Transient accelerated expansion and double quintessence

David Blais

Laboratoire de Mathématiques et Physique Théorique, UMR 6083 CNRS, Université de Tours, Parc de Grandmont,
37200 Tours, France
and Laboratoire de Physique Mathématique et Théorique, UMR 5825 CNRS, Université de Montpellier II,
34095 Montpellier, France

David Polarski

Laboratoire de Physique Mathématique et Théorique, UMR 5825 CNRS, Université de Montpellier II, 34095 Montpellier, France

We consider Double Quintessence models for which the Dark Energy sector consists of two coupled scalar fields. We study, in particular, the possibility to have a transient acceleration in these models. In both Double Quintessence models studied here, it is shown that, if acceleration occurs, it is necessarily transient. We consider also the possibility to have transient acceleration in two one-field models, the Albrecht-Skordis model and the pure exponential. Using separate conservative constraints (marginalizing over the other parameters) on the effective equation of state w_{eff}, the relative density of the Dark Energy $\Omega_{Q,0}$ and the present age of the Universe, we construct scenarios with a transient acceleration that has already ended at the present time, and even with no acceleration at all, but a less conservative analysis using the cosmic microwave background data rules out the last possibility. The scenario with a transient acceleration ended by today can be implemented for the range of cosmological parameters $\Omega_{m,0} \gtrsim 0.35$ and $h \lesssim 0.68$.

I. INTRODUCTION

The release of type Ia Supernovae data independently by two groups, the Supernovae Cosmology Project and the High-z Survey Project [1], confirmed in more recent work [2], indicating that our Universe might be presently accelerating, has profound implications on the current paradigm in cosmology. It was found that, assuming flatness, the best-fit Universe with a cosmological constant Λ is given by the set of cosmological relative densities $(\Omega_{\Lambda,0}, \Omega_{m,0}) = (0.72, 0.28)$. Here, m stands for usual pressureless matter including (cold) dark matter and baryonic matter. These data are also in surprising agreement with the location of the first acoustic (Doppler) peak of the cosmic microwave background (CMB) temperature anisotropy multipoles. If these data are confirmed in the future, they imply a radical departure from usual textbook Friedman-Lemaître-Robertson-Walker (FLRW) cosmology [3]. Indeed, a perfect isotropic fluid cannot lead to accelerated expansion unless it has a sufficiently negative pressure while the data seem to imply that such a kind of smoothly distributed matter, called Dark Energy, constitutes about two-thirds of the whole matter budget of our Universe.

Obviously, a pure cosmological constant Λ could be responsible for this acceleration. However, its amplitude has to be exceedingly small, about 123 orders of magnitude too small in order to be explained in a "natural" way. While this possibility is actually in good agreement with observations and attractive as it seems to make many pieces of our present understanding of structure formation fit into a consistent picture, in view of the above-mentioned theoretical problems research has focused on other, more elaborate, Dark Energy candidates that would mimic a pure cosmological constant.

A Λ term is equivalent to a perfect isotropic fluid with constant equation of state $p_{\Lambda} = -\rho_{\Lambda}$, equivalently $w_{\Lambda} = -1$. Provided enough of the total energy density is stored in this component, the expansion will be accelerated. A first generalization could be some phenomenological scaling Dark Energy with constant equation of state and $w < -\frac{1}{3}$. Some interesting insight can be gained by considering most of the Universe energy stored in such a component (see, e.g., [4]).

Another more elaborate alternative to a pure cosmological constant Λ is some effective, slowly varying, cosmological constant term that will start driving the Universe expansion at low redshifts. The prominent candidate in this respect is some minimally coupled scalar field Φ, often termed quintessence, slowly rolling down its potential so that it has a negative pressure, $p_{\Phi} = w_{\Phi}\rho_{\Phi}$, $w_{\Phi} < -\frac{1}{3}$ [5–8]. Later on tracking solutions were introduced [9], a substantial improvement with respect to the initial conditions problem, however without solving the cosmic coincidence problem. Actually, this is precisely the mechanism which drives the inflationary stage in the early Universe and here, too, one can consider several potentials and investigate how well they fit the observational data. If a scenario of this kind is the correct one, then it is possible in principle to reconstruct (the relevant part of) its potential $V(\Phi)$ and the corresponding equation of state characterized by w_{Φ}, using luminosity-distance measurements in function of redshift z. This

procedure can be extended to more elaborate models of gravity like so-called generalized, or extended, quintessence models in the framework of scalar-tensor theories of gravity [10,11]. Note that the latter case can represent a possible physical realization of so-called phantom energy, with $w < -1$ [10] and it has been intensively investigated (see, e.g., [12]). One can also consider other modifications to the theory of gravity or to the coupling of scalar fields (see, e.g., [13]).

A Chaplygin gas is another example of more exotic Dark Energy candidates that was proposed; in the course of the Universe expansion, this gas undergoes a transition from a dustlike to a cosmological constantlike equation of state [14].

Actually, as the Dark Energy sector is still unknown, it is interesting to explore all possible models that can pass successfully the observational constraints and we want to investigate Double Quintessence models for which this sector consists of two coupled scalar fields. Two scalar fields models give rise to exchange of kinetic energy between the two fields and produce a nontrivial time evolution of the Dark Energy equation of state. Some models of two-fields quintessence have been introduced and studied in the literature [15–18]. We show with simple two-fields models how the introduction of an auxiliary field can bring accelerated expansion of our Universe to an end. Though our models have natural parameters, the cosmological coincidence however is not solved here. We note that many-fields inflationary models can have signatures [19] which distinguish them from single-field inflationary models, in the first place the possibility to have a characteristic scale in the primordial fluctuations spectrum (see, e.g., [20]). In this respect, the situation is here more contrasted and we will return to this point in the conclusion.

The existence of an event horizon in the case of a de Sitter phase is an obstacle to the implementation of string theory because the S-matrix formulation is no longer possible, hence eternal acceleration, leading to a de Sitter space in the asymptotic future is problematic [21]. Therefore, transient acceleration of our Universe is certainly a welcome feature in this respect. Even more, we will show that we can have, in a fairly natural way, a transient acceleration which has already ended at the present time and is still in accordance with the observations. We explore the window in parameter space for which this intriguing scenario is realized. It is even possible, though this is a marginal possibility, to construct scenarios where there is no acceleration at all, neither at the present time nor in the past of our Universe. Of course, agreement with the observational data still requires that the Dark Energy sector dominate for some time and has an effective equation of state satisfying $w < 0$. We will see that in our models the quantity w, which will be called eos (equation of state)

parameter, is not constant and is actually strongly varying on the redshift interval $0 < z < 2$ relevant for (past) luminosity-distance measurements. Actually, it was already found earlier that luminosity-distance measurements are rather insensitive to large variations of the eos parameter at low redshifts [4,22]. This is why, very interestingly, in our Double Quintessence models, it is possible to construct scenarios where the standard interpretation of a presently accelerating Universe can be challenged.

In Sec. II, we give the basic equations of our system and introduce the relevant quantities and notations. In Sec. III, we list the observational constraints on our models and the associated quantities. In Sec. IV, our models are presented and investigated numerically in Sec. V, and regions in parameter space leading to viable models are explored. We will take $8\pi G = M_p^{-2} = 1$ (where M_p is the reduced Planck mass) and $\hbar = c = 1$ in the following.

II. COSMOLOGICAL EVOLUTION

Let us introduce now the background evolution equations of our system. The equations of motion for a spatially flat FLRW cosmology with two coupled homogeneous scalar fields Φ and Ψ, and Hubble parameter H, are conveniently written in the following way:

$$\dot{\rho}_b = -3H\gamma_b\rho_b \tag{1}$$

$$\ddot{\Phi} = -3H\dot{\Phi} - \partial_\Phi V \tag{2}$$

$$\ddot{\Psi} = -3H\dot{\Psi} - \partial_\Psi V \tag{3}$$

$$\dot{H} = -4\pi G(\gamma_m\rho_m + \gamma_r\rho_r + \gamma_Q\rho_Q), \tag{4}$$

subject to the constraint equation:

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_Q) - \frac{k}{a^2}. \tag{5}$$

Here a dot denotes a derivative with respect to the cosmic time t, the subscript b refers to the dominant background quantity, either dust (m) or radiation (r), while Q refers to the Dark Energy sector, here the two quintessence scalar fields. We have further

$$p_b = (\gamma_b - 1)\rho_b, \tag{6}$$

with $\gamma_m = 1$ and $\gamma_r = \frac{4}{3}$ where, for any component i, we have introduced for convenience the quantity

$$\gamma_i \equiv 1 + w_i. \tag{7}$$

Finally the quintessence fields with potential V have the following energy density and pressure:

$$\rho_Q = \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}\dot{\Psi}^2 + V(\Phi, \Psi) \tag{8}$$

$$p_Q = \frac{1}{2}\dot{\Phi}^2 + \frac{1}{2}\dot{\Psi}^2 - V(\Phi, \Psi) \qquad (9)$$

with $p_Q = (\gamma_Q - 1)\rho_Q$. It is convenient to define the following new variables:

$$X_\Phi = \sqrt{\frac{8\pi G}{3H^2}}\frac{\dot{\Phi}}{\sqrt{2}}, \qquad X_\Psi = \sqrt{\frac{8\pi G}{3H^2}}\frac{\dot{\Psi}}{\sqrt{2}},$$
$$X_V = \sqrt{\frac{8\pi G}{3H^2}}\sqrt{V}. \qquad (10)$$

The above equations are then written in the following way :

$$\Phi' = (8\pi G)^{-(1/2)}\sqrt{6}X_\Phi \qquad (11)$$

$$\Psi' = (8\pi G)^{-(1/2)}\sqrt{6}X_\Psi \qquad (12)$$

$$X'_\Phi = -3X_\Phi - \sqrt{\frac{3}{2}}(8\pi G)^{-(1/2)}X_V^2\partial_\Phi \ln V + \frac{3}{2}X_\Phi F \quad (13)$$

$$X'_\Psi = -3X_\Psi - \sqrt{\frac{3}{2}}(8\pi G)^{-(1/2)}X_V^2\partial_\Psi \ln V + \frac{3}{2}X_\Psi F \quad (14)$$

$$X'_V = \sqrt{\frac{3}{2}}(8\pi G)^{-(1/2)}X_V(X_\Phi\partial_\Phi \ln V + X_\Psi\partial_\Psi \ln V)$$
$$+ \frac{3}{2}X_V F \qquad (15)$$

with

$$F(X_\Phi, X_\Psi, X_V, N) = (1 - X_\Phi^2 - X_\Psi^2 - X_V^2)$$
$$\times\left(\frac{\gamma_m\rho_m + \gamma_r\rho_r}{\rho_m + \rho_r}\right) + 2(X_\Phi^2 + X_\Psi^2) \qquad (16)$$

where a prime denotes a derivative with respect to the quantity N, the number of e-folds with respect to the present time,

$$N \equiv \ln\frac{a}{a_0}, \qquad (17)$$

and we have also $H = \dot{N}$. It is straightforward to relate the quantity N to the redshift z:

$$1 + z = e^{-N}. \qquad (18)$$

The fields are expressed in units of the reduced Planck mass and all quantities above are dimensionless. Moreover we have the following standard evolution for matter and radiation densities

$$\rho_m = \rho_{m,0}\exp(-3N), \qquad (19)$$

$$\rho_r = \rho_{r,0}\exp(-4N), \qquad (20)$$

and more generally for constant w_X,

$$\rho_X = \rho_{X,0}\exp(-3[1 + w_X]N). \qquad (21)$$

Hereafter and in (17), the subscript 0 refers to the value of any quantity today and the subscript i refers to its value at some initial time t_i.

