra of neutron stars. In order to study this effect, we fixed the asymptotic value of the scalar field to $\varphi_0 = 0.0$ and we constructed a sequence of stellar models with $M = 1.4M_{\odot}$ by varying β . In Fig. 3, the effect of varying β on the frequencies of the fluid becomes immediately apparent. For values $\beta \lesssim -4.35$ the frequency gets a sharp change and increases linearly with decreasing β signaling the “spontaneous scalarization.” The effect becomes more pronounced for the higher p -modes as $\partial\omega/\partial(-\beta)$ increases with the order of the mode showing the dramatic impact of the “spontaneous

scalarization” on the spectrum. By studying the first 30 modes for each equation of state we have actually found that

$$\frac{\partial \omega_n}{\partial(-\beta)} \approx \frac{n}{4} \quad \text{for } \beta \lesssim -4.35, \quad (72)$$

where ω_n is the frequency of the n th mode. This relation seems to be independent of the equation of state and suggests that a possible tracing (via electromagnetic or gravitational observations) of the higher p -modes will signal the existence of a scalar field even for small deviations from general relativity.

The idea of a gravitational wave asteroseismology [28,29] was based on empirical relations that can be drawn for the relation of the stellar parameters to the eigenfrequencies of an oscillating neutron star. These empirical relations were derived by taking into account data for a dozen or more EOS, and it was shown that through these relations, one can extract the stellar parameters by analyzing the gravitational wave signal of an oscillating neutron star. The easiest relation to be understood on intuitive physical grounds is the one between the fundamental oscillation mode, the f -mode, and the average density. This relation emerges naturally by combining the time that a perturbation needs to propagate across the star and the sound speed; this boils down to a linear relation between the period of oscillation and the average density or the star or better $\omega^2 \sim M/R^3$. This was the reason that we have normalized the frequencies in Fig. 3 with the average density. According to [28], the empirical relation between the f -mode frequency and the average density of typical neutron stars is

$$f_{f\text{-mode}}(\text{kHz}) \approx 0.78 + 1.63 \left(\frac{M}{1.4M_\odot} \right)^{1/2} \left(\frac{R}{10\text{km}} \right)^{-3/2}. \quad (73)$$

Almost all EOS follow this empirical relation (including the two EOS used in this article). This observation suggests a unique way in estimating the average density of a star via its f -mode frequency, and it can be a very good observational test for the neutron stars in scalar-tensor theories. In Fig. 4, we draw the frequency as function of the averaged density. The actual relations of the frequency of the f -mode as functions of the average density for general relativistic neutron stars are shown as thick solid lines in both panels of Fig. 4. Both solid lines correspond to normal neutron stars with $\beta = 0$ and follow the empirical relation (73) with quite small error. The introduction of a scalar field even in moderate central densities alters completely the behavior of the f -mode for both EOS. The frequencies grow considerably faster as functions of the average density, for $\beta \lesssim -4.35$, and the three examples that we have chosen ($\beta = -6.0, -7.0$ and -8.0) show exactly this behavior. The change is quite dramatic even for typical neutron stars with average

