Linearized Bekenstein varying α models

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We study the simplest class of Bekenstein-type, varying α models, in which the two available free functions (potential and gauge kinetic function) are Taylor-expanded up to linear order. Any realistic model of this type reduces to a model in this class for a certain time interval around the present day. Nevertheless, we show that no such model is consistent with all existing observational results. We discuss possible implications of these findings, and, in particular, clarify the ambiguous statement (often found in the literature) that "the Webb results are inconsistent with Oklo."

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I. INTRODUCTION

In theories with additional spacetime dimensions [1] there are typically many light or massless degrees of freedom, which can give rise to a number of observable cosmological consequences. Noteworthy among these are variations of the fundamental couplings [2–4] (with the ensuing violations of the Equivalence Principle [5,6]) and contributions to the energy density budget of the Universe.

In recent years there has been a growing body of evidence for the presence of these two effects. Type Ia supernovae [7], the Cosmic Microwave Background (CMB) [8] and lensing data [9] are all consistent with the existence of the so-called dark energy component, whose gravitational behavior is very similar to that of a cosmological constant, and which indeed appears to have become the dominant component in the energy budget of the Universe at a redshift $z \sim 1$.

On the other hand there is some (somewhat more controversial) evidence for the spacetime variation of the fine-structure constant α, coming from both quasar absorption systems (at redshifts $z \sim 1 - 3$, [10–12]) and the Oklo natural nuclear reactor ($z \sim 0.14$, [13,14]). There is also a further claim of a varying proton to electron mass ratio, also at $z \sim 3$ [15]. While it is conceivable that hidden systematic effects are still contaminating some of these measurements, an unprecedented effort is being made by a number of independent groups and using a range of techniques, to search for such variations at various key cosmological epochs, which should soon clarify the situation. There is also a range of other constraints, either local [16] (from atomic clocks) or at low [17] (from Rhenium decay in meteorites) or high redshift [18–20] (from the CMB and Big Bang Nucleosynthesis), with much stringent ones forthcoming [21,22].

It goes without saying that from the point of view of a fundamental theory there is more than ample freedom to allow the dark energy and the varying couplings to be due to different degrees of freedom in the theory, and even to emerge through different physical mechanisms. Nevertheless, it is useful to study the simplest case in which the two have a common origin, as this will in principle have the fewest free parameters and can therefore be better constrained.

In what follows we will discuss a simple toy model for this which, although simpler than others already available in the literature, has the advantage of having a minimal number of free parameters, and hence it is highly predictive and easy to compare with observations, thus providing physical insights that can be valuable when trying to derive more realistic models from fundamental physics. The basic idea is to consider Bekenstein-type models [23], and reduce the freedom in the two free functions (the potential $V(\phi)$ and the gauge kinetic function $B_F(\phi)$) by Taylor-expanding them around the present day, and retaining only terms up to linear order. In fact, we will see that its free parameters are in some sense too few, so that the model is very tightly constrained by existing observations, and indeed ruled out if all of them are correct. Still the model can be useful in providing some guidance for the likely requirements of successful, fundamental theory inspired models. Other interesting analyses of this class of models can be found in [24–29].

We will start in Sec. II with a brief overview of the Bekenstein-type models. We then consider in more detail the linearized regime that interests us (Sec. III) and discuss it in the context of existing observational data (Sec. IV). Finally Sec. V summarizes our results and briefly discusses further prospects. Throughout this paper we shall use fundamental units with $\hbar = c = G = 1$.

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II. OVERVIEW OF BEKENSTEIN MODELS

The variation of the fine-structure constant in Bekenstein-type theories [23] is due to the coupling of a scalar field ϕ to the electromagnetic field tensor $F_{\mu\nu}$, through a term of the generic form

$$S_{em} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}B_F\left(\frac{\phi}{m}\right)F_{\mu\nu}F^{\mu\nu}\right], \qquad (1)$$

where $m \sim m_{Pl} = 1$ and B_F, known as the gauge kinetic function, is the effective dielectric permittivity. This should be explicitly specified in a fundamental theory, but can be phenomenologically taken to be a free function. The fine-structure constant is then given by

$$\alpha(\phi) = \frac{\alpha_0}{B_F(\phi/m)}, \qquad (2)$$

and at the present day one has $B_F(\phi_0/m) = 1$.

The inclusion of an interaction term such as $B_F(\phi)F^2$ that is nonrenormalizable in 4D requires, at the quantum level, the existence of an ultraviolet momentum cutoff. Any particle physics motivated choice of this cutoff will destabilize the quintessence potential, since it will yield a mass term much larger than the quintessence one (recall that typically $m_q \sim H_0$). This is therefore a further fine-tuning problem, akin to the cosmological constant one. In our phenomenological approach (and in common with all previous work on these models) we will ignore this problem, assuming that any mechanism that solves the cosmological constant problem will also solve this one. (Such a mechanism must exist if the dark energy of the universe is indeed dynamical.)

