Modulated fluctuations from hybrid inflation

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Inflation universally produces classical almost scale free Gaussian inhomogeneities of any light scalars. Assuming the coupling constants at the time of inflation depend on some light moduli fields, we encounter the generation of modulated cosmological fluctuations from (p)reheating. This is an alternative mechanism to generate observable (almost) scale free adiabatic metric perturbations. We extend this idea to the class of hybrid inflation, where the bifurcation value of the inflaton is modulated by the spatial inhomogeneities of the couplings. As a result, the symmetry breaking after inflation occurs not simultaneously in space but with the time laps in different Hubble patches inherited from the long-wavelength moduli inhomogeneities. To calculate modulated fluctuations we introduce techniques of general relativistic matching conditions for metric perturbations at the time hypersurface where the equation of state after inflation undergoes a jump, without evoking the detailed microscopic physics, as far as it justifies the jump. We apply this theory to the modulated fluctuations from the hybrid and chaotic inflations. We discuss what distinguishes the modulated from the inflation-driven fluctuations, in particular, their spectral index, modification of the consistency relation, and the issue of weak non-Gaussianity.

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I. INTRODUCTION

One of the generic predictions of inflation lies in the fact that vacuum fluctuations of all light scalar fields, χ_a, minimally coupled to gravity and with a mass smaller than the Hubble parameter ($m \ll H$), are universally unstable and appear after inflation as classical random *a priori* Gaussian inhomogeneities with (almost) scale free spectrum $H/\sqrt{2k^3}$. The wavelengths of the fluctuations, $\delta\chi_a(t, \vec{x})$, of such a light scalar field are stretched by inflation and exceed the Hubble patch after inflation.

One can relate $\delta\chi_a(t, \vec{x})$ to the cosmological scalar metric perturbations in different ways, depending on the composition of the underlying theory. Indeed, the simplest and most studied possibility is to assume that there is a single light scalar field that is the inflaton itself, φ. The inflaton fluctuations $\delta\varphi$ are transferred to the scalar metric perturbations through gravitational interaction [1].

On general grounds, we may however expect many scalar fields playing roles during inflation. There is a broad range of multiple field inflationary models with different motivations behind them, like, e.g., double inflation [2] or hybrid inflation [3], where different fields dominate at different stages of the cosmological evolution. Multiple fields were also evoked to design departures from the standard inflationary predictions: existence of isocurvature modes [3,4], nonscale free spectrum of primordial fluctuations [5], or deviation from Gaussianity [6,7]. In these cases some fields unnecessarily give a dominant contribution to the background evolution, but during some time give a dominant contribution to the perturbations. Another corner of the multiple field parameter space is related to the curvaton scenario [8]. There, a newly introduced scalar field, the curvaton, that is indeed not the inflaton, should be light during inflation, plus dominate after inflaton decay, plus decay after inflaton but prior to big-bang nucleosynthesis, plus give a dominant contribution to the metric fluctuations.

A new and more economic idea to generate cosmological perturbations from modulated fluctuations of couplings was proposed recently [9–11]. In the context of multiple scalar field theories, it is assumed in these models that some of the light fields never give a dominant contribution neither into background nor in perturbations during inflation, but contribute to the coupling constants

$$\alpha = \alpha(\chi_a). \tag{1}$$

Indeed, in string theory [12] couplings are in fact the

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vacuum expectation value (VEV) of moduli fields and, in supersymmetry (SUSY) theories couplings can also depend on scalars. As a result, fluctuations of the moduli $\delta\chi_a$ generated during inflation will manifest themselves in spatial inhomogeneities of the couplings

$$\delta\alpha = \frac{\partial\alpha}{\partial\chi_a}\delta\chi_a. \tag{2}$$

The interaction is not as important during inflation as it is after inflation. Consider, for example, simple chaotic inflation. The background scalar field rolls towards the minimum of the potential, where it begins to oscillate. Because of the coupling to the other fields the inflaton decays into radiation in the process of (p)reheating [13]. Because of the large scale coupling inhomogeneities, Eq. (2), at scales much larger than the causal horizon after inflation, transition from the matter dominated regime of inflaton oscillations to radiation occurs in different causal patches not simultaneously, which leads to small adiabatic metric perturbations after (p)reheating. After the moduli χ_a get pinned down to their minima, the spatial variations of coupling constants in the late time universe will be erased. However, the large scale metric fluctuations that are produced due to interactions survive as a memory of the primordial moduli inhomogeneities.