III. OBSERVATIONAL CONSTRAINTS

We consider now all the relevant quantities pertaining to our system which will enter the observational constraints.

The relative energy density for matter, radiation and quintessence, Ω_m, Ω_r and Ω_Q, where we have for each component labeled by the subscript i, $\Omega_i \equiv \rho_i/\rho_c$ with ρ_c the critical density, are given by

$$\Omega_m = (1 - X_\Phi^2 - X_\Psi^2 - X_V^2)\frac{\Omega_{m,0}}{\Omega_{m,0} + \Omega_{r,0}e^{-N}}, \qquad (22)$$

$$\Omega_r = (1 - X_\Phi^2 - X_\Psi^2 - X_V^2)\frac{\Omega_{r,0}}{\Omega_{r,0} + \Omega_{m,0}e^{N}}, \qquad (23)$$

$$\Omega_Q = X_\Phi^2 + X_\Psi^2 + X_V^2. \qquad (24)$$

The equation of state (eos) parameter w_Q for the Dark Energy (Double Quintessence) sector, and the effective eos parameter w_{eff} [23] read

$$w_Q \equiv \frac{p_Q}{\rho_Q} = \frac{X_\Phi^2 + X_\Psi^2 - X_V^2}{X_\Phi^2 + X_\Psi^2 + X_V^2}, \qquad (25)$$

$$w_{\text{eff}} = \frac{\int_0^{a_0} da' w_Q(a')\Omega_Q(a')}{\int_0^{a_0} da'\Omega_Q(a')}$$
$$= \frac{\int_{-\infty}^0 dN' e^{N'}(X_\Phi^2 + X_\Psi^2 - X_V^2)}{\int_{-\infty}^0 dN' e^{N'}(X_\Phi^2 + X_\Psi^2 + X_V^2)}. \qquad (26)$$

The case where Dark Energy consists of one field is straightforwardly recovered from the above equations. Finally, the deceleration parameter q, the Hubble-parameter-free luminosity distance D_L and the age of the Universe t_0 are, respectively, given in terms of N by

$$q \equiv -\frac{\ddot{a}}{aH^2} = \frac{1}{2}\sum_i \Omega_i(1 + 3w_i), \qquad (27)$$

$$= \frac{1}{2}(1 + \Omega_r + 3w_Q\Omega_Q), \qquad (28)$$

$$D_L \equiv H_0 d_L = (1 + z)\int_0^z dz'\frac{H_0}{H(z')}$$
$$= e^{-N}\int_N^0 dN' e^{-N'}\frac{H_0}{H}, \qquad (29)$$

$$H_0 t_0 = \int_0^{+\infty} dz\frac{H_0}{(1 + z)H(z)} = \int_{-\infty}^0 dN\frac{H_0}{H}, \qquad (30)$$

with

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{m,0}e^{-3N} + \Omega_{r,0}e^{-4N}}{1 - X_\Phi^2 - X_\Psi^2 - X_V^2}. \quad (31)$$

Let us consider two universes with the same "history," in the sense that they share the quantity $h(z) \equiv [H(z)]/H_0$ while they differ in the present value H_0. As can be seen immediately from (29), the quantity D_L depends only on $h(z)$ and will therefore be the same for both universes. It will be interesting to compare luminosity distances $d_L(z)$ for such universes; we have, in particular, when comparing two universes with same $h(z)$ but different Hubble constant H_0 and $H_{0,\Lambda}$

$$d_L(z; H_0) = \frac{H_{0,\Lambda}}{H_0}d_L(z; H_{0,\Lambda}) = \frac{h_\Lambda}{h}d_L(z; H_{0,\Lambda})$$

$$\equiv e_\Lambda d_L(z; H_{0,\Lambda}), \quad (32)$$

where $h \equiv H_0/(100\ \text{km/s/Mpc})$, respectively, $h_\Lambda \equiv H_{0,\Lambda}/(100\ \text{km/s/Mpc})$. We have further for any two universes (dropping the argument z)

$$\frac{d_L(h)}{d_{L,\Lambda}}(h_\Lambda) = e_\Lambda \frac{D_L}{D_{L,\Lambda}}, \quad (33)$$

which shows the relative variation of $d_L(z; H_0)$ when varying H_0 with respect to some *fixed* value $H_{0,\Lambda}$. We will make use of (33) in Fig. 1.

The traditional sign convention for the "deceleration" parameter q gives a positive q for a decelerating universe. A cosmological constant Λ corresponds to the particular case $w_\Lambda = -1$ and $\Omega_\Lambda = \Lambda/(3H^2)$. For flat universes, the relative energy densities Ω_i satisfy $\sum_i \Omega_i = \Omega_m + \Omega_r + \Omega_Q = 1$ at all times. In particular, at late times, the energy density of radiation can be neglected and (27) and (28) give

$$q \simeq \tfrac{1}{2}\Omega_m + \tfrac{1}{2}\Omega_Q(1 + 3w_Q) \simeq \tfrac{1}{2}(1 + 3w_Q\Omega_Q). \quad (34)$$

We see, in particular, from (27) that a decelerated expansion at late times requires

$$w_Q > -\tfrac{1}{3}\Omega_Q^{-1}. \quad (35)$$

In this work, we will consider universes where the quantity q (re)changes sign, from negative to positive, i.e., accelerating universes resuming a decelerated expansion, in some cases even before the present time. We will call henceforth t_{end} the time at which the transient accelerated stage ends:

$$q(t \geq t_{\text{end}}) \geq 0. \quad (36)$$

For our system, the eos parameter w_Q is constrained between $-1 < w_Q < 1$ while the energy density of the Dark Energy sector (like the energy density of any component) is bounded according to $0 < \Omega_Q < 1$. The pa-
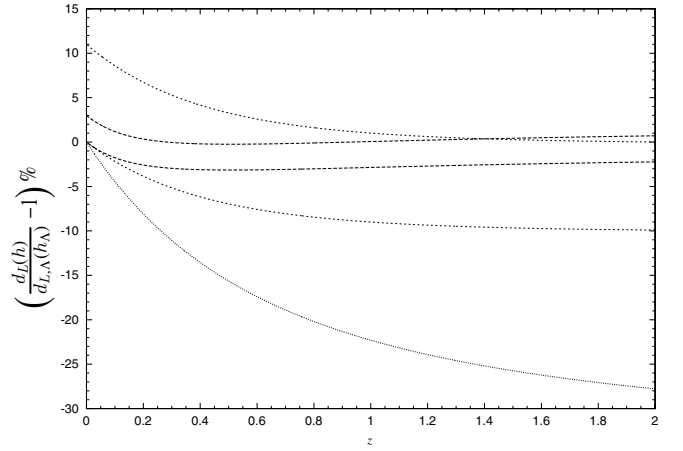

FIG. 1. The relative error for the luminosity-distance $d_L(z)$ with respect to the Λ model with $\Omega_{\Lambda,0} = 0.66$ is shown. One has $[d_L(h)]/[d_{L,\Lambda}(h_\Lambda)] - 1 = e_\Lambda(D_L/D_{L,\Lambda}) - 1$. The following models are plotted: the Einstein-de Sitter Universe with $e_\Lambda = 1$ (dotted line), the potential Eq. (80) for $P_0 = 0.164$ of Table II and accelerated expansion ended by today, with $e_\Lambda = 1$ (lower long-dashed line) and $e_\Lambda = 1.03$ (upper long-dashed line), the pure exponential potential Eq. (49) for parameter values $\lambda = 1.84$ and $X_{V,i}^2 = 5.8 \times 10^{-113}$ and *no acceleration at all*, with $e_\Lambda = 1$ (lower short-dashed line) and $e_\Lambda = 1.11$ (upper short-dashed line). As seen from the figure, by varying e_Λ, and $e_\Lambda > 1$, we improve agreement with the SNIa data, except for small $z \lesssim 0.3$ where the relative departure can be large. Clearly, this is impossible with the Einstein-de Sitter model.

rameter q has to be negative at the present time if the Universe is accelerating today, viz. $q_0 < 0$.

The nucleosynthesis bound is the most stringent one [24]

$$\Omega_Q(N \sim -23) \lesssim 0.045 \qquad \text{at } 2\sigma, \quad (37)$$

while we have at last scattering [24]

$$\Omega_Q(N \sim -8) \lesssim 0.39 \qquad \text{at } 2\sigma. \quad (38)$$

We adopt the following range $0.6 \lesssim h \lesssim 0.8$ and we consider universes satisfying the conservative bounds [2,25–30]

$$0.2 \lesssim \Omega_{m,0} \lesssim 0.45, \quad (39)$$

$$\Omega_{r,0}h^2 = 4.3069 \times 10^{-5}, \quad (40)$$

as well as

$$0.55 \lesssim \Omega_{Q,0} \lesssim 0.8. \quad (41)$$

In [2,25,26], a constant eos parameter w_Q was considered. The quantity w_{eff} was introduced in [23] in order to account for a varying equation of state of the Dark Energy sector. Note that for constant eos parameter one has $w_{\text{eff}} = w_Q$ and it was shown in [25] that constraints

on w_Q can be replaced by constraints on w_{eff} so we consider the observational constraint :

$$w_{\mathrm{eff}} \lesssim -0.70. \qquad (42)$$

The conservative bound (42) is required by the CMB as well as the SNIa and the large scale structure data [25–29].