Assuming that the cosmological change in the scalar field ϕ is small (at least in recent epochs), one can expand all couplings around their present-day values, in particular

$$B_F\left(\frac{\phi}{m}\right) = 1 + \zeta_F\frac{\phi-\phi_0}{m} + \frac{1}{2}\xi_F\left(\frac{\phi-\phi_0}{m}\right)^2 + \ldots, \qquad (3)$$

corresponding to a variation of α (again relative to the present day)

$$-\frac{\Delta\alpha}{\alpha} = \zeta_F\frac{\phi-\phi_0}{m} + \frac{1}{2}(\xi_F - 2\zeta_F)\left(\frac{\phi-\phi_0}{m}\right)^2 + \ldots. \qquad (4)$$

Given that the classical predictions of the model will be independent of the particular choice for the mass m, we shall take $m = m_{Pl} = 1$ throughout. In addition to the variation of the fine-structure constant, this coupling is responsible for an effective nonuniversality of the gravitational force, which through Equivalence Principle tests [5,6] leads to the constraint

$$|\zeta_F| < 5\times 10^{-4}; \qquad (5)$$

note, for example, that in Bekenstein's original theory one has $\zeta_F = -2$.

The evolution of the scalar field is then typically of the form

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = -\zeta_m\rho_m, \qquad (6)$$

where ρ_m is the matter density of the universe. Given a complete particle theory, ζ_F will be specified and it should be possible to calculate the coupling of the scalar field to matter, ζ_m. Typically the scalar field will only evolve significantly during the matter era—a result that is well known from the study of scalar-tensor theories [30,31]. A nonzero ζ_F is in principle sufficient to ensure a cosmological variation of ϕ, driven by the electromagnetic part of the baryon mass density, and hence a variation of α. However, the resulting change will typically be small if this is the only source. For example, in the original Bekenstein model, one can only fit the Webb results at the cost of having a huge violation of the weak Equivalence Principle; conversely if one wants to satisfy these constraints the typical allowed variation is only $\Delta\alpha/\alpha \sim 10^{-10}$. This constraint can only be evaded by saying that ϕ couples only (or predominantly) to the dark matter [24].

In addition to the Equivalence Principle constraint, there are a number of bounds or detections which restrict the cosmological evolution of α, as already discussed above. Moreover, if one assumes that ϕ is also providing the dark energy its evolution will be further constrained through its present contribution for the energy budget and the evolution of its equation of state [32–34].

In passing, we should also point out that spatial inhomogeneities in the context of these models can also be studied [35,36]. The relevance here is that it is conceivable that significant clustering of the dark energy could be a way out of avoiding some of the above constraints. It is well known in the literature that spatial variations of the fine-structure constant are proportional to the gravitational potential. However, the proportionality constant is itself constrained to be small by Equivalence Principle tests. This means that these spatial variations are typically far too small to be detected directly with present-day technology, except perhaps in the vicinity of compact objects with strong gravitational fields. So although this effect is not directly relevant for our discussion, it will have to be considered in other contexts.

III. ANALYSIS OF THE LINEARIZED CASE

Let us consider the class of models of a neutral scalar field coupled to the electromagnetic field with

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_{\phi F} + \mathcal{L}_{\text{other}}, \qquad (7)$$

where

$$\mathcal{L}_{\phi} = \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - V(\phi), \tag{8}$$

$$\mathcal{L}_{\phi F} = -\frac{\alpha_0}{4\alpha}F_{\mu\nu}F^{\mu\nu}, \tag{9}$$

and $\mathcal{L}_{\rm other}$ is the Lagrangian density of the other fields. We will make the simplifying assumption that both $V(\phi)$ and α are linear functions of ϕ, namely

$$V(\phi) = V(\phi_0) + \frac{dV}{d\phi}(\phi - \phi_0), \tag{10}$$

and

$$\alpha = \alpha_0 + \frac{d\alpha}{d\phi}(\phi - \phi_0), \tag{11}$$

with both $dV/d\phi$ and $d\alpha/d\phi$ assumed to be constants. One then has

$$\frac{\Delta\alpha}{\alpha} = (B_F^{-1} - 1) = \frac{1}{\alpha_0}\frac{d\phi}{d\alpha}(\phi - \phi_0). \tag{12}$$

We will further assume that $dV/d\phi < 0$. Assuming that the interpretation of the Webb *et al.* results as evidence for a variation of the fine-structure constant is correct this implies a smaller value of α in the past and consequently $d\alpha/d\phi > 0$, though if the claimed Oklo detection is also true there must be oscillations. Let us first comment on these assumptions. In any model both $V(\phi)$ and $\alpha(\phi)$ can be taken as linear functions of ϕ for some limited period of time around today. *Thus any Bekenstein-type model will reduce to our model close to today.* For how long that assumption holds is of course model-dependent. In this paper we will be considering a particular class of models for which this assumption is valid for a considerable time, possibly even all the way from the epoch of nucleosynthesis up to the present time. In other words, we are effectively *testing* the validity of this assumption for the class of Bekenstein models, *assuming* the validity of the claimed low-redshift detections of a varying fine-structure constant.