It is actually not mandatory for scalar fields to be light during inflation. In fact, in the context of $N = 1$ supergravity with the minimal Kähler potential during inflation the scalars typically acquire the mass $m_a \sim H$. Cosmological fluctuations neither in the inflaton sector nor in the moduli sector are produced unless special care is taken to make at least some of them light. This, together with the options to build up inflationary models, motivates the study of different mechanisms of generations of cosmological perturbations.

In this article, we investigate this mechanism of *modulated cosmological fluctuations*. Original papers on the modulated fluctuations [10,11] discussed metric perturbations generated from the decay of inflaton oscillations. However, there is another important class of hybrid inflationary models, which typically emerges in supergravity and the string theory/brane cosmology. The two field (φ, σ) hybrid inflation has the effective potential

$$V_{\rm eff} = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{1}{2}g^2\varphi^2\sigma^2 + V(\varphi), \tag{3}$$

where $V(\varphi)$ is the inflaton potential. This potential contains two couplings λ and g^2, which define the end point of inflation and the field dynamics. Assuming these couplings are moduli dependent, we encounter modulated fluctuations in the hybrid inflation. This is the main novel idea of the paper which we describe and develop in Sec. II.

One may think of different aspects of the theory of modulated fluctuations, related to the nature of the moduli and dependencies, Eq. (1), details of preheating, thermalization, moduli evolution, etc. Here we concentrate on the cosmological general relativistic part of the story, namely, how to derive cosmological metric perturbations from the coupling perturbations, Eq. (2). In the chaotic inflation typically inflaton oscillations decay through the nonperturbative effect of parametric resonance of particle creation [13]. This leads to copious production of particles in an out-of-equilibrium state. Further interactions between particles relax them towards thermal equilibrium. Perturbative regime of particle interactions takes place at the latest stages of transition from inflation towards hot radiation [14]. In hybrid inflation, preheating after inflation has the character of the tachyonic preheating [15], accompanying the symmetry breaking after inflation [16]. Tachyonic preheating leads to the generation of particles out of equilibrium with the subsequent thermalization of them. Coupling constants of interaction appears at different stages of preheating and thermalization. However, for the production of modulated cosmological perturbations it is essential that couplings are responsible for the change of the equation of state. If the equation of state is not changing, modulated perturbations are not generated. In this respect we shall concentrate at the first instance where the equation of state after inflation is changing due to the couplings. In the case of chaotic inflation it happens when an effective matter equation of state of the coherent inflaton oscillations is replaced by the radiation in the very fast process of preheating. In the case of hybrid inflation, the vacuum-like (inflation) equation of state is replaced by radiation in the very quick process of tachyonic preheating.

So far the formalism of the modulated cosmological fluctuations was considered for the toy model of slow perturbative reheating [10,11,17,18], often with the Yukawa type interaction $\phi\bar{\psi}\psi$ between inflatons and fermions. This is fair enough to see in principle that metric fluctuations are generated from the moduli inhomogeneities. However, the modern theory of the transition between inflation and radiation is described by the theory of preheating. Even for the case of the fermions their production is nonperturbative and significantly modified by preheating [19]. In this paper, on the technical side, we suggest the method to treat the generation of modulated cosmological fluctuations in the inflationary models with preheating. We will use the fact that in all cases preheating is very short, and from the general relativistic point of view can be considered as the instant jump of the equation of state. It is then convenient to use the formalism of matching conditions of the geometrical quantities at the time of the transition [20]. For this, we do not need to know the microscopic details of preheating and thermalization.