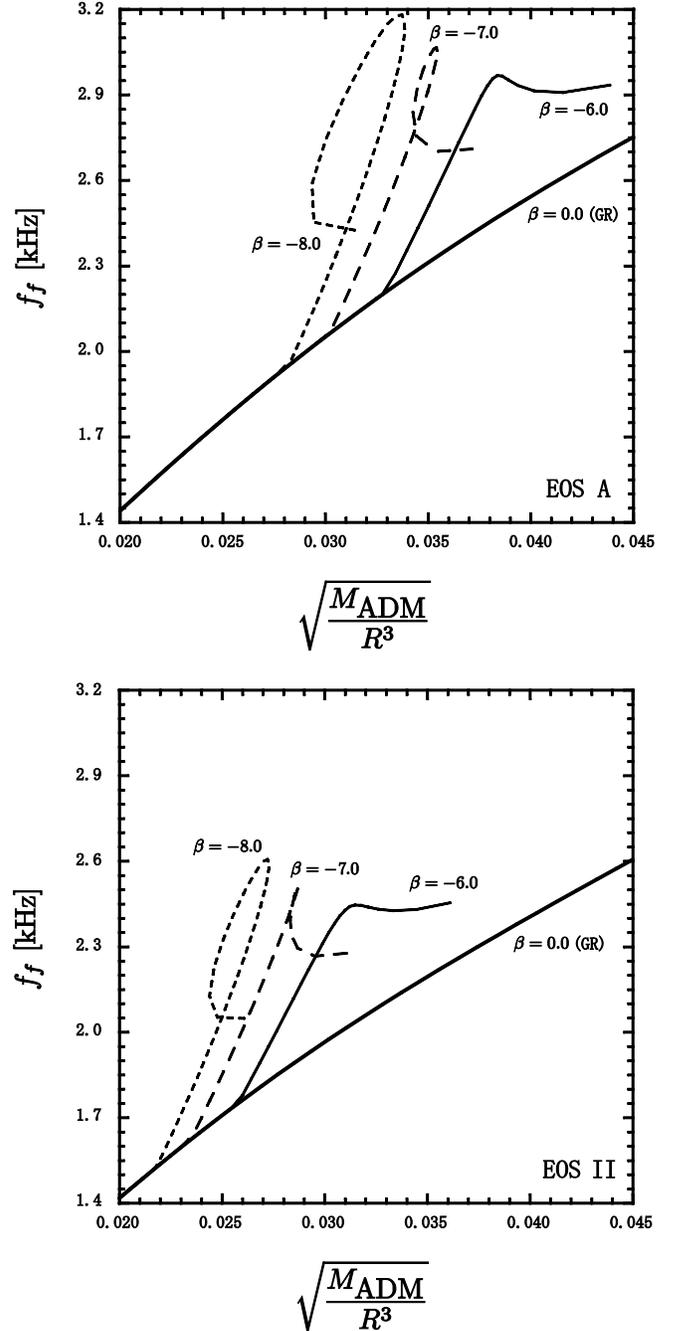


FIG. 4. The frequency of the f -mode as function of the averaged density $(M_{\text{ADM}}/R^3)^{1/2}$ of the star (notice that $f_f = \omega_f/2\pi$). The thick solid line corresponds to the values of the mode for $\beta = 0.0$ (GR) while we also show the effect of the scalar field for three values of the scalar parameter β ($-8.0, -7.0$, and -6.0). Here we have assumed that $\varphi_0 = 0.0$. The left panel corresponds to EOS A and the right panel to EOS II.

density. Depending on the value of the parameter β , they become 30-50% larger than those of a general relativistic neutron star. This can be an observable effect, since the detection of frequencies which are higher than