In a spatially flat Friedman-Robertson-Walker universe the equations of motion are given approximately by

$$H^2 = H_0^2(\Omega_{m0}a^{-3} + \Omega_{r0}a^{-4} + \Omega_{\Lambda 0} + \Omega_{\phi}), \tag{13}$$

$$\frac{\ddot{a}}{a} = -H_0^2\left[\frac{\Omega_{m0}}{2}a^{-3} + \Omega_{r0}a^{-4} - \Omega_{\Lambda 0} + \frac{\Omega_{\phi}}{2}(1 + 3w_{\phi})\right], \tag{14}$$

where

$$\Omega_{\phi} = \frac{8\pi[\dot{\phi}^2/2 + V(\phi)]}{3H_0^2}, \tag{15}$$

and

$$\omega_{\phi} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}. \tag{16}$$

Since any variation of the fine-structure constant from the epoch of nucleosynthesis onwards is expected to be very small [18,20] we have also neglected the minor contribution that such a variation has in the evolution of the baryon density (included in Ω_{m0}). The equation of motion for the field ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} - \frac{\alpha_0}{4\alpha^2}\frac{d\alpha}{d\phi}F_{\mu\nu}F^{\mu\nu}. \tag{17}$$

It is crucial to discuss the relative importance of the last two terms. We shall assume that the main contribution to the last term comes from baryons. Given that $F^{\mu\nu}F_{\mu\nu} = 2(B^2 - E^2) < 0$ the last two terms in Eq. (17) have opposite signs. It has been shown that the time variations of the fine-structure constant induced by the last term are too small to ever be observed, if Equivalence Principle constraints are to be obeyed [25]. Indeed, some authors have used this as an excuse to neglect this term altogether in the scalar field equation.

However, this does not happen with the first term. Hence, in order to have interesting variations of α the first term needs to dominate at recent times which will happen if

$$\left(\frac{dV}{d\alpha}\alpha\right)_0 \gg \left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)_0 \sim 10^{-4}\rho_{c0}, \tag{18}$$

where ρ_{c0} is the present-day critical density. Note that the last term can be neglected at the present time but it becomes important at early times since the main contribution to this term comes from baryons whose energy density varies as a^{-3}. In summary, in this context, a particular model is fully specified by the parameters $\dot{\phi}_0$, V_0, $dV/d\phi$ and $d\alpha/d\phi$, in addition to the nominal cosmological parameters.

Let us start with some order of magnitude constraints on the value of $\delta\phi \equiv \phi(z=0) - \phi(z=1)$, with $z \sim 1$ being singled out as the approximate redshift at which the matter domination epoch ends (do not confuse this particular $\delta\phi$ with the generic $\Delta\phi$ that has already been introduced above). If our theory is to produce interesting variations of the fine-structure constant with redshift the first term in the right hand side of (17) needs to be the dominant one. Hence we have

$$\delta\phi H^2 \sim \frac{\delta V}{\delta\phi} \lesssim \frac{\rho_{c0}}{\delta\phi}, \tag{19}$$

which implies that $\delta\phi \lesssim 10^{-1}$. On the other hand if we want to have a variation in the fine-structure constant of $\delta\alpha/\alpha \sim 10^{-6}$ without violating the Equivalence Principle one needs

$$\frac{\delta\alpha}{\alpha} = \frac{\delta\phi}{\alpha}\frac{d\alpha}{d\phi} \sim 10^{-6}. \tag{20}$$

Given that the Equivalence Principle tests give the constraint

$$\frac{d\alpha}{d\phi} \lesssim 10^{-5}, \tag{21}$$

one has necessarily $\delta\phi \lesssim 10^{-3}$. These two constraints give a limit on $\delta\phi$ of

$$10^{-3} \lesssim \delta\phi \lesssim 10^{-1}, \tag{22}$$

which will be further tightened by future tests of the Equivalence Principle. A related limit on $dV/d\phi$ gives:

$$-H_0^2 \lesssim \frac{dV}{d\phi} \lesssim -10^{-2}H_0^2, \tag{23}$$

In order for the lower limit on $\delta\phi$ to be satisfied one needs $w_\phi(z=0) - 1 \gtrsim 10^{-4}$. It is also straightforward to verify that $w \longrightarrow -1$ very rapidly as we move backwards in time.