In Sec. II, we first discuss the basics of the modulation of the couplings and some model building aspects that have to be fulfilled for such a mechanism to be efficient.

The construction of an explicit model in a supergravity context is proposed in Sec. III. After recalling the basics of the matching conditions in Sec. IV, we then apply them to the standard single field inflationary case in Sec. V. We then turn to the case of modulated fluctuations in Sec. VI. We will discuss the main features of the mechanism in Sec. VII and, in particular, emphasize that it allows one to extend the standard consistency relation of inflation.

II. MODULATED FLUCTUATIONS OF COUPLINGS IN HYBRID INFLATION

Consider a model of hybrid inflation. The basic shape of the effective potential is given by Eq. (3), where φ is the inflaton and σ is another scalar field, which is massive during inflation but whose effective mass changes sign at the critical value of the inflaton value

$$\varphi_c = \frac{\sqrt{\lambda}v}{g}. \tag{4}$$

The point where $\varphi = \varphi_c$, $\sigma = 0$ is a bifurcation point. For $\varphi > \varphi_c$ the squares of the effective masses of both fields are positive and the potential has a minimum at $\sigma = 0$. For $\varphi < \varphi_c$ the potential has a maximum at $\sigma = 0$. The global minimum is located at $\varphi = 0$ and $|\sigma| = v$. However, at $\varphi > \varphi_c$ the effective potential has a valley along $\sigma = 0$. In this model, inflation occurs while the φ field rolls slowly in this valley from large values towards the bifurcation point. At the bifurcation point the symmetry breaking occurs. Recall, however, that immediately after the bifurcation point the field σ has a negative mass square. Hence, dynamics of the symmetry breaking is accompanied by the tachyonic instability of inhomogeneous modes. It results in the very rapid decay of the homogeneous fields into inhomogeneous modes in the nonlinear regime of tachyonic preheating [15,16] much before the global potential minimum is reached. For what we are interested in this bath of inhomogeneous modes essentially behaves like a radiation fluid. In the following we will simply assume the transition between inflation and radiation domination to be instantaneous.

In general in hybrid models φ and σ can be viewed as scalar fields coming from a much larger scalar sector of the theory. We then assume that there exists a set of light scalar fields, χ_a and that the couplings λ and g absorb the dependence on these scalars as

$$\lambda = \lambda(\chi_a), \qquad g = g(\chi_a) \Rightarrow \varphi_c = \varphi_c(\chi_a). \tag{5}$$

λ and g^2 are determined by the VEVs of these fields. Since the fields χ_a are light, they develop super-Hubble inhomogeneities so that the symmetry breaking that terminates the inflationary stage does not occur at the same time everywhere and is modulated over space. The fluctuations of the light fields modulate the couplings λ and g^2 which then have spatial fluctuations in a way that depends on their dependence on the χ_a VEVs,

$$\delta\lambda \approx \sum_a \frac{\partial\lambda}{\partial\chi_a}\delta\chi_a; \tag{6}$$

$$\delta g^2 \approx \sum_a \frac{\partial g^2}{\partial\chi_a}\delta\chi_a. \tag{7}$$

It follows that our model consists of the inflaton, φ, a Higgs-like field, σ, and the light fields, χ_a, with a potential of the form

$$V_{\rm eff} = \frac{1}{4}\lambda(\chi_a)(\sigma^2 - v^2)^2 + \frac{1}{2}g^2(\chi_a)\varphi^2\sigma^2 + V(\varphi). \tag{8}$$

In principle, moduli may have potentials $U(\chi_a)$, which for simplicity are assumed to be negligible during inflation, or included into (8), as we will see below.

