

Quantum inflaton, primordial perturbations, and CMB fluctuations

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We compute the primordial scalar, vector and tensor metric perturbations arising from *quantum field* inflation. Quantum field inflation takes into account the *nonperturbative quantum* dynamics of the inflaton consistently coupled to the dynamics of the (classical) cosmological metric. For chaotic inflation, the quantum treatment avoids the unnatural requirements of an initial state with all the energy in the zero mode. For new inflation it allows a consistent treatment of the explosive particle production due to spinodal instabilities. Quantum field inflation (under conditions that are the quantum analog of slow-roll) leads, upon evolution, to the formation of a condensate starting a regime of effective classical inflation. We compute the primordial perturbations taking the dominant quantum effects into account. The results for the scalar, vector and tensor primordial perturbations are expressed in terms of the classical inflation results. For a N -component field in a $O(N)$ symmetric model, adiabatic fluctuations dominate while isocurvature or entropy fluctuations are negligible. The results agree with the current Wilkinson Microwave Anisotropy Probe observations and predict corrections to the power spectrum in classical inflation. Such corrections are estimated to be of the order of $\frac{m^2}{NH^2}$, where m is the inflaton mass and H the Hubble constant at the moment of horizon crossing. An upper estimate turns to be about 4% for the cosmologically relevant scales. This quantum field treatment of inflation provides the foundations to the classical inflation and permits to compute quantum corrections to it.

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I. INTRODUCTION

Inflation is a stage of accelerated expansion in the very early Universe [1,2]. The present observations make inflationary cosmology the leading theoretical framework to explain the homogeneity, isotropy and flatness of the Universe, as well as the observed features of the cosmic microwave background. The Wilkinson Microwave Anisotropy Probe (WMAP) results [3–5] confirm the basic tenets of the inflationary paradigm.

There are many different models for inflation and most (if not all) of them invoke one or several scalar fields, the inflaton(s), whose dynamical evolution coupled with the space-time geometry leads to an inflationary epoch. The inflaton is a scalar field which provides an effective description for field condensates in the grand unified theories (GUT). The inflaton field is just an effective description of the particle dynamics and may not correspond to any real particle (even unstable). Fortunately, we do not need to know the detailed microscopical description given by the GUT to get the cosmological evolution. Indeed, a more precise description should be possible from a microscopic GUT. Somehow, the inflaton is to

the microscopic GUT theory like the Ginzburg-Landau theory of superconductivity is to the microscopic BCS superconductivity theory.

Most treatments of inflation study the evolution of the inflaton as a homogeneous *classical* scalar field. The quantum field theory interpretation is that this classical homogeneous field configuration is the expectation value of a quantum field operator in a translational invariant quantum state. In the classical treatments, the evolution of this coherent field configuration is studied through classical equations of motion, while fluctuations of the scalar field around this classical value are treated perturbatively and quantum mechanically, and provide the seeds for the scalar density perturbations of the metric [1].

However, since the energy scale of inflation is so high (the GUT scale), it is necessary a full *quantum* field theory description for the matter. Only such a quantum treatment permits a consistent description of particle production and particle decays.

An important class of inflationary models, the “large field” models [2], produce inflation starting from a large field amplitude configuration that rolls down the potential (for example: chaotic inflation). Another important class of inflationary models, the “small field” models [2], produce inflation starting from a small field amplitude configuration near the false vacuum of a spontaneously broken symmetry potential (for example: new inflation).

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The inflaton background dynamics for these models is usually studied in a classical framework, and in order to have a long inflationary period it is necessary that the field rolls down very slowly: for these models various conditions have been obtained which are different realizations of what we will call here the *classical slow-roll condition*,

$$\dot{\tilde{\varphi}}^2 \ll |m^2| \tilde{\varphi}^2. \quad (1.1)$$

This condition guarantees that there is inflation ($\ddot{a} > 0$) and that it lasts long enough. ($\tilde{\varphi}$ is the classical inflaton field, m its mass, and the dot denotes cosmic time derivative.)

One of the *shortcomings* of the *classical* chaotic inflationary scenarios is the need of an initial state with quite unnatural restrictions. In classical inflation the energy is dominated by the zero mode which leads the dynamics. In quantum inflation all modes contribute to the energy and one can choose, in particular, initial states with zero expectation value for the inflaton. Therefore, the initial state may not break the $\varphi \rightarrow -\varphi$ symmetry of the potential, while this symmetry is always broken in classical inflation. On the other hand, classical inflation scenarios do not allow a consistent treatment of the particle creation.

In order to overcome the restrictions on chaotic inflation and provide a consistent description of new inflation, a *quantum* treatment for the inflaton dynamics is needed [6–9]. This quantum treatment must be *nonperturbative* in order to consistently include the contribution of the excited modes since the energy is proportional to the inverse of the coupling. The nonperturbative method we use here is the large N expansion. Thus, we consider a N component inflaton field with an $O(N)$ invariant interaction. The presence of the $O(N)$ symmetry simplifies the calculation of the large N limit but is not a conceptual restriction.

The goal of this article is to compute the primordial perturbations produced in the *quantum* inflation scenarios, and show its relation with those produced by classical inflation.

We consider the case where the gravitational and the inflaton backgrounds are homogeneous, and the metric background is of the flat Friedman-Robertson-Walker (FRW) type,

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2). \quad (1.2)$$

We present a quantum field framework for inflation that takes into account the nonperturbative quantum dynamics of the inflaton consistently coupled to the dynamics of the (classical) metric. This generalized framework avoids the shortcomings of the classical chaotic and new inflation scenarios. It also clarifies *how and when* a classical effective scenario emerges.

Quantum gravity corrections can be neglected during inflation because the energy scale of inflation $\sim m \sim M_{\text{GUT}} \sim 10^{-6} M_{\text{Planck}}$. That is, quantum gravity effects are at most $\sim 10^{-6}$ and can be neglected in this context.

Quantum field inflation (under conditions that are the quantum analogue of slow-roll) leads, upon evolution, to the formation of a *condensate* starting a regime of effective classical field inflation. That is, the N -component quantum inflaton becomes an effective N -component classical inflaton, which can be directly expressed in terms of an effective *single*-field classical inflation scenario. The action structure, parameters (mass and coupling) and initial conditions for the effective classical quantum field inflation. This condensate description allows an easy computation of the primordial perturbations that takes into account the dominant quantum effects. That is, the effects of the quantum nature of the inflaton background (which is absent in classical inflation), and the effects due to the quantum nature of the inflaton and metric perturbations. This condensate description provides the primordial perturbation spectrum for quantum field inflation in terms of the well-known classical inflation results. We show that quantum inflation allows a consistent computation of the background and of the primordial perturbations with results in agreement with the observations.

For a $O(N)$ symmetric model, adiabatic fluctuations dominate while isocurvature and entropy fluctuations are negligible in agreement with the WMAP observations. Therefore, the presence of a large symmetry in multi field models is supported by observations. Nonsymmetric multi field models produce sizable isocurvature and entropy fluctuations.

Furthermore, the classical inflation scenario emerges as an effective description of the post-condensate inflationary period both for the *background* and for the *perturbations*. Therefore, the quantum treatment of inflation provides the *foundations* for classical inflation.

The cosmologically relevant density fluctuations [i.e., for the cosmic microwave background (CMB) anisotropies] in quantum inflation exhibit corrections compared with classical inflation. We find as the main source of such corrections the quantum changes in the effective mass felt by the cosmological fluctuations. An estimate for such quantum corrections to the power spectrum yields $\mathcal{O}(\frac{m^2}{NH^2})$ at the moment of horizon crossing which turns to be about 4% for the cosmologically relevant scales [and $N = O(1)$].

This paper is organized as follows: in Sec. II, we briefly present the single-field classical inflation scenario. In Sec. III, we present the results in quantum field inflation dynamics for the inflaton and the scalar factor (Sec. III A), and for the scalar, vector and tensor perturbations (Sec. III B). Both quantum chaotic and quantum

new inflation are treated. The conclusions are developed in Sec. IV. Finally, two appendices are devoted to the computation of scalar and tensor perturbations in multiple-field inflation.

II. CLASSICAL INFLATION

Let us briefly present first the single-field classical inflation scenario. The action for the *classical inflaton dynamics* is

$$\tilde{S}_{cl} = \tilde{S}_g + \tilde{S}_m + \delta\tilde{S}_g + \delta\tilde{S}_m, \quad (2.1)$$

where $\tilde{S}_{gr} + \tilde{S}_m$ describes the dynamics of the background for the metric and the inflaton, respectively, and $\delta\tilde{S}_{gr} + \delta\tilde{S}_m$ describe their perturbations. In the classical framework only the perturbations are quantized. We use the tilde, $\tilde{\cdot}$, to denote the quantities in classical inflation.

The gravitational action and its perturbation are

$$\tilde{S}_{gr} + \delta\tilde{S}_{gr} = -\frac{1}{16\pi G} \int \sqrt{-g} d^4x R, \quad (2.2)$$

where G is the universal gravitational constant, and R is the Ricci scalar for the complete metric $g_{\mu\nu}$. By expanding $g_{\mu\nu}$ in terms of the background FRW metric and its perturbation, $g_{\mu\nu} = g_{\mu\nu}^{(FRW)} + \delta g_{\mu\nu}$, the \tilde{S}_{gr} terms (those which do not contain $\delta g_{\mu\nu}$) and the $\delta\tilde{S}_{gr}$ terms can be identified. These terms take account of the dynamics of the FRW background and of the dynamics of the metric perturbations, respectively. (Detailed expressions can be found in [10].)

The classical matter action and its perturbation are

$$\tilde{S}_m + \delta\tilde{S}_m = \int \sqrt{-g} d^4x \left[\frac{1}{2} \partial_\alpha \tilde{\chi} \partial^\alpha \tilde{\chi} - \tilde{V}(\tilde{\chi}) \right] \quad (2.3)$$

$$= \int d^4x a^3(t) \left[\frac{1}{2} (\dot{\tilde{\chi}})^2 - \frac{1}{2} \frac{(\nabla \tilde{\chi})^2}{a^2(t)} - \tilde{V}(\tilde{\chi}) \right]. \quad (2.4)$$

We consider here a potential of the form

$$\tilde{V}(\tilde{\chi}) = \frac{1}{2} \tilde{m}^2 \tilde{\chi}^2 + \frac{\tilde{\lambda}}{4} \tilde{\chi}^4 + \frac{\tilde{m}^2}{4\tilde{\lambda}} \frac{1-\alpha}{2}, \quad (2.5)$$

with $\tilde{\alpha} \equiv \text{sign}(\tilde{m}^2) = \pm 1$,

where $\tilde{m}^2 > 0$ describes chaotic inflation, and $\tilde{m}^2 < 0$ describes new inflation. Expanding the inflaton field as $\tilde{\chi} = \tilde{\varphi} + \delta\tilde{\varphi}$, \tilde{S}_m stands for the terms without $\delta\tilde{\varphi}$ and takes account of the field background dynamics, while $\delta\tilde{S}_m$ containing the $\delta\tilde{\varphi}$ terms describes the field perturbations dynamics.

