

Distance duality relation from x-ray and Sunyaev-Zel'dovich observations of clusters

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X-ray and Sunyaev-Zel'dovich data of clusters of galaxies enable to construct a test of the distance duality relation between the angular and luminosity distances. We argue that such a test on large cluster samples may be of importance, as a consistency check, while trying to distinguish between various models accounting for the acceleration of the universe. The analysis of a data set of 18 clusters shows no significant violation of this relation for a Λ -Cold Dark Matter (CDM) model. The origin and amplitude of systematic effects and the possibility to increase the precision of this method are discussed.

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I. INTRODUCTION

Most cosmological observations provide compelling evidences that our universe is undergoing a late time acceleration phase [1]. However, there are still several debates about the physical interpretation of these observations. While it seems clear that the Friedmann equations for a universe only composed of normal matter (i.e. radiation and dust) even including dark matter cannot explain the current data, there are different ways of facing this fact. Either one can conclude that the interpretation of the cosmological data are not correct (i.e. we do not accept the evidence for the acceleration of the universe, see Ref. [1] for a recent critical review and, e.g., Ref. [2]) or one tries to introduce new degrees of freedom in the cosmological model. In this latter case, these extra degrees of freedom, often referred to as *dark energy*, can be introduced as a new kind of matter or as a new property of gravity.

In the first approach one assumes that gravitation is described by general relativity while introducing new forms of gravitating components, beyond the standard model of particle physics, which must have some effective negative pressure to explain the acceleration of the universe. Various candidates such as a cosmological constant, quintessence [1,3] with many potentials, K-essence [4] etc. have been proposed. But, one is still left with the cosmological constant problem [5] (why is the density of

vacuum energy expected from particle physics so small?) as well as the time coincidence problem (why does the dark energy starts dominating today?) unsolved. From a cosmological point of view, these models are characterized by their equation of state which can be reconstructed from the function $E(a) = H^2(a)/H_0^2$ where H_0 is the Hubble constant at present and a the scale factor, either using the observation of background quantities or the growth of cosmic structures [6].

The other route is to allow for modification of gravity. This means that the only long-range force that cannot be screened is assumed to be not described by general relativity. Once such a possibility is considered, many classes of models exist (see, e.g., Ref. [7]). For instance, a light scalar field can couple to matter leading to models of extended quintessence [8] and more generally to scalar-tensor types of theories. Such theories have some difficulties to explain the current cosmological observations [9] without a quintessencelike potential for the scalar field or a cosmological constant. This scalar field may also be at the origin of some variation of the fundamental constants, depending on its couplings, and violation of the universality of free fall (see Ref. [10] for a review). Other possibilities include braneworld models in which the standard model fields are localized on a 3-dimensional brane embedded in a higher dimensional spacetime. Among braneworld models, a subclass of models have the property of allowing for deviation from 4-dimensional Einstein gravity on large scales. This is, for example, the case of some multibrane models [11], multigravity [12], brane-induced gravity [13] or simu-

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lated gravity [14]. In such models, gravity is not mediated only by massless gravitons; one therefore expects to have deviations from Newton inverse square law on large scales. Testing the Poisson equation on large scales may be a way to distinguish between these alternatives [15,16].

The different types of models are summarized schematically on Fig. 1. A diagnostic of the cause of the acceleration of the universe will require to make many tests. In particular, the reconstruction of the function $E(a)$ (or equivalently measuring the effective equation of state of the dark energy) will not be sufficient to distinguish between many models. It is thus important to simultaneously check for the Poisson equation, the growth of structures, and the variation of the constants.