In Eq. (8) the dependencies of the coupling constant on the moduli fields χ_a have a quite different status. Let us inspect the conditions in the theory (8) which would keep the moduli light during inflation. When $\varphi > \varphi_c$ and $\sigma = 0$ the equations for the fields χ_a are

$$\ddot\chi_a + 3H\dot\chi_a + \frac{v^4}{4}\frac{\partial\lambda}{\partial\chi_a} = 0, \tag{9}$$

where a dot refers to a derivation with respect to cosmic time. Equations for its fluctuations $\delta\chi_a e^{i\vec{k}\cdot\vec{x}}$ are

$$\delta\ddot\chi_a + 3H\delta\dot\chi_a + \frac{k^2}{a^2}\delta\chi_a + \frac{v^4}{4}\frac{\partial^2\lambda}{\partial\chi_a\partial\chi_b}\delta\chi_b = -\frac{v^4}{2}\frac{\partial\lambda}{\partial\chi_a}\Phi + 4\dot\chi_a\dot\Phi. \tag{10}$$

During the inflationary stage $\varphi > \varphi_c$ the effective potential in which χ_a evolves is $\lambda(\chi_a)v^4/4$.

If the background value of χ_a is the order of M and the natural argument of coupling is χ_a/M then the fluctuations of the fields χ_a have a mass $m_a^2 = (v^4/4)\partial\lambda/\partial\chi_a \sim H^2M_p^2/M^2$, where $M_p = 8\pi G$ is the Planck mass. But as we will see in the next section, the right amplitude of modulated fluctuations is achievable if the mass scale M is of order or smaller than M_p. Thus, $\delta\chi_a$ are heavy unless the coupling λ has no dependence on moduli. There is no such reservation for g^2. In the following we will see that supergravity D-term inflation precisely provides us with such a pattern for the moduli dependence.

When the mass of χ_a is small, and its contribution to the background geometry is negligible, its kinetic energy is also small, e.g., $\dot\chi_a \approx 0$. In this case the fluctuations $\delta\chi_a$ can be considered as the light test field at a given background driven by inflaton φ. It is easy to see that χ_a also do not influence the evolution of the inflaton field and fluctuations during inflation. Indeed we have equations for inflaton fluctuations $\delta\phi$ and metric fluctuations Φ (in the longitudinal gauge) during this phase

$$\dot\Phi + H\Phi = \frac{1}{2M_p}(\dot\varphi\delta\varphi + \dot\chi_a\delta\chi_a), \tag{11}$$

$$\delta\ddot{\varphi} + 3H\delta\dot{\varphi} + \frac{k^2}{a^2}\delta\varphi = -2V'\Phi + 4\dot{\varphi}\,\dot{\Phi} - V''\delta\varphi, \quad (12)$$

where a prime refers to a derivation with respect to φ. It follows that the fluctuations of the light fields χ_a do not contribute to the metric fluctuations in Eq. (11) during inflation, so that the set of Eqs. (11) and (12) reduces to the one of the inflaton coupled to metric perturbation, as in single field inflation.

To summarize, since $\sigma = 0$ during inflation, the function $g(\chi_a)$ does not enter the perturbation equations during inflation so that the light fields can be considered as test fields. As anticipated, the light fields influence only the end of inflation by modulating over space the time at which the symmetry breaking occurs, imprinted in the hypersurface $\varphi = \varphi_c$. They are subdominant with respect to the VEVs of φ and σ that drive the evolution of the background and do not affect the generation of metric perturbations during inflation. We expect the cumulative fluctuations after inflation to inherit both from the metric fluctuation during inflation and from the modulated transition, hence opening the possibility of nontrivial phenomenological consequences.

III. MODULI IN COUPLINGS OF SUGRA *D*-TERM INFLATION

In this section we slightly step aside of the main topic of the paper, modulated cosmological fluctuations in the generic hybrid inflation. We consider a specific example of the sugra D-term inflation, to illustrate how the dependence of couplings on moduli can be originated. Although in general, D-term inflation is similar to the hybrid inflation, there are some differences. In particular, the slow-roll regime of inflation may be ended even before ϕ reaches the bifurcation point [21,22]. D-term inflation is reduced to the hybrid inflation in the limit of small g^2. All our results are immediately applicable for this limit of D-term inflation. In a more general case of D-term inflation the equation of state may be changed twice, at ϕ_c and even before at some hypersurface $\phi_e > \phi_c$, but ϕ_e is still spatially varying due to the moduli dependence. As a result the magnitude of modulated fluctuations may be even greater than our estimations below (where only a single jump in the equation of state is considered). We notice but do not consider these effects in the paper.