The initial state for chaotic inflation is a highly excited field state, i.e., a state with large $|\tilde{\varphi}|$, while for new inflation is a lowly excited state, i.e., with small $|\tilde{\varphi}|$.

In order to have a long inflationary period, it is necessary that the field rolls down towards the minimum very slowly. For these models various conditions have been

obtained that are different realizations of what we call here the *classical slow-roll condition*:

$$\dot{\tilde{\varphi}}^2 \ll |m^2| \tilde{\varphi}^2, \quad (2.6)$$

this condition guarantees that there is inflation and that it lasts long enough.

We will denote $|\tilde{\delta}_k^{(S)}(\tilde{m}^2, \tilde{\lambda})|^2$ and $|\tilde{\delta}_k^{(T)}(\tilde{m}^2, \tilde{\lambda})|^2$, the spectrum of primordial scalar and tensor perturbations, respectively, for classical inflation. These spectrums have been only computed in the literature [1,10–12] for *classical* inflation, but not for *quantum* inflation (see, however, Ref. [8]).

III. QUANTUM FIELD INFLATION

On the other hand, the action for *quantum field inflaton* is

$$S_q = \tilde{S}_g + S_m + \delta\tilde{S}_g + \delta S_m \quad (3.1)$$

where $\tilde{S}_g + S_m$ describes the dynamics of the background, and $\delta\tilde{S}_g + \delta S_m$ that of the perturbations. The important difference with classical inflation is that the dynamics of the inflaton background (S_m) is computed here in quantum field theory.

The gravitational terms have the same expressions as in the classical inflaton dynamics [Eq. (2.2)].

In our treatment we consider semiclassical gravity: the geometry is classical and the metric obeys the semiclassical Einstein equations where the right-hand side is the expectation value of the quantum energy-momentum tensor. (Quantum gravity corrections are at most of order $\sim m/M_{\text{Planck}} \sim M_{\text{GUT}}/M_{\text{Planck}} \sim 10^{-6}$ and can be neglected.)

In order to implement a nonperturbative treatment, we consider a N -component inflaton field $\tilde{\chi}$. The matter action, besides of being quantum, is for a N component inflaton $\tilde{\chi}$ and displays a $1/N$ factor in the $(\tilde{\chi}^2)^2$ term in order to allow a consistent implementation of the large N limit.

$$S_m + \delta S_m = \int \sqrt{-g} d^4x \left[\frac{1}{2} \partial_\alpha \tilde{\chi} \partial^\alpha \tilde{\chi} - V(\tilde{\chi}) \right] \quad (3.2)$$

$$= \int d^4x a^3(t) \left[\frac{1}{2} (\dot{\tilde{\chi}})^2 - \frac{1}{2} \frac{(\nabla \tilde{\chi})^2}{a^2(t)} - V(\tilde{\chi}) \right], \quad (3.3)$$

where $\tilde{\chi} = (\chi_1, \dots, \chi_N)$

$$V(\tilde{\chi}) = \frac{1}{2} m^2 \tilde{\chi}^2 + \frac{\lambda}{8N} (\tilde{\chi}^2)^2 + \frac{Nm^4}{2\lambda} \frac{1-\alpha}{2},$$

with $\alpha \equiv \text{sign}(m^2) = \pm 1$, (3.4)

For positive m^2 the $O(N)$ symmetry is unbroken while it is spontaneously broken for $m^2 < 0$. The first case describes chaotic inflation and the second one corresponds to new inflation.

The quantum field $\vec{\chi}$ can be expanded as its expectation value $\langle\vec{\chi}(x)\rangle$ plus quantum contributions which are in general large and cannot be linearized (except for k/a much larger than the effective mass). We therefore split the quantum contribution $\vec{\chi} - \langle\vec{\chi}(x)\rangle$ into large quantum contributions $\vec{\varphi}(x)$ (or background), plus small quantum contributions $\delta\vec{\varphi}(x)$. Thus, we express the N -component quantum scalar field $\vec{\chi}$ as

$$\vec{\chi}(x) = \langle\vec{\chi}(x)\rangle + \vec{\varphi}(x) + \delta\vec{\varphi}(x). \quad (3.5)$$

The dynamics of $\delta\vec{\varphi}(x)$ can be then linearized, and includes the cosmologically relevant fluctuations, that is those which had exited the horizon during the last $N_e \simeq 60$ e -folds of inflation.

Without loss of generality, we can chose the “1”-axis in the direction of the expectation value $\langle\vec{\chi}(x)\rangle$ of the inflaton, and collectively denote by $\vec{\chi}_\perp$ its $N - 1$ perpendicular directions. That is,

$$\vec{\chi}(x) = [\chi_\parallel(x), \vec{\chi}_\perp(x)], \quad \langle\vec{\chi}(x)\rangle = \left[\sqrt{N}\varphi(t), \vec{0} \right], \quad (3.6)$$

then, Eq. (3.5) reads

$$\vec{\chi}(x) = \left[\sqrt{N}\varphi(t) + \varphi_\parallel(x), \vec{\varphi}_\perp(x) \right] + [\delta\varphi_\parallel(x), \delta\vec{\varphi}_\perp(x)]. \quad (3.7)$$

$\varphi(t)$, $\varphi_\parallel(x)$ and $\vec{\varphi}_\perp(x)$ are the inflaton background contributions which come from the quantum expectation value and from the quantum fluctuations respectively; $\delta\vec{\varphi}(x)$ is the perturbation contribution. (The factor \sqrt{N} is made explicit for convenience.)

After expanding Eq. (3.2) using Eq. (3.5), S_m stands for the terms without $\delta\vec{\varphi}$ and describes the inflaton background dynamics, while δS_m stands for the remaining terms which describe the inflaton perturbation dynamics.

Both $(\varphi_\parallel, \vec{\varphi}_\perp)$ and $(\delta\varphi_\parallel, \delta\vec{\varphi}_\perp)$ represent quantum fluctuations of the field around its expectation value, and both can be expanded in Fourier modes. The field modes which contribute to the observable primordial perturbations (those that had exited the horizon during the last $N_e \simeq 60$ e -folds) are part of the perturbation $(\delta\varphi_\parallel, \delta\vec{\varphi}_\perp)$; while the field modes with larger spatial scales are part of the background $(\varphi_\parallel, \vec{\varphi}_\perp)$. In momentum space, let us call Λ the k -scale that separates the perturbation from the background. Namely, Λ must be smaller than the characteristic k -scale at which the modes exited the horizon $N_e \simeq 60$ e -folds before the end of inflation and larger than the k -scales that dominate the background.

The mode expansions for the background inflaton $(\varphi_\parallel, \vec{\varphi}_\perp)$ and the inflaton perturbations $(\delta\varphi_\parallel, \delta\vec{\varphi}_\perp)$ are then

$$\varphi_\parallel(\vec{x}, t) = \frac{1}{\sqrt{2}} \int_0^\Lambda \frac{d^3k}{(2\pi)^3} [b_k g_k(t) e^{i\vec{k}\cdot\vec{x}} + b_k^\dagger g_k^*(t) e^{-i\vec{k}\cdot\vec{x}}], \quad (3.8)$$

$$\vec{\varphi}_\perp(\vec{x}, t) = \frac{1}{\sqrt{2}} \int_0^\Lambda \frac{d^3k}{(2\pi)^3} [\vec{a}_k f_k(t) e^{i\vec{k}\cdot\vec{x}} + \vec{a}_k^\dagger f_k^*(t) e^{-i\vec{k}\cdot\vec{x}}], \quad (3.9)$$

$$\delta\varphi_\parallel(\vec{x}, t) = \frac{1}{\sqrt{2}} \int_\Lambda^\infty \frac{d^3k}{(2\pi)^3} [b_k g_k(t) e^{i\vec{k}\cdot\vec{x}} + b_k^\dagger g_k^*(t) e^{-i\vec{k}\cdot\vec{x}}], \quad (3.10)$$

$$\delta\vec{\varphi}_\perp(\vec{x}, t) = \frac{1}{\sqrt{2}} \int_\Lambda^\infty \frac{d^3k}{(2\pi)^3} [\vec{a}_k f_k(t) e^{i\vec{k}\cdot\vec{x}} + \vec{a}_k^\dagger f_k^*(t) e^{-i\vec{k}\cdot\vec{x}}], \quad (3.11)$$

with b_k , \vec{a}_k and b_k^\dagger , \vec{a}_k^\dagger being annihilation and creation operators, respectively, satisfying the canonical commutation relations. The background $(\varphi_\parallel, \vec{\varphi}_\perp)$, includes the modes with $k < \Lambda$, while the perturbations $(\delta\varphi_\parallel, \delta\vec{\varphi}_\perp)$ include the modes with $k > \Lambda$.

For asymptotic values of k (hence $k > k_{\text{Planck}}$) the modes tend to the vacuum modes, ensuring the finiteness of the total energy. The scale Λ is well above the k -modes that dominate the bulk of the energy, and well below the cosmologically relevant modes. The results are independent of the precise value of Λ .

This is due to the fact that modes with $k \gg m$ cannot be significantly excited since the energy density of the universe during inflation must be of the order $\geq 10m^2 M_{\text{Planck}}^2$.

On the other hand, relevant modes for the large scale structure and the CMB are today in the range from 0.1 Mpc to 10^3 Mpc. These scales at the beginning of inflation correspond to physical wavenumbers in the range

$$e^{N_T-60} 10^{16} \text{ GeV} < k < e^{N_T-60} 10^{20} \text{ GeV},$$

where N_T stands for the total number of e -folds (see, for example, Ref. [13]).

Therefore, there is an intermediate k -range of modes which are neither relevant for the background nor for the observed perturbation. Λ is inside this k -range, and the results are independent of its particular value. In usual cases we can safely choose for Λ ,

$$10m \lesssim \Lambda \lesssim 10^3 e^{N_T-60} m.$$

A. Quantum field inflation dynamics

We now describe the main features of the *background dynamics*, i.e., the a , φ and $(\varphi_\parallel, \vec{\varphi}_\perp)$ dynamics. We treat the inflaton as a full quantum field, and we study its dynamics in a self-consistent classical space-time metric (consistent with inflation at a scale well below the Planck

energy density). The dynamics of the space-time metric is determined by the semiclassical Einstein equations, where the source term is given by the expectation value of the energy-momentum tensor of the quantum inflaton field $G_{\mu\nu} = 8\pi m_{\text{pl}}^{-2} \langle T_{\mu\nu} \rangle$. Hence we solve *self-consistently* the coupled evolution equations for the classical metric and the quantum inflaton field.

The amplitude of the quantum fluctuations for a set of modes can be large (in quantum chaotic inflation due to the initial state, and in new inflation due to spinodal instabilities). This implies the need of a nonperturbative treatment of the evolution of the quantum state, and therefore we use the large N limit method.

