

$B - L$ cosmic strings and baryogenesisPijushpani Bhattacharjee,^{2,*} Narendra Sahu,^{1,†} and Urjit A. Yajnik^{1,‡}¹*Department of Physics, Indian Institute of Technology, Bombay, Mumbai 400076, India*²*Indian Institute of Astrophysics, Bangalore-560 034, India*

(Received 4 June 2004; published 29 October 2004)

Cosmic strings arising from breaking of the $U(1)_{B-L}$ gauge symmetry that occurs in a wide variety of unified models can carry zero modes of heavy Majorana neutrinos. Decaying and/or repeatedly self-interacting closed loops of these “ $B - L$ ” cosmic strings can be a nonthermal source of heavy right-handed Majorana neutrinos whose decay can contribute to the observed baryon asymmetry of the Universe (BAU) via the leptogenesis route. The $B - L$ cosmic strings are expected in grand unified theory (GUT) models such as $SO(10)$, where they can be formed at an intermediate stage of symmetry breaking well below the GUT scale $\sim 10^{16}$ GeV; such light strings are not excluded by the cosmic microwave background (CMB) anisotropy data and may well exist. We estimate the contribution of $B - L$ cosmic string loops to the baryon-to-photon ratio of the Universe in the light of current knowledge on neutrino masses and mixings implied by atmospheric and solar neutrino measurements. We find that $B - L$ cosmic string loops can contribute significantly to the BAU for $U(1)_{B-L}$ symmetry-breaking scale $\eta_{B-L} \gtrsim 1.7 \times 10^{11}$ GeV. At the same time, in order for the contribution of decaying $B - L$ cosmic string loops not to exceed the observed baryon-to-photon ratio inferred from the recent Wilkinson microwave anisotropy probe (WMAP) results, the lightest heavy right-handed Majorana neutrino mass M_1 must satisfy the constraint $M_1 \leq 2.4 \times 10^{12} (\eta_{B-L}/10^{13} \text{ GeV})^{1/2}$ GeV. This may have interesting implications for the associated Yukawa couplings in the heavy neutrino sector and consequently for the light neutrino masses generated through seesaw mechanism.

DOI: 10.1103/PhysRevD.70.083534

PACS numbers: 98.80.Cq, 12.10.Dm, 14.60.St

I. INTRODUCTION

A very attractive scenario of origin of the baryon (B) asymmetry of the Universe (BAU) is that it arose from a lepton (L) asymmetry [1–3]. The conversion of the L -asymmetry to the B -asymmetry occurs via the high temperature behavior of the $B + L$ anomaly of the standard model [4]. This is an appealing route for several reasons. First, the extremely small neutrino masses, suggested by the atmospheric neutrino data [5], solar neutrino data [6], and KamLAND experiment data [7], point to the possibility of Majorana masses for the neutrinos; such small neutrino mass can be generated, for example, through the seesaw mechanism [8] that involves heavy right-handed neutrinos whose interactions involve L violation in a natural way. Second, most particle physics models incorporating the above possibility demand new Yukawa couplings and also possibly scalar self-couplings; these are the kind of couplings which, unlike gauge couplings, can naturally accommodate adequate CP violation, one of the necessary ingredients [9] for generating the BAU.

Most proposals along these lines rely on out-of-equilibrium decay of the thermally generated right-handed heavy Majorana neutrinos in the early Universe to generate the L -asymmetry. The simplest possibility to implement this scenario is to extend the standard model

(SM) by the inclusion of a right-handed neutrino, ν_R . A more appealing alternative is to consider this within the context of unified models with an embedded $U(1)_{B-L}$ gauge symmetry. For example, it can be the left-right symmetric model [10,11] where $B - L$ is naturally required to be a gauge charge, or it can be a grand unified theory (GUT) based on $SO(10)$ gauge group. Because $B - L$ is a gauge charge in such models, no primordial $B - L$ can exist as long as the $U(1)_{B-L}$ gauge symmetry remains unbroken. However, spontaneous breaking of the $U(1)_{B-L}$ gauge symmetry gives heavy Majorana mass to the right-handed neutrinos, and a net $B - L$ can be dynamically generated through out-of-equilibrium decay of these heavy right-handed Majorana neutrinos. Rapid violation of $B + L$ by the high temperature sphaleron transitions erases any $B + L$ generated earlier. These sphaleron transitions, however, conserve $B - L$. Thus, in this scenario the final BAU is related to the $B - L$ produced after the $U(1)_{B-L}$ symmetry-breaking phase transition.

One of the interesting features of any $U(1)$ gauge symmetry-breaking phase transition in the early universe is the possible formation of cosmic strings [12,13]. It has been noted earlier by several authors [14–18] that decaying, collapsing, or repeatedly self-intersecting closed loops of such cosmic strings can be a *nonthermal* source of massive particles that “constitute” the string, and that the decay of these massive particles can give rise to the observed BAU or at least can give significant contribution to it. Cosmic strings formed at a phase transition can also influence the nature of a subsequent phase transition that

*Electronic address: pijush@iitb.res.in

†Electronic address: narendra@phy.iitb.ac.in

‡Electronic address: yajnik@phy.iitb.ac.in

may have important implications for the generation of BAU [19,20].

In the present context, the “ $B - L$ ” cosmic strings associated with the $U(1)_{B-L}$ symmetry-breaking phase transition mentioned above are of particular interest [17] because they can carry zero modes [21,22] of the heavy right-handed neutrinos ν_R . This is possible because the Higgs field involved in the cosmic string solution arising from the spontaneous breaking of the $U(1)_{B-L}$ is the same Higgs that gives heavy Majorana mass to the ν_R through Yukawa coupling. It has, therefore, been suggested [17] that decaying closed loops of such cosmic strings can be an additional, nonthermal, source of the ν_R 's, whose subsequent decay can contribute to the BAU through the leptogenesis route.