Combining Eqs. (35) and (41), *decelerated* expansion today requires the necessary condition

$$w_{Q,0} \gtrsim -0.606. \qquad (43)$$

We note immediately that conditions (42) and (43) are incompatible for a constant equation of state, i.e., $w_{\mathrm{eff}} = w_Q = \mathrm{const}$. Both conditions (42) and (43) can be met today when the equation of state varies strongly at low redshifts as we will see with concrete models studied in this work (see Figs. 2 and 3). The lower bound (43) is always necessary but only sufficient when $\Omega_{Q,0} = 0.55$. When $\Omega_{Q,0} = 0.8$, the condition sufficient for decelerated expansion is tighter, namely $w_{Q,0} \geq -\frac{5}{12} \approx -0.416$. Clearly, the lower $\Omega_{Q,0}$ the easier it is to implement decelerated expansion today. Finally, we impose that the age of the Universe satisfies [31]

$$t_0 \gtrsim 13 \text{ Gyr}, \qquad (44)$$

which translates into a constraint on the quantity H_0, or equivalently on h, as can be seen from (30). One has
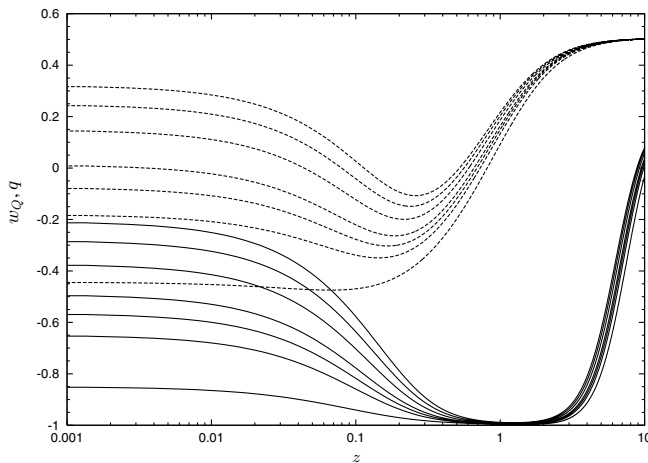

FIG. 2. The evolution of the quantities w_Q (solid line) and q (dashed line) are shown versus redshift for the potential Eq. (80) with the fixed parameters $\lambda = 10$, $\Phi_c = 23.8$ while P_0 takes the values, from bottom to top, $0.160, 0.162, 0.163, 0.164, 0.166, 0.168, 0.170$ ($\Phi_i = 0$ and $\Psi_i = 5$), see Table II. Note that the third model from the bottom, $P_0 = 0.163$, satisfies the necessary condition for decelerated expansion today; however, this is not sufficient because $\Omega_{Q,0} > 0.55$. The models with $P_0 = 0.164, 0.166, 0.168, 0.170$ produce a transient acceleration which ends before today ($q_0 > 0$).
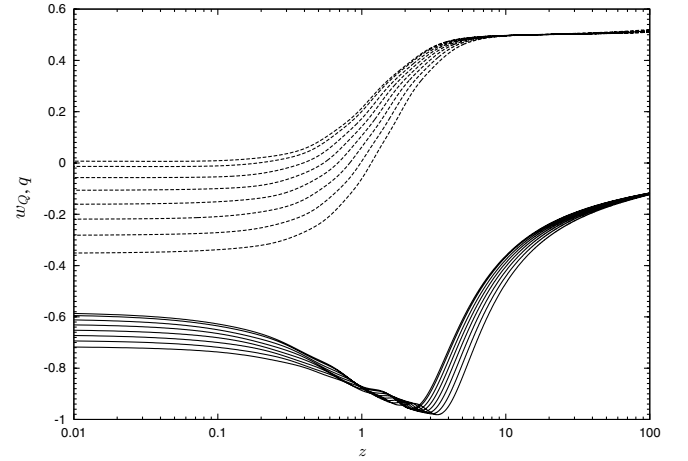

FIG. 3. The evolution of w_Q (solid line) and of the deceleration parameter q (dashed line) is shown for the potential Eq. (81) with the same parameters as in Fig. 10, and with growing values of λ from bottom to top. For the model $\lambda = 1.85$, there is no accelerated expansion at all. Note that w_Q exhibits oscillations in the range $z \sim 1$–3 because the auxiliary field Ψ reaches, and oscillates around, 0.

$$H_0^{-1} = 3.0856 \times 10^{17} h^{-1} \text{ s} = 9.7776 \, h^{-1} \text{ Gyr}, \qquad (45)$$

so that the constraint (44) can be rewritten as

$$H_0 t_0 \gtrsim 1.330 h. \qquad (46)$$

Note that, in the past, the inclusion of a cosmological constant was invoked precisely in order to reconcile an "old" universe with a high value for H_0 while today it is motivated by completely different observations. The bound (44) is conservative and corresponds to the present age of an Einstein-de Sitter universe with $h = 0.65$. In our simulations we will assume equipartition of the energy

$$\tfrac{1}{2}\dot{\Phi}_i^2 \sim \tfrac{1}{2}\dot{\Psi}_i^2 \sim V_i \qquad (47)$$

initially at the end of the primordial inflationary stage ($z_i \sim 10^{29}$), but the initial time could as well be taken at nucleosynthesis or at matter-radiation equality [32].

IV. MODELS

We will be interested in universes where the accelerated expansion is brought to an end, or even does not take place. If the Universe is accelerating indefinitely it will exhibit an event horizon such that

$$D_H(t') = \int_{t'}^{\infty} \frac{dt}{a(t)} \qquad (48)$$

is finite. In the case of a constant eos parameter w_Q and a quintessence domination ($\Omega_Q \gg \Omega_m, \Omega_r$), we have $a \propto t^{2/[3(1+w_Q)]}$ and if $w_Q < -1/3$ the integral (48) is finite.

It is well known that the problem of initial conditions for the quintessence field can be considerably alleviated by the so-called tracking behavior whereas the relative density Ω_Q follows the density Ω_b of the dominating background component, with some specific evolution of w_Q. We will consider two one-field models and two Double Quintessence models.

Let us stress first what will be relevant in the framework of either one-field, or two-field quintessence models with transient acceleration that we study here. There are several ways to obtain transient acceleration:

   (i) The tracking behavior of the scalar potential is not reached until today; namely, fields are frozen in the past with $w_Q \simeq -1$, $\Omega_Q \ll 1$, while once the tracking behavior is reached, Eq. (35) must be satisfied. In Sec. IV A, we will see that a pure exponential potential can exhibit such a behavior as studied in [33−36]. Moreover, in [37] it is shown that a FLRW universe filled with a scalar field only can undergo a stage of transient acceleration. As we show in Sec. IV D, even the quintessence domination can be avoided because of the varying coupling constant of the model studied there.

  (ii) The existence of a minimum or a local flatness in the Φ direction of the potential can produce an accelerated expansion but the minimum has to disappear in order to make this acceleration transient. In the model of Albrecht and Skordis [38] (Sec. IV B) for which a feature is introduced in a pure exponential potential, the parameters have to be fitted in order for the field to roll over the barrier. In the model IV C, the minimum is dynamical and disappears. So in these cases, the actual acceleration takes place outside the scaling regime, which was reached early on in the Universe evolution, when the field Φ approaches his minimum; see Secs. IV B and IV C.

 (iii) Still another possibility to obtain transient acceleration is to cancel the scalar potential V: This is the case in the hybrid inverse power-law potential $V(\Phi, \Psi) = \Psi^2 M^{4+n}\Phi^{-n}$ studied in [15]. On the contrary, the inverse power-law potential cannot produce a transient acceleration because $w_Q \to -1$, $\Omega_Q \to 1$ in the future [5,9]. Other models where V cancels can produce transient acceleration like the oscillatory Dark Energy model with a double exponential potential $V(\Phi) = [A\exp(\frac{1}{2}\lambda\Phi) - B\exp(-\frac{1}{2}\lambda\Phi)]^2$ [39] and also the power-law potential $V \propto \Phi^{2n}$, $n = 1, 2, \ldots$.

If $V \leq 0$, even a flat universe can undergo a transient acceleration followed by a big crunch [40,41], and an ever decelerating universe can also be considered in a closed FLRW ($k = +1$) [42]. In the following we will restrict ourselves to positive potentials $V \geq 0$ and flat space

$k = 0$. In the transient acceleration picture, it is interesting to consider whether the acceleration finishes in a time comparable to H_0^{-1}, namely, $t_{\text{end}} \gtrsim t_0$ or $t_{\text{end}} \lesssim t_0$, much larger than H_0^{-1}, i.e., $t_{\text{end}} \gg t_0$, or if the acceleration does not occur at all.

A. Pure exponential potential

We consider a scalar field Φ with an exponential potential

$$V(\Phi) = M^4 e^{-\lambda\Phi}, \qquad (49)$$

already widely investigated in the past [5−7,43] and motivated in [44]. For this potential an attractor solution exists: either a scaling solution such that

$$\Omega_Q = \frac{3\gamma_b}{\lambda^2} \quad \text{and} \quad \gamma_Q = \gamma_b \quad \text{if } \lambda^2 > 3\gamma_b, \qquad (50)$$

or else a scalar field dominated solution

$$\Omega_Q = 1 \quad \text{and} \quad \gamma_Q = \frac{\lambda^2}{3} \quad \text{if } 0 < \lambda^2 < 3\gamma_b. \qquad (51)$$

The nucleosynthesis bound $\Omega_Q(1 \text{ MeV}) \lesssim 0.045$ at the 2σ level [24] implies from (50) that $\lambda \gtrsim 9$ during the scaling regime. If we want to take advantage of the attractor property of this potential, it is impossible to have reasonable $\Omega_{Q,0}$ without violating the nucleosynthesis bound, and anyway w_Q would mimic the background component eos, $w_Q = w_m = 0$ thereby preventing a past and/or actual acceleration.

In [33−35] the authors present a way to circumvent these arguments and revive the pure exponential potential, using the attractor property in the future instead of the past of the Universe by fine-tuning the mass scale $M^2 \sim M_p H_0$ (in this section we will sometimes reput M_p for clarity). Contrary to the claim in [33], $\dot{\Phi}_i^2 \gg V_i$ is not necessary, only $V_i \sim M_p^2 H_0^2$ has to be imposed initially. Apart from λ and M, two extra values Φ_i and $\dot{\Phi}_i$ must be specified. We can always take $\Phi_i = 0$ by a redefinition of M which corresponds to a rescaling of the problem.