Among these tests it has recently been pointed out in Ref. [17] that the reciprocity relation, and the distance duality relation that derives from it, have also to be checked. The *reciprocity relation* is a relation between the source angular distance, r_s , and the observer area distance, r_o . The former is defined by considering a bundle of null geodesics diverging from the source and which subtends a solid angle $d\Omega_s$ (see Fig. 2). This bundle has a cross section dS_s and the source angular distance is defined by the relation

$$dS_s = r_s^2 d\Omega_s. \quad (1)$$

The observer area distance r_o is defined analogously by considering a null geodesic bundle converging at the observer by

$$dS_o = r_o^2 d\Omega_o. \quad (2)$$

It can be shown that if photons travel along null geodesics and the geodesic deviation equation holds then these two distances are related by the reciprocity relation (see Ref. [18] for a derivation)

$$r_s^2 = r_o^2 (1+z)^2, \quad (3)$$

regardless of the metric and matter content of the space-time. Unfortunately, the solid angle $d\Omega_s$ cannot be measured so that r_s is not an observable quantity. But, it can be shown that, if the number of photons is conserved, the source angular distance is related to the luminosity distance, D_L , by the relation [18]

$$D_L = r_s (1+z). \quad (4)$$

It follows that there exists a *distance duality relation*

$$D_L = D_A (1+z)^2 \quad (5)$$

that holds between the angular distance D_A , the luminosity distance D_L , and the redshift z . This relation can be checked observationally.

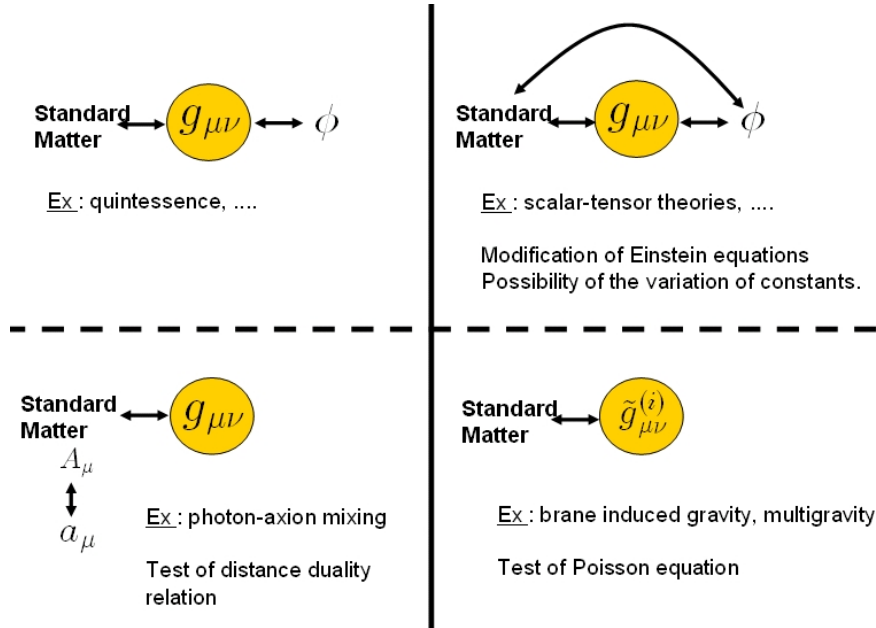


FIG. 1 (color online). Summary of the different classes of models and of the specific tests that can help distinguish between them (besides the equation of state and the growth of cosmic structures). The classes differ according to the kind of new fields and to the way they couple to the metric $g_{\mu\nu}$ and to the standard matter fields. Upper-left class consists of models in which a new kind of gravitating matter is introduced, e.g., quintessence. In the upper-right class, a light field induces a long-range force so that gravity is not described by a spin-2 graviton only. This is the case of scalar-tensor theories of gravity. In this class, Einstein equations are modified and there may be a variation of the fundamental constants. The lower-right class corresponds to models in which there may exist massive gravitons, such as in some class of braneworld scenarios. These models predict a modification of the Poisson equation on large scales. In the last class (lower-left), the distance duality relation may be violated.