In the large N limit, the longitudinal quantum contributions φ_{\parallel} are subleading by a factor $1/N$ [6,14,15]. Thus, the evolution equations for the inflaton background at leading order in large N can be expressed in terms of its expectation value, $\varphi(t)$, and the mode functions $f_k(t)$ of the transversal quantum contributions $\tilde{\varphi}_{\perp}$. In the large N limit, the evolution equations for the inflaton background are

$$\ddot{\varphi} + 3H\dot{\varphi} + \mathcal{M}^2\varphi = 0, \quad (3.12)$$

$$\ddot{f}_k + 3H\dot{f}_k + \left(\frac{k^2}{a^2} + \mathcal{M}^2\right)f_k = 0, \quad (3.13)$$

$$\text{with } \mathcal{M}^2 = m^2 + \frac{\lambda}{2}\varphi^2 + \frac{\lambda}{2}\int_R \frac{d^3k}{2(2\pi)^3} |f_k|^2, \quad (3.14)$$

and for the scale factor ($H \equiv \dot{a}/a$),

$$\begin{aligned} H^2 &= \frac{8\pi}{3m_{\text{pl}}^2} \rho; \\ \frac{\rho}{N} &= \frac{1}{2}\dot{\varphi}^2 + \frac{\mathcal{M}^4 - m^4}{2\lambda} + \frac{m^4}{2\lambda} \frac{1 - \alpha}{2} \\ &\quad + \frac{1}{4}\int_R \frac{d^3k}{(2\pi)^3} \left(|\dot{f}_k|^2 + \frac{k^2}{a^2} |f_k|^2 \right). \end{aligned} \quad (3.15)$$

where $\rho = \langle T^{00} \rangle$ is the energy density. The pressure ($p\delta_i^j = \langle T_i^j \rangle$) is given by

$$\frac{p + \rho}{N} = \dot{\varphi}^2 + \frac{1}{2}\int_R \frac{d^3k}{(2\pi)^3} \left(|\dot{f}_k|^2 + \frac{k^2}{3a^2} |f_k|^2 \right). \quad (3.16)$$

The index R denotes the renormalized expressions of these integrals [6]. This means that we must subtract the appropriate asymptotic ultraviolet behavior in order to make convergent the integrals in Eqs. (3.14)-(3.16):

$$\begin{aligned} |f_k|^2 &\stackrel{k^2 \gg B(t)}{\approx} \frac{1}{ka^2(t)} \left[1 - \frac{B(t)}{2k^2} + \frac{1}{8k^4} \left\{ 3B^2(t) \right. \right. \\ &\quad \left. \left. + a(t) \frac{d}{dt} [a(t)\dot{B}(t)] \right\} + \mathcal{O}\left(\frac{B^3(t)}{k^6}\right) \right], \\ |\dot{f}_k|^2 &\stackrel{k^2 \gg B(t)}{\approx} \frac{k}{a^4(t)} \left[1 + \frac{1}{2k^2} [B(t) + 2\dot{a}^2(t)] \right. \\ &\quad \left. - \frac{1}{8k^4} [B^2(t) + a^2(t)\ddot{B}(t) - 3a(t)\dot{a}(t)\dot{B}(t) \right. \\ &\quad \left. + 4\dot{a}(t)B] + \mathcal{O}\left(\frac{B^3(t)}{k^6}\right) \right], \end{aligned} \quad (3.17)$$

where

$$B(t) = a^2(t) \left[\mathcal{M}^2(t) - \frac{\mathcal{R}(t)}{6} \right]$$

and the scalar curvature is

$$\mathcal{R}(t) = 6 \left[\frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} \right].$$

Equations. (3.12)-(3.13) for the expectation value and for the field modes are analogous to damped oscillator equations, and the inflationary period ($\ddot{a} > 0$) corresponds to the overdamped regime of these damped oscillators.

We consider here two typical classes of quantum inflation models:

- (i) *Quantum chaotic inflation*, where inflation is produced by the dynamical *quantum* evolution of a excited initial pure state with large energy density (more details and the generalization to mixed states can be found in [6]). This state is formed by a distribution of excited modes. It can be shown that the initial conditions for a general pure state are given by fixing the complex values of $f_k(0)$ and $\dot{f}_k(0)$. Among these four real (two complex) numbers for each k mode, one is an arbitrary global phase, and another is fixed by the wronskian. The two remaining degrees of freedom fix the occupation number for each mode and the relative phase between $f_k(0)$ and $\dot{f}_k(0)$. The coherence between different k modes turns out to be determined by such relative phases.
- (ii) *Quantum new inflation*, where inflation is produced by the dynamical *quantum* evolution of a state with small inflaton expectation value, and small occupation numbers for the quantum modes, evolving with a spontaneously broken symmetry potential. (More details can be found in [7,8].)

The two classes of quantum inflation models have important differences in their initial state and in their background and perturbation dynamics (e.g., spinodal instabilities are present in new inflation and not in chaotic inflation). However, we stress here the common features

which allow a unified treatment of the computation for the primordial perturbations generated in these models.

In this quantum field inflation framework we have found the following *generalized slow-roll condition*

$$\dot{\varphi}^2 + \int_R \frac{d^3k}{2(2\pi)^3} |\dot{f}_k|^2 \ll m^2 \left(\varphi^2 + \int_R \frac{d^3k}{2(2\pi)^3} |f_k|^2 \right) \quad (3.18)$$

which guarantees inflation ($\ddot{a} > 0$) and that it lasts long (for both scenarios). (This condition includes the classical slow-roll condition $\dot{\varphi}^2 \ll m^2 \varphi^2$ as a particular case.) There is a wide class of quantum initial conditions satisfying Eq. (3.18) and leading to inflation that lasts long enough [6].

The quantum field dynamics considered here leads to *two inflationary epochs*, separated by a condensate formation:

- (1) *The pre-condensate epoch*—During this epoch the term $D \equiv \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{a^2} |f_k|^2$ in Eq. (3.15) has an important contribution to the energy density while it quickly decreases due to the exponential redshift of the excitations ($k/a \rightarrow 0$). This epoch ends at a time τ_A when the D contribution to the energy density becomes negligible, i.e., the k^2/a^2 contribution in the background evolution equations is negligible at $\tau = \tau_A$.

After outward horizon crossing, the time dependence of the modes factorizes and becomes k independent. The k^2/a^2 term in Eq. (3.13) becomes negligible, and all the modes satisfy the same damped oscillator equation. For $m^2 > 0$ the modes decrease (due to the damping), while for $m^2 < 0$ they grow (due to spinodal instabilities). At the end of this epoch ($t = \tau_A$) all the relevant modes for the background dynamics have exited the horizon, and the time dependence factorization allows to consider them as a zero mode condensate.

- (2) *The post-condensate quaside Sitter epoch*—The enormous redshift of the previous epoch assembles the quanta into a zero mode condensate, $\tilde{\varphi}_{\text{eff}}$, given by [7,8]

$$\tilde{\varphi}_{\text{eff}}^1(t) = \sqrt{N} \varphi(t), \quad (3.19)$$

$$\tilde{\varphi}_{\text{eff}}^i(t) = \sqrt{\int \frac{d^3k}{2(2\pi)^3} |f_k(t)|^2} \quad \text{for } i = 2, \dots, N,$$

with constant direction in the field space [due to the $O(N)$ invariance of the potential], and modulus

$$\tilde{\varphi}_{\text{eff}}(t) = \sqrt{N} \sqrt{\varphi^2(t) + \int \frac{d^3k}{2(2\pi)^3} |f_k(t)|^2}. \quad (3.20)$$

The modulus $\tilde{\varphi}_{\text{eff}}$ verifies the classical equations of motion,

$$\ddot{\tilde{\varphi}}_{\text{eff}} + 3H\dot{\tilde{\varphi}}_{\text{eff}} + \tilde{m}^2 \tilde{\varphi}_{\text{eff}} + \tilde{\lambda} \tilde{\varphi}_{\text{eff}}^3 = 0, \quad (3.21)$$

$$H^2 = \frac{8\pi}{3m_{\text{Pl}}^2} \rho, \quad \rho = \frac{1}{2} \dot{\tilde{\varphi}}_{\text{eff}}^2 + \frac{1}{2} \tilde{m}^2 \tilde{\varphi}_{\text{eff}}^2 + \frac{\tilde{\lambda}}{4} \tilde{\varphi}_{\text{eff}}^4, \quad (3.22)$$

$$\text{with } \tilde{\lambda} = \frac{\lambda}{2N}, \quad \tilde{m}^2 = m^2, \quad (3.23)$$

$$\tilde{\alpha} \equiv \text{sign}(\tilde{m}^2) = \pm 1.$$

The pressure is given by

$$p + \rho = \dot{\tilde{\varphi}}_{\text{eff}}^2. \quad (3.24)$$

Therefore, the background evolution in this period can be *effectively* described by a *classical* scalar field obeying the evolution Eq. (3.21) and with initial conditions defined at $t = \tau_A$. Moreover, it is important to stress that the *initial conditions for $\tilde{\varphi}_{\text{eff}}$ are fixed by the quantum state*:

$$\tilde{\varphi}_{\text{eff}}(\tau_A) = \sqrt{N} \sqrt{\varphi^2(\tau_A) + \int \frac{d^3k}{2(2\pi)^3} |f_k(\tau_A)|^2}. \quad (3.25)$$

Also the value of τ_A depends on the full quantum evolution *before* the formation of the condensate. τ_A is therefore a function of the coupling, the mass and the quantum initial conditions [6].

The previous result shows that after the formation of the condensate (both for chaotic and for new inflation), the background dynamics can be described by an effective classical background inflation whose action structure, parameters (mass and coupling) and initial conditions are fixed by those of the underlying quantum field inflation.

Let us call $N(t)$ the e -folds remaining at time t till the end of inflation:

$$N(t) \equiv \log \left[\frac{a_{\text{end}}}{a(t)} \right] = \int_t^{t_{\text{end}}} dt' H(t'). \quad (3.26)$$

In particular, the total number of e -folds is given by $N_T = \log \left[\frac{a_{\text{end}}}{a_{\text{initial}}} \right]$ which must be larger than $N_e \simeq 60$.

One particular consequence of quantum inflation, is that it can change the total number of e -folds N_T . In chaotic inflation N_T *decreases* if the initial state had excited modes with nonzero wavenumber (for constant initial energy). For example, if the initial energy is concentrated in a shell of wavenumber k_0 and for simplicity the quadratic term dominates the potential Eq. (3.4), we have [6]

$$N_T \simeq \frac{4\pi}{m_{\text{Pl}}^2 m^2} \frac{\rho_0}{1 + (k_0/m)^2} \quad (3.27)$$

(where the classical result is recovered at $k_0 = 0$). We have shown [6] that there are enough e -folds even for

$k_0 \sim 80m$ for reasonable choices of the initial energy density ($\rho_0 = 10^{-2}m_{\text{Pl}}^4$) and of the parameters (for instance, $Nm^2/[\lambda m_{\text{Pl}}^2] = 2 \times 10^5$).

B. Primordial perturbations in quantum field inflation

The relevant primordial perturbations are those that exited the horizon during the last e -folds of inflation [1]. As we have seen in the previous subsection the background (a , φ , φ_{\parallel} and $\tilde{\varphi}_{\perp}$) dynamics during the last e -folds in the quantum field inflation scenarios (both chaotic and new) are effectively classical. This will allow to compute the relevant primordial perturbations for these scenarios and express them in terms of the known perturbations for the corresponding single-field classical scenarios.