Let us recall how potentials of the form of Eq. (3) can be obtained from particle physics models. Such potentials are indeed prototypes of models of inflation motivated by supergravity (including low-energy string theory) such as F- or D-term inflations (see, e.g., Ref. [23] for a review). It might be worth keeping in mind that the more generic P-term inflation has both these models as limiting cases [24]. Note also that brane-antibrane D-$\overline{D}$ systems in superstring theory produce four-dimensional effective potentials like (3).

In F-term inflation we always have $g^2 = \lambda$ and no modulated fluctuations can be generated. We will not consider this case any further. On the contrary a D-term inflation driven by a nonzero Fayet-Iliopoulos D term does not lead to any specific relation between g^2 and λ.

For further discussion recall that a generic $N = 1$ supergravity Lagrangian including interaction with matter and Yang-Mills fields in $3 + 1$ dimensions is built from three arbitrary functions: the Kähler potential $K(\chi_a^*, \chi_a)$ which encodes the kinetic term of the scalar fields, the superpotential $W(\chi_a)$, and the kinetic terms $f_{\alpha\beta}(\chi_a)$ for the vector multiplet fields, $[\mathrm{Re} f_{\alpha\beta}(\chi_a)] F^{\alpha}_{\mu\nu} F^{\beta\mu\nu}$. We will use notation and the form of the supergravity Lagrangian, see Eq. (5.15) of Ref. [25], adapted to cosmology.

The simplest model of D-term hybrid inflation [22] consists of three (left) chiral superfields Φ_i: the inflaton and two fields of opposite charge under a local U(1). The potential for this model comes from the superpotential

$$W = \sqrt{2} g \Phi \Phi_+ \Phi_-, \quad (13)$$

and the D term

$$D = \frac{\sqrt{\lambda}}{2}(2|\Phi_+|^2 - 2|\Phi_-|^2 - v^2), \quad (14)$$

where the fields $\Sigma_\pm$ have charges $\pm\sqrt{\lambda}$ and the Fayet-Iliopoulos term is $\sqrt{\lambda}v^2/2$. We may choose the fields to be real and define $\sigma_\pm \equiv \sqrt{2}|\Phi_\pm|$ and $\phi \equiv \sqrt{2}|\Phi|$, allowing for canonical kinetic terms. In terms of these fields, in the global SUSY limit the potential reduces to the form

$$\begin{aligned} V &= V_D + V_F \\ &= \frac{\lambda}{4}(\sigma_+^2 - \sigma_-^2 - v^2)^2 + \frac{g^2}{2}(\phi^2\sigma_+^2 + \phi^2\sigma_-^2 \\ &\quad + \sigma_+^2\sigma_-^2). \end{aligned} \quad (15)$$

When ϕ, which plays the role of the inflaton, is large enough both σ_+ and σ_- have a large positive mass and are forced to be zero. Supersymmetry is however broken and the one-loop corrections give an extra term in the potential, which is only ϕ dependent so that it rolls down toward $\phi = 0$. This model exactly matches Eq. (3) noting that during the whole evolution σ_- is forced to be zero.

In a more general context of supergravity, the D term has a prefactor $[\mathrm{Re} f_{\alpha\beta}(\chi_a)]^{-1}$ absorbed in the coupling λ, $\lambda(\chi_a) \to \lambda \times [\mathrm{Re} f(\chi_a)]^{-1}$. Thus, a large mass of χ_a can be avoided by simply assuming that $\mathrm{Re} f_{\alpha\beta} = 1$. The effective coupling g^2 can then be made dependent on χ_a through the Kähler potential.