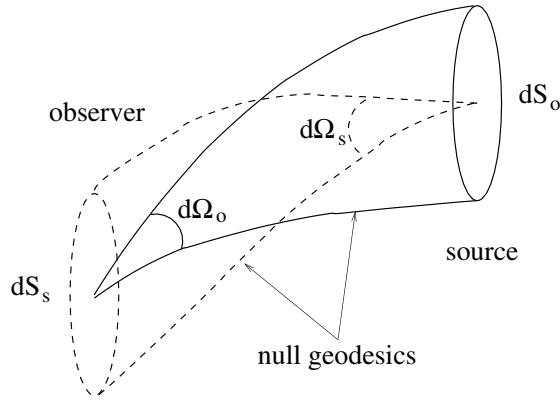


FIG. 2. A bundle of null geodesics diverging from the source (dashed line) subtending a solid angle $d\Omega_s$ has a cross section dS_s at the observer while a bundle converging at the observer (solid line) subtending a solid angle $d\Omega_o$ has a cross section dS_o at the source.

While the reciprocity relation holds as soon as photons follow null geodesic and that the geodesic deviation equation is valid, the distance duality relation will hold if the reciprocity relation is valid and the number of photon is conserved. In fact, one can show that in a metric theory of gravitation, if Maxwell equations are valid, then both the reciprocity relation and the area law are satisfied and so is the distance duality relation (see Ref. [18]).

There are many possibilities for one of these conditions to be violated. For instance the nonconservation of the number of photons can arise from absorption by dust, but more exotic models involving photon-axion oscillation in an external magnetic field [19] can also be a source of violation [20]. Note also that in principle both the reciprocity and distance duality relations hold for infinitesimal light bundles so that gravitational lensing may be a source of violation for macroscopic bodies. More drastic violations would arise from theories in which gravity is not described by a metric theory and in which photons do not follow null geodesic.

In this paper, we propose and explore a potential new test of the distance duality relation based on Sunyaev-Zel'dovich [21] (SZ) and X-ray measurements of clusters of galaxies. In Sec. II, we first show that when the relation (5) does not hold, cluster data do not give a measurement of the angular distance but of D_A/η^2 where

$$\eta(z) \equiv \frac{D_A}{D_L}(1+z)^2. \quad (6)$$

In Sec. III, we use existing cluster data to search for any hint that $\eta = 1$ may be excluded and then we discuss in Sec. IV the possibility to improve the accuracy of the test.

II. RECIPROcity RELATION FROM GALAXY CLUSTER OBSERVATIONS

Galaxy clusters are known as the largest gravitationally bound systems in the universe. They contain large quantities of hot and ionized gas which temperatures are typically 10^{7-8} K. The spectral properties of intracluster gas show that it radiates through bremsstrahlung in the X-ray domain. Therefore, this gas can modify the Cosmic Microwave Background (CMB) spectral energy distribution through inverse Compton interaction of photons with free electrons. This is the so-called SZ effect. It induces a decrement in the CMB brightness at low frequencies and an increment at high frequencies.

The possibility of using the SZ effect together with X-ray emission of galaxy clusters to measure angular distances was suggested soon after the SZ effect was pointed out (see, for example, Ref. [22]). Used jointly, they provide an independent method to determine distance scales and thus to measure the value of the Hubble constant (e.g. Ref. [23,24] for details).

In brief, the method is based on the fact that the CMB temperature (i.e. brightness) decrement due to the SZ effect is given by

$$\Delta T_{SZ} \sim \overline{Ln_e T_e} \quad (7)$$

where the bar refers to an average over the line of sight and L is the typical size of the line of sight in the cluster. T_e is the electron temperature, and n_e is the electron density. Besides, the total X-ray surface brightness is given by

$$S_X \sim \frac{V}{4\pi D_L^2} \overline{n_e n_p T_e^{1/2}} \quad (8)$$

where the volume V of the cluster is given in terms of its angular diameter by $V = D_A^2 \theta^2 L$. It follows that

$$S_X \sim \frac{\theta^2}{4\pi} \frac{D_A^2}{D_L^2} \overline{Ln_e n_p T_e^{1/2}}. \quad (9)$$