The more general metric perturbation $\delta g_{\mu\nu}$ can be decomposed as usual in scalar $\delta g_{\mu\nu}^{(S)}$, vector $\delta g_{\mu\nu}^{(V)}$ and tensor $\delta g_{\mu\nu}^{(T)}$ components [10,16]

$$g_{\mu\nu} = g_{\mu\nu}^{(\text{FRW})} + \delta g_{\mu\nu} = g_{\mu\nu}^{(\text{FRW})} + \delta g_{\mu\nu}^{(S)} + \delta g_{\mu\nu}^{(V)} + \delta g_{\mu\nu}^{(T)}, \quad (3.28)$$

with

$$g_{\mu\nu}^{(\text{FRW})} = a^2(\mathcal{T}) \begin{pmatrix} 1 & 0 \\ 0 & -\delta_{ij} \end{pmatrix}, \quad (3.29)$$

$$\delta g_{\mu\nu}^{(S)} = a^2(\mathcal{T}) \begin{pmatrix} 2\Phi & -\partial_i B \\ -\partial_i B & 2[\Psi\delta_{ij} - \partial_i\partial_j E] \end{pmatrix}, \quad (3.30)$$

$$\delta g_{\mu\nu}^{(V)} = -a^2(\mathcal{T}) \begin{pmatrix} 0 & -S_i \\ -S_i & \partial_i F_j + \partial_j F_i \end{pmatrix}, \quad (3.31)$$

$$\delta g_{\mu\nu}^{(T)} = -a^2(\mathcal{T}) \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix}. \quad (3.32)$$

Here, \mathcal{T} is the conformal time, and we define

$$\mathcal{H} \equiv \frac{1}{a} \frac{da}{d\mathcal{T}} = aH, \quad \mathcal{T} \equiv \int^t \frac{d\bar{t}}{a(\bar{t})},$$

Ψ , Φ , E , B , S_i , F_i and h_{ij} are functions of space and time. Ψ , Φ , E and B are scalars, S_i and F_i are three-vectors and h_{ij} is a three-tensor. i, j are three-spatial indexes raised and lowered with δ^{ij} and its inverse δ_{ij} , respectively. The following constraints are imposed,

$$\partial^i S_i = \partial^i F_i = 0, \quad h_i^i = 0, \quad \partial^j h_{ij} = 0,$$

in order to guarantee that S_i and F_i do not contain pieces that transform as scalars, and that h_{ij} do not contain pieces that transform as scalars or vectors. The gauge independence of the physical results allows us to choose the longitudinal gauge ($E = 0$, $B = 0$) for the scalar perturbations, and the vector gauge ($F_i = 0$) for the vector perturbations [10,16]. The advantage of these gauges is

that the equations have the same form as with the gauge invariant quantities

$$\begin{aligned} \Phi^{(gi)} &\equiv \Phi + \frac{1}{a} \frac{d}{d\mathcal{T}} \left[\left(B - \frac{dE}{d\mathcal{T}} \right) a \right]; \\ \Psi^{(gi)} &\equiv \Psi - \mathcal{H} \left(B - \frac{dE}{d\mathcal{T}} \right); \\ S_i^{(gi)} &\equiv S_i - \frac{dF_i}{d\mathcal{T}}. \end{aligned} \quad (3.33)$$

On the other hand, as the space-space perturbations of the energy-momentum tensor for the inflaton satisfies $\delta T_j^i \propto \delta_j^i$ we have $\Psi = \Phi$.

The perturbations are usually described in terms of the following spectral quantities

$$|\delta_k^{(S)}|^2 \equiv \frac{2k^3}{9\pi^2} \langle \Phi_k^2 \rangle; \quad (3.34)$$

$$|\delta_k^{(V)}|^2 \equiv \frac{2k^3}{9\pi^2} \langle S_i(k) S^i(k) \rangle; \quad (3.35)$$

$$|\delta_k^{(T)}|^2 \equiv \frac{2k^3}{9\pi^2} \langle h_{ij}(k) h^{ij}(k) \rangle; \quad (3.36)$$

which are the spectra for scalar, vector, and tensor perturbations, respectively. The index k in $\Phi_k \equiv \Phi(k)$, $S_i(k)$, $h^{ij}(k)$, ... denotes the k Fourier component of the respective perturbation, defined as

$$h^{ij}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[h^{ij}(k) e^{i\vec{k}\cdot\vec{x}} + h^{ij*}(k) e^{-i\vec{k}\cdot\vec{x}} \right]. \quad (3.37)$$

The scalar and tensor spectral indexes, n_S and n_T and their respective runnings $\frac{dn_S}{d\ln k}$, $\frac{dn_T}{d\ln k}$, are defined in the environment of a momentum scale k_0 by

$$\begin{aligned} |\delta_k^{(S)}|^2 &\equiv |\delta_{k_0}^{(S)}|^2 \left(\frac{k}{k_0} \right)^{n_S - 1 + \frac{1}{2} \frac{dn_S}{d\ln k} \ln\left(\frac{k}{k_0}\right)}, \\ |\delta_k^{(T)}|^2 &\equiv |\delta_{k_0}^{(T)}|^2 \left(\frac{k}{k_0} \right)^{n_T + \frac{1}{2} \frac{dn_T}{d\ln k} \ln\left(\frac{k}{k_0}\right)}. \end{aligned} \quad (3.38)$$

In the linear approximation, scalar, vector, and tensor perturbations evolve independently and thus can be considered separately [10].

I. Scalar perturbations

We now compute the *scalar metric perturbations* to the background, these are tightly coupled to the inflaton perturbations, and therefore, both perturbations have to be studied together [10,11].

We have shown that the background dynamics for quantum field inflation can be separated in two epochs: before and after the formation of the condensate. As the first one is short, in the more natural scenarios the last $N_e \simeq 60$ e -folds take place after the formation of the condensate. Thus, the cosmologically relevant scales of

the perturbations exit the horizon when the condensate was already formed.

Therefore, the dynamics of the perturbations after the formation of the condensate is well approximated by that given by the effective classical inflation background, Eqs. (3.21)-(3.25). Recall that the form of the effective classical evolution, and the values of the parameters and of the initial conditions for the condensate are determined by the underlying quantum field theory through the relations shown in the previous Sec. III A.

The precondensate period determines the initial conditions of the cosmological relevant modes for the post-condensate epoch at time $t = \tau_A$. These initial conditions are different from the vacuum ones. We choose in general,

$$\begin{aligned} f_k(0) &= \frac{1}{\sqrt{\Omega_k}}, \\ \dot{f}_k(0) &= -[i\Omega_k + H(0) + \beta_k \omega_k(0)]f_k(0), \end{aligned} \quad (3.39)$$

where Ω_k and β_k are functions of k that characterize the initial state and $\omega_k(t) = \sqrt{k^2 + \mathcal{M}^2(t)}$. The initial modes for vacuum initial conditions (as those customary used in classical inflation) are

$$\begin{aligned} f_k^{\text{cl inf}}(0) &= \frac{1}{\sqrt{\omega_k(0)}}, \\ \dot{f}_k^{\text{cl inf}}(0) &= -[i\omega_k(0) + H(0)]f_k^{\text{cl inf}}(0), \end{aligned}$$

However, the finiteness of the energy density [Eq. (3.15)] imposes that the difference between these initial conditions and the vacuum ones must asymptotically vanish for $k \rightarrow \infty$. We see from Eqs. (3.15) and (3.17) that the mode functions $|f_k(0)|^2$ and $|f_k^{\text{cl inf}}(0)|^2$ can differ asymptotically at most as

$$\mathcal{O}\left(\frac{a^5(t)m^5}{k^5}\right), \quad (3.40)$$

where we used the inflaton mass as scale for the effective inflaton mass.

A further source of quantum inflation effects arises from the evolution Eqs. (3.13). The effective mass squared \mathcal{M}^2 differs from the one used in classical inflation due to the last term in Eq. (3.14) which contains the integral over the square modulus of the mode functions. The contribution of the modes $k < \Lambda$ is taken into account during the post-condensate epoch by $\tilde{\varphi}_{\text{eff}}$. The modes $k > \Lambda$ contribute to $\mathcal{M}^2(t)$ as

$$\begin{aligned} \delta\mathcal{M}_\Lambda^2(t) &\equiv [\text{contribution to } \mathcal{M}^2(t) \text{ from modes } k > \Lambda] \\ &\sim \lambda a^2(t)m^4 \int_\Lambda^\infty \frac{d^3k}{k^5} \sim \lambda \frac{a^2(t)m^4}{\Lambda^2}. \end{aligned} \quad (3.41)$$

Furthermore, contributions from higher orders in $1/N$ will take the form,

$$\begin{aligned} \delta\mathcal{M}_N^2(t) &\equiv [\text{contribution to } \mathcal{M}^2(t) \text{ from } \frac{1}{N} \text{ corrections}] \\ &\sim \frac{m^2}{N}. \end{aligned} \quad (3.42)$$

Contributions $\delta\mathcal{M}^2(t)$ to the effective mass squared induce corrections in the square modulus of the mode functions $|f_k(t)|^2$ of the order $\mathcal{O}([a^2\delta\mathcal{M}^2]/k^3)$ as one sees from their large- k behavior Eq. (3.17). As a result of these three quantum effects: initial conditions Eq. (3.40), $\delta\mathcal{M}_\Lambda^2(t)$ in Eq. (3.41) and $\delta\mathcal{M}_N^2(t)$ in Eq. (3.42), the mode functions result,

$$\begin{aligned} \frac{|f_k(t)|^2}{|f_k^{\text{cl inf}}(t)|^2} &\stackrel{k \gg am}{\approx} 1 + \mathcal{O}\left(\frac{a^5(t)m^5}{k^5}\right) + \mathcal{O}\left(\frac{a^2(t)}{k^2} \delta\mathcal{M}_\Lambda^2(t)\right) \\ &\quad + \mathcal{O}\left(\frac{a^2(t)}{k^2} \delta\mathcal{M}_N^2(t)\right). \end{aligned} \quad (3.43)$$

Using the above estimates for $\delta\mathcal{M}_\Lambda^2(t)$ and $\delta\mathcal{M}_N^2(t)$ Eqs. (3.41)-(3.42) and expressing k in terms of the time where the mode exited the horizon through $k \sim aH$ we obtain,

$$\begin{aligned} \frac{|f_k(t)|^2}{|f_k^{\text{cl inf}}(t)|^2} &\stackrel{H \gtrsim m}{\approx} 1 + \mathcal{O}\left(\frac{m^5}{H^5}\right) + \mathcal{O}\left(\frac{\lambda a^2(t)m^4}{H^2 \Lambda^2}\right) \\ &\quad + \mathcal{O}\left(\frac{m^2}{NH^2}\right). \end{aligned} \quad (3.44)$$

The ratio $\frac{m}{H}$ is time dependent and depends on the inflationary scenario considered. In chaotic inflation $\frac{m}{H}$ decreases with time and takes a value $\frac{m}{H} \sim \frac{1}{5}$ when cosmologically relevant scales cross the horizon, that is about 50 e -folds before the end of inflation. Hence, we can make an order of magnitude estimate using the asymptotic approximation Eq. (3.43) at horizon crossing. Explicit calculations have to be performed in order to obtain the time dependence of these corrections. We can however state that the corrections in Eq. (3.44) are multiplied by functions of time [which are $\mathcal{O}(1)$].