In this paper we revisit this scenario of generating the BAU through the decay of ν_R 's released from $B - L$ cosmic string loops in the light of recent ideas about neutrino masses and mixings implied by the solar and atmospheric neutrino data. We find that, $B - L$ cosmic string loops can contribute significantly to the BAU for $U(1)_{B-L}$ symmetry-breaking scale $\eta_{B-L} \gtrsim 1.7 \times 10^{11}$ GeV. At the same time we show that, in order for the contribution of decaying $B - L$ cosmic string loops not to exceed the observed baryon-to-photon ratio inferred from the recent Wilkinson microwave anisotropy probe (WMAP) results [23], the lightest heavy right-handed Majorana neutrino mass M_1 must satisfy the constraint $M_1 \leq 2.4 \times 10^{12} (\eta_{B-L}/10^{13} \text{ GeV})^{1/2}$ GeV.

The rest of this paper is organized as follows. In Sec. II we briefly discuss some examples of symmetry-breaking schemes in unified models with an embedded gauged $U(1)_{B-L}$ which potentially allow cosmic string solutions, and discuss an explicit example of a cosmic string solution in the context of a simple extension of the SM, namely, the gauge group $SM \otimes U(1)_{B-L}$ spontaneously broken to SM. The nature of the neutrino zero modes in presence of such a cosmic string is then discussed. In Sec. III we briefly review the evolution of cosmic strings with particular attention to formation of closed loops and their subsequent evolution, and the production of massive particles from decaying and/or repeatedly self-intersecting cosmic string loops. We also discuss the observational constraints on the relevant cosmic string parameters. We then estimate, in Sec. IV, the contribution of the $B - L$ cosmic string loops to the BAU, and discuss the constraint on the lightest heavy right-handed Majorana neutrino mass M_1 . Sec. V concludes the paper with a brief summary of our main results. Throughout this paper we use natural units with $\hbar = c = k_B = 1$.

II. $U(1)_{B-L}$ COSMIC STRINGS AND NEUTRINO ZERO MODES

There are several realistic particle physics models where a gauged $B - L$ symmetry exists and breaks at a

certain scale. Since $SO(10)$ minimally incorporates $B - L$ gauge symmetry we consider the models embedded in $SO(10)$. The following breaking schemes can potentially accommodate cosmic strings. One of the breaking schemes, motivated by supersymmetric $SO(10)$ [24,25], involves the intermediate left-right symmetric model:

$$\begin{aligned} SO(10) &\xrightarrow{54+45} SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \\ &\xrightarrow{126+\overline{126}} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_2 \\ &\xrightarrow{10+10'} SU(3)_C \otimes U(1)_Q \otimes Z_2. \end{aligned} \quad (1)$$

During the first phase of symmetry breaking, presumably at a GUT scale of $\sim 10^{16}$ GeV, monopoles are formed. However, during the second and third phases of symmetry-breaking cosmic strings are formed since $\pi_1(\frac{3221}{321}) = \pi_1(\frac{321}{31}) = Z_2$, where the numbers inside the parentheses symbolize the group structures. The monopole problem in this model can be solved by using a hybrid inflation ending at the left-right symmetric phase of the Universe [25] thus inflating away the monopoles. The formation of cosmic strings in the later phases is of great interest since these “light” (i.e., lighter than GUT scale) cosmic strings do not conflict with any cosmological observations. The Z_2 strings of the low energy theory was investigated in an earlier work [26].

Another scheme is to break supersymmetric $SO(10)$ directly to $SU(3)_C \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L}$ with the inclusion of extra $54 + 45$. The first 45 acquires a vacuum expectation value along the direction of $B - L$. However, the latter 45 acquires a vacuum expectation value along the direction of T_{3R} :

$$\begin{aligned} SO(10) &\xrightarrow{54+45+54'+45'} SU(3)_C \otimes SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} \\ &\xrightarrow{126+\overline{126}} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_2 \\ &\xrightarrow{10+10'} SU(3)_C \otimes U(1)_Q \otimes Z_2. \end{aligned} \quad (2)$$

For the present purpose it is sufficient to consider a model based on the gauge group $SM \otimes U(1)_{B-L}$ which is spontaneously broken to SM. Existence of cosmic strings and the related zero modes in this model can be established as follows. Let the gauge field corresponding to the $U(1)_{B-L}$ symmetry be denoted by C_μ , and the symmetry be broken by a SM singlet χ . Let $\langle \chi \rangle$ be η_{B-L} below the critical temperature T_{B-L} . In a suitable gauge a long cosmic string oriented along the z axis can be represented (in cylindrical polar coordinates) by the ansatz [13]

$$\chi = \eta_{B-L} f(r) e^{in\theta}, \quad (3)$$

$$C_\mu = \frac{ng(r)}{\alpha r} \delta_\mu^\theta, \quad (4)$$

where n is an integer giving the winding number of the

phase of the complex Higgs field χ , and α is the gauge coupling constant for the group $U(1)_{B-L}$. In order for the solution to be regular at the origin we set $f(0) = g(0) = 0$. Also requiring the finiteness of energy of the solution, we set $f(r) = g(r) = 1$ as $r \rightarrow \infty$. It turns out that both $f(r)$ and $g(r)$ take their asymptotic values everywhere outside a small region of the order of η_{B-L}^{-1} around the string. Thus away from the string $\langle \chi \rangle = \eta_{B-L}$ up to a phase, and C_μ is a pure gauge. The mass scale of the string is fixed by the energy scale of the symmetry-breaking phase transition η_{B-L} at which the strings are formed. Then the mass per unit length of a cosmic string, μ , is of order $\eta_{B-L}^2 \sim T_{B-L}^2$:

The Lagrangian for the right-handed neutrino is

$$\mathcal{L}_{\nu_R} = i\bar{\nu}_R \sigma^\mu D_\mu \nu_R - \frac{1}{2} [ih\bar{\nu}_R \chi \nu_R^c + \text{H.c.}], \quad (5)$$

where h is the Yukawa coupling constant, $\sigma^\mu = (-I, \sigma^i)$, and $\nu_R^c = i\sigma^2 \nu_R^*$ defines the Dirac charge conjugation operation. The resulting equations of motion have been shown [21] to possess $|n|$ normalizable zero modes in winding number sector n . The field equations in the $U(1)$ example are [27]

$$\begin{pmatrix} -e^{i\theta} \left[\partial_r + \frac{i}{r} \partial_\theta + \frac{ng(r)}{2r} \right] & \partial_z + \partial_t \\ \partial_z - \partial_t & e^{-i\theta} \left[\partial_r - \frac{i}{r} \partial_\theta - \frac{ng(r)}{2r} \right] \end{pmatrix} \nu_R - M_R e^{in\theta} \nu_R^* = 0, \quad (6)$$

where the expressions (3) and (4) have been substituted for χ and C_μ , and $M_R = h\eta_{B-L}$. In the winding number sector n the normalizable zero modes obey $\sigma^3 \psi = \psi$ and are of the form

$$\nu_R(r, \theta) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} [U(r)e^{i\theta} + V^*(r)e^{i(n-1)\theta}], \quad (7)$$

where $U(r)$ and $V(r)$ are well behaved functions at the origin and have the asymptotic behavior $\sim \exp(-M_R r)/\sqrt{r}$. When nontrivial z and t dependences are included, these modes have solutions that depend on $z + t$ and are right movers. For $n < 0$, normalizable solutions obey $\sigma^3 \psi = -\psi$, and form the zero-energy set of a Left moving spectrum. On a straight string these modes are massless. However on wiggly strings they are expected to acquire effective masses proportional to the inverse radius of the string curvature.

III. EVOLUTION OF COSMIC STRINGS: FORMATION AND EVOLUTION OF CLOSED LOOPS AND PRODUCTION OF MASSIVE PARTICLES

A. Scaling solution and closed loop formation

The evolution of cosmic strings in the expanding Universe has been studied extensively, both analytically

as well as numerically; for a textbook review, see the monograph [13]. Here we briefly summarize only those aspects of cosmic string evolution that are relevant for the subject of the present paper, namely, the formation and subsequent evolution of closed loops of strings and production of massive particles from them. This closely follows the discussion in Sec. 6.4 of Ref. [28].

Immediately after their formation at a phase transition, the strings would in general be in a random tangled configuration. One can characterize the string configuration in terms of a coarse-grained length scale ξ_s such that the overall string energy density ρ_s is given by $\rho_s = \mu/\xi_s^2$. Initially, the strings move under a strong damping force due to friction with the ambient thermal plasma. In the friction dominated epoch a curved string segment of radius of curvature r acquires a terminal velocity $\propto 1/r$. As a result the strings tend to straighten out so that the total length of the strings decreases. Thus the overall energy density in the form of strings decreases as the Universe expands. This in turn means that the length scale ξ_s increases. Eventually, ξ_s reaches the causal horizon scale $\sim t$. After the damping regime ends (when the background plasma density falls to a sufficiently low level as the Universe expands), the strings start to move relativistically. However, causality prevents the length scale ξ_s from exceeding the horizon size $\sim t$. Analytical studies supported by extensive numerical simulations show that the subsequent evolution of the system is such that the string configuration reaches a ‘‘scaling regime’’ in which the ratio $\frac{\xi_s}{t} \equiv x$ remains a constant. Numerical simulations generally find the number x to lie approximately in the range $\sim 0.4 - 0.7$. This is called the scaling regime because then the energy density in the form of strings scales as, and remains a constant fraction of, the energy density of radiation in the radiation dominated epoch or the energy density of matter in the matter dominated epoch both of which scale as t^{-2} .

The fundamental physical process that maintains the string network in the scaling configuration is the formation of *closed loops* which are pinched off from the network whenever a string segment curves over into a loop, intersecting itself. In the ‘‘standard’’ picture [13], the closed loops so formed have average length at birth

$$L_b = K\Gamma G\mu t, \quad (8)$$

and they are formed at a rate (per unit volume per unit time) which, in the radiation dominated epoch, is given by

$$\frac{dn_b}{dt} = \frac{1}{x^2} (\Gamma G\mu)^{-1} K^{-1} t^{-4}, \quad (9)$$

where $\Gamma \sim 100$ is a geometrical factor that determines the average loop length, and K is a numerical factor of order unity.

The whole string network consisting of closed loops as well as long strands of strings stretched across the horizon gives rise to density fluctuations in the early Universe which could potentially contribute to the process of formation of structures in the Universe. More importantly, they would produce specific anisotropy signatures in the cosmic microwave background (CMB). Using a large-scale cosmic string network simulation and comparing the resulting prediction of CMB anisotropies with observations, a recent analysis [29] puts an upper limit on the fundamental cosmic string parameter μ , giving $G\mu \lesssim 0.7 \times 10^{-6}$. This translates to an upper limit, $\eta \lesssim 1.0 \times 10^{16}$ GeV, on the symmetry-breaking energy scale of the cosmic string-forming phase transition. This probably rules out cosmic string formation at a typical GUT scale $\sim 10^{16}$ GeV. However, lighter cosmic strings arising from symmetry breaking at lower scales, such as the $B - L$ cosmic strings in the case of the SO(10) model discussed in the previous section, are not ruled out.

It should be noted here that, in the standard scenario of cosmic string evolution described above, the loops are formed on a length scale that is a constant fraction of the horizon length, as given by Eq. (8). Thus, the average size of the newly formed loops increases with time. At the relevant times of interest, these loops, although small in comparison to the horizon scale, would still be of macroscopic size in the sense that they are much larger than the microscopic string width scale $w \sim \eta^{-1} \sim \mu^{-1/2}$.