Before we investigate the possible role of the scalar field responsible for the variation of the fine-structure constant as a quintessence candidate it is instructive to study analytic solutions for the evolution of the scalar field, assuming that its contribution to the dynamics of the universe is subdominant. From the equations above it is easy to show that

$$f(a)\frac{d^2\phi}{da^2} + g(a)\frac{d\phi}{da} = \frac{1}{H_0^2}\left(-\frac{dV}{d\phi} - \frac{\Theta}{a^3}\right), \tag{24}$$

where

$$f(a) = \Omega_{m0}a^{-1} + \Omega_{\Lambda 0}a^2 + \Omega_{r0}a^{-2}, \tag{25}$$

$$g(a) = \frac{5}{2}\Omega_{m0}a^{-2} + 4\Omega_{\Lambda 0}a + 2\Omega_{r0}a^{-3}, \tag{26}$$

and the last term has been expressed as a function of the behavior of the matter density (with the constant Θ absorbing the additional parameters). Assuming that $dV/d\phi$ is a constant one can find the following asymptotic solutions in the radiation, matter and Λ-dominated epochs

$$\phi_r = A_r + B_r a^{-1} + \frac{1}{\Omega_{r0}H_0^2}\left(-\frac{1}{20}\frac{dV}{d\phi}a^4 - \frac{\Theta}{2}a\right), \tag{27}$$

$$\phi_m = A_m + B_m a^{-3/2} + \frac{1}{\Omega_{m0}H_0^2}\left(-\frac{2}{27}\frac{dV}{d\phi}a^3 - \frac{2\Theta}{3}\ln a\right), \tag{28}$$

$$\phi_\Lambda = A_\Lambda + B_\Lambda a^{-3} + \frac{1}{3\Omega_{\Lambda 0}H_0^2}\left[-\frac{dV}{d\phi}\ln a + \Theta a^{-3}\left(\ln a + \frac{1}{3}\right)\right]. \tag{29}$$

If we neglect the decaying mode it is possible to match the solutions deep in the matter era with the solution deep in the radiation era in such a way that both ϕ and $d\phi/da$ are continuous functions of the redshift. We can then find A_Λ, B_Λ, A_m as

$$A_\Lambda = A_m - \frac{1}{\Omega_{m0}H_0^2}\left[\frac{1}{27}\frac{dV}{d\phi}\left(\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right)\left(1 - 6ln\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right) + \frac{\Theta}{3}\left(2 + ln\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right)\right], \tag{30}$$

$$B_\Lambda = -\frac{1}{\Omega_{m0}H_0^2}\left[\frac{1}{27}\frac{dV}{d\phi}\left(\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right)^2\left(1 + 3ln\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right) - \frac{\Theta}{3}\frac{\Omega_{m0}}{\Omega_{\Lambda 0}}\right], \tag{31}$$

As expected [24,30,31], we find that the evolution of ϕ is negligible during the radiation dominated epoch but significant during the matter one, and that the onset of cosmological constant domination damps this evolution. On the other hand, the cosmological evolution in the case where the dark energy of the universe is provided by ϕ itself rather than by a cosmological constant can be approximately inferred from the above analysis, since it is observationally known that the gravitational behavior of the former must be very close to the latter.

IV. DISCUSSION

We now discuss the full problem again, assuming that the field ϕ contributes to the dynamics of the Universe. In Fig. 1 we plot the evolution of the fine-structure constant as a function of redshift. Given that $\dot{\phi}_0$ is proportional to $dV/d\phi$ and the dynamics of the universe near the present time is constrained to be very close to that of a universe with $\Omega_\phi^0 \sim 0.7$ and $\omega_\phi \sim -1$, the shape of the evolution curve of $\Delta\alpha/\alpha$ as a function of redshift, z, is unambiguously predicted by our model up to a normalization factor

$$\zeta_\alpha \equiv -\frac{dV}{d\phi}\frac{\zeta_F}{H_0^2}. \tag{32}$$

We see that for a value of $\zeta_\alpha \sim -10^{-6}$ (take for example $\zeta_F = -10^{-4}$ and $dV/d\phi = -10^{-2}H_0^2$) one has