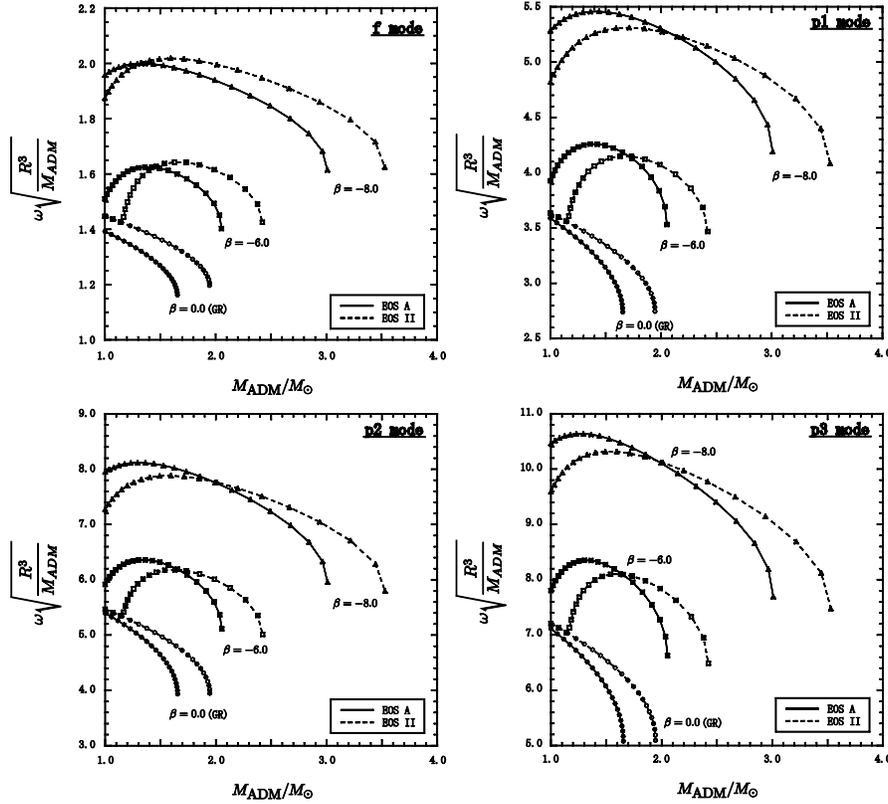


FIG. 5. The normalized frequencies of the first four fluid modes f (upper-left panel), p_1 (upper-right panel), p_2 (lower-left panel), and p_3 (lower right panel) are plotted as functions of the ADM mass. The results of the two EOS are shown for three values of the scalar parameter β (0, -6 and -8). The asymptotic value of the scalar field is $\varphi_0 = 0$.

those expected from a typical neutron star would signal in a unique way the presence of a scalar field.

A careful study of Fig. 4 suggests that the observed higher frequencies predicted by scalar-tensor gravity might be attributed to a denser neutron star. This possibility cannot be ruled out if only one mode is observed, but as one can easily find out from Figs. 3 and 5, the effect of the scalar field on the higher fluid modes, e.g., p_1 , p_2 , etc., are more dramatic, and a synchronous observation of a few modes will not only probe the presence of a scalar field but it might provide a direct estimation of its strength. Concluding the discussion related to Fig. 4, we should admit that there is no apparent explanation why the f -mode frequencies after reaching a maximum either

do not increase as the density increases ($\beta = -6$) or even move towards lower frequencies ($\beta = -7.0$ and -8.0).

In the two preceding Figs. 3 and 4, we have shown the effect of the varying scalar factor β on the frequency of the modes. In Fig. 3, the ADM mass and the asymptotic value of φ_0 were fixed, while in Fig. 4, we have studied only one mode (the f -mode) for fixed φ_0 . The next figures show the effect of the scalar field on the frequencies of the f , p_1 , p_2 , and p_3 modes for varying ADM mass or/and the asymptotic value of φ_0 . In Fig. 5, we plot the value of the normalized frequency of the mode as function of the ADM mass. One can easily observe that the differences due to the presence of the scalar field are really impressive. The models have been chosen to span a range of

TABLE I. The stellar parameters for models with maximum ADM mass (EOS A).

β	φ_0	M_{ADM}/M_\odot	$\bar{\rho}_c$ (g/cm 3)	R (km)	φ_c	M_{ADM}/R	z
0.00	—	1.655	3.742×10^{15}	8.56	—	0.285	0.527
-6.00	0.00	2.054	3.048×10^{15}	11.64	0.2676	0.261	0.445
-6.00	0.03	2.462	3.199×10^{15}	13.67	0.3269	0.266	0.462
-8.00	0.00	3.010	3.294×10^{15}	16.49	0.3338	0.2695	0.473
-8.00	0.03	3.792	3.317×10^{15}	20.61	0.3755	0.2717	0.480

TABLE II. The stellar parameters for models with maximum ADM mass (EOS II).