The usual approach [22], is to assume the distance duality relation ($\eta = 1$) so that forming the ratio $\Delta T_{SZ}^2/S_X$ eliminates n_e . Then, using a measurement of the angular diameter of the cluster and Eq. (14) one gets an estimate of the angular diameter distance and thus the Hubble constant. As a first conclusion, we point out that this method determines the angular distance only if the distance duality relation is valid. Therefore one needs to be careful when using such data to test the distance duality relation.

To make this point more precise, let us come back to the details of the method assuming the classical β -model for the galaxy cluster [25], that is assuming that the electron density of the hot intracluster gas has a profile of the form

$$n_e(r) = n_0 \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-3\beta/2}. \quad (10)$$

for $0 < r < R_{\text{cluster}}$ and 0 otherwise, R_{cluster} being the maximum extension of the cluster. The temperature decrement due to the SZ effect in the Rayleigh-Jeans part of the spectrum is given by

$$\Delta T_{\text{SZ}}(\theta) = -2 \frac{kT_0}{m_e c^2} \sigma_T \int_{-\ell_{\text{max}}}^{\ell_{\text{max}}} n_e d\ell \quad (11)$$

where we have assumed that the temperature of the hot gas, T_e , is independent of r [$T_0 \equiv T_e(r=0)$]. $2\ell_{\text{max}}$ is the length of the path along the line of sight inside the halo of the cluster and θ is the angular radial position projected on the celestial sphere from the cluster center. The X-ray emission is due to thermal bremsstrahlung and the surface brightness in a beam of angular diameter $\delta\theta$ takes the form

$$S_X(\theta) = \frac{\delta\theta^2}{4\pi} \frac{D_A^2}{D_L^2} \int_{-\ell_{\text{max}}}^{\ell_{\text{max}}} \frac{dL_X}{dV} d\ell \quad (12)$$

where the emissivity in the frequency band $[\nu_1, \nu_2]$ is given by

$$\frac{dL_X}{dV} = \alpha(T_e, \nu_1, \nu_2, z) n_e^2. \quad (13)$$

$\alpha(T_e, \nu_1, \nu_2, z)$ is a function that depends on the properties of the free-free emission for ions, on the mass fraction of hydrogen and on the gas temperature (see, e.g., Refs. [23,24] for its expression). Introducing the angle θ_c by

$$\theta_c = r_c/D_A, \quad (14)$$

where r_c is the cluster core radius, and using the profile (10) we obtain in the limit $R_{\text{cluster}} \rightarrow \infty$

$$\Delta T_{\text{SZ}}(\theta) = -2 \frac{kT_0}{m_e c^2} \sigma_T n_0 r_c B\left(\frac{3\beta-1}{2}, \frac{1}{2}\right) \left[1 + \left(\frac{\theta}{\theta_c} \right)^2 \right]^{(1-3\beta)/2} \quad (15)$$

and

$$S_X(\theta) = \frac{\delta\theta^2}{4\pi} \frac{D_A^2}{D_L^2} \alpha n_0^2 r_c B\left(\frac{6\beta-1}{2}, \frac{1}{2}\right) \left[1 + \left(\frac{\theta}{\theta_c} \right)^2 \right]^{(1-6\beta)/2}, \quad (16)$$

where B is the Euler beta function. Using the definition of η from Eq. (6), this latter expression rewrites as

$$S_X(\theta) = \frac{\delta\theta^2}{4\pi} \frac{\eta^2(z)}{(1+z)^4} \alpha n_0^2 r_c B\left(\frac{6\beta-1}{2}, \frac{1}{2}\right) \left[1 + \left(\frac{\theta}{\theta_c} \right)^2 \right]^{(1-6\beta)/2}. \quad (17)$$

As expected, $\Delta T_{\text{SZ}}^2/S_X$ eliminates n_e and gives a measurement of the core radius r_c from which we can deduce

the angular diameter distance through Eq. (14). When $\eta \neq 1$, what is thus extracted from the data is an estimate of $\tilde{r}_c = r_c/\eta^2$.