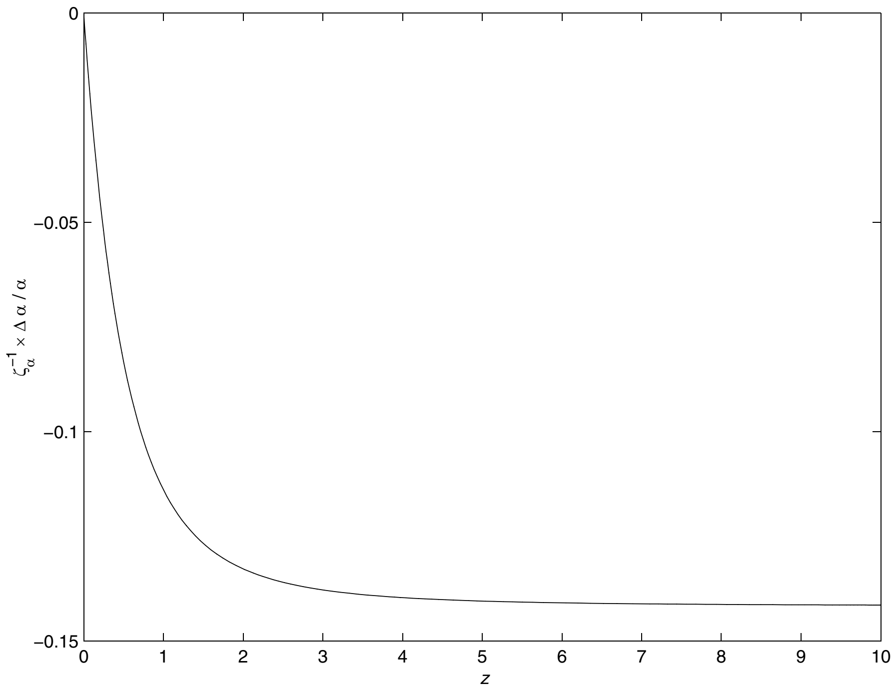

FIG. 1. The evolution of the value of $\Delta\alpha/\alpha$ as a function of redshift, in rescaled units with $\zeta_\alpha \equiv \frac{1}{\alpha H_0^2}\frac{dV}{d\alpha} = -\zeta_F \frac{1}{H_0^2}\frac{dV}{d\phi}$.

interesting variations of α which we would expect to be able to detect some time in the near future. However, note that the evolution of α with redshift is still quite significant at very low redshifts, which indicates that it is not possible to reconcile the Oklo [13,14] or meteorite [17] results (at $z = 0.14$ and $z = 0.45$ respectively) with the Webb/Murphy results [10,11] in the context of our model. If we take the Oklo and meteorite limits seriously the maximum variation of α that is allowed in this class of models by a redshift $z \sim 2$ say is about 10^{-7}, clearly below the Webb/Murphy results, though perhaps compatible with Chand *et al.* [12]. More on this below.

Examples of two particular models are displayed in Fig. 2. In the top panel we plot the evolution of the value of $\Delta\alpha/\alpha$ as a function of redshift for $dV/d\phi = -10^{-6}H_0^2$ (solid line) and $dV/d\phi = 0$ (dashed line) with $\zeta_F = -5 \times 10^{-4}$ and $\Theta = 10^{-8}H_0^2$. For a nearly flat scalar field potential the electromagnetic term in Eq. (17) is the main source of a variation of the fine-structure constant, favoring a larger value of α in the past. However, in that case the variations are too small to be of any cosmological significance. In the bottom panel we plot the evolution of the value of $\Delta\alpha/\alpha$ as a function of redshift for $dV/d\phi = -10^{-2}H_0^2$ showing that no cosmological significant variation of α beyond $z = 10$.

Given a value of H_0, the parameters V_0 and $\dot{\phi}_0$ determine in a unique way $w_{\phi 0}$ and $\Omega_{\phi 0}$. Hence, the equation of state $w_\phi(z)$ is a function of $dV/d\phi$, $\Omega_{\phi 0}$ and $\omega_{\phi 0}$ only and will evolve very rapidly towards a cosmological constant with $\omega_\phi \rightarrow -1$ when we move backwards in time. This is clearly shown in Fig. 3 (again, note that $w_{\phi 0}$ and $\Omega_{\phi 0}$ are constrained to be very close to -1 and 0.7 respectively).

As an aside, we mention that [37] studies effects of the dark energy equation of state and the coupling of α to the matter fields on the spacetime evolution of α, but the dark energy field (which they call ϕ) is different from the α-varying field (which they call ψ). Hence the relation between α and the dark energy equation of state is not explicit (as in our case or [29]), but only indirect, through the different evolution of the background.

Let us now go back to the issue of the comparison with observational data, which is plotted in Fig. 4 against two typical models. Note that the Oklo [13,14] and meteorite [17] data are plotted as points, since they apply to specific redshifts. However, for the quasar absorption data [11,12] we have chosen to plot them as bands, rather than plotting the individual observational points (or some binning thereof). This choice is partially motivated by the fact that the error bars are still much larger than those from Oklo and the meteorites, and also because the quasar "distilled" results are often quoted as a single number that is supposed to apply to a range of redshifts. Note however that this practice can be misleading. For example, a number of authors quote the latest results by Webb and collaborators [11] as

$$\frac{\Delta\alpha}{\alpha} = (-0.54 \pm 0.12) \times 10^{-5}, \quad 0.2 < z < 3.7, \quad (33)$$

ignoring the fact that even though the sample spans all that redshift range, the data only prefers a value of α different from today's beyond redshift $z \sim 1$—hence our