β	φ_0	M_{ADM}/M_\odot	$\bar{\rho}_c$ (g/cm 3)	R (km)	φ_c	M_{ADM}/R	z
0.00	—	1.946	2.543×10^{15}	10.90	—	0.264	0.454
-6.00	0.00	2.423	2.282×10^{15}	14.00	0.2724	0.256	0.430
-6.00	0.03	2.896	2.423×10^{15}	16.32	0.3307	0.262	0.450
-8.00	0.00	3.529	2.523×10^{15}	19.54	0.3366	0.267	0.464
-8.00	0.03	4.442	2.543×10^{15}	24.42	0.3792	0.269	0.470

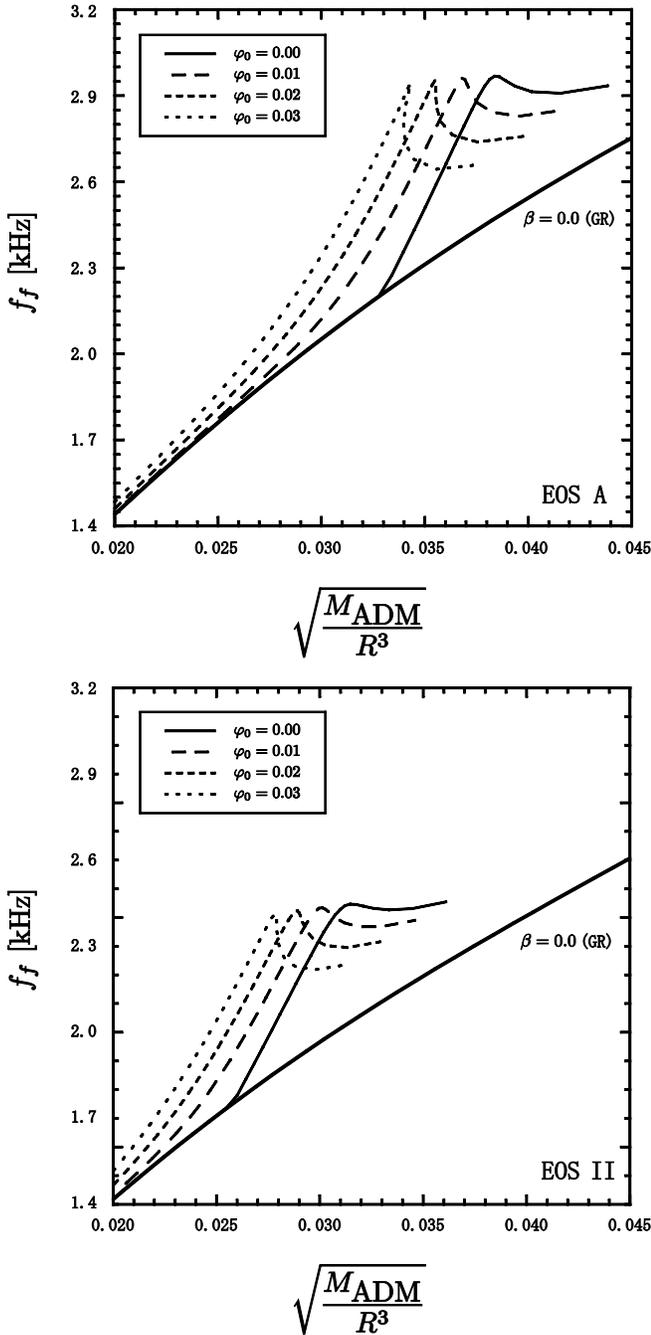


FIG. 6. The frequency of the f -mode as function of the averaged density $(M_{\text{ADM}}/R^3)^{1/2}$ of the star (notice that $f_f = \omega_f/2\pi$). The thick solid line corresponds to the values of the mode for $\beta = 0.0$ (GR); the thinner continuous and dashed lines show the effect of the asymptotic value of the scalar field φ_0 for a fixed value of the scalar parameter $\beta = -6.0$. The left panel corresponds to EOS A and the right panel to EOS II.

masses from $1M_\odot$ up to the maximum allowed mass from each theory. The normalized frequencies in these diagrams show that the imprint of the scalar field is not only apparent, but it can influence the spectrum in a