It follows from this analysis that, if we do not assume the distance duality relation to hold, what is in fact determined is $D_A^{\text{data}}(z) = \tilde{r}_c/\theta_c$ which differs from the angular distance. We thus have access to

$$D_A^{\text{data}}(z) = D_A(z)/\eta^2(z) \quad (18)$$

which reduces to the angular diameter distance only when the distance duality relation holds.

III. METHOD AND DATA ANALYSIS

Our method is straightforward once we have made the previous remark. Using a data set of angular distances determined from the combination of X-ray and SZ measurements, we have access to $\{z, D_A(z)/\eta^2(z)\}$. To get η one needs to know the angular diameter distance. The first possibility would be to use another set of observations, such as, e.g., type Ia supernovae (SNIa) data, to get either D_A or D_L . Another possibility, and probably the most robust, is to estimate it from its theoretical expression in a given cosmological model. While the test is completely general, it is then restricted to the class of chosen models. In a Friedmann-Lemaître universe

$$D_A^{\text{Th}}(z) = \frac{1}{1+z} f_K \left[\int_{1/(1+z)}^1 \frac{d}{x} x^2 E(x) \right] \quad (19)$$

where $x = 1/(1+z)$ and f_K is defined by

$$f_K(u) = \left(\frac{\sin\sqrt{K}u}{\sqrt{K}}, u, \frac{\sinh\sqrt{-K}u}{\sqrt{-K}} \right) \quad (20)$$

respectively for $K = H_0^2(1 - \Omega_{\text{mat}}^0 - \Omega_{\Lambda}^0)/c^2$ positive, null and negative. The function $E^2(x) = H(x)/H_0$ is explicitly given, for a Λ -CDM model by

$$E^2(x) = \Omega_{\text{mat}}^0 x^{-3} + \Omega_{\Lambda}^0 + (1 - \Omega_{\text{mat}}^0 - \Omega_{\Lambda}^0) x^{-2}, \quad (21)$$

where Ω_{mat}^0 and Ω_{Λ}^0 are, respectively, the present matter and cosmological constant density parameters.

We estimate $\eta(z)$ as

$$\eta(z) = \sqrt{D_A^{\text{Th}}/D_A^{\text{data}}}. \quad (22)$$

The error bars on this quantity will be estimated by a combination of the data error bars and of the one σ error bars on the cosmological parameters as obtained from Ref. [26]

$$\Omega_{\text{mat}}^0 = 0.29 \pm 0.07, \quad \Omega_{\Lambda}^0 = 0.73 \pm 0.05, \quad (23)$$

$$h = 0.73 \pm 0.04.$$

Note that in the case of a detection of $\eta \neq 1$, the interpretation of the signal is not trivial since a varying equation of state may be undistinguishable from a violation of the distance duality relation. In such a case going

back to SNIa data to get D_L and of X-ray data to get D_A/η^2 may help to break this degeneracy.

We use the catalog by Reese *et al.* [27] that contains 18 galaxy clusters, with redshifts ranging from 0.142 to 0.784, all observed in X-ray and SZ (see Table I). Combining these data as discussed in Sec. II together with the theoretical estimate of the angular distance, we get a measurement of $\eta(z)$ for each cluster, using Eq. (22). The result is summarized on Table I and is depicted on Fig. 3.