The third and fourth terms in Eq. (3.43) and (3.44) give k -dependent corrections when we replace the time dependence present in both terms using $k \sim a(t)H(t)$ at horizon crossing.

Since $\frac{m}{H} \sim \frac{1}{5}$, the second term in Eq. (3.44) yields corrections of the order $\sim 10^{-4}$ and can be neglected. The third term gives very small corrections for times about 50 e -folds before the end of inflation of the order $\sim 10^{-9} \frac{m^2}{\Lambda^2}$ where we used that $\lambda \sim 10^{-12}$ and $\Lambda \gg m$. The validity of the small k -large k decomposition [Eqs. (3.5)-(3.11)] is confirmed by the smallness of this third contribution, and by the smallness of the correction induced by $\delta\mathcal{M}_\Lambda^2(t)$ [Eq. (3.41)] in the metric background through the Friedmann equations.

In conclusion, we find that the quantum inflaton corrections to the power spectrum are of the order

$$\frac{m^2}{NH^2} \sim 4\%. \quad (3.45)$$

This value corresponds to $N = O(1)$ and gives an upper estimate to the corrections.

Moreover, the corrections to the effective mass squared given by Eqs. (3.41)-(3.42) induce changes in the metric background through the Friedmann Eq. (3.15). Such changes on $H(t)$ and $a(t)$ produce changes of the order $\frac{1}{N}$ in the mode functions. Such effects are larger than those in Eqs. (3.44) and (3.45). However, they could be similar to those appearing in classical inflation since they arise from a change of the metric background.

Indeed, detailed calculations are needed to obtain a precise evaluation of the quantum inflation effects on density fluctuations.

It must be noticed that a complete study of the cosmologically relevant modes $f_k(t)$, ($k > \Lambda$) must include the graviton in a gauge invariant way. The order of magnitude of the estimates given above [Eqs. (3.43)-(3.44)] is not changed by that effect.

During the condensate period further quantum effects can also come from the quantum interaction between the quantum inflaton background and the quantum perturbations. However, these effects are expected to be diluted due to the large difference in k .

In addition, the precondensate period enters in the computation of the cosmologically relevant perturbations through the determination of the initial state for the condensate at $t = \tau_A$.

In summary, we compute the scalar perturbations dynamics using the effective classical inflaton background and considering initial quantum vacuum conditions for the modes responsible of the formation of cosmic structure. As we have shown this treatment takes account of the dominant quantum effects: the quantum nature of the inflaton background (absent in classical inflation), and the quantum nature of the inflaton and metric perturbations.

The most general scalar density perturbation is the sum of an adiabatic and an isocurvature (or entropy) perturbation [11,16].

$$\Phi_k = \Phi_{k \text{ ad}} + \Phi_{k \text{ iso}}. \quad (3.46)$$

The fluctuations in the different field components generate entropy or isocurvature perturbations which is characteristic of multi field models.

A generic multi field model has all three contributions: adiabatic, isocurvature and mixture of them, all of the same order. However, if the potential is completely symmetric for $O(N)$ rotations in the internal space, the slow-roll trajectories are straight lines in field space. We show below that in this case the isocurvature density perturba-

tions are negligible and the adiabatic contributions dominate. The adiabatic density perturbations are then the same as in the *single-field case*.

It is obvious that straight trajectories exist for $O(N)$ invariant interactions, that is, one can always assume a solution moving in a fixed direction in internal space. However, nonstraight trajectories can also exist for $O(N)$ invariant interactions. For such trajectories at least two components of the field should be nonzero and therefore, the ‘‘isospin’’ tensor $\chi^a \Pi_\chi^b - \chi^b \Pi_\chi^a$ will be nonzero. We have restricted ourselves to $O(N)$ states with zero isospin here as well as in refs.[6–8] since the universe as a whole should be expected to be in a $O(N)$ invariant state.

Adiabatic perturbations are produced by the field fluctuations parallel to the background inflaton trajectory in phase-space, and have nonzero total energy density. On the other hand, isocurvature perturbations are related to the field perturbations in other directions (thus, they require a multicomponent inflaton), and have vanishing total energy density.

In our case, due to the $O(N)$ invariance of the potential, the background inflaton solution does not change its direction in field space. In this case, we show in Appendix A that the *isocurvature density perturbations* are *negligible*

$$\Phi_{k \text{ iso}} = 0, \quad (3.47)$$

implying

$$|\delta_{k \text{ iso}}^{(S)}(m^2, \lambda)|^2 \equiv \frac{2k^3}{9\pi^2} \langle \Phi_{k \text{ iso}}^2 \rangle = 0, \quad (3.48)$$

$$|\delta_{k \text{ mix}}^{(S)}(m^2, \lambda)|^2 \equiv \frac{2k^3}{9\pi^2} (\langle \Phi_{k \text{ iso}} \Phi_{k \text{ ad}} \rangle + \langle \Phi_{k \text{ ad}} \Phi_{k \text{ iso}} \rangle) = 0. \quad (3.49)$$

The symmetric multi field models (as here invariant under $O(N)$) are consistent with the last CMB data from WMAP [3–5], as they indicate that the adiabatic contribution dominates and give an upper bound for isocurvature contributions. Initial conditions are consistent with being purely adiabatic.

It must be noticed that *nonsymmetric* models with different masses or couplings for the different components of the field, would lead to *non negligible* isocurvature density perturbations, analogously to the classical case.

As a consequence of the straight trajectory of the background inflaton in field space (see Appendix A), the power spectrum of *adiabatic scalar perturbations* for the quantum field inflation is

$$|\delta_{k \text{ ad}}^{(S)}(m^2, \lambda)|^2 = \left| \tilde{\delta}_{k \text{ ad}}^{(S)}\left(m^2, \frac{\lambda}{2N}\right) \right|^2, \quad (3.50)$$

where $|\tilde{\delta}_k^{(S)}(m^2, \tilde{\lambda})|^2$ is the power spectrum of scalar per-

turbations for the *single-field* classical background inflation [Eqs. (3.21)-(3.25)].

The result Eq. (3.50) express the scalar density perturbations for the quantum field inflation in terms of the associated effective classical background inflation, whose action structure, parameters and initial conditions are determined by the underlying quantum field inflation. The relation [Eq. (3.50)] allows to link to the primordial perturbations from classical field inflation. In terms of the slow-roll parameters,

$$\begin{aligned} |\delta_{k\text{ ad}}^{(S)}(m^2, \tilde{\lambda})|^2 &= \frac{1}{8\pi^2 M_{\text{Pl}}^2} \frac{H_{\mathcal{H}}^2}{\epsilon_{\mathcal{H}}} [1 - 2\epsilon_{\mathcal{H}} + 2(2 - \gamma - \ln 2)(2\epsilon_{\mathcal{H}} + \eta_{\mathcal{H}}) + \mathcal{O}(\epsilon_{\mathcal{H}}^2, \eta_{\mathcal{H}}^2, \eta_{\mathcal{H}}\epsilon_{\mathcal{H}})] \\ &= \frac{1}{12\pi^2 M_{\text{Pl}}^6} \frac{V^3}{V'^2} \left\{ 1 - \frac{5}{6} M_{\text{Pl}}^2 \left(\frac{V'}{V} \right)^2 + M_{\text{Pl}}^2 (2 - \gamma - \ln 2) \left[\left(\frac{V'}{V} \right)^2 + 2 \frac{V''}{V} \right] + \mathcal{O} \left(\left[M_{\text{Pl}} \frac{V'}{V} \right]^4 \right) \right\}. \end{aligned} \quad (3.52)$$

with $\gamma = 0.57721\dots$ the Euler constant. All quantities are evaluated at the time of horizon crossing, when $\mathcal{H} = k$, i.e., $Ha = k$. This is stressed by the subscript \mathcal{H} . Here, $M_{\text{Pl}}^2 = \frac{1}{8\pi G} = \frac{m_{\text{Pl}}^2}{8\pi}$ and primes denote derivatives with respect to the inflaton field $\tilde{\varphi}_{\text{eff}}$ which satisfies,

$$\dot{\tilde{\varphi}}_{\text{eff}} = -2M_{\text{Pl}}^2 H'(\tilde{\varphi}_{\text{eff}}), \quad H^2 \left(1 - \frac{\epsilon}{3} \right) = \frac{1}{3M_{\text{Pl}}^2} V(\tilde{\varphi}_{\text{eff}}). \quad (3.53)$$

Using the relation Eq. (3.50) also allows to express the scalar spectral index and its running [Eq. (3.38)] in terms of the slow-roll parameters as

$$\begin{aligned} n_S &= 1 - 4\epsilon_{\mathcal{H}} + 2\eta_{\mathcal{H}} = 1 - 6\epsilon_{\mathcal{H}} + 2\eta_V, \\ \frac{dn_S}{d \ln k} &= 10\epsilon_{\mathcal{H}}\eta_{\mathcal{H}} - 8\epsilon_{\mathcal{H}}^2 - 2\xi_{\mathcal{H}} \\ &= 16\epsilon_{\mathcal{H}}\eta_V - 24\epsilon_{\mathcal{H}}^2 - 2\xi_V, \end{aligned} \quad (3.54)$$

or in terms of the effective classical potential as

$$\begin{aligned} n_S &= 1 - 3M_{\text{Pl}}^2 \left(\frac{V'}{V} \right)^2 + 2M_{\text{Pl}}^2 \frac{V''}{V}, \\ \frac{dn_S}{d \ln k} &= -2M_{\text{Pl}}^4 \frac{V'V'''}{V^2} - 6M_{\text{Pl}}^4 \left(\frac{V'}{V} \right)^4 + 8M_{\text{Pl}}^4 \frac{V''V'^2}{V^3}. \end{aligned} \quad (3.55)$$

taken at the value of the field when the scale of interest exited the horizon.

Expressions (3.52), (3.54) and (3.55) get in addition quantum inflaton corrections of the order 4% [see Eq. (3.45)].

For example, in chaotic inflation when $\tilde{\varphi} \ll m/\sqrt{\lambda}$, the potential [Eq. (3.22)] is dominated by the quadratic term, while for $\tilde{\varphi} \gg m/\sqrt{\lambda}$ the quartic term dominates. Thus, for these limiting cases the potential has the form

$$V = \beta \tilde{\varphi}^b, \quad (3.56)$$

(where β and b are positive constants) implying that at N_e

$$\begin{aligned} \epsilon &\equiv 2M_{\text{Pl}}^2 \left(\frac{H'}{H} \right)^2 = \frac{1}{2} M_{\text{Pl}}^2 \left(\frac{V'}{V} \right)^2, & \eta &\equiv 2M_{\text{Pl}}^2 \frac{H''}{H}, \\ \eta_V &\equiv M_{\text{Pl}}^2 \frac{V''}{V} = \eta + \epsilon, & \xi &\equiv 4M_{\text{Pl}}^4 \frac{H'H'''}{H^2}, \\ \xi_V &\equiv M_{\text{Pl}}^4 \frac{V'V'''}{V^2} = \xi + 3\epsilon\eta. \end{aligned} \quad (3.51)$$

the adiabatic scalar perturbations can be expressed as

e -folds before the end of inflation $\tilde{\varphi}_{N_e}$ has the value

$$\tilde{\varphi}_{N_e}^2 \simeq 2N_e b M_{\text{Pl}}^2. \quad (3.57)$$

where we have used Eq. (3.26), i. e., $N_e = \frac{1}{M_{\text{Pl}}^2} \int_{\tilde{\varphi}_{\text{end}}}^{\tilde{\varphi}} \frac{V}{V'} d\tilde{\varphi}$ with $\tilde{\varphi}_{\text{end}} \ll \tilde{\varphi}$.