We give a toy model example where we want to observe that the moduli dependence can appear in g^2 while it does not appear in λ. The Kähler potential intervenes in the F-term part of the potential

$$V_F = e^{K/M_P^2}\left[K^{j^* i}\mathcal{D}_{j^*}W\mathcal{D}_i W - 3\frac{W^2}{M_P^2}\right], \tag{16}$$

where $\mathcal{D}_i = 1/M_P^2 \partial K/\partial\Phi_i + \partial/\partial\Phi_i$, and $K^{j^* i}$ is the inverse matrix to $K_{ij^*} \equiv \partial^2 K/\partial\Phi_i\partial\Phi_j^*$. The D term on the other hand is given by

$$D = \frac{\sqrt{\lambda}}{2}(q_i K_i \phi_i - v^2), \tag{17}$$

where $K_i \equiv \partial K/\partial\Phi_i$, and q_i is the charge of the corresponding scalar.

For a standard simple string theory toroidal compactification scheme [26] for which

$$K = -3\log\left(t + t^* - \sum_i |\Phi_i|^2\right), \tag{18}$$

t being a bulk modulus, and in the case the VEVs of the fields (and in particular that of Φ_0 which is nonvanishing during inflation) are small compared to $|t|$ the computations can be easily completed. We have $K_i \approx 6\phi_i^*/(t + t^*)$ and $K_{ij^*} \approx 6/(t + t^*)\delta_{ij}$. This latter matrix can easily be inverted. As a result the fields $\tilde{\Phi}_i \equiv \sqrt{3/(t + t^*)}\Phi_i$ are the fields that have a standard kinetic term in the low-energy limit. We see that in this case the t dependence in Eq. (17) drops. Unfortunately this is also the case in the expression of V_F.

If however the compactification is made in such a way that there are three different moduli directions t_i then the expression of V_F can be made dependent on the moduli values. This is the case for instance if

$$K = -\log\left(t_1 + t_1^* - \sum_i |\Phi_i|^2\right) - \log(t_2 + t_2^*) - \log(t_3 + t_3^*). \tag{19}$$

In this case $K_i = 2\Phi_i^*/(t_1 + t_1^*)$ and $K_{ij^*} = 2/(t_1 + t_1^*)\delta_{ij}$. The kinetically regularized fields are $\tilde{\Phi}_i \equiv \sqrt{1/(t_1 + t_1^*)}\Phi_i$ from which we get

$$V_F = \tilde{g}^2[|\tilde{\Phi}\tilde{\Phi}_+|^2 + |\tilde{\Phi}\tilde{\Phi}_-|^2 + |\tilde{\Phi}_-\tilde{\Phi}_+|^2], \tag{20}$$

$$\tilde{g}^2 = \frac{(t_1 + t_1^*)^2}{(t_2 + t_2^*)(t_3 + t_3^*)}g^2,$$

which indeed leads to a moduli dependent effective g^2. Note that if each Φ is associated with a different moduli direction the dependence in g^2 also vanishes. We see that unless the Kähler potential has very specific features, the effective coupling constant g^2 is dependent on the moduli fields. This is what we were looking for. The picture we obtained here is not as simple as the one described in the introduction since it implies that the inflaton potential, $V(\phi)$ here, coming from the radiative correction to the potential induced by the breaking of the boson-fermion mass equalities, depends on g, therefore on the moduli fields. It means that the inflaton is actually a combination of Φ and of the moduli. The picture sketched in the beginning will be recovered if the dependence with the latter is small enough. Note also that the theory we present is incomplete because it does not provide for a stabilization mechanism for the VEVs of the moduli. The problem of stabilization in the context of D-term inflation is discussed, e.g., in [27], and references therein. We will then not exploit this specific model any further. We value it however because it shows that modulated inflation should be rather generic in realistic models of sugra.

In the following sections we derive the geometrical theory of modulated cosmological fluctuations which is applicable for the hybrid inflation, as well as from the chaotic inflation.

IV. JUNCTION CONDITIONS FOR METRIC PERTURBATIONS AT THE TIME HYPERSURFACE

In this and the following sections we turn to a technical part of the paper. We will deal with the general relativistic aspects of the cosmological metric fluctuations. First, in this section we will remind one of the general formalism of the matching conditions [28] of two geometries on the different sides (past and future) of a time hypersurface. This spacelike hypersurface, Σ, also divides the matter contents in the Universe, which has different equations of state on different sides of Σ. In subsequent sections, we apply this general formalism to different situations.