The question is then whether this data set is compatible with $\eta = 1$ or not. As can be seen from the original data (see Ref. [27]), the error bars on D_A^{data} are not symmetric. To derive the distribution of η we proceed in the following way. We first assume that the data points are independent so that the likelihood $L = P(\eta_1^{\text{data}} \dots \eta_n^{\text{data}} | \eta)$ can be factorized as $L = \prod_i P_i(\eta_i^{\text{data}} | \eta)$. We then introduce S defined as

$$S = -2 \ln L. \quad (24)$$

If the probabilities P_i are Gaussian, S reduces to the standard χ^2 . In one dimension, a variation $\Delta S = 1$ around the minimum of S will give the 1σ error bar. To proceed, we need to know the probabilities P_i . Without any further information, we assume that they follow a Gaussian distribution that is that $\eta_i^{\pm \delta_i^{\pm}}$ corresponds to a

TABLE I. The 18 clusters of the Reese *et al.* catalog with their redshift used in our analysis. D_A^{data} refers to the angular distance determined in Ref. [27] assuming that the distance duality relation holds. It leads, once this hypothesis is relaxed, to a measurement of η with one σ error bars.

cluster	redshift	D_A^{data} (Mpc)	η
MS 1137.5 + 6625	0.784	3179^{+1103}_{-1640}	$0.689^{+0.352}_{-0.127}$
MS 0451.6 - 0305	0.550	1278^{+265}_{-299}	$1.001^{+0.198}_{-0.136}$
Cl 0016 + 16	0.546	2041^{+484}_{-514}	$0.796^{+0.167}_{-0.116}$
RX J1347.5 - 1145	0.451	1221^{+368}_{-343}	$0.977^{+0.227}_{-0.161}$
Abell 370	0.374	4352^{+1388}_{-1245}	$0.489^{+0.115}_{-0.083}$
MS 1358.4 + 6245	0.327	866^{+248}_{-310}	$1.049^{+0.316}_{-0.166}$
Abell 1995	0.322	1119^{+247}_{-282}	$0.918^{+0.189}_{-0.124}$
Abell 611	0.288	995^{+325}_{-293}	$0.936^{+0.225}_{-0.159}$
Abell 697	0.282	998^{+298}_{-250}	$0.928^{+0.189}_{-0.149}$
Abell 1835	0.252	1027^{+194}_{-198}	$0.878^{+0.140}_{-0.108}$
Abell 2261	0.224	1049^{+306}_{-272}	$0.831^{+0.174}_{-0.131}$
Abell 773	0.216	1450^{+361}_{-332}	$0.697^{+0.129}_{-0.010}$
Abell 2163	0.202	828^{+181}_{-205}	$0.899^{+0.179}_{-0.119}$
Abell 520	0.202	723^{+270}_{-236}	$0.962^{+0.258}_{-0.176}$
Abell 1689	0.183	688^{+172}_{-163}	$0.948^{+0.181}_{-0.136}$
Abell 665	0.182	466^{+217}_{-179}	$1.149^{+0.374}_{-0.240}$
Abell 2218	0.171	1029^{+339}_{-352}	$0.754^{+0.213}_{-0.127}$
Abell 1413	0.142	573^{+171}_{-151}	$0.936^{+0.198}_{-0.148}$

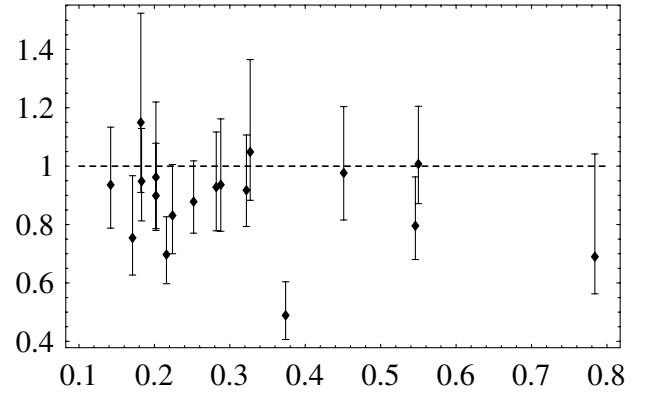


FIG. 3. η as a function of the redshift for the 18 clusters of the Reese *et al.* [27] catalog. The error bars include the observational error bars as determined by Reese *et al.* and the uncertainties in the cosmological parameters.