The Universe experiences first a kination regime such that $V \ll \dot{\Phi}^2 \propto a^{-6}$ with $w_Q \simeq 1$ followed by a regime during which $V \gg \dot{\Phi}^2$ and while $M_p^2 m_\Phi^2 \sim V \ll M_p^2 H^2$, the field Φ gets frozen with $w_Q \simeq -1$ until now where $\dot{\Phi}^2 \sim M_p^2 m_\Phi^2 \sim V \sim M_p^2 H_0^2$ allowing the attractor regime to be reached in the near future and we are left with a fixed ratio, either $\Omega_Q = 1$ and $w_Q = \lambda^2/3 - 1$ if $\lambda^2 < 3$ or else $\Omega_Q = 3/\lambda^2$ and $w_Q = 0$ if $\lambda^2 > 3$, in the presence of dust [33−35].

In contrast to the statement in [34], the value $\lambda > \sqrt{3}$ gives a viable model provided the scaling regime is not yet reached and it produces intriguing possibilities for the evolution of the Universe, namely, an acceleration which ends before today and even *no acceleration at all*.

An acceptable quintessence density $\Omega_Q > \Omega_{Q,0} \gtrsim 0.55$ implies, using (50), $\lambda \lesssim 2.3$. If $\lambda < \sqrt{2}$, acceleration is eternal because the attractor solution (in the future) is characterized by $\Omega_Q = 1$ and $w_Q < -1/3$ as seen from (51). Therefore transient acceleration implies $\sqrt{2} < \lambda \lesssim 2.3$ and an eternal quintessence density domination ($\Omega_Q > \Omega_m$). Numerical results of section V A can tighten even more the upper bound. This model has one fine-tuned parameter M, which has to satisfy $M^2 \sim M_p H_0$.

It is possible to produce analogous scenarios containing multiple scalar fields with an exponential potential [45] of the type

$$V = M^4 \sum_{i=1}^{n} e^{-\lambda_i \Phi_i}. \qquad (52)$$

In the context of assisted inflation, though each field is unable to support separately an inflationary stage, together they are able to do so.

But we are interested in a transient acceleration whether already finished or not, hence in no inflationary solutions at late times, which requires the condition

$$\sum_{i=1}^{n} \frac{1}{\lambda_i^2} < \frac{1}{2}, \qquad (53)$$

because both attractor solutions (50) and (51) remain valid with the change

$$\lambda^2 \rightarrow \left(\sum_{i=1}^{n} \frac{1}{\lambda_i^2} \right)^{-1}. \qquad (54)$$

If each $\Phi_i = 0$ initially, then M has to satisfy $M \sim \sqrt{M_p H_0}$ with some level of fine-tuning.

Models containing coupled scalar fields with an exponential potential [46] of the type

$$V = M^4 e^{-\sum_{j=1}^{m} \lambda_j \Phi_j} \qquad (55)$$

can produce the same scenarios with λ^2 replaced by $\sum_{j=1}^{m} \lambda_j^2$ in the expressions (50) and (51). So if

$$2 < \sum_{j=1}^{m} \lambda_j^2, \qquad (56)$$

transient acceleration will occur even though for each slope separately $\lambda_j < \sqrt{2}$. Again if each $\Phi_j = 0$ initially, then M has to satisfy $M \sim \sqrt{M_p H_0}$.

It is possible to generalize the two last cases [47] with the potential

$$V = M^4 \sum_{i=1}^{n} e^{-\sum_{j=1}^{m_i} \lambda_{ij} \Phi_{ij}}. \qquad (57)$$

It can be shown that, in the presence of a barotropic fluid, the two late-time attractor solutions are either a scaling solution,

$$\Omega_Q = \frac{3\gamma_b}{\lambda_r^2} \quad \text{and} \quad \gamma_Q = \gamma_b \quad \text{if } \lambda_r^2 > 3\gamma_b, \qquad (58)$$

or else a scalar field dominated solution,

$$\Omega_Q = 1 \quad \text{and} \quad \gamma_Q = \frac{\lambda_r^2}{3} \quad \text{if } 0 < \lambda_r^2 < 3\gamma_b \qquad (59)$$

with

$$\frac{1}{\lambda_r^2} \equiv \sum_{i=1}^{n} \frac{1}{\sum_{j=1}^{m_i} \lambda_{ij}^2}. \qquad (60)$$

For example if each $\lambda_{ij} = \lambda$ and $m_i = m$ then $\lambda_r = \sqrt{\frac{m}{n}}\lambda$. Thus assisted inflation tends to lower λ_r and the coupled part to increase it but, when

$$2 < \frac{1}{\sum_{i=1}^{n} \frac{1}{\sum_{j=1}^{m_i} \lambda_{ij}^2}}, \qquad (61)$$

the late-time solution produces a decelerated expansion with the fine-tuning $M \sim \sqrt{M_p H_0}$ if initially each $\Phi_{ij} = 0$.

B. Albrecht and Skordis potential

Albrecht and Skordis [38] proposed an interesting model of quintessence. As noted by [48], this model contains solutions for which there is a transient acceleration of our Universe. We would like to emphasize even more that acceleration which has already ended by today is also a possibility. The model (denoted AS in the following) has the following potential:

$$V(\Phi) = M^4 e^{-\lambda \Phi}[P_0 + (\Phi - \Phi_c)^2]. \qquad (62)$$

The potential Eq. (62) has a small minimum in order for the field to be trapped at $\Phi_\pm = \Phi_c + (1 \pm \sqrt{1 - \lambda^2 P_0})/\lambda$. If $\lambda^2 P_0 < 1$ the minimum exists and the field plays for a while the role of a quasicosmological constant term (Φ slows down but does not oscillate); however, if the field has enough kinetic energy it can roll over the barrier. If $\lambda^2 P_0 > 1$ there is no minimum and the potential has to be flattened sufficiently for acceleration to occur and Ω_Q can reach 0.55. In Sec. V B, an accurate interval will be given for $\lambda^2 P_0$. The only fine-tuning in this model is the value of Φ_c which expresses the cosmological coincidence problem: Φ_c is roughly the minimum of the potential and so defines the moment when acceleration begins. Once Φ_c is fixed, the beginning of the accelerated stage is given. Apart from that, all parameters (M, λ, P_0, Φ_c) take natural values.

To summarize, we have to take $\lambda \gtrsim 9$ in order to satisfy the nucleosynthesis bound, $\lambda^2 P_0 \sim 1$ in order for the acceleration to be transient and possibly ending before the present time, and finally Φ_c has to be fine-tuned so that quintessence dominates today.

C. Pseudoexponential potential

For the AS potential Eq. (62) the minimum, when it exists, is fixed. Hence, the acceleration will be eternal for most parameter values, see Fig. 4. With a straightforward generalization, allowing a simple coupling between two scalar fields, we can obtain a transient acceleration, possibly ending before today, whenever acceleration takes place. The introduction of an auxiliary field Ψ will control the presence or not of a minimum for Φ. The idea is to have initially on one hand a minimum of the potential in the Φ direction responsible for the acceleration of our Universe, and on the other hand an evolution of the Ψ field such that this minimum disappears in the course of time allowing the resumption of matter domination. We will study a two-fields potential of the form

$$V(\Phi, \Psi) = M^4 e^{-\lambda\Phi}[P_0 + f(\Psi)(\Phi - \Phi_c)^2 + g(\Psi)].$$
(63)

The AS model [38] is recovered when $f \equiv 1$ and $g \equiv 0$. For the potential Eq. (63) the minimum is now located at

$$\Phi_\pm = \Phi_c + \frac{1}{\lambda}\left(1 \pm \sqrt{1 - \lambda^2 \frac{P_0 + g(\Psi)}{f(\Psi)}}\right).$$
(64)

The function g ($g > 0$) can describe a mass term for Ψ of the form $g \propto \Psi^2$, but it is not essential for the dynamics of the model. Thus, we will take $g \equiv 0$ for simplicity.
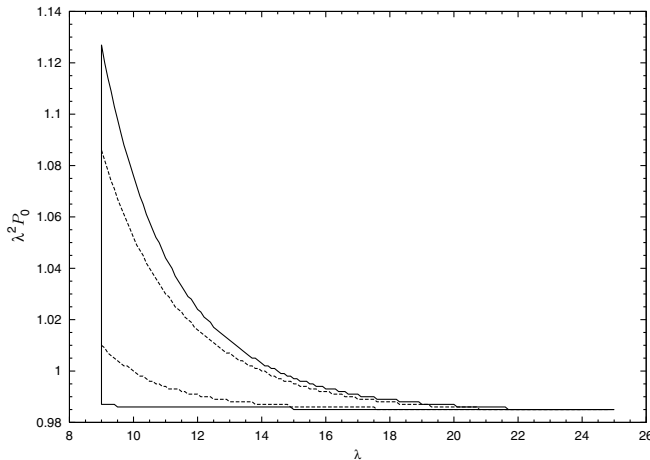

FIG. 4. The allowed window (the area between the solid curves) in the parameter space $(\lambda, \lambda^2 P_0)$ is shown for the AS potential Eq. (62) when the acceleration is transient *and* all observational constraints, Eq. (69), are met. The nucleosynthesis bound implies $\lambda \gtrsim 9$ while transient acceleration implies $\lambda \lesssim 25$. The observations constrain the remaining parameter of the model Φ_c to be in the range $237.5 \lesssim \lambda\Phi_c \lesssim 239$ for the initial condition $\Phi_i = 0$ at $N_i = -67$. Above the lower dashed line, q_0 can be negative as well as positive; above the upper dashed line $q_0 > 0$ always. For $\lambda^2 P_0$ values below the window, acceleration is eternal as Φ settles at its minimum forever.

The minimum (64) will disappear provided we have

$$f(\Psi) < \lambda^2 P_0 \equiv f(\Psi_c).$$
(65)

We note in passing that the potential can be rewritten in the form $V(\Phi, \Psi) = M^4/\lambda^2 e^{-\lambda\Phi}[f(\Psi_c) + f(\Psi) \times (\lambda\Phi - \lambda\Phi_c)^2]$. We will use for f a positive, continuous function which is monotonic in the region $\Psi > 0$ and/or $\Psi < 0$. As can be seen from the condition (65), if f is decreasing, respectively, increasing, then for Ψ_i smaller, respectively, larger, than Ψ_c acceleration is possible because the minimum (64) exists.