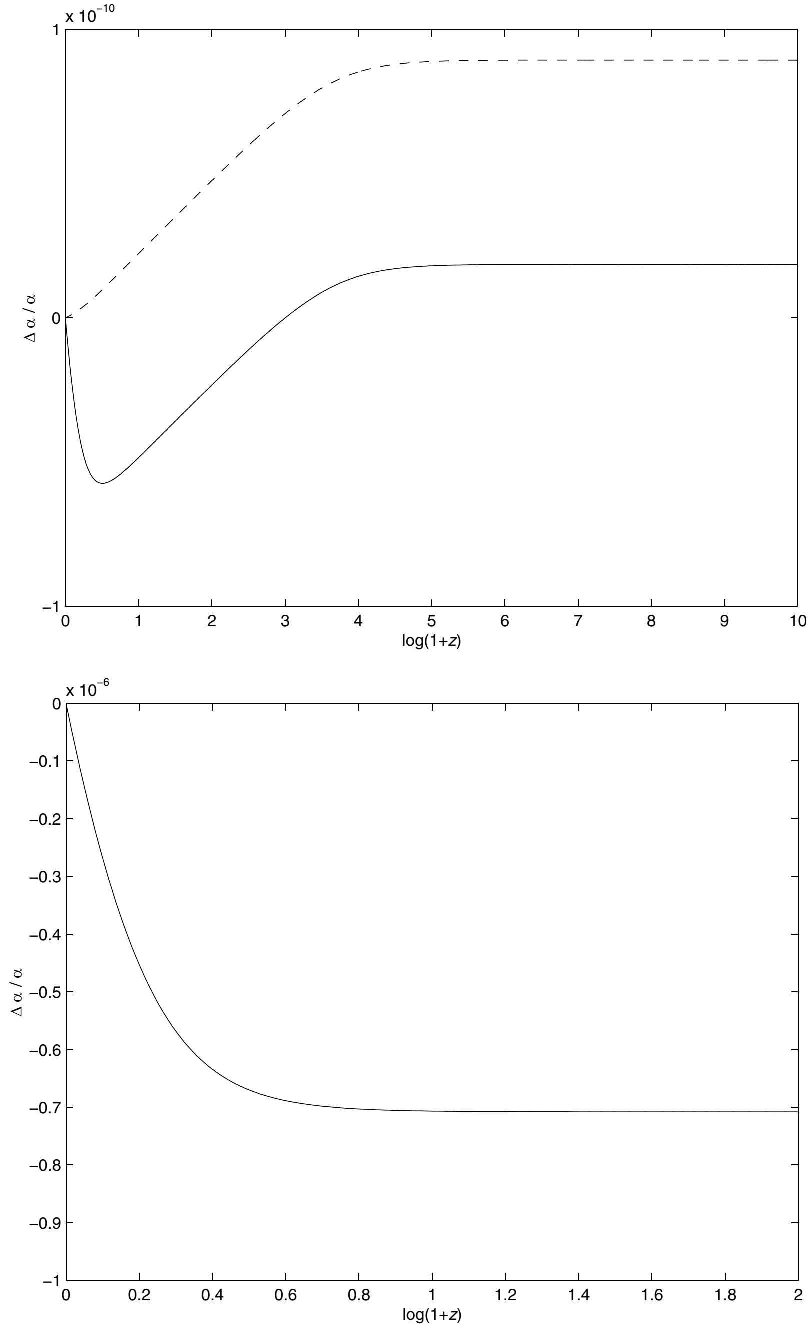

FIG. 2. *(Top Panel)*: The evolution of the value of $\Delta\alpha/\alpha$ as a function of redshift for $dV/d\phi = -10^{-6}H_0^2$ (solid line) and $dV/d\phi = 0$ (dashed line) with $\zeta_F = -5 \times 10^{-4}$ and $\Theta = 10^{-8}H_0^2$. The electromagnetic term gives a negligible contribution to the variation of α if $|dV/d\phi|$ is large enough favoring a larger value of α in the past. *(Bottom Panel)*: The evolution of the value of $\Delta\alpha/\alpha$ as a function of redshift for $dV/d\phi = -10^{-2}H_0^2$. We clearly see that no cosmological significant variation of α exists beyond $z = 10$.

choice for the horizontal range of the (dark gray) band. For the Chand *et al.* results (light gray bands) we have included their two possible results,

$$\frac{\Delta\alpha}{\alpha} = (-0.06 \pm 0.06) \times 10^{-5}, \quad 0.4 < z < 2.3, \quad (34)$$

assuming terrestrial isotopic abundances (case 1), or

$$\frac{\Delta\alpha}{\alpha} = (-0.36 \pm 0.06) \times 10^{-5}, \quad 0.4 < z < 2.3, \quad (35)$$

assuming low-metalicity isotopic abundances (case 2). The true result is expected to lie somewhere between the two. Note that a lower abundance of heavier isotopes (e.g. of Mg) at high redshift decreases the value of α.