probability distribution function of the form

$$P(x) = \sqrt{\frac{2}{\pi}} \frac{1}{(\delta_i^+ + \delta_i^-)} \begin{cases} e^{-(x-\eta_i)^2/2\delta_i^+} & x > \eta_i \\ e^{-(x-\eta_i)^2/2\delta_i^-} & x \leq \eta_i \end{cases} \quad (25)$$

(see, e.g., Ref. [28]).

We perform this analysis using two data sets. The first set, labeled 1, contains all the clusters, while in the second, labeled 2, we have removed the point at $z = 0.374$ that lies outside of the other data points. This point corresponds to Abell 370 that clearly shows an apparent bimodal shape in optical and X-ray data, making its modelling as a single spherical potential a likely oversimplification for our purpose. The result is summarized on the plot 4 where we have displayed the function S , its minimum, and the 1σ confidence level. Figure 5 compares the probability distribution function of η to a Gaussian distribution fitted to the data. We obtain from

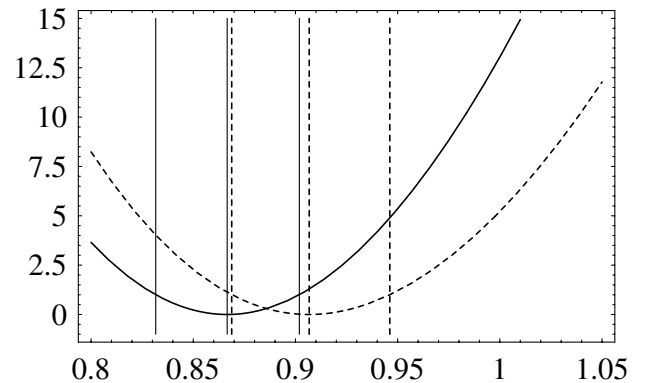


FIG. 4. The function S as a function of η for the two data sets (one in solid line and two in dashed lines). The vertical bars indicates for each set the position of the minimum and the 1σ confidence interval defined by $\Delta S = 1$.

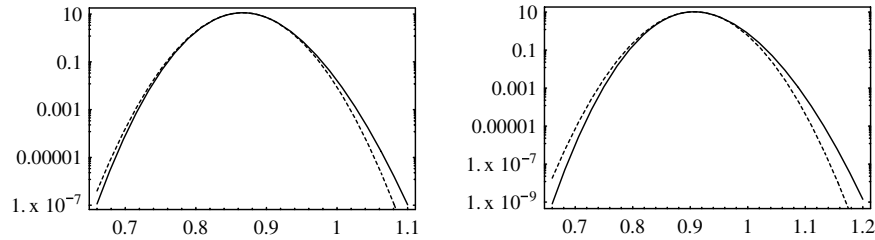


FIG. 5. Distribution of η obtained from the data of Fig. 3 compared with a Gaussian fit (dashed line) (left=set 1 and right= set 2).

our analysis that

$$\eta = 0.87_{-0.03}^{+0.04} \quad (26)$$

for the first data set and

$$\eta = 0.91_{-0.04}^{+0.04} \quad (27)$$

for the second.

Additionally we have analyzed, for the second data set (i.e. without Abell 370), separately the low redshift ($z < 0.3$) data and the high redshift data. We find for the low redshift set $\eta = 0.89_{-0.05}^{+0.05}$ and for the high redshift set $\eta = 0.95_{-0.07}^{+0.07}$. It is noteworthy that the largest departure from $\eta = 1$ is at small redshifts. These results and the one obtained for the whole data set suggest that there is no significant violation of the distance duality relation from combined X-ray and SZ measurements.