During the evolution of the Universe, the field Φ rolls down its potential, which is dominated by the exponential part, such that $M_p^2 m_\Phi^2 \sim M_p^2 m_\Psi^2 \sim V \sim \dot{\Phi}^2 \sim M_p^2 H^2$ with $\Omega_Q = 4/\lambda^2$, $w_Q = 1/3$ during radiation domination, while $\Omega_Q = 3/\lambda^2$, $w_Q = 0$ during matter domination until Φ approaches its minimum. As long as the field Φ is trapped at its minimum (64) and oscillates around it, the Universe undergoes an accelerated expansion with $V \gg \dot{\Phi}^2$ and therefore $w_Q \simeq -1$ until Ψ satisfies $\Psi \leq \Psi_c$, respectively $\Psi \geq \Psi_c$, if f is increasing, respectively decreasing. At that moment the minimum (64) disappears, allowing Φ to continue to roll freely towards larger values and hence the matter dominated regime is resumed. For a given set of parameters $(\lambda, P_0, \dots)$, the further the initial condition Ψ_i from Ψ_c, the longer the evolution of Ψ toward Ψ_c and so the longer the accelerated regime of our Universe. When Ψ is initially larger (smaller) than Ψ_c, provided f is increasing (decreasing) Ψ passes through Ψ_c because $\partial_\Psi V = M^4 e^{-\lambda\Phi}(\Phi - \Phi_c)^2 \frac{df}{d\Psi}$ is positive (negative), acceleration occurs which is always transient; if on the contrary $\Psi_i \leq \Psi_c$ ($\Psi_i \geq \Psi_c$) quintessence domination is not possible. The critical value Ψ_c controls the presence or not of the minimum for Φ. Contrary to [16,17] where acceleration is always permanent, or [18] where acceleration can be either transient or permanent, acceleration here is *always* transient as in [15,39]. Of course the absence of acceleration is also possible in principle for all these potentials but this possibility has to be rejected by observations.

For the numerical computations presented in Sec. V C, we will use a very simple function f without any additional parameter, namely,

$$f(\Psi) = \Psi^2,$$
(66)

for which the minimum of Φ disappears if $-\Psi_c \leq \Psi \leq \Psi_c$ with $\Psi_c \equiv \lambda\sqrt{P_0}$. Analogously to the AS model, we have to take $\lambda \gtrsim 9$ in order to satisfy the nucleosynthesis bound and, once Φ_i is fixed, Φ_c has to be fine-tuned in order for quintessence domination to occur today.

D. Pure exponential potential with a varying coupling constant

Starting from a pure exponential potential $V(\Phi) \propto e^{-\lambda\Phi}$, we now allow λ to depend on the auxiliary field Ψ and to vary in time, i.e., we make the generalization $\lambda \to \lambda_{\text{eff}}(\Psi)$, and we consider the potential

$$V(\Phi, \Psi) = M^4 e^{-\lambda_{\text{eff}}(\Psi)\Phi}. \tag{67}$$

A similar idea was used in [38] where $\lambda_{\text{eff}}(\Phi)$ depends on Φ in order to create a minimum and to produce an eternal acceleration.

We will assume that λ_{eff} has a global minimum equal to λ in $\Psi = \Psi_{\text{min}} = 0$. Whatever the precise form of λ_{eff}, $\partial_\Phi \ln V = -\lambda_{\text{eff}} < 0$ for all Ψ which implies that Φ is always growing and $\partial_\Psi \ln V = -\Phi[(d\lambda_{\text{eff}})/(d\Psi)]$ is positive, respectively, negative, if Φ and Ψ have opposite signs, respectively, the same sign. Hence if initially $\Phi_i < 0$ and $\Psi_i > 0$, respectively $\Psi_i < 0$, (we could take $\Phi_i > 0$ as well with $\lambda_{\text{eff}} \to -\lambda_{\text{eff}}$) Φ grows towards positive values while Ψ decreases, respectively, grows, until it reaches, and oscillates around, zero. While $\Phi < 0$ and $\Psi \sim 0$, $\lambda_{\text{eff}} \to \lambda$ and $V \to e^{-\lambda\Phi}$. Hence, as for the pure exponential $\lambda \lesssim 2.3$ implies sufficient quintessence domination ($\Omega_{Q,0} \gtrsim 0.55$) after the matter dominated stage. Once $\Phi > 0$, Ψ moves away from zero and the function λ_{eff} is again growing, allowing for the resumption of matter domination ($\Omega_Q \ll 1$). The current quintessence domination begins when Ψ goes to zero and consequently when λ_{eff} reaches its minimum. Thus $\partial_\Phi V$ will only depend on λ and so will the dynamics of Φ. The lower λ, the slower the evolution of Φ toward zero and hence the longer the acceleration and/or the quintessence domination regime of our Universe. Clearly the more remote Φ_i from 0, the stronger this effect.

The quantity M is solely determined by the conditions to have a realistic evolution of our Universe and turns out to be roughly equal to the energy scale today $M \sim \sqrt{M_p H_0}$. In this potential $|\lambda_{\text{eff}}\Phi| \leq -\ln M^4/3H^2$ because $X_V^2 \leq 1$. So the potential (67) allows us to use the scaling property of the exponential potential as early as the end of inflation without necessarily having kination followed by a stage where the fields are frozen. If $X_{V,i}^2 \ll 1$, then after a kination regime ($w_Q \sim 1$), we have $\rho_Q \ll \rho_b$ and the evolution of the fields is frozen since $m_\Phi^2 \sim m_\Psi^2 \sim M_p^{-2}V \ll H^2$ until $V \sim M_p^2 H^2$. For potential (67), as for the potential (63), acceleration if it takes place is necessarily transient. Moreover, the full range $0 < \lambda \lesssim 2.3$ gives viable models, either with transient acceleration or without acceleration because for this potential the domination of quintessence is transient.

We will take for numerical simulations in Sec. V D:

$$\lambda_{\text{eff}}(\Psi) = \lambda(1 + \alpha\Psi^2), \tag{68}$$

with $\alpha > 0$. To summarize, for the potential (67), Ψ

controls the beginning of quintessence domination while Φ controls its end. Observational constraints require the following two conditions: $\lambda \lesssim 2.3$ and $M^2 \sim M_p H_0$.

V. NUMERICAL RESULTS

We summarize briefly the observational constraints and the initial conditions used in our numerical calculations. We will use the following conservative constraints (41), (42), and (46) on the quintessence density today $\Omega_{Q,0}$, the effective eos parameter w_{eff} and the age of the Universe t_0:

$$0.55 \leq \Omega_{Q,0} \leq 0.80, \qquad w_{\text{eff}} \leq -0.70,$$
$$H_0 t_0 \geq 1.330 h. \tag{69}$$

From the equipartition of energy, natural initial conditions suggest that $\Omega_{Q,i} \sim 10^{-3} - 10^{-4}$ [9]. When $\Omega_{Q,i} \lesssim 1\%$, the parameter window allowed by observations is unchanged and so we will take

$$X_{\Phi,i} = X_{\Psi,i} = X_{V,i} = 10^{-2} \qquad \text{at } N_i = -67, \tag{70}$$

implying $\Omega_{Q,i} = 3 \times 10^{-4}$ except for the pure exponential potential, Sec. VA, where we take $X_{\Phi,i}^2 = X_{\Psi,i}^2 = 10^{-4}$ and $X_{V,i}^2$ has to be adjusted.

A. Pure exponential potential

The exponential potential Eq. (49) is a viable candidate for quintessence provided the attractor solution is not yet reached as explained in Sec. IVA, and, in particular, it can produce transient acceleration. In addition to the constraint $\lambda > \sqrt{2}$ in order to have transient acceleration (see Sec. IVA), imposing the observational constraints (69) only $\lambda \leq 1.975$ is allowed. In [34], only the case $\lambda < \sqrt{3}$ was considered because they used the constraints $w_{Q,0} \leq -0.60$ and $\Omega_{Q,0} \geq 0.60$ which lead to a stronger constraint on λ.

Thus in order to have transient acceleration, λ is constrained as $\sqrt{2} < \lambda \leq 1.975$ resulting in $w_{\text{eff}} \gtrsim -0.86$. Here $3.7 \lesssim X_{V,i}^2 \times 10^{113} \lesssim 9.2$ with $\Phi_i = 0$ at $N_i = -67$.

Different possibilities arise: If $(1.82-1.837) \leq \lambda \leq 1.837$, the acceleration ends before today with a low quintessence density $\Omega_{Q,0} \lesssim 0.62$ and $w_{\text{eff}} \gtrsim -0.75$. Surprisingly there is *no acceleration at all* in the interval $1.838 \leq \lambda \leq 1.975$ while $\Omega_{Q,0} \lesssim 0.61$ and $w_{\text{eff}} \gtrsim -0.75$. If the observations would constrain w_{eff} so that $w_{\text{eff}} \lesssim -0.86$, only permanent acceleration would be possible (i.e., $\lambda < \sqrt{2}$).

In Fig. 5, the ratio t_{end}/t_0 is shown as a function of λ.