In contrast, results of certain Abelian Higgs (AH) model simulations of cosmic string evolution [30] seem to indicate that scaling configuration of the string network is maintained primarily by loops formed at the smallest fixed length scale in the problem, namely, on the scale of the width $w \sim \eta^{-1} \sim \mu^{-1/2}$ of the string. These microscopic ‘‘loops’’ quickly decay into massive particles (quanta of gauge bosons, Higgs bosons, heavy fermions, etc.) that constitute the string. In other words, in this scenario, there is essentially no macroscopic loop formation at all; instead, the scaling of the string network is maintained essentially by massive particle radiation. In order for the scaling configuration of the string network to be maintained by this process, the microscopic loops must be formed at a rate

$$\left(\frac{dn_b}{dt}\right)_{\text{AH}} = \frac{1}{x^2} \mu^{1/2} t^{-3}. \quad (10)$$

The above scenario of cosmic string evolution in which massive particle radiation rather than gravitational radiation plays the dominant role is, however, currently a subject of debate [31]. One of the major problems hindering a resolution of the issues involved is the insufficient dynamic range possible in the currently available AH model simulations and the consequent need for extrapolation of the simulation results to the relevant cosmological scales, which is not straightforward. In this paper, we

shall primarily restrict ourselves to consideration of the standard macroscopic loop formation scenario described by Eqs. (8) and (9) above, although we shall have occasions to refer to the massive particle radiation scenario below (see, in particular, Sec. III.B.2).

B. Fate of the closed loops and massive particle production

The behavior of the closed loops after their formation may be broadly categorized into following two classes:

1. Slow death

Any closed loop of length L in its center of momentum frame has an oscillation period $L/2$ [32]. However, a loop may be either in a self-intersecting or non-self-intersecting configuration. In general, a closed loop configuration can be represented as a superposition of waves consisting of various harmonics of sin’s and cos’s. Some explicit low harmonic number analytical solutions of the equations of motion of closed loops representing non-self-intersecting loops are known in literature [32–35], and it is possible that there exists a large class of such non-self-intersecting solutions. Indeed, numerical simulations, while limited by spatial resolution, do seem to indicate that a large fraction of closed loops are born in non-self-intersecting configurations.

A non-self-intersecting loop oscillates freely. As it oscillates, it loses energy by emitting gravitational radiation, and thereby shrinks. When the radius of the loop becomes of the order of its width $w \sim \eta^{-1} \sim \mu^{-1/2}$, the loop decays into massive particles. Among these particles will be the massive gauge bosons, Higgs bosons, and in the case of the $B - L$ strings, massive right-handed neutrinos (ν_R) which were trapped in the string as fermion zero modes. We shall hereafter collectively refer to all these particles as X particles. We are, of course, interested here only in the ν_R ’s. In addition to those directly released from the loop’s final decay, there will also be some ν_R ’s coming from the decays of the gauge and Higgs bosons released in the final loop decay. It is difficult to calculate exactly the total number of ν_R ’s so obtained from each loop, but we may expect that it would be a number of order unity. For the purpose of this paper we shall assume that each final demise of a loop yields a number $N_N \sim \mathcal{O}(1)$ of heavy right-handed Majorana neutrinos; we shall keep this number N_N as a free parameter in the problem.

The rate of release of ν_R ’s at any time t by the above process can be calculated as follows. The lifetime of a loop of length L due to energy loss through gravitational wave radiation is

$$\tau_{\text{GW}} \sim (\Gamma G\mu)^{-1} L. \quad (11)$$

Eqs. (8) and (11) thus show that loops born at time t have a lifetime $\sim Kt \gtrsim H^{-1}(t)$, where $H^{-1}(t) \sim t$ is the Hubble expansion time scale. It is thus a slow process. From the

above, we see that the loops that are disappearing at any time t are the ones that were formed at the time $(K + 1)^{-1}t$. Taking into account the dilution of the number density of loops due to expansion of the Universe between the times of their birth and final demise, Eq. (9) gives the number of loops disappearing due to this “slow death” (SD) process per unit time per unit volume at any time t (in the radiation dominated epoch) as

$$\begin{aligned} \frac{dn_{\text{SD}}}{dt} &= f_{\text{SD}} \frac{1}{x^2} (\Gamma G \mu)^{-1} \frac{(K + 1)^{3/2}}{K} t^{-4} \\ &= f_{\text{SD}} (K + 1)^{3/2} \frac{dn_b}{dt}, \end{aligned} \quad (12)$$

where f_{SD} is the fraction of newly born loops which die through the SD process.

The rate of release of heavy right-handed neutrinos (we shall hereafter denote them by N ; see Sec. IV below) due to SD process can then be written as

$$\begin{aligned} \left(\frac{dn_N}{dt}\right)_{\text{SD}} &= N_N \frac{dn_{\text{SD}}}{dt} \\ &= N_N f_{\text{SD}} \frac{1}{x^2} (\Gamma G \mu)^{-1} \frac{(K + 1)^{3/2}}{K} t^{-4}. \end{aligned} \quad (13)$$

2. Quick death

Some fraction of the loops may be born in configurations with waves of high harmonic number. Such string loops have been shown [36] to have a high probability of self-intersecting. Ref. [36] gives the self-intersecting probability of a loop as

$$P_{\text{SI}} = 1 - e^{-\alpha - \beta N}, \quad (14)$$

where $\alpha = 0.4$, $\beta = 0.2$, and N is the harmonics number.

A self-intersecting loop would break up into two or more smaller loops. The process of self-intersection leaves behind “kinks” on the loops, which themselves represent high harmonic configurations. So, the daughter loops would also further split into smaller loops. If a loop does self-intersect, it must do so within its one oscillation period, since the motion of a loop is periodic. Under this circumstance, since smaller loops have smaller oscillation periods, it can be seen that a single initially large loop of length L can break up into a debris of tiny loops of size η^{-1} (at which point they turn into the constituent massive particles) on a time scale $\sim L$. Equation (8) then implies that a loop born at the time t in a high harmonic configuration decays, due to repeated self-intersection, into massive particles on a time scale $\tau_{\text{QD}} \sim K \Gamma G \mu t \ll H^{-1}(t)$. It is thus a “quick death” (QD) process—the loops die essentially instantaneously (compared to cosmological time scale) as soon as they are formed. Equation (9), therefore, directly gives the rate at which loops die through this quick death process:

$$\frac{dn_{\text{QD}}}{dt} = f_{\text{QD}} \frac{dn_b}{dt}, \quad (15)$$

where f_{QD} is the fraction of newly born loops that undergo QD.