Our convention for the amplitude of scalar perturbations $|\delta_{k\text{ ad}}^{(S)}(m^2, \lambda)|^2$ Eqs. (3.50)-(3.52) is the same as the one used by the WMAP collaboration [3–5] (called $\Delta_{\mathcal{R}}^2(k)$ by them, WMAP only uses the leading order). The quoted WMAP_{ext} + 2dFGRS + Lyman α data [3–5] for the overall primordial spectrum amplitude is

$$A(k_0 = 0.002 \text{ Mpc}^{-1}) = 0.75_{-0.09}^{+0.08} (68\% CL),$$

or $0.71_{-0.11}^{+0.10} (68\% CL)$ using WMAP data alone. $A(k)$ is related to $|\delta_{k\text{ ad}}^{(S)}(m^2, \lambda)|^2$ by

$$\begin{aligned} |\delta_{k\text{ ad}}^{(S)}(m^2, \lambda)|^2 &\equiv \Delta_{\mathcal{R}}^2(k) = 800\pi^2 \left(\frac{5}{3} \right)^2 \frac{1}{T_{\text{CMB}}^2} A(k) \\ &\simeq 2.95 \times 10^{-9} A(k). \end{aligned}$$

where $T_{\text{CMB}} = 2.725K$. This factor comes from the relation between the CMB multipole coefficients (C_l) and $|\delta_{k\text{ ad}}^{(S)}(m^2, \lambda)|^2$. They are connected in the conventions of the CMBFAST code used by WMAP as

$$C_l = 4\pi T_{\text{CMB}}^2 \int \frac{9}{25} |\delta_{k\text{ ad}}^{(S)}(m^2, \lambda)|^2 [g_l(k)]^2 \frac{dk}{k},$$

with $g_l(k)$ being the radiation transfer function.

Using Eq. (3.52), this implies,

$$\frac{V^3}{V'^2} = \frac{\beta}{b^2} \tilde{\varphi}^{b+2} \simeq 2.62 \cdot 10^{-7} M_{\text{Pl}}^6, \quad (3.58)$$

for the scale when $k_0 = 0.002 \text{ Mpc}^{-1}$ exited the horizon.

For example, when the potential is dominated by the quadratic term, Eq. (3.58) implies

$$\begin{aligned} m &= 6.02 \cdot 10^{-6} M_{\text{Pl}} \approx 1.45 \cdot 10^{13} \text{ GeV}, \\ \varphi_{N_e} &= 15.5 M_{\text{Pl}} \approx 3.72 \cdot 10^{19} \text{ GeV}, \end{aligned} \quad (3.59)$$

while if the potential is dominated by the quartic term, Eq. (3.58) implies

$$\begin{aligned} \lambda &= 3.34 \cdot 10^{-13} N, \\ \varphi_{N_e} &= 21.9 M_{\text{Pl}} \approx 5.26 \cdot 10^{19} \text{ GeV}. \end{aligned} \quad (3.60)$$

Here the potentials have been evaluated at the field value Eq. (3.57) for $N_e = 60$ e -folds and $M_{\text{Pl}} = \frac{m_{\text{Pl}}}{\sqrt{8\pi}} = 2.4 \cdot 10^{18}$ GeV.

Plugging Eqs. (3.56)-(3.57) in Eqs. (3.55), we obtain

$$n_S = 1 - \frac{b+2}{2N_e}, \quad \frac{dn_S}{d \ln k} = -\frac{1}{2N_e^2}(b+2). \quad (3.61)$$

Evaluating these expressions $N_e = 60$ e -folds before the end of inflation (which correspond to a typical scale of astrophysical interest), if the quadratic term dominates we obtain

$$n_S = 0.97, \quad \frac{dn_S}{d \ln k} = -5.5 \cdot 10^{-4}, \quad (3.62)$$

while if the quartic term dominates we have

$$n_S = 0.95, \quad \frac{dn_S}{d \ln k} = -8.3 \cdot 10^{-4}, \quad (3.63)$$

The values obtained from these examples are compatible with the current WMAP data [3–5]. An exact scale-invariant spectrum (i.e., $n_S = 1, \frac{dn_S}{d \ln k} = 0$) is not yet excluded at more than 2σ level by WMAP data.

2. Vector perturbations

The vector metric perturbations [Eq. (3.31)] do not have any source in their evolution equation, because the energy-momentum tensor for a scalar field does not lead to any vector perturbation. The $(0i)$ components of the Einstein equations, in the absence of vector perturbation sources, gives

$$\Delta S_i = 0, \quad (3.64)$$

implying that there cannot be any space-dependent vector perturbations [in Fourier space $k^2 S_i(k) = 0$]. Therefore, the *vector perturbations* are *negligible* (as for classically driven inflation [16]).

$$|\delta_k^{(V)}(m^2, \lambda)|^2 = 0. \quad (3.65)$$

3. Tensor perturbations

As the energy-momentum tensor of a scalar field do not have tensor perturbations, the tensor metric perturbations [Eq. (3.32)] do not have any source in their equation. Therefore, the amplitude of tensor perturbations is determined only by the background evolution, which after the

condensate formation has an *effective single-field classical description*. Thus, the tensor perturbations for the quantum inflation scenario are

$$|\delta_k^{(T)}(m^2, \lambda)|^2 = \left| \tilde{\delta}_k^{(T)}\left(m^2, \frac{\lambda}{2N}\right) \right|^2 \quad (3.66)$$

where $|\tilde{\delta}_k^{(T)}(\tilde{m}^2, \tilde{\lambda})|^2$ is the power spectrum of tensor perturbations for *single-field* classical inflation.

It can be expressed in terms of the slow-roll parameters [see Appendix] as

$$\begin{aligned} |\tilde{\delta}_k^{(T)}(\tilde{m}^2, \tilde{\lambda})|^2 &= \frac{2}{\pi^2 M_{\text{Pl}}^2} H_{\mathcal{H}}^2 [1 + 2(1 - \gamma - \ln 2)\epsilon_{\mathcal{H}}] \\ &= \frac{2}{3} \frac{V}{\pi^2 M_{\text{Pl}}^4} \left[1 + \left(\frac{7}{6} - \gamma - \ln 2\right) M_{\text{Pl}}^2 \left(\frac{V'}{V}\right)^2 \right]. \end{aligned} \quad (3.67)$$

Using the relation Eq. (3.66) we can compute the tensor spectral index from the single-field classical inflation result, which in terms of the slow-roll parameters is:

$$\begin{aligned} n_T &= -2\epsilon, \\ \frac{dn_T}{d \ln k} &= 4\epsilon\eta_V - 8\epsilon^2 = -n_T(n_S - 1 - n_T), \\ n_T &= -M_{\text{Pl}}^2 \left(\frac{V'}{V}\right)^2, \end{aligned} \quad (3.68)$$

$$\frac{dn_T}{d \ln k} = -2M_{\text{Pl}}^4 \left(\frac{V'}{V}\right)^2 \left[\left(\frac{V'}{V}\right)^2 - \frac{V''}{V} \right].$$

taken at the value of the field when the scale of interest exited the horizon.

The tensor perturbations (3.67) and (3.68) only get quantum inflaton corrections from the changes in the metric background. These corrections, induced by $\delta \mathcal{M}^2$ [Eqs. (3.41)-(3.42)] through the Friedmann equation, are of the order $1/N$. They could be similar to those appearing in classical inflation since they arise from a change of the metric background.

For example, for chaotic inflation in the limiting cases $\tilde{\varphi} \ll m/\sqrt{\lambda}$ and $\tilde{\varphi} \gg m/\sqrt{\lambda}$, the potential and the inflaton field are given by Eqs. (3.56) and (3.57), respectively. Plugging these results in Eq. (3.68) yields,

$$n_T = -\frac{b}{2N_e}, \quad \frac{dn_T}{d \ln k} = -\frac{b}{2N_e^2}. \quad (3.69)$$

Evaluating this expression at $N_e = 60$ e -folds before the end of inflation, if the quadratic term dominates we obtain

$$n_T = -0.017, \quad \frac{dn_T}{d \ln k} = -0.0003; \quad (3.70)$$

while if the quartic term dominates we have

$$n_T = -0.033, \quad \frac{dn_T}{d \ln k} = -0.0005. \quad (3.71)$$

Future measurements for the amplitude and spectral index of tensor perturbations are important because the relation between n_T and the tensor to scalar amplitude ratio is model dependent, and therefore it will allow to discriminate between inflationary models (see below).

4. Tensor to scalar amplitude ratio

The tensor to scalar ratio r is defined as

$$r \equiv \frac{|\delta_k^{(T)}|^2}{|\delta_{k\text{ad}}^{(S)}|^2}. \quad (3.72)$$

From the previous expressions for the spectra of tensor [Eq. (3.66)] and adiabatic scalar perturbations [Eq. (3.50) and (3.66)], respectively, it follows that

$$r(m^2, \lambda) = \frac{|\delta_k^{(T)}(m^2, \lambda)|^2}{|\delta_{k\text{ad}}^{(S)}(m^2, \lambda)|^2} = \frac{|\tilde{\delta}_k^{(T)}(m^2, \frac{\lambda}{2N})|^2}{|\tilde{\delta}_{k\text{ad}}^{(S)}(m^2, \frac{\lambda}{2N})|^2} = \tilde{r}(m^2, \frac{\lambda}{2N}) \quad (3.73)$$

where $\tilde{r}(m^2, \lambda)$ is the tensor to scalar amplitude ratio for single-field classical inflation.

Expressing this result in terms of the slow-roll parameters Eqs. (3.51) we obtain

$$\begin{aligned} r &= 16\epsilon_{\mathcal{H}}[1 - 2(2 - \gamma - \ln 2)(\epsilon_{\mathcal{H}} + \eta_{\mathcal{H}})] \\ &\quad + \mathcal{O}(\epsilon_{\mathcal{H}}^2, \eta_{\mathcal{H}}\epsilon_{\mathcal{H}}, \eta_{\mathcal{H}}^2) \\ &= 8M_{\text{Pl}}^2 \left(\frac{V'}{V}\right)^2 \left[1 - 2(2 - \gamma - \ln 2)M_{\text{Pl}}^2 \frac{V''}{V}\right], \end{aligned} \quad (3.74)$$

and the following consistency relation at leading order

$$n_T = -\frac{r}{8}, \quad \frac{dn_T}{d\ln k} = \frac{r}{8} \left[n_S - 1 + \frac{r}{8} \right], \quad (3.75)$$

which is the same as for single-field classical inflation [16].