We should stress that the constraints we have adopted are conservative and a more refined analysis would restrain the viability of our models. Let us consider the following constraints at 1σ from the Wilkinson Microwave Anisotropy Probe (WMAP) alone [26]:
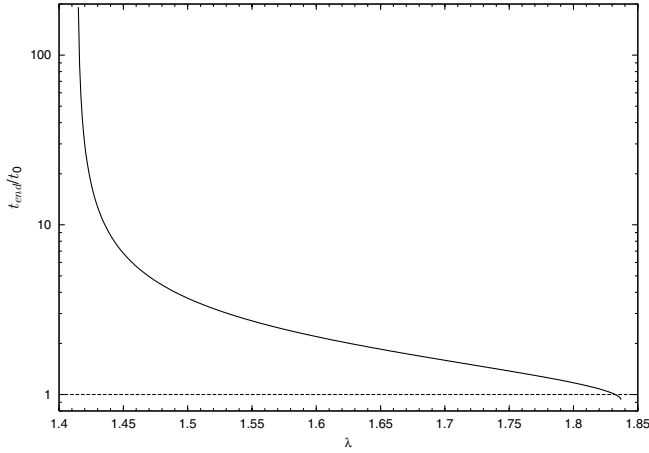
FIG. 5. The ratio t_{end}/t_0 is displayed as a function of λ for the pure exponential potential Eq. (49). For $t > t_{end}$, decelerated expansion is recovered ($q > 0$). All models displayed have a transient acceleration; for those below the dashed line the acceleration is ended by today. Initially, at $N_i = -67$, we take $X_{V,i}^2 = V_i/\rho_{c,i} = M^4/\rho_{c,i} = 5.8 \times 10^{-113}$. For this initial value $X_{V,i}$, the range $1.833 \leq \lambda \leq 1.837$ yields an accelerated stage which ends before today while the range $1.838 \leq \lambda \leq 1.975$ produces *no acceleration at all*, the range $\lambda > 1.975$ being excluded by observations, Eq. (69). Note that $t_{end} \rightarrow +\infty$ when $\lambda \rightarrow \sqrt{2}$.

$$13.2 \text{ Gpc} \leq 3h^{-1} \int_0^{z_{dec}} \frac{dz}{h(z)} \leq 14.2 \text{ Gpc}, \qquad (71)$$

$$0.12 \lesssim \Omega_{m,0}h^2 \lesssim 0.16, \qquad (72)$$

$$0.67 \leq h \leq 0.77, \qquad (73)$$

$$13.1 \leq t_0 \leq 13.7 \text{ Gyr}. \qquad (74)$$

We have finally from the SNIa data at 1σ [49]

$$0.26 \leq \Omega_{m,0} \leq 0.34. \qquad (75)$$

It is easily checked that the range (39) corresponds to (72) with the lower, respectively upper, bound corresponding to $h \approx 0.81$, respectively $h \approx 0.58$. However in a more refined analysis the quantities h and $\Omega_{m,0}$ are no longer independent.

In Fig. 6 the quantity $D_L(z)$ is plotted for an ever decelerating universe with $\lambda = 1.84$, $X_{V,i}^2 = 5.8 \times 10^{-113}$. This model, representative of the scenario without acceleration at all, is characterized by the following cosmological parameters $\Omega_{Q,0} = 0.583$, $w_{Q,0} = -0.568$, $w_{eff} = -0.722$, $H_0 t_0 = 0.843$. With a correction factor $1.11 \leq e_\Lambda \leq 1.18$, the luminosity distances agree with the SNIa data, with uncertainties taken at 1σ. Taking the fiducial value $h_\Lambda = 0.72$, we would get $0.61 \leq h \leq 0.65$. For this model we have further $\int_0^{z_{dec}} \frac{dz}{h(z)} = 2.668$ so that the constraint (71) yields $0.56 \leq h \leq 0.61$ and
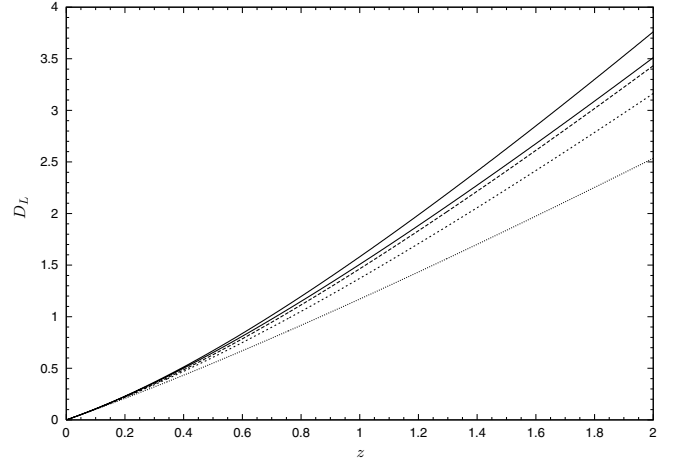

FIG. 6. The following models are plotted: the SNIa data with 1σ errors corresponding to flat Λ models with $\Omega_{\Lambda,0} = 0.74$ (upper solid line), $\Omega_{\Lambda,0} = 0.66$ (lower solid line), the Einstein-de Sitter Universe (dotted line), the potential Eq. (80) for $P_0 = 0.164$ of Table II and accelerated expansion ended by today (long-dashed line), the pure exponential potential Eq. (49) for parameter values $\lambda = 1.84$, $X_{V,i}^2 = 5.8 \times 10^{-113}$ and *no acceleration at all* (short-dashed line). The luminosity distances $d_L(z)$ for a given model with H_0 can be compared by varying e_Λ, $e_\Lambda D_L(z) \equiv (h_\Lambda/h)D_L(z) = H_{0,\Lambda}d_L(z)$, where $H_{0,\Lambda}$ (and the corresponding h_Λ) is some fixed fiducial value. The correction factor $1.03 \leq e_\Lambda \leq 1.10$ will bring the first model (pseudoexponential) inside the SNIa data (1σ uncertainties), $1.11 \leq e_\Lambda \leq 1.18$ is needed for the pure exponential.

$0.133 \leq \Omega_{m,0}h^2 \leq 0.153$ and an age $13.51 \leq t_0 \leq 14.72$ Gyr. So this extreme scenario is already in marginal agreement with the WMAP data alone due to the low value of h, a value $h \simeq 0.61$ being slightly below the 2σ error from the WMAP data alone.

Let us consider now when WMAP is combined with other CMB data and other data probing the power spectrum of the perturbations, we get at 1σ [26]

$$13.7 \text{ Gpc} \leq 3h^{-1} \int_0^{z_{dec}} \frac{dz}{h(z)} \leq 14.2 \text{ Gpc}, \qquad (76)$$

$$0.126 \lesssim \Omega_{m,0}h^2 \lesssim 0.143, \qquad (77)$$

$$0.68 \leq h \leq 0.75, \qquad (78)$$

$$13.5 \leq t_0 \leq 13.9 \text{ Gyr}. \qquad (79)$$

We get now the tighter bounds $0.56 \leq h \leq 0.58$, $0.133 \leq \Omega_{m,0}h^2 \leq 0.142$. The value of h is too low and the model without any acceleration is clearly in trouble.

We should however make the following important remark: The uncertainties are obtained from the data assuming a ΛCDM model and a specific model for the perturbations, a constant spectral index n_s for (71)–(74) and a running spectral index for (76)–(79). Our models

are not, by definition, ΛCDM models so that, strictly speaking, the data should be processed specifically for each of our models.

B. Albrecht and Skordis potential

For the AS potential Eq. (62), only a tiny interval for $\lambda^2 P_0$, namely $0.985 \lesssim \lambda^2 P_0 \lesssim 1.127$, allows a transient acceleration if we impose the constraints (69), see Fig. 4. If $\lambda^2 P_0 \lesssim 0.985$, the field Φ stays at its minimum and acceleration lasts forever while if $\lambda^2 P_0 \gtrsim 1.127$ acceleration can occur but $\Omega_{Q,0}$ will never reach the value 0.55 because the minimum does not exist and the evolution of the field is too fast. If the transient acceleration ends before today, again with the constraints (69) the parameter space $(\lambda, \lambda^2 P_0)$ is even more restricted, see Fig. 4. The parameter λ is constrained to be $\lambda \gtrsim 9$ because of the nucleosynthesis bound if the scaling behavior starts earlier than nucleosynthesis, while transient acceleration implies $\lambda \lesssim 25$. For this interval, we have $237.5 \lesssim \lambda\Phi_c \lesssim 239$ when we start with $\Phi_i = 0$ at $N_i = -67$. Moreover $M \sim 10^{-2}\rho_{c,i}^{1/4}$ if we impose $X_{V,i}^2 = 10^{-4}$.

In Table I some examples of a transient acceleration are given with fixed parameters $\lambda = 10$, $\lambda\Phi_c = 238.5$ and we vary P_0. All the points for which the transient acceleration is ended by today have $w_{\text{eff}} \gtrsim -0.88$. If w_{eff} is constrained from observations to satisfy $w_{\text{eff}} \lesssim -0.88$, acceleration ended by today is not possible in the AS potential. Nevertheless transient acceleration ended by today with the AS potential is easier to achieve than any transient acceleration using the pure exponential with $w_{\text{eff}} \gtrsim -0.86$.

Whatever λ and P_0, the maximum value for t_{end}/t_0 is 3 because P_0 is bounded from below (see also Fig. 4).

At the end of Sec. V C, a model representative of all four models with acceleration ended by today will be analyzed in the light of the constraints (71)–(79).

C. Pseudoexponential potential

We can illustrate the considerations concerning the potential Eq. (63) (with $g \equiv 0$) using the function defined in Eq. (66). The potential reads then

$$V = \frac{M^4}{\lambda^2} e^{-\lambda\Phi} [\Psi_c^2 + \Psi^2(\lambda\Phi - \lambda\Phi_c)^2], \quad (80)$$

and the minimum for Φ disappears if $-\Psi_c \leq \Psi \leq \Psi_c$ with $\Psi_c = \lambda\sqrt{P_0}$. Here we will use the following initial conditions $\Phi_i = 0$ and $\Psi_i = 5$. Once Φ_i and Ψ_i are given, we can choose Ψ_c so that $0 < \Psi_c < \Psi_i$ while it is always possible to find an appropriate Φ_c. In a way analogous to the numerical calculations with the AS potential, we keep the parameters $\lambda = 10$, $\lambda\Phi_c = 238$ fixed and we vary Ψ_c. Also, as for the AS model, we have $M \sim 10^{-2}\rho_{c,i}^{1/4}$ if we impose $X_{V,i}^2 = 10^{-4}$.

In Fig. 7, the evolution of densities is plotted for the parameter $P_0 = 0.164$ ($\Psi_c \simeq 4.05$). This set of parameters induces a transient acceleration which ends before today. In the scaling regime, $\Omega_Q \simeq 0.04$ during radiation domination and $\Omega_Q \simeq 0.03$ during matter domination, while $\Omega_{Q,0} \simeq 0.661$.
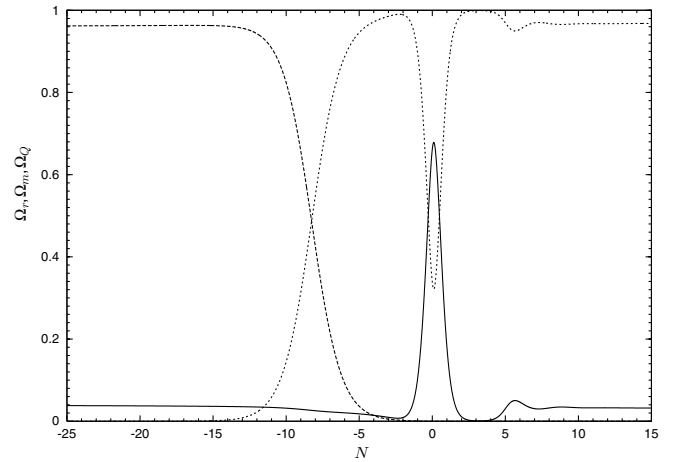
TABLE I. Models for the AS potential (62) with fixed parameters $\lambda = 10$ and $\lambda\Phi_c = 238.5$ are tabulated. All models have transient acceleration; for those in the upper part of the table the expansion is already decelerated today ($t_{\text{end}} < t_0$). Note that the model at the top of the lower part of the table, $\lambda^2 P_0 = 1.03$, satisfies the necessary condition for decelerated expansion today (43); however, this condition is not sufficient as $\Omega_{Q,0} = 0.637$.