Note that, since these loops at each stage self-intersect and break up into smaller loops before completing one oscillation, they would lose only a negligible amount of energy in gravitational radiation. Thus, almost the entire original energy of these loops would eventually come out in the form of massive particles.

Assuming again, as we did in the SD case, that each segment of length $\sim w \sim \mu^{-1/2}$ of the loop yields a number $N_N \sim \text{O}(1)$ of heavy right-handed Majorana neutrinos, we can write, using Eqs. (15), (9), and (8), the rate of release of the N 's due to QD process as

$$\left(\frac{dn_N}{dt}\right)_{\text{QD}} = N_N f_{\text{QD}} \frac{1}{x^2} \mu^{1/2} t^{-3}. \quad (16)$$

It is interesting to note here that if all loops were to die through this QD process, i.e., if we take $f_{\text{QD}} = 1$ in Eqs. (15) and (16), then the situation is in effect exactly equivalent to the microscopic loop formation scenario described by Eq. (10), although the primary loops themselves are formed with macroscopic size given by Eq. (8).

While the important issue of whether or not massive particle radiation plays a dominant role in cosmic string evolution remains to be settled, the standard model may, of course, still allow a small but finite fraction, $f_{\text{QD}} \ll 1$, of quickly dying loops. There already exist, however, rather stringent astrophysical constraints [28,37] on f_{QD} from the observed flux of ultrahigh cosmic rays (UHECR) above 10^{11} GeV [38] and the cosmic diffuse gamma ray background in the energy region 10 MeV–100 GeV measured by the EGRET experiment [39]. This comes about in the following way:

The massive X particles released from the string loops would decay to SM quarks and leptons. The hadronization of the quarks gives rise to nucleons and pions with energy up to $\sim M_X$, the mass of the relevant X particle. The neutral pions decay to photons. These extremely energetic nucleons and photons, after propagating through the cosmic radiation background, can survive as ultrahigh energy particles. The observed flux of UHECR, therefore, puts constraints on the rate of release of the massive X particles, thereby constraining f_{QD} . The most stringent constraint on f_{QD} , however, comes from the fact that the electromagnetic component (consisting of photons and electrons/positrons) of the total energy injected in the Universe from the decay of the X particles initiates an electromagnetic cascade process due to interaction of the high energy electrons/positrons and photons with the photons of the various cosmic background radiation fields (such as the radio, the microwave and the infrared/optical backgrounds); see, e.g., Ref. [28] for a review. As a result,

a significant part of the total injected energy cascades down to lower energies. The measured flux of the cosmic gamma ray background in the 10 MeV–100 GeV energy region [39] then puts the constraint [28]

$$f_{\text{QD}} \eta_{16}^2 \leq 9.6 \times 10^{-6}, \quad (17)$$

where $\eta_{16} \equiv (\eta/10^{16} \text{ GeV})$. For GUT scale cosmic strings with $\eta_{16} = 1$, for example, the above constraint implies that $f_{\text{QD}} \leq 10^{-5}$, so that most loops should be in non-self-intersecting configurations, consistent with the standard scenario of cosmic string evolution. Note, however, that f_{QD} is not constrained by the above considerations for cosmic strings formed at a scale $\eta \lesssim 3.1 \times 10^{13} \text{ GeV}$.

In this context, it is interesting to note that there is no equivalent constraint on the corresponding parameter f_{SD} for the slow death case from gamma ray background consideration. The reason is that, unlike in the QD case where the entire initial energy of a large loop goes into X particles, only \sim one X particle is released from a initially large loop in the SD case. This in turn makes the time dependence of the rate of release of massive particles $\propto t^{-4}$ in the SD case [see Eq. (13)], while it is $\propto t^{-3}$ in the QD case [see Eq. (16)]. Thus, while the SD process dominates at sufficiently early times, the QD process can dominate at relatively late times and can potentially contribute to the nonthermal gamma ray background.

IV. CALCULATION OF BARYON ASYMMETRY

A. Decay of heavy right-handed Majorana neutrinos and L -asymmetry

The heavy right-handed Majorana neutrino decays to a SM lepton (ℓ) and Higgs (ϕ) through the Yukawa coupling

$$\mathcal{L}_Y = f_{ij} \bar{\ell}_i \phi \nu_{Rj} + \text{H.c.}, \quad (18)$$

where f_{ij} is the Yukawa coupling matrix, and $i, j = 1, 2, 3$ for three flavors.

We shall work in a basis in which the right-handed Majorana neutrino mass matrix M is diagonal, $M = \text{diag}(M_1, M_2, M_3)$. In this basis the right-handed Majorana neutrino is given by $N_j = \nu_{Rj} \pm \nu_{Rj}^c$, which satisfies $N_j^c = \pm N_j$. The standard seesaw mechanism then gives the corresponding light neutrino mass eigenstates ν_1, ν_2, ν_3 with masses m_1, m_2, m_3 , respectively; these are mixtures of flavor eigenstates ν_e, ν_μ, ν_τ , and are also Majorana neutrinos, i.e., $\nu_i = \nu_i^c$.