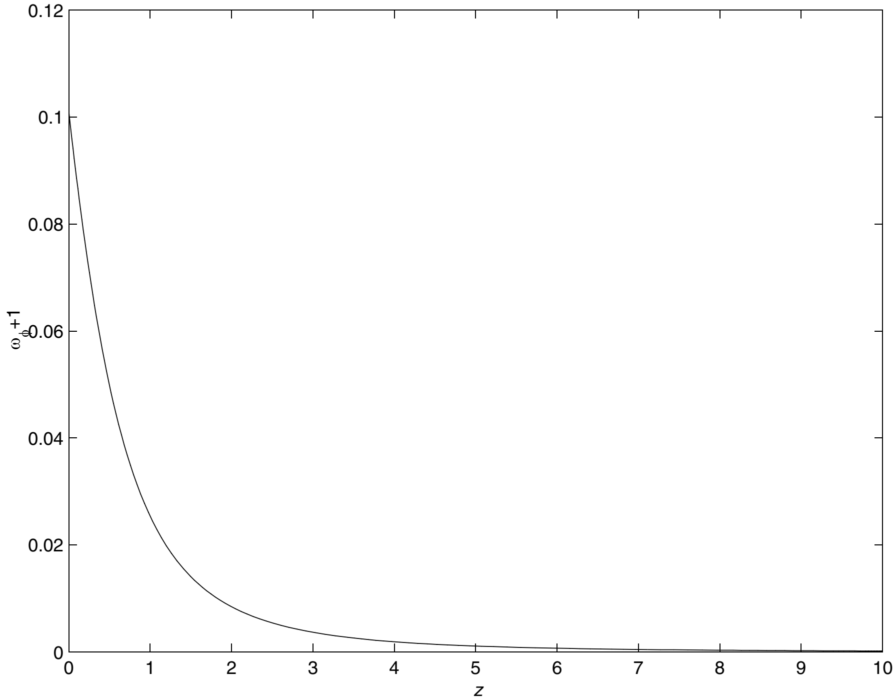

FIG. 3. The evolution of the value of $w_\phi + 1$ with redshift, z, for $dV/d\phi = -0.35$ assuming $\Omega_{\phi 0} \sim 0.7$ and $w_\phi^0 \sim -1$. Note that $w_\phi + 1$ evolves very rapidly toward zero when one moves backward in time.

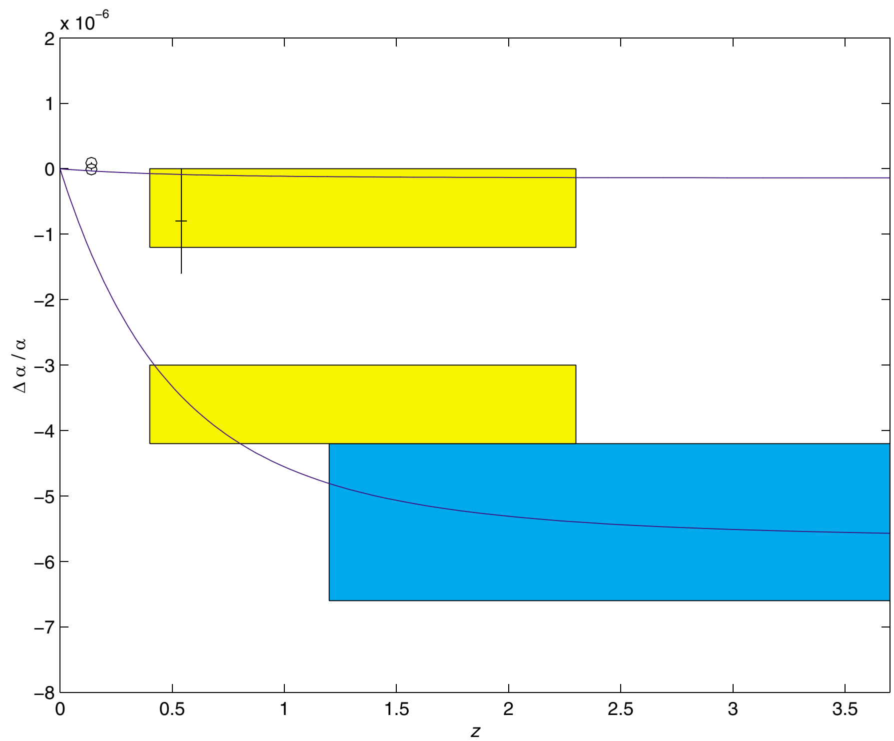

FIG. 4 (color online). Comparison of two typical examples of this class of models with observational data: Oklo ([13], circles), Rhenium decay from meteorites ([17], vertical bar) and quasar data (Murphy *et al.* [11] dark shaded box, Chand *et al.* [12] light shaded box). Either this class of models is not valid up to redshifts about $z \sim 3$, or some of the data is strongly affected by systematics. See main text for further discussion.