This consistency relation is model dependent, therefore simultaneous measurement of r and n_T will select between inflationary models. In particular the consistency relation Eq. (3.75) will change when isocurvature scalar perturbations are present.

For example, for chaotic inflation in the limiting cases $\tilde{\varphi} \ll m/\sqrt{\lambda}$ and $\tilde{\varphi} \gg m/\sqrt{\lambda}$, the potential and the field are given by Eqs. (3.56) and (3.57), respectively. Using the relation (3.75) and the result in Eq. (3.69) yields,

$$r = \frac{8b}{2N_e}. \quad (3.76)$$

Evaluating this expression 60 e -folds before the end of inflation, if the quadratic term dominates we obtain

$$r = 0.13; \quad (3.77)$$

while if the quartic term dominates we have

$$r = 0.27. \quad (3.78)$$

From the WMAP_{ext} + 2dFGRS + Lyman α data[3–5],

the upper bound for r is

$$r(k_0 = 0.002 \text{ Mpc}^{-1}) < 0.90 \quad (95\%CL).$$

The maximum likelihood single-field inflationary model for the WMAP_{ext} + 2dFGRS + Lyman α data set has $r = 0.42$. Detection and measurement of gravity wave power spectrum will be a further key test for inflation.

The no-prior r -limit $r < 0.90$ along with the $2 - \sigma$ upper limit on the amplitude $A(k_0 = 0.002 \text{ Mpc}^{-1}) = 0.75_{-0.09}^{+0.08}$ (68%CL), implies that the energy scale of inflation is:

$$V^{\frac{1}{4}} < 3.3 \times 10^{16} \text{ GeV}$$

at 95% confidence level.

The two limiting examples of the quadratic and quartic potentials (for which $\eta_V = \epsilon = 2(\frac{M_{\text{Pl}}}{\varphi})^2$ and $\eta_V = \frac{3}{2}\epsilon$ respectively), fall in the class B potentials models ($0 \leq \eta_V \leq 2\epsilon$) in WMAP classification[3–5]. The WMAP data analysis give for this class

$$\begin{aligned} 0.94 \leq n_S \leq 1.01, \quad -0.02 \leq \frac{dn_S}{d\ln k} \leq 0.01, \\ 0.007 \leq r \leq 0.26. \end{aligned}$$

A pure monomial quartic potential (minimally coupled) is disfavored at more than $3 - \sigma$ by WMAP data [3–5] since a too large r is produced.

We want to stress that excluding the quadratic mass term in the potential $V(\varphi)$ implies a nongeneric choice which is only justified at isolated points (critical points in statistical mechanics). Therefore, from a purely theoretical point of view, the pure quartic potential is a weird choice implying to fine tune to zero the coefficient of the quadratic term.

As stated at the beginning of the Introduction, the inflaton field must be considered an effective description of matter in the GUT scale in a Ginsburg-Landau approach. Therefore, in this context the inflaton potential $V(\varphi)$ should be generically a polynomial of degree four. [Higher degree terms should be irrelevant]. Moreover, shifting the minimum of $V(\varphi)$ to the origin implies that the lowest order term must be quadratic. A cubic term cannot be excluded in general but most potentials are assumed to be even for symmetry. This argument lefts us with a quadratic plus quartic polynomial as in Eq. (2.5).

IV. CONCLUSIONS

We present in this article, *both* for the background and the perturbations, a complete quantum field treatment of inflation that takes into account the nonperturbative quantum dynamics of the inflaton consistently coupled to the dynamics of the (classical) metric. We avoid in quantum inflation the unnatural requirements of an initial state with all the energy in the zero mode and breaking

the $\vec{\varphi} \rightarrow -\vec{\varphi}$ symmetry of the potential. For new inflation this quantum framework allows a consistent treatment of the explosive particle production due to spinodal instabilities.

Quantum field inflation (under conditions that are the quantum analog of slow-roll) leads, upon evolution, to the formation of a condensate starting a regime of effective classical field inflation. That is, the N -component quantum inflaton becomes an effective N -component classical inflaton, which can be directly expressed in terms of an effective *single*-field classical inflation scenario. The action structure, parameters (mass and coupling) and initial conditions for the effective classical field description are fixed by those of the underlying quantum field inflation.

We show that this effective description allows an easy computation of the primordial perturbations which takes into account the dominant quantum effects (quantum inflaton background and quantum inflaton and metric perturbations). The computation gives the primordial perturbations for quantum field inflation in terms of the classical inflation results.

In particular, isocurvature scalar perturbations are absent (at first order of slow-roll) due to the $O(N)$ invariance of the potential in agreement with the WMAP data. More general nonsymmetric potentials with different masses or couplings for the different components of the field would lead to non negligible isocurvature density perturbations. It is thus the presence of a large symmetry in multi field models that make them compatible with the present observations.

Quantum field inflation provides enough e -folds of inflation provided the generalized slow-roll condition is fulfilled. In the case of chaotic quantum field inflation the number of e -folds is lower than in classical inflation when modes with nonzero wavenumber are excited initially as shown in Eq. (3.27). As in classical inflation, the primordial spectrum of perturbations turns to be independent of the details of the initial quantum state. Quantum corrections to the power spectrum turn to be approximately of the order of $\frac{m^2}{NH^2} \sim 4\%$ for chaotic inflation. [This value is an upper estimate corresponding to $N = O(1)$].

In summary, the classical inflationary scenario emerges as an effective description of the post-condensate inflationary period both for the *background* and for the *perturbations*. Therefore, this generalized inflation provides the *quantum field foundations* for classical inflation, which is in agreement with CMB anisotropy observations [3–5,17].

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APPENDIX A: SCALAR PERTURBATIONS IN MULTIPLE-FIELD INFLATION

We treat here the generation of scalar density perturbations by multi field inflatons.

The approach of Ref. [16] is particularly useful in the model under study. The procedure begins defining a basis in the internal $\vec{\varphi}$ field space which allows a physical interpretation of the various scalar field components. The first basis vector, \vec{e}_1 , is the direction of the velocity of the field

$$\vec{e}_1 = \frac{\dot{\vec{\varphi}}}{|\dot{\vec{\varphi}}|}, \quad (\text{A1})$$

where the dot stands for the derivative with respect of the cosmic time.

Next, \vec{e}_2 is the direction of that part of the field acceleration $\ddot{\vec{\varphi}}$ which is perpendicular to \vec{e}_1 , and this procedure is extended to higher order derivatives to define the other basis vectors. The most important projectors defined by this basis are

$$\mathbf{P}^{\parallel} \equiv \vec{e}_1 \vec{e}_1^T; \quad \mathbf{P}^{\perp} \equiv \mathbf{1} - \mathbf{P}^{\parallel}; \quad (\text{A2})$$

(superindex T meaning dual).

In multifield inflation the leading slow-roll functions are given by

$$\epsilon \equiv -\frac{\dot{H}}{H^2}; \quad \vec{\eta} \equiv \frac{\ddot{\vec{\varphi}}}{H|\dot{\vec{\varphi}}|}. \quad (\text{A3})$$

The later can be decomposed in the components of $\vec{\eta}$ parallel and perpendicular to the field velocity ($\dot{\vec{\varphi}}$), i.e.,

$$\eta^{\parallel} \equiv \vec{e}_1 \cdot \vec{\eta} \quad \text{and} \quad \eta^{\perp} \equiv \vec{e}_2 \cdot \vec{\eta} \quad (\text{A4})$$

(by construction there are no other components).

Next, it is convenient to define the gauge invariant variable \vec{q} ,

$$\vec{q} \equiv a \left(\delta \vec{\varphi} + \frac{\Phi}{\mathcal{H}} \frac{d\vec{\varphi}}{d\mathcal{T}} \right), \quad (\text{A5})$$

where $\delta \vec{\varphi}$ is the inflaton perturbation, Φ is the scalar metric perturbation [Eq. (3.30)], and $\mathcal{H} = aH$. In terms of \vec{q} the evolution equation for the perturbations in terms of the conformal time takes the form [16]

$$\frac{d^2 \vec{q}_{\vec{k}}}{d\mathcal{T}^2} + (k^2 + \mathcal{H}\Omega) \vec{q}_{\vec{k}} = 0, \quad (\text{A6})$$

where $\vec{q}_{\vec{k}}$ is defined by

$$\vec{q} = \frac{1}{(2\pi)^{3/2}} \int d^3 k [\vec{q}_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}} + \vec{q}_{\vec{k}}^* e^{-i\vec{k}\cdot\vec{x}}],$$

and

$$\begin{aligned} \Omega \equiv & \frac{1}{H^2} \frac{\partial^2 V}{\partial \varphi^a \partial \varphi^b} - (2 - \epsilon)\mathbf{1} - 2\epsilon[(3 + \epsilon)\mathbf{P}^\parallel \\ & + \vec{e}_1 \vec{\eta}^T + \vec{\eta} \vec{e}_1^T] \end{aligned} \quad (\text{A7})$$

ϵ , \mathbf{P} and $\vec{\eta}$ are defined by Eqs. (A2) and (A3).

After quantizing the fields we can expand \vec{q}_k as

$$\vec{q}_k = \mathbf{Q}(\mathcal{T}) \vec{c}_k^\dagger + \mathbf{Q}^*(\mathcal{T}) \vec{c}_k, \quad (\text{A8})$$

with \vec{c}_k^\dagger ; \vec{c}_k constant creation and annihilation operator vectors and $\mathbf{Q}(\mathcal{T})$ a matrix function of time \mathcal{T} which satisfies Eq. (A6). The initial conditions for the cosmological relevant modes are vacuum initial conditions.