The decays of the heavy right-handed Majorana neutrinos can create a nonzero L -asymmetry (which is ultimately converted to B -asymmetry) only if their decay violates CP . The CP -asymmetry parameter in the decay

of N_j is defined as

$$\epsilon_j \equiv \frac{\Gamma(N_j \rightarrow \ell \phi) - \Gamma(N_j \rightarrow \ell^c \phi^c)}{\Gamma(N_j \rightarrow \ell \phi) + \Gamma(N_j \rightarrow \ell^c \phi^c)}. \quad (19)$$

Assuming a mass hierarchy in the heavy neutrino sector, $M_1 < M_2 < M_3$, it is reasonable to expect that the final lepton asymmetry is produced mainly by the decay of the lightest right-handed neutrino N_1 . Any asymmetry produced by the decay of N_2 and N_3 will be washed out by the lepton number violating interactions mediated by the N_1 . As the Universe expands, the temperature of the thermal plasma falls. Below a temperature $T_F \sim M_1$, all L -violating scatterings mediated by N_1 freeze-out, thus providing the out-of-equilibrium situation [9] necessary for the survival of any net L -asymmetry generated by the decay of the N_1 's. The final L -asymmetry is, therefore, given essentially by the CP asymmetry parameter ϵ_1 .

An accurate calculation of the net L -asymmetry can only be done by numerically solving the full Boltzmann equation that includes all lepton number violating interactions involving all the N_j 's present at any time, including the N_j 's of nonthermal origin such as the ones produced from the decaying cosmic string loops, as well as those of thermal origin. This is beyond the scope of the present paper; here we shall simply assume that below the temperature $T_F = M_1$, all interactions except the decay of the N_1 are unimportant, so that each N_1 released from cosmic strings additively produces a net L -asymmetry ϵ_1 when it decays.

To fix the value of ϵ_1 , we note that there is an upper bound [40] on ϵ_1 , which is related to the properties of the light neutrino masses. In a standard hierarchical neutrino mass scenario with $m_3 \gg m_2 > m_1$, this upper limit is given by [40,41]

$$|\epsilon_1| \leq \frac{3}{16\pi} \frac{M_1 m_3}{v^2}, \quad (20)$$

where $v \simeq 174 \text{ GeV}$ is the electroweak symmetry-breaking scale. Furthermore, the above upper limit is in fact *saturated* [41] in most of the reasonable neutrino mass models, which we shall assume to be the case.

The atmospheric neutrino data [5] indicate $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with nearly maximal mixing ($\theta_{\text{atm}} \simeq 45^\circ$) and a mass squared difference $\Delta m_{\text{atm}}^2 \equiv |m_3^2 - m_2^2| \approx 2.6 \times 10^{-3} \text{ eV}^2$. The solar neutrino [6] and KamLAND [7] data, on the other hand, can be explained by $\nu_e \leftrightarrow \nu_\mu$ oscillations with large mixing angle (LMA) ($\theta_{\text{sol}} \simeq 32^\circ$) and $\Delta m_{\text{sol}}^2 \equiv |m_2^2 - m_1^2| \approx 7.13 \times 10^{-5} \text{ eV}^2$. Assuming, again, the standard light neutrino mass hierarchy, the above numbers give $m_3 \simeq (\Delta m_{\text{atm}}^2)^{1/2} \simeq 0.05 \text{ eV}$. In our calculations below, we shall use

$$\epsilon_1 \simeq 9.86 \times 10^{-4} \left(\frac{M_1}{10^{13} \text{ GeV}} \right) \left(\frac{(\Delta m_{\text{atm}}^2)^{1/2}}{0.05 \text{ eV}} \right). \quad (21)$$

The L -asymmetry is partially converted to a B -asymmetry by the rapid nonperturbative sphaleron transitions which violate $B + L$ but preserve $B - L$. Assuming that sphaleron transitions are ineffective at temperatures below the electroweak transition temperature (T_{EW}), the B -asymmetry is related to L -asymmetry by the relation [42]

$$B = p(B - L) = \frac{p}{p - 1}L \simeq -0.55L, \quad (22)$$

where we have taken $p = 28/79$ appropriate for the particle content in SM [42]. If sphaleron transitions continue to be effective below T_{EW} , then the above relation between B and L is slightly modified; we can however ignore this at the level of accuracy aimed at in the present paper.

The net baryon asymmetry of the Universe is defined as

$$Y_B = \frac{n_B - n_{\bar{B}}}{s}, \quad (23)$$

where

$$s \simeq (2\pi^2/45)g_*T^3 \simeq 43.86\left(\frac{g_*}{100}\right)T^3 \quad (24)$$

is the entropy density, g_* being the number of relativistic degrees of freedom contributing to the entropy at the temperature T . At temperatures in the early Universe relevant for the process of baryon asymmetry generation, $g_* \simeq 100$ in SM.

Observationally, the BAU is often expressed in terms of the baryon-to-photon ratio $\eta \equiv (n_B - n_{\bar{B}})/n_\gamma$, whose present-day value η_0 is related to that of Y_B through the relation

$$\eta_0 \simeq 7.0Y_{B,0}. \quad (25)$$

The observed value of η_0 inferred from the WMAP data is [23]

$$\eta_0^{\text{WMAP}} = (6.1_{-0.2}^{+0.3}) \times 10^{-10}. \quad (26)$$

We now proceed to estimate the contribution to the BAU from the two cosmic string loop processes discussed in the previous section.

B. Slow death case

The contribution of the SD process to η_0 can be written as

$$\eta_0^{\text{SD}} \simeq 7.0 \times 0.55\epsilon_1 \int_{t_F}^{t_0} \frac{1}{s} \left(\frac{dn_N}{dt}\right)_{\text{SD}} dt, \quad (27)$$

where t_F is the cosmic time corresponding to the temperature $T_F \simeq M_1$ and t_0 is the present age of the Universe. Using Eqs. (13) and (24), and the standard time-temperature relation in the early Universe,

$$t \simeq 0.3g_*^{-1/2} \frac{M_{\text{Pl}}}{T^2}, \quad (28)$$

where $M_{\text{Pl}} \simeq 1.22 \times 10^{19}$ GeV is the Planck mass, we see that the dominant contribution to the integral in Eq. (27) comes from the time $t_F \ll t_0$, i.e., from the epoch of temperature $T_F \simeq M_1$, giving

$$\begin{aligned} \eta_0^{\text{SD}} &\simeq 2.0 \times 10^{-7} N_N \left(\frac{M_1}{10^{13} \text{ GeV}}\right)^4 \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}}\right)^{-2} \\ &= 2.0 \times 10^{-7} N_N h_1^4 \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}}\right)^2, \end{aligned} \quad (29)$$

where we have defined the Yukawa coupling $h_1 \equiv M_1/\eta_{B-L}$, used Eq. (21) for ϵ_1 with $(\Delta m_{\text{atm}}^2)^{1/2} = 0.05$ eV, and also taken $x = 0.5$, $\Gamma = 100$, $K = 1$, and $f_{\text{SD}} = 1$ in Eq. (13).