$\lambda^2 P_0$	$\Omega_{Q,0}$	w_{eff}	$H_0 t_0$	t_{end}/t_0	$w_{Q,0}$	q_0
1.07	0.556	-0.782	0.838	0.879	-0.250	0.291
1.06	0.573	-0.804	0.849	0.907	-0.303	0.240
1.05	0.592	-0.828	0.862	0.936	-0.364	0.176
1.04	0.613	-0.854	0.878	0.966	-0.437	0.098
1.031	0.634	-0.879	0.895	0.997	-0.516	0.009
1.03	0.637	-0.882	0.897	1.001	-0.526	-0.002
1.02	0.663	-0.913	0.920	1.042	-0.634	-0.131
1.01	0.691	-0.946	0.948	1.098	-0.768	-0.296
1.0	0.719	-0.977	0.978	1.194	-0.914	-0.486
0.99	0.733	-0.993	0.995	1.478	-0.996	-0.595
0.986	0.732	-0.994	0.994	2.306	-1.0	-0.598

FIG. 7. The evolution of the densities Ω_r (dotted line), Ω_m (dashed line) and Ω_Q (solid line) are shown for the potential Eq. (80) with $\lambda = 10$, $P_0 = 0.164$, $\Phi_c = 23.8$ ($\Phi_i = 0$ and $\Psi_i = 5$). The quintessence density today is $\Omega_{Q,0} \simeq 0.661$. The age of the Universe is $H_0 t_0 \simeq 0.912$ and accelerated expansion stops at $t_{\text{end}} \simeq 0.996 t_0$. The scaling behavior of the quintessence field Φ, Eq. (50), is obvious when the background is radiation or matter dominated, in the past as well as in the future.

In Fig. 8, the eos parameter is plotted with $w_{Q,0} \simeq -0.491$ and $w_{\text{eff}} = -0.874$ and the scaling behavior is evident: $w_Q \simeq 1/3$ during radiation domination, $w_Q \simeq 0$ during matter domination. When the field Φ settles at its minimum, $w_Q \simeq -1$ and this stage is followed by a stage of kination and freezing out of Φ once the minimum is left. The deceleration parameter q is also plotted implying an actual deceleration as $q_0 \simeq 0.013$. Acceleration ($q < 0$) begins at $z \simeq 0.658$ and finishes at $z \simeq 0.0035$ when the age of the Universe is $t_{\text{end}}/t_0 \simeq 0.996$ while the present age of the Universe satisfies $H_0 t_0 \simeq 0.912$.

In Fig. 9, the ratio t_{end}/t_0 is plotted as a function of the ratio Ψ_0/Ψ_c. Clearly, the more remote Ψ_i (Ψ_0) from Ψ_c, the longer the acceleration regime. As soon as the minimum disappears, Φ stops oscillating and continues to roll down its potential. Contrary to the AS potential t_{end}/t_0 can be as large as we want because P_0, or equivalently Ψ_c, is not bounded from below.

In Fig. 2, w_Q and q are plotted against redshift: the cases $P_0 = 0.164, 0.166, 0.168, 0.170$ induce an acceleration ended by today. Viable models where the acceleration ends before today have $w_{\text{eff}} \gtrsim -0.88$.

In Table II, some examples for this model are shown with $\Phi_i = 0$, $\Psi_i = 5$ and again $\lambda = 10$, $\Phi_c = 23.8$ and we vary P_0 (or equivalently Ψ_c).

In Fig. 6, the quantity D_L is plotted for a model for which the acceleration is already finished ($\lambda = 10$, $P_0 = 0.164$, $\Phi_c = 23.8$, $\Phi_i = 0$, $\Psi_i = 5$, $\Omega_{m,0} = 0.339$, $H_0 t_0 = 0.912$). With a correction factor $1.03 \leq e_\Lambda \leq 1.10$, the luminosity distances agree with the SNIa data, with uncertainties taken at 1σ. Taking the fiducial value $h_\Lambda = 0.72$, we would get $0.65 \leq h \leq 0.70$.

FIG. 9. The ratio t_{end}/t_0, where t_{end} is the time at which accelerated expansion ends, is shown for the potential Eq. (80) versus the ratio Ψ_0/Ψ_c with $\Psi_c = \lambda\sqrt{P_0}$. The displayed models have the fixed parameters $\lambda = 10$, $\Phi_c = 23.8$ with the initial conditions $\Phi_i = 0$ and $\Psi_i = 5$, while we vary the parameter Ψ_c, or equivalently, P_0. All models displayed have transient acceleration but only those below the dashed line have a decelerated expansion *today*. The range of values $\Psi_0/\Psi_c \lesssim 0.95$ is excluded by the observations for this set of parameter values.

We can repeat for this model a more refined analysis of the kind done for the other model (pure exponential) shown in Figs. 1 and 6, see Sec. VA. We have $\int_0^{z_{\text{dec}}} \frac{dz}{h(z)} = 2.992$ so that the constraint (71) yields $0.63 \leq h \leq 0.68$, $0.135 \leq \Omega_{m,0}h^2 \leq 0.157$ and an age $13.11 \leq t_0 \leq 14.11$ Gyr. So this model is in good agreement with the
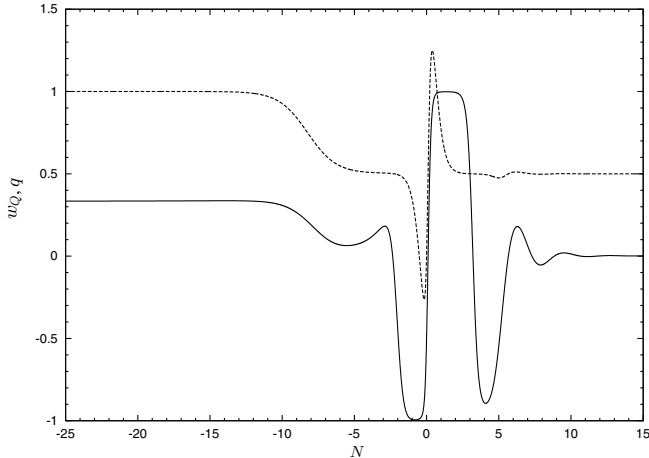

FIG. 8. The evolution of w_Q (solid line) and the deceleration parameter q (dashed line) are shown for the model of Fig. 7. We obtain here $w_{Q,0} \simeq -0.491$, $q_0 = 0.013$ and $w_{\text{eff}} \simeq -0.874$. Again, we note the scaling behavior of w_Q, Eq. (50). In the future we have a kination regime ($w_Q \sim 1$) and a freezing regime ($w_Q \sim -1$) of the field Φ.
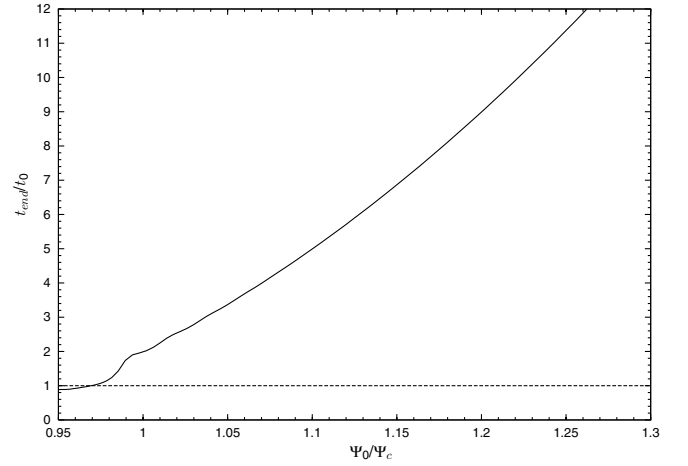
TABLE II. Models for the potential (80) with fixed parameters $\lambda = 10$ and $\lambda\Phi_c = 238$ are tabulated. The four upper models of the table have a transient acceleration already ended by today, see Fig. 2. The model at the top of the lower part of the table, $P_0 = 0.163$, satisfies the necessary condition for decelerated expansion today (43); however, this condition is not sufficient as $\Omega_{Q,0} = 0.679$.

P_0	$\Omega_{Q,0}$	w_{eff}	$H_0 t_0$	t_{end}/t_0	$w_{Q,0}$	q_0
0.17	0.575	-0.771	0.846	0.889	-0.207	0.321
0.168	0.600	-0.801	0.863	0.915	-0.280	0.248
0.166	0.628	-0.835	0.885	0.954	-0.372	0.149
0.164	0.661	-0.874	0.912	0.996	-0.491	0.013
0.163	0.679	-0.894	0.929	1.02	-0.564	-0.075
0.162	0.699	-0.916	0.949	1.05	-0.649	-0.181
0.16	0.740	-0.960	0.995	1.15	-0.850	-0.444
0.15	0.746	-0.989	1.008	2.49	-0.993	-0.611
0.14	0.716	-0.990	0.978	3.62	-0.995	-0.569
0.13	0.681	-0.990	0.947	5.02	-0.996	-0.518
0.12	0.643	-0.989	0.917	6.80	-0.998	-0.463
0.11	0.602	-0.988	0.889	9.06	-0.998	-0.401
0.1	0.559	-0.987	0.862	11.98	-0.997	-0.336

WMAP data alone and the SNIa data. If we consider the tighter constraints (76), we obtain $0.63 \le h \le 0.66$, $0.135 \le \Omega_{m,0} h^2 \le 0.145$ and an age $13.51 \le t_0 \le 14.11$ Gyr, still allowed by (76) and (77).

D. Pure exponential potential with a varying coupling constant

We now consider the potential Eqs. (67) and (68)

$$V = M^4 \exp[-\lambda(1 + \alpha \Psi^2)\Phi], \quad (81)$$

for which quintessence domination occurs when $\lambda \lesssim 2.3$ and acceleration today takes place provided $M \sim \sqrt{M_p H_0}$. We have taken $\alpha\lambda = 12.44$ with $\Phi_i = -5$ and $\Psi_i = 2$ and initially $X_{V,i}^2 = 10^{-4}$. Thus the dynamics of the fields starts initially, that is, there is no freezing out $(V \sim H^2)$.