IV. PERSPECTIVES AND CONCLUSIONS

Testing for the distance duality relation and/or the reciprocity relation can give some insight on the puzzling apparent acceleration of the universe derived from cosmological observations. To distinguish between various models, one needs several complementary tests to the reconstruction of the Hubble parameter as a function of the scale factor (or equivalently of the equation of state). Examples of such tests are the test of the Poisson equation on large scales, the test of the constancy of the fundamental constants, and the test of the reciprocity relation. Figure 1 illustrates how the combination of these tests can help in identifying classes of models.

We have then shown that observations of galaxy clusters offer a test of the distance duality relation. In particular, using SZ and X-ray measurements of the same clusters give an estimate of D_A/η^2 . An important consequence is that X-ray/SZ combined analysis does not give a measurement of the angular distances when the distance duality relation is violated.

Testing the distance duality relation was already proposed in Ref. [17]. In that work, different sets of data were used such as type Ia supernovae data to get the luminosity distance, and the FRIBb radio galaxies, X-ray clusters and compact radio data to derive angular distances. Interestingly, a general three parameter form of $\eta(z)$ was proposed in Ref. [17], based on general arguments

about the violation of the conservation of the number of photons. Using their data, [17] found a two σ violation of the distance duality relation, mainly caused by an excess brightening of SNIa at redshift larger than 0.5. This analysis also allowed to put constraints on systematic effects, such as SNIa extinction or evolution, that may bias apparent magnitudes.

The analysis of the Reese *et al.* [27] cluster catalogue has shown that $\eta = 1$ is marginally consistent with the data. In our study, we have not searched for a fit of a general expression for $\eta(z)$. Our main concern was first to use X-ray and SZ combined measurements of galaxy clusters to check whether the critical value $\eta = 1$ was compatible with the data. Although we found that a value of η slightly lower than one is favored, drawing any conclusion on the possible discrepancy between the distances as predicted in the concordance model and those determined by our X-ray/SZ combined analysis is premature. There are indeed several systematic effects that may bias our derivation of $\eta(z)$, like oversimplification of cluster symmetry (substructures, triaxiality), or of their temperature and luminosity radial profiles. For example, the clusters that deviate most from the $\eta(z) = 1$ line on Fig. 2 are those that clearly show bimodal structures from X-ray, SZ and optical images (Abell 370, Abell 773, and Abell 1689). In contrast, those showing a single emission region with spherical shape lie very close to $\eta(z) = 1$. It is interesting to note that when the three most bimodal clusters are removed from the sample, the Reese *et al.* [27] remaining clusters lead to

$$\eta = 0.93_{-0.04}^{+0.05} \quad (28)$$

which is compatible to $\eta = 1$ at a two σ level. The shape, temperature distribution, etc., are key points to control in order to reduce the systematics that limit the accuracy of this test.

Therefore, we will have to make sure that the marginal trend¹ $\eta(z) < 1$ survives further explorations of this method with new data. This trend is indeed related to the fact that X-ray/SZ analysis systematically favors a rather low value of the Hubble constant. Besides, the

¹Note also that this trend is opposite to the one obtained in Ref. [17] (see their Fig. 1), which strengthens that there is in fact no systematic trend toward $\eta < 1$.

analysis of the low and high redshift subsets and the fact that the largest departure from $\eta = 1$ is at low redshift suggest that there is no violation of the distance duality relation. In particular, parametric forms, such as the one proposed in Ref. [17], predict a cumulative effect with redshift.

More specifically, a larger number of clusters spread over the whole redshift range and showing simple apparent geometry (i.e. as compact and spherical as possible) must be selected carefully. It will improve to lower systematics, to reduce errors bars on cluster data and, in turn, to provide much better angular distance estimates, mak-

ing the test of the distance duality relation from X-ray and SZ measurement an efficient method.

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