The Yukawa couplings are generally thought to be less than unity. With $h_1 \leq 1$, we see from (29) and (26) that the cosmic string loop slow death process can produce the observed BAU only for $B - L$ phase transition scale

$$\eta_{B-L}^{\text{SD}} \gtrsim 5.5 \times 10^{11} N_N^{-1/2} \text{ GeV}. \quad (30)$$

Assuming $N_N \leq 10$, say, we see that cosmic string loop SD process can contribute to BAU if $\eta_{B-L} \gtrsim 1.7 \times 10^{11}$ GeV; lower values of η_{B-L} are relevant only if we allow $h_1 > 1$.

At the same time, for a given η_{B-L} satisfying (30), in order that the contribution (29) not exceed the highest allowed observed value of η_0 given by Eq. (26), the Yukawa coupling h_1 must satisfy the constraint

$$h_1^{\text{SD}} \leq 0.24 \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}}\right)^{-1/2} N_N^{-1/4}, \quad (31)$$

which, in terms of the lightest heavy right-handed Majorana neutrino mass M_1 , reads

$$M_1^{\text{SD}} \leq 2.4 \times 10^{12} N_N^{-1/4} \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}}\right)^{1/2} \text{ GeV}. \quad (32)$$

Note the rather weak dependence of the above constraints on N_N . Also, the 4th power dependence on M_1 of Eq. (29) and the rather narrow range of the observed value of η_0 given by Eq. (26) together imply that, in order for the SD process to explain the observed BAU, M_1 (and equivalently h_1) cannot be much smaller than their respective values saturating the above constraints.

C. Quick death case

Replacing $(\frac{dn_N}{dt})_{\text{SD}}$ in Eq. (27) by $(\frac{dn_N}{dt})_{\text{QD}}$ given by Eq. (16), and following the same steps as in the SD case above, we get the contribution of the QD process to η_0 as

$$\begin{aligned} \eta_0^{\text{QD}} &\simeq 5.17 \times 10^{-13} N_N f_{\text{QD}} \left(\frac{M_1}{10^{13} \text{ GeV}}\right)^2 \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}}\right) \\ &= 5.17 \times 10^{-13} N_N f_{\text{QD}} h_1^2 \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}}\right)^3. \end{aligned} \quad (33)$$

From (33) and (26) we see that, considering the most optimistic situation with $f_{\text{QD}} = 1$, the QD process is relevant for BAU only for

$$\eta_{B-L}^{\text{QD}} \gtrsim 1.1 \times 10^{14} N_N^{-1/3} \text{ GeV}; \quad (34)$$

lower values of η_{B-L} are relevant only if we allow $h_1 > 1$. On the other hand, the constraint (17) allows $f_{\text{QD}} = 1$ only if $\eta_{B-L} \leq 3.1 \times 10^{13} \text{ GeV}$. This can be reconciled with the above constraint (34) only for $N_N > 45$ or so. Such a large value of N_N seems unlikely.

In general, using the constraint (17) on f_{QD} in (33) we get

$$\begin{aligned} \eta_0^{\text{QD}} &\leq 5.0 \times 10^{-12} N_N \left(\frac{M_1}{10^{13} \text{ GeV}} \right)^2 \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}} \right)^{-1} \\ &= 5.0 \times 10^{-12} N_N h_1^2 \left(\frac{\eta_{B-L}}{10^{13} \text{ GeV}} \right). \end{aligned} \quad (35)$$

Comparing again with the observed value of η_0 , we now see that, for $h_1 \leq 1$, the QD process can be relevant for BAU only for

$$\eta_{B-L}^{\text{QD}} \gtrsim 1.2 \times 10^{15} N_N^{-1} \text{ GeV}. \quad (36)$$

For values of η_{B-L} satisfying the above constraint (36), the QD process can produce the observed value of BAU for

$$h_1^{\text{QD}} \lesssim 0.36 \left(\frac{\eta_{B-L}}{10^{16} \text{ GeV}} \right)^{-1/2} N_N^{-1/2}, \quad (37)$$

which in terms of M_1 now reads

$$M_1^{\text{QD}} \lesssim 3.6 \times 10^{15} N_N^{-1/2} \left(\frac{\eta_{B-L}}{10^{16} \text{ GeV}} \right)^{1/2} \text{ GeV}. \quad (38)$$

From the above discussions we see that, as far as their contributions to the BAU is concerned, the QD process becomes important only at relatively higher values of the symmetry-breaking scale η_{B-L} compared to the SD process.

V. SUMMARY AND CONCLUSIONS

A wide class of unified theories with an embedded $U(1)_{B-L}$ gauge symmetry allows formation of “ $B-L$ ” cosmic strings at the $U(1)_{B-L}$ symmetry-breaking phase transition at a symmetry-breaking scale η_{B-L} well below the GUT scale ($\sim 10^{16} \text{ GeV}$). Such light cosmic strings are not currently excluded by CMB anisotropy data. The

$B-L$ cosmic strings can carry zero modes of heavy right-handed Majorana neutrinos (N), and the latter can be released from closed loops of these cosmic strings when the loops eventually disappear. The decay of the N 's can give rise to a L -asymmetry which is partially converted to B -asymmetry via nonperturbative sphaleron transitions.