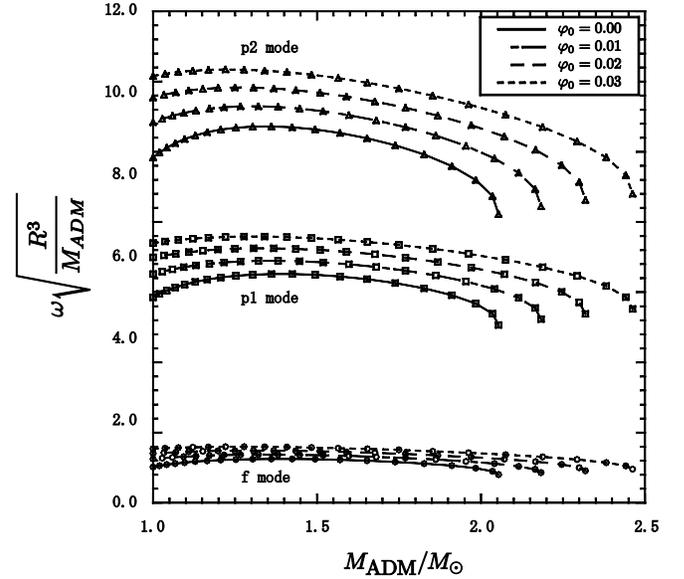


FIG. 7. The dependence of eigenfrequencies on the value of φ_0 , where it is fixed $\beta = -6$ for EOS A. The range of ADM mass is from $1.0M_\odot$ to maximum ADM mass for each φ_0 . The open circles, squares, and triangles represent the f , p_1 and p_2 modes, respectively.

dramatic way. For example, for all four oscillation modes (f , p_1 , p_2 , and p_3) the normalized frequency of the maximum mass model in GR is *half* of the corresponding frequency to an equal mass model with $\beta = -8$ or about 40% smaller for $\beta = -7$. A very interesting observation is that in contrast to the redshift of the atomic lines[25] which for the maximum mass models is typically smaller than 10% (see Tables I and II) the difference in the frequencies between the models with $\beta = 0(\text{GR})$ and $\beta = -8$ is larger than 50%. This figure suggests that a possible observation of more than one mode can clearly signal the existence of the scalar field.

In all previous discussion about the effect of the scalar field on the eigenmode frequencies of the star we have shown results for stellar models for which the asymptotic value of the scalar field was assumed zero. It is still an important question whether the asymptotic value of φ can be traced via the neutron star asteroseismology. The results of a varying φ_0 are shown in Figs. 6 and 7. Figure 6 is similar to Fig. 4 but here we have fixed the scalar factor to $\beta = -6$ and we vary φ_0 . In Fig. 7, we show the dependence of f , p_1 , and p_2 modes for varying φ_0 for the EOS A; the results for EOS II are similar. The last two figures show that the imprint of an asymptotically nonvanishing scalar field can be observed, while it becomes more pronounced for the higher modes. Varying φ_0 from 0 to 0.03, we observe frequency variations larger than 10-20%; variations of this order can provide additional constraints on the asymptotic value of the scalar field.

IV. CONCLUSIONS

In this paper we have discussed the effect of the scalar-tensor theories on the oscillation spectra of neutron stars. Scalar-tensor theories of gravity are generalizations of general relativity and provide a natural connection to superstring theories [8,40], as well as to inflation [41]. The presence of a scalar field in the strong-field regime has already been constrained by weak-field experiments and here we provide an additional way of testing/constraining their existence via the gravitational wave asteroseismology.

We show that the oscillation frequencies of neutron stars carry very “clean” imprints of the presence of the scalar field. An observation of the neutron star oscillation spectrum via gravitational waves or via electromagnetic signals emanating from or around the surface of a neutron star will not only probe the existence of the scalar field, but might also provide a measurement of its asymptotic value.

In this study we have used the Cowling approximation which, although a restricted way of studying stellar oscillation, has proven to be a very accurate and useful tool in asteroseismology. Still, more detailed study is needed

for proper modeling of the effect. The inclusion of metric and scalar field perturbations will result to additional information in the gravitational wave spectrum. The combination of this extra information should provide more accurate constraints on the existence of the scalar field.

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