A number of interesting points can be inferred from the figure. As a first remark, let us point out that even though the Webb/Murphy and the Chand *et al.* results seem to be statistically inconsistent, the discrepancy may be much smaller than one could guess by simply comparing (33) and (34). Nevertheless, the question remains as to whether there are hidden systematics contaminating one or both of the data or the analysis pipelines.

We have already pointed out that the distinguishing feature of this class of models is that significant variations of α occur relatively near the present epoch. (As a side remark we note that in this context this justifies the commonly used assumption of a uniform value of α throughout the last scattering epoch when constraining variation of the fine-structure constant with CMB observations [18–22].) However, this late variation has dramatic consequences. Roughly speaking, depending on the model parameters one can divide models in this class into two different types: they can either be consistent with Oklo + meteorites+(34) but be inconsistent with (33), or else be consistent with (33)+(35), but inconsistent with Oklo + meteorites.

In other words, if we assume that our linearized class of models holds true up to at least redshift $z \sim 3$ or so, then the Webb/Murphy results are indeed inconsistent with Oklo/meteorites. We note that a number of authors have in the past made the unqualified statement that "the Webb results are inconsistent with Oklo". Such a statement is not meaningful *per se*, since any such comparison is necessarily model-dependent: one needs to specify a timescale as well as a model for the redshift evolution of α (see the discussion in [2]). Indeed one can build models where the two can be made compatible—an example is [27]. Having said that, here we do find that the two are inconsistent for the models we considered.

This therefore calls for improvements on the existing observational results. Following the controversy generated by the quasar data results, at least five (to our knowledge) independent groups are currently working on the subject, using a variety of different methods, so there is hope that the situation will be clarified soon. No similar interest exists for the Oklo or meteorite data, though independent confirmation of both of these results would be much welcome since as we have seen they are quite more constraining, particularly for the class of models that we have discussed. We note that measurements of α using quasar data are, notwithstanding the possible sources of observational systematics, quite straightforward in the sense that one measures α directly. On the other hand, measurements using Oklo and meteorite data are indirect: what one measures directly here is some combination of various couplings, and using them to obtain constraints on α requires either assuming that other couplings do not vary (which is almost certainly unrealistic) or assuming some (necessarily model-dependent) relations between them. It is therefore important to check how robust these constraints are to the specific assumptions being made to obtain them

Of course, if both of these observational results survive further scrutiny, then our toy model cannot be correct. We emphasize again that any realistic model will reduce to a model in this class for some period of time close to today, so that would indicate that our linearized approximation will break down very close to today, arguably much earlier than one would have thought. Note also that this class of models, with a linearized behavior for the scalar field, are arguably the *simplest* possible models for a varying α. Certainly models where α has a linear dependence on redshift or on cosmic time (which have been explicitly or implicitly assumed by a number of authors) are much more unnatural, and it is hard to see how such could be obtained from a sensible particle physics theory an a way that would be consistent with other observational and experimental constraints.

V. CONCLUSIONS

Over the past few years the issue of possible variations of the fine-structure constant has been a very hot topic, both on the observational and on the theoretical side. Despite the efforts of a number of observers, it is clear that the existing observational data is not as yet conclusive. In this state of affairs, the task of phenomenological model-building has to be tackled with caution. We take the view that for the moment the role of a phenomenological model is not really to fit the data but to sharpen the questions. By this we mean that introducing a new model with a number of additional free parameters that one has to tune to fit all available data is almost certainly a pointless exercise, since the data will almost certainly change on a very fast timescale. At this stage it is more productive to try to understand what general trends seem to be emerging from the data, and what mechanisms could (or could not) be at play.

With these ideas in mind, we have studied the simplest class of Bekenstein-type, varying α models, and compared them to existing observational constraints. These are models in which the two available free functions (the potential and the gauge kinetic function) are Taylor-expanded around present-day values, with terms kept only up to linear order. Despite their apparent simplicity, they are interesting to the extent that any realistic model of this type should reduce to a model in this class for a certain time interval around the present day. Nevertheless, their simplicity means that very specific predictions ensue, that can be compared with existing data. We have shown that no such model is consistent with all the existing observational results. Hence either some of these observations are dominated by unknown systematics or our linearity assumption breaks down on a time-scale significantly smaller than a Hubble time.

Given that a scalar field that produces a varying fine-structure constant can also make a significant contribution towards the dark energy of the universe, it is interesting to speculate on the possible relation between the above observation and hints for a time-varying equation of state of dark energy. Indeed in the latter context it has been argued that something analogous seems to happen: observational data seem to disfavor not only a constant equation of state, but even a mildly varying one, say with a linear dependence in redshift [38–40]. It is unclear if the two things are somehow related, but it has been said that a coincidence is always worth noticing—one can always discard it later if it turns out to be just a coincidence.

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