In Fig. 10, the quintessence density evolution is plotted for different values $\lambda = 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.85$. The effect of the value of λ is shown where, for fixed initial conditions, the models today are roughly equivalent, but the duration of the transient acceleration depends strongly on the value of λ.

In Fig. 3, w_Q and q are shown and we note a permanently decelerating universe for $\lambda = 1.85$; however, all of the models with no acceleration at all have $w_{\text{eff}} \gtrsim -0.74$
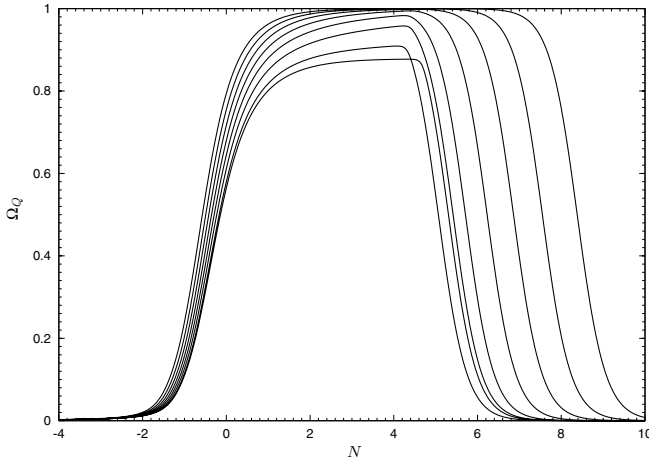

FIG. 10. The evolution of the quintessence density is shown for the potential Eq. (81) and same models as in Table III with, from top to bottom, $\lambda = 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.85$. For the models $\lambda = 1.8, 1.85$ ($\lambda > \sqrt{3}$), there is an intermediate stage where $\Omega_Q \to (3/\lambda^2)\gamma_b$ [cf. Equation (50)] starting around $N \simeq -1.4$ ($z \simeq 3$); for the models $\lambda = 1.2, 1.3, 1.4, 1.5, 1.6, 1.7$ ($\lambda < \sqrt{3}$), there is an intermediate stage where $\Omega_Q \to 1$ [cf. Equation (51)]. For all models displayed here, $\Omega_Q \to 0$ asymptotically. Note that for $\alpha = 0$ (pure exponential), the models with $\lambda = 1.2, 1.3, 1.4$ [$\lambda < \sqrt{2}$, see Eq. (51)] would yield eternal acceleration and domination of the quintessence energy density. We see that the smaller λ, the longer quintessence dominates.

TABLE III. Models for the potential Eq. (81) with fixed parameter values $\alpha\lambda = 12.44$, and initial conditions $\Phi_i = -5$, $\Psi_i = 2$, are shown here. All models have transient acceleration except for the model at the bottom of the table which has no acceleration at all. Note that the necessary condition (43) for decelerated expansion today is satisfied for $\lambda = 1.8$ but it is not sufficient as $\Omega_{Q,0} = 0.580$.

λ	$\Omega_{Q,0}$	w_{eff}	$H_0 t_0$	t_{end}/t_0	$w_{Q,0}$	q_0
1.2	0.794	−0.806	1.016	46.42	−0.715	−0.352
1.3	0.755	−0.792	0.973	56.31	−0.691	−0.282
1.4	0.717	−0.778	0.937	63.94	−0.669	−0.220
1.5	0.680	−0.765	0.907	3.62	−0.649	−0.161
1.6	0.645	−0.753	0.883	2.19	−0.627	−0.107
1.7	0.611	−0.741	0.861	1.61	−0.608	−0.057
1.8	0.580	−0.730	0.843	1.20	−0.590	−0.013
1.85	0.564	−0.724	0.835	$q > 0 \; \forall \, t$	−0.582	0.007

and are marginally viable. Note that the oscillations of the auxiliary field Ψ around $z \sim 1$–3 translate into oscillations in w_Q, but it is impossible to see them in the luminosity distance d_L [4,22].

In Table III below some examples for this model are shown with $\Phi_i = -5$, $\Psi_i = 2$, $\alpha\lambda = 12.44$ and we vary λ.

Scenarios with no acceleration at all in model D are of the same type as for the pure exponential, model A. The analysis performed at the end of Sec. VA is therefore representative for model D.

VI. CONCLUSION

Recent observational data suggest that some unknown component, called Dark Energy, contributes to about two-thirds of the present total energy density filling our Universe. Though the accuracy of the existing observations allows one to constrain already at the present stage to some extent the possible Dark Energy candidates, a very large number of models are still permitted. It is therefore interesting to investigate all possible scenarios and we have also investigated in this work specific models where the Dark Energy sector is made of two coupled scalar fields. We have studied numerically two one-field models, the pure exponential in Sec. VA and the AS model in Sec. V B, and two Double Quintessence models, which can be seen as extensions of the corresponding one-field model, in Secs. V C and V D. We were interested, in particular, in scenarios for which the (recent) stage of accelerated expansion is transient.

Constraining the models with the observations, we have found the allowed window in the corresponding parameter space of each model. Investigation of these models has revealed that the following three possibilities can arise: The present acceleration is transient and still going on; some accelerated expansion did take place in

TABLE IV.   The possible scenarios for our one-field models (upper part of the table) and our Double Quintessence models (lower part) are summarized. As can be seen, for the one-field models, eternal acceleration covers most of the allowed parameter space. Note that for the Double Quintessence models, the acceleration is necessarily transient. All four models can produce scenarios satisfying (69) with a transient acceleration already ended at the present time.

Models	Scenarios	Parameters	Fine-tuning
Pure exponential	Eternal acceleration	$0 \leq \lambda \leq \sqrt{2}$	$V_0 = M^4 e^{-\lambda \Phi_0} \sim M_p^2 H_0^2$
$V = M^4 e^{-\lambda \Phi}$	Transient acceleration	$\sqrt{2} < \lambda \lesssim 1.837$	
	No acceleration at all	$1.838 \lesssim \lambda \lesssim 1.975$	
AS	Eternal acceleration	$9 \lesssim \lambda$	Φ_c depends on initial conditions
$V = M^4 e^{-\lambda \Phi}[P_0 + (\Phi - \Phi_c)^2]$		$\lambda^2 P_0 \lesssim 0.985$	
	Transient acceleration	$9 \lesssim \lambda \lesssim 25$	
		$0.985 \lesssim \lambda^2 P_0 \lesssim 1.127$	
AS generalization	Transient acceleration	$9 \lesssim \lambda$	Φ_c depends on initial conditions
$V = M^4 e^{-\lambda \Phi}[P_0 + \Psi^2(\Phi - \Phi_c)^2]$		$\lambda^2 P_0 \lesssim \Psi_0 \lesssim \Psi_i$	
Exponential with a varying λ	Transient acceleration	$0 < \lambda \lesssim 1.84$	$M^4 \sim M_p^2 H_0^2$
$V = M^4 e^{-\lambda(1 + \alpha \Psi^2)\Phi}$	No acceleration at all	$1.84 \lesssim \lambda \lesssim 2$	

the recent past but is already finished by today; finally no acceleration at all, this latter possibility being marginal. The two first scenarios, i.e., a transient acceleration either finished or not by today, can be obtained in all four models. A scenario with no acceleration at all is obtained only for the two one-field models V A and V D provided the cosmological parameters take their values at the edge of the allowed range, $\Omega_{m,0} \gtrsim 0.4$ and $h \lesssim 0.65$. We summarize the possible dynamics of our models in Table IV.

In both Double Quintessence models studied in Secs. V C and V D, for the allowed window in parameter space where accelerated expansion takes place, the dynamics of the model is such that the acceleration is necessarily transient; hence, for these two models acceleration if it takes place will eventually come to an end either before today or in the future. In the model of Sec. V C, in addition to the tracking field Φ, an auxiliary field Ψ is introduced which controls the presence of a minimum for the Φ field and the duration of the Universe transient acceleration. In the model of Sec. V D, it is the auxiliary field Ψ, and hence λ_{eff}, which induces accelerated expansion when it reaches its minimum. In contrast to our Double Quintessence models where acceleration is necessarily transient, for the one-field models we have studied the acceleration is typically eternal and covers most of the allowed parameter space. In view of the theoretical problems posed by eternal acceleration, all viable scenarios with transient acceleration constitute a welcome alternative. On the other hand, the cosmological coincidence is not solved here and requires some amount of fine-tuning on one of the free parameters. We insist that all the scenarios studied here are in agreement with observations, in particular, with the Hubble diagram $H(z)$, or the luminosity distances $d_L(z)$ as a function of redshift, as reconstructed from the Supernovae data lead-ing to the possible interpretation of a flat universe with $\Omega_{\Lambda,0} \simeq 0.72$.

As mentioned in the introduction, it is well known that many-fields inflationary models can produce primordial fluctuation spectra with a characteristic scale. The question arises naturally whether similar effects can be produced here. In fact, the luminosity distances can exhibit a characteristic scale if the equation of state of dark energy undergoes a phase transition. Such cases were considered in [50], where the quantity $w(z)$ was taken with a steplike structure. We have checked numerically that such models could indeed exhibit a characteristic scale in their luminosity distances. In our Double Quintessence models where accelerated expansion is ended at the present time, we have also a large variation of $w(z)$ at low redshifts, typically from $w(z) \simeq -1$ up to some higher value. However, as can be seen from Figs. 2 and 3 this variation takes place at very low redshifts $0 \leq z \lesssim 0.3$ and, though significant, this variation is not sharp enough. This is the reason why no characteristic scale is seen in the corresponding luminosity distances. Actually, it was already emphasized that the luminosity distances are not very sensitive to large, smooth, variations of the equation of state, or equivalently of the quantity $w(z)$, at low redshifts z. Results obtained here are a particular illustration of this property.

In light of our results, it is clear that eternal acceleration could be challenged in two ways, either the acceleration is transient and will end at some time in the future, or it has already ended by today. Results obtained here show that the second possibility must be taken seriously if the observations allow a rather high matter content today, viz. $\Omega_{m,0} \gtrsim 0.35$ and also large variations of the eos parameter $w(z)$ at low redshifts. In particular, if it turns out that the data require $w_{Q,0} \lesssim -0.5$, this possibility is ruled out.

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