In this paper we have studied the contribution to the baryon asymmetry of the Universe due to decay of heavy right-handed Majorana neutrinos released from closed loops of $B-L$ cosmic strings in the light of current ideas on light neutrino masses and mixings implied by atmospheric and solar neutrino measurements. We have estimated the contribution to BAU from cosmic string loops which disappear through the process of (a) slow shrinkage due to energy loss through gravitational radiation—which we call slow death (SD), and (b) repeated self-intersections—which we call quick death (QD). We find that for reasonable values of the relevant parameters, the SD process dominates over the QD process as far as their contribution to BAU is concerned. We find that $B-L$ cosmic string loop SD process can contribute significantly to, and can in principle produce, the observed BAU for $U(1)_{B-L}$ symmetry-breaking scale $\eta_{B-L} \gtrsim 1.7 \times 10^{11} \text{ GeV}$. The QD process, on the other hand, becomes relevant for BAU only for relatively higher values of $\eta_{B-L} \gtrsim 10^{14} \text{ GeV}$.

We have also found that, in order for the contribution of decaying $B-L$ cosmic string loop SD process not to exceed the observed baryon-to-photon ratio inferred from the recent WMAP results, the lightest heavy right-handed Majorana neutrino mass M_1 must satisfy the constraint $M_1 \leq 2.4 \times 10^{12} (\eta_{B-L}/10^{13} \text{ GeV})^{1/2} \text{ GeV}$. This result may have interesting implications for the associated Yukawa couplings in the heavy neutrino sector and consequently for the light neutrino masses generated through seesaw mechanism.

We conclude that processes involving closed loops of cosmic strings formed at a $U(1)_{B-L}$ symmetry-breaking phase transition may make significant contribution to the observed BAU and should be included in considerations of baryon generation processes in general. A full Boltzmann equation calculation of the baryogenesis process including the heavy right-handed neutrinos of cosmic string origin as well of thermal origin is in progress and will be reported elsewhere.

-
- [1] M.Fukugita and T.Yanagida, Phys. Lett. B **174**, 45 (1986).
 [2] M. A. Luty, Phys. Rev. D **45**, 455 (1992).
 [3] M. Plümacher, Z. Phys. C **74**, 549 (1997).

- [4] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985).
 [5] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5656 (2001).

- [6] SNO Collaboration, S. N. Ahmed *et al.*, Phys. Rev. Lett. **92** 181301 (2004); Q. R. Ahmed *et al.*, Phys. Rev. Lett. **89**, 011301 (2002).
- [7] KamLAND collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
- [8] See, e.g., R. N. Mohapatra and P. B. Pal, *Massive Neutrinos in Physics and Astrophysics* 2nd ed. (World Scientific, Singapore, 1998).
- [9] A. D. Sakharov, JETP Lett. **5**, 24 (1967).
- [10] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); **11**, 2558 (1975).
- [11] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981).
- [12] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
- [13] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).
- [14] P. Bhattacharjee, T. W. B. Kibble, and N. Turok, Phys. Lett. B **119**, 95 (1982).
- [15] R. H. Brandenberger, A. C. Davis, and M. Hindmarsh, Phys. Lett. B **263**, 239 (1991).
- [16] H. Lew and A. Riotto, Phys. Rev. D **49**, 3837 (1994).
- [17] R. Jeannerot, Phys. Rev. Lett. **77**, 3292 (1996).
- [18] P. Bhattacharjee, Phys. Rev. Lett. **81**, 260 (1998).
- [19] S. Bhowmik Duari and U. A. Yajnik, Phys. Lett. B **326**, 21 (1994).
- [20] R. Brandenberger, A.-C. Davis, and M. Trodden, Phys. Lett. B **335**, 123 (1994).
- [21] R. Jackiw and P. Rossi, Nucl. Phys. **B190**, 681 (1981).
- [22] E. J. Weinberg, Phys. Rev. D **24**, 2669 (1981).
- [23] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003).
- [24] R. Jeannerot and A.-C. Davis, Phys. Rev. D **52**, 7220 (1995).
- [25] R. Jeannerot, Phys. Rev. D **53**, 5426 (1996).
- [26] U. A. Yajnik, H. Widyana, D. Choudhari, S. Mahajan, and A. Mukherjee, Phys. Rev. D **59**, 103508 (1999).
- [27] S. C. Davis, Int. J. Theor. Phys. **38**, 2889 (1999).
- [28] P. Bhattacharjee and G. Sigl, Phys. Rep. **327**, 109 (2000).
- [29] M. Landriau and E. P. S. Shellard, Phys. Rev. D **69**, 023003 (2004).
- [30] G. Vincent, N. Antunes, and M. Hindmarsh, Phys. Rev. Lett. **80**, 2277 (1998).
- [31] J. N. Moore, E. P. S. Shellard, and C. J. A. P. Martins, Phys. Rev. D **65**, 023503 (2002).
- [32] T. W. B. Kibble and N. Turok, Phys. Lett. B **116**, 141 (1982).
- [33] N. Turok, Nucl. Phys. **B242**, 520 (1984).
- [34] A. L. Chen, D. A. DiCarlo and S. A. Hotes, Phys. Rev. D **37**, 863 (1988).
- [35] D. DeLaney, K. Engle, and X. Scheick, Phys. Rev. D **41**, 1775 (1990).
- [36] X. A. Siemens and T. W. B. Kibble, Nucl. Phys. **B438**, 307 (1995).
- [37] P. Bhattacharjee and N. C. Rana, Phys. Lett. B **246**, 365 (1990).
- [38] AGASA Collaboration, M. Takeda *et al.*, Astropart. Phys. **19**, 447 (2003); HiRes Collaboration, R. U. Abbasi, *et al.*, Phys. Rev. Lett. **92**, 151101 (2004).
- [39] P. Sreekumar *et al.*, Astrophys. J. **494**, 523 (1998).
- [40] S. Davidson and A. Ibarra, Phys. Lett. **B535**, 25 (2002).
- [41] W. Buchmuller, P. Di Bari, and M. Plumacher, Nucl. Phys. **B643**, 367 (2002).
- [42] J. A. Harvey and M. S. Turner, Phys. Rev. D **42**, 3344 (1990).