Applying this procedure, the adiabatic and isocurvature contributions to $\hat{\Phi}_{\vec{k}} = \hat{\Phi}_{\vec{k} \text{ ad}} + \hat{\Phi}_{\vec{k} \text{ iso}}$ at leading order of the slow-roll approximation are

$$\hat{\Phi}_{\vec{k} \text{ ad}} = \frac{3}{5} \frac{\kappa}{2k^{3/2}} \frac{H_{\mathcal{H}}}{\sqrt{\epsilon_{\mathcal{H}}}} (\vec{e}_1^T + \vec{U}_{Pe}^T) \mathbf{E}_{\mathcal{H}} \vec{c}_k^\dagger + c.c., \quad (\text{A9})$$

$$\hat{\Phi}_{\vec{k} \text{ iso}} = \frac{1}{6} \frac{3}{5} \frac{\kappa}{2k^{3/2}} \frac{H_{\mathcal{H}}}{\sqrt{\epsilon_{\mathcal{H}}}} \vec{V}_e^T \mathbf{E}_{\mathcal{H}} \vec{c}_k^\dagger + c.c. \quad (\text{A10})$$

(*c.c.* meaning the adjoint of the previous terms) where,

$$\begin{aligned} \mathbf{E}_{\mathcal{H}} & \equiv (1 - \epsilon_{\mathcal{H}})\mathbf{1} + [2 - \gamma - \ln 2] \delta_{\mathcal{H}}, \\ \delta & \equiv \epsilon \mathbf{1} - \frac{1}{3H^2} \frac{\partial^2 V}{\partial \tilde{\varphi}^a \partial \tilde{\varphi}^b} + 2\epsilon \vec{e}_1 \vec{e}_1^T, \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} \vec{U}_{Pe}^T & \equiv 2\sqrt{\epsilon_{\mathcal{H}}} \int_{t_{\mathcal{H}}}^{t_e} dt' H \frac{\eta^\perp}{\sqrt{\epsilon}} \frac{a_{\mathcal{H}}}{a} \vec{e}_2^T \mathbf{Q} \mathbf{Q}_{\mathcal{H}}^{-1}, \\ \vec{V}_e^T & \equiv \sqrt{\epsilon_{\mathcal{H}}} \frac{\sqrt{\epsilon_e} \eta^\perp}{\epsilon_e + \eta^\parallel} \frac{a_{\mathcal{H}}}{a_e} \vec{e}_2^T \mathbf{Q}_e \mathbf{Q}_{\mathcal{H}}^{-1}, \end{aligned} \quad (\text{A12})$$

with $\gamma = 0.57721 \dots$ the Euler constant, $\kappa^2 \equiv 8\pi G = 8\pi/m_{\text{Pl}}^2$, the subscript e meaning that the quantity has to be evaluated at the end of inflation, at a time, t_e , while the subscript \mathcal{H} means it has to be evaluated at the time of horizon crossing, $t_{\mathcal{H}}$, when $\mathcal{H} = k$ (i.e., $Ha = k$) during inflation.

Scalar density perturbations in multi field inflation has been considered in refs. [16,18,19]. In the general case, the spectra of scalar perturbations arising from adiabatic, isocurvature and mixture of adiabatic and isocurvature contributions, are

$$\begin{aligned} |\tilde{\delta}_{\vec{k} \text{ ad}}^{(S)}|^2 & \equiv \frac{2k^3}{9\pi^2} \langle \hat{\Phi}_{\vec{k} \text{ ad}}^2 \rangle \\ & = \frac{\kappa^2}{50\pi^2} \frac{H_{\mathcal{H}}^2}{\epsilon_{\mathcal{H}}} [(1 - 2\epsilon_{\mathcal{H}})(1 + \vec{U}_{Pe}^T \vec{U}_{Pe}) + 2(2 - \gamma - \ln 2)(2\epsilon_{\mathcal{H}} + \eta_{\mathcal{H}}^\parallel + 2\eta_{\mathcal{H}}^\perp e_2^T \vec{U}_{Pe} + \vec{U}_{Pe}^T \delta_{\mathcal{H}} \vec{U}_{Pe})], \\ |\tilde{\delta}_{\vec{k} \text{ mix}}^{(S)}|^2 & \equiv \frac{2k^3}{9\pi^2} (\langle \hat{\Phi}_{\vec{k} \text{ iso}} \hat{\Phi}_{\vec{k} \text{ ad}} \rangle + \langle \hat{\Phi}_{\vec{k} \text{ ad}} \hat{\Phi}_{\vec{k} \text{ iso}} \rangle) \\ & = \frac{1}{6} \frac{\kappa^2}{50\pi^2} \frac{H_{\mathcal{H}}^2}{\epsilon_{\mathcal{H}}} [(1 - 2\epsilon_{\mathcal{H}}) \vec{U}_{Pe}^T V_e + 2(2 - \gamma - \ln 2)(\eta_{\mathcal{H}}^\perp e_2^T V_e + \vec{U}_{Pe}^T \delta_{\mathcal{H}} V_e)] |\tilde{\delta}_{\vec{k} \text{ iso}}^{(S)}|^2 \\ & \equiv \frac{2k^3}{9\pi^2} \langle \hat{\Phi}_{\vec{k} \text{ iso}}^2 \rangle \\ & = \frac{1}{36} \frac{\kappa^2}{50\pi^2} \frac{H_{\mathcal{H}}^2}{\epsilon_{\mathcal{H}}} [(1 - 2\epsilon_{\mathcal{H}}) V_e^T V_e + 2(2 - \gamma - \ln 2) V_e^T \delta_{\mathcal{H}} V_e]. \end{aligned} \quad (\text{A13})$$

Coupling between perturbations in the different field components generate entropy or isocurvature perturbations.

A generic multi field model has all three contributions of the same order. However, if the potential is completely symmetric for $O(N)$ rotations in the internal space, the slow-roll trajectories are straight lines in field space. We show below that in this case the isocurvature density perturbations are negligible and the adiabatic contributions dominate. The adiabatic density perturbations are then the same as in the *single-field case*.

Scalar perturbations for a straight trajectory of the background field

We particularize in this section the previous results to the case when the trajectory of the background field is a straight trajectory in the internal field space. In this case $\vec{\eta}$ Eqs. (A3) and (A4) has

$$\eta^\perp = 0, \quad \eta^\parallel \equiv \eta, \quad (\text{A14})$$

that implies

$$\vec{U}_{Pe}^T = 0; \quad \vec{V}_e^T = 0, \quad (\text{A15})$$

using Eq. (A9)-(A10),

$$\hat{\Phi}_{\vec{k}\text{ ad}} = \frac{3}{5} \frac{\kappa}{2k^{3/2}} \frac{H_{\mathcal{H}}}{\sqrt{\epsilon_{\mathcal{H}}}} \vec{e}_1^T \mathbf{E}_{\mathcal{H}} \vec{c}_{\vec{k}}^\dagger + c.c., \quad (\text{A16})$$

$$\hat{\Phi}_{\vec{k}\text{ iso}} = 0. \quad (\text{A17})$$

Therefore, the scalar isocurvature perturbations are negligible at leading order of slow-roll when the bulk inflaton trajectory is a straight line, and we have

$$|\tilde{\delta}_{k\text{ iso}}^{(S)}|^2 \equiv \frac{2k^3}{9\pi^2} \langle \hat{\Phi}_{k\text{ iso}}^2 \rangle = 0, \quad (\text{A18})$$

$$|\tilde{\delta}_{k\text{ mix}}^{(S)}|^2 \equiv \frac{2k^3}{9\pi^2} (\langle \hat{\Phi}_{k\text{ iso}} \hat{\Phi}_{k\text{ ad}} \rangle + \langle \hat{\Phi}_{k\text{ ad}} \hat{\Phi}_{k\text{ iso}} \rangle) = 0. \quad (\text{A19})$$

From Eqs. (A11), (A15) and (A16), the scalar adiabatic perturbations are

$$\begin{aligned} |\tilde{\delta}_{\vec{k}\text{ ad}}^{(S)}|^2 &\equiv \frac{25k^3}{18\pi^2} \langle \hat{\Phi}_{\vec{k}\text{ ad}}^2 \rangle \\ &= \frac{1}{8\pi^2 M_{\text{Pl}}^2} \frac{H_{\mathcal{H}}^2}{\epsilon_{\mathcal{H}}} \{1 - 2\epsilon_{\mathcal{H}} + 2[2 - \gamma - \ln 2] \\ &\quad \times (2\epsilon_{\mathcal{H}} + \eta_{\mathcal{H}}) + \mathcal{O}(\epsilon_{\mathcal{H}}^2, \eta_{\mathcal{H}}^2, \eta_{\mathcal{H}}\epsilon_{\mathcal{H}})\}, \end{aligned} \quad (\text{A20})$$

i.e., the adiabatic perturbations are the same as for a single-field inflation which only takes into account the field component in the direction of \vec{e}_1 . (Recall that the subscript \mathcal{H} means that the quantity has to be evaluated at the time of horizon crossing, when $Ha = k$.) We used here the same normalization convention as the WMAP collaboration [3–5]. $|\tilde{\delta}_{k\text{ ad}}^{(S)}(\tilde{m}^2, \tilde{\lambda})|^2$ in Eq. (3.52) corresponds to the WMAP amplitude $\Delta_{\mathcal{R}}^2(k)$. WMAP only uses the leading order in slow-roll.

APPENDIX B: TENSOR PERTURBATIONS IN MULTIPLE-FIELD INFLATION

In the general multi field inflaton case, the energy-momentum of the inflaton does not have tensor perturbations, therefore the equation for the tensor metric perturbations do not have any source [16]. Thus, the tensor perturbations are determined only by the background evolution.

The Einstein equations for the tensor perturbations are:

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \Delta h_{ij} = 0. \quad (\text{B1})$$

h_{ij} is symmetric, transverse and traceless. Thus, each of its Fourier modes only has two independent components, and they can be decomposed as

$$h_{ij}(\vec{k}) = h^+(\vec{k})e_{ij}^+(\vec{k}) + h^\times(\vec{k})e_{ij}^\times(\vec{k}), \quad (\text{B2})$$

where e_{ij}^+ and e_{ij}^\times are the polarization tensors. (In a coordinate system where \vec{k} points along the z -axis, the nonzero components are $e_{xx}^+ = -e_{yy}^+ = 1$ and $e_{xy}^\times = e_{yx}^\times = 1$.)

$h_{ij}(\vec{k})$ can be quantized and expressed as

$$h_{ij}(\vec{k}) = \sum_{A=+,\times} \frac{\sqrt{2}\kappa}{a} e_{ij}^A(\vec{k}) [\psi_A(\vec{k}) a_{A\vec{k}}^\dagger + c.c.], \quad (\text{B3})$$

with the creation and annihilation operators satisfying the relations,

$$[a_{A\vec{k}}, a_{B\vec{k}'}^\dagger] = \delta_{AB} \delta(\vec{k} - \vec{k}'). \quad (\text{B4})$$

The equation of motion for the mode functions $\psi_A(\vec{k})$ is,

$$\frac{d^2 \psi_{A\vec{k}}}{d\mathcal{T}^2} + \left(k^2 - \frac{1}{a} \frac{d^2 a}{d\mathcal{T}^2} \right) \psi_{A\vec{k}} = 0. \quad (\text{B5})$$

After imposing vacuum initial conditions for these modes at large k , the previous equations give [16]

$$\begin{aligned} h_{ij}(\vec{k}) &= \frac{\kappa}{k^{3/2}} H_{\mathcal{H}} [1 + \epsilon_{\mathcal{H}}(1 - \gamma - \ln 2)] \\ &\quad \times \sum_{A=+,\times} e_{ij}^A a_{A\vec{k}}^\dagger + c.c.. \end{aligned} \quad (\text{B6})$$

Therefore, the spectrum of tensor perturbations for multi field inflation results

$$\begin{aligned} |\tilde{\delta}_k^{(T)}(\tilde{m}^2, \tilde{\lambda})|^2 &= \left(\frac{9}{4} \right) \frac{2k^3}{9\pi^2} \langle h_{ij}(k) h^{ij}(k) \rangle \\ &= \frac{2H_{\mathcal{H}}^2}{\pi^2 M_{\text{Pl}}^2} [1 + 2(1 - \gamma - \ln 2)\epsilon_{\mathcal{H}}], \end{aligned} \quad (\text{B7})$$

where the factor $\frac{9}{4}$ corresponds to the WMAP normalization convention.

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