We begin with the derivation of the matching conditions in the cosmological context. We assume that the transition between two eras, e.g., inflation and radiation dominated, takes place on a three-dimensional spacelike hypersurface defined by

$$\Sigma = \{q = \text{const}\},$$

where q is a scalar to be specified; see Fig. 1. We focus on scalar and tensor modes of metric perturbations and

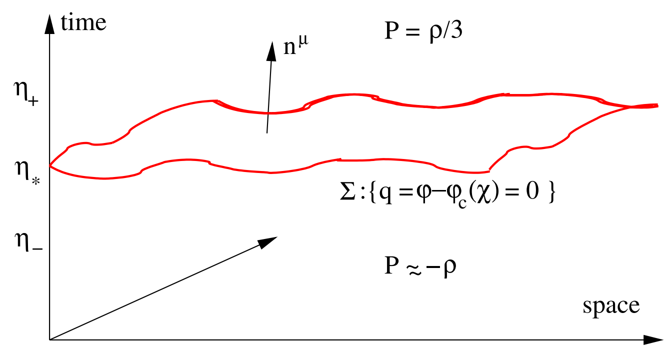

FIG. 1 (color online). The inflationary period is matched to a radiation dominated era on a spacelike hypersurface Σ defined by $q(\varphi, \chi_a) \equiv \varphi - \varphi_c(\chi_a) = 0$.

assume that the spatial sections of the Universe are flat and work in longitudinal gauge. It follows that the metric of spacetime takes the form

$$ds^2_\pm = a^2_\pm(\eta_\pm)[-(1+2\Phi_\pm)d\eta^2_\pm + \{(1-2\Phi_\pm)\delta_{ij} + h^\pm_{ij}\}dx^i dx^j], \tag{21}$$

where the indexes $-$ and $+$ refer, respectively, to the two eras, before and after Σ. h_{ij} is a symmetric traceless ($h^i_i = 0$) transverse ($\partial_i h^{ij} = 0$) tensor describing the gravitational waves. Note that the conformal times $\eta_\pm$ in both eras are *a priori* different. We also split q as

$$q = \bar{q} + \delta q, \tag{22}$$

with δq being the perturbation in longitudinal gauge.

The junction conditions [28] reduce to the continuity of the induced three-dimensional metric on Σ and of the extrinsic curvature of Σ; see Refs. [20,29,30]. The normal unit vector to the hypersurface Σ is given by

$$n_\mu = \frac{\partial_\mu q}{\sqrt{-\partial_\alpha q \partial^\alpha q}} \tag{23}$$

so that the induced three-dimensional metric on Σ takes the form

$$\gamma^a_b = a^2\left[\left(1 - 2\left(\Phi + \mathcal{H}\frac{\delta q}{q'}\right)\right)\delta^a_b + h^a_b\right] \tag{24}$$

and its extrinsic curvature

$$K^a_b = \frac{1}{a}\left\{-\mathcal{H}\delta^a_b + \left[\mathcal{H}\Phi + \Phi' + (\mathcal{H}' - \mathcal{H}^2)\frac{\delta q}{q'}\right]\delta^a_b + \partial^a\partial_b\frac{\delta q}{q'} + \frac{1}{2}h'^a_b\right\}, \tag{25}$$

where a, b run from 1 to 3, and the prime is the derivative with respect to η, $\mathcal{H} = \frac{a'}{a}$. It follows that the matching conditions for the background geometry reduce to

$$[a]_\pm = 0, \qquad [\mathcal{H}]_\pm = 0, \tag{26}$$

where $[X]_\pm \equiv X_+ - X_-$. In other words, the scalar factor and its time derivative are continuous through Σ.

Matching conditions for perturbations are split into

$$[\Phi]_\pm = 0, \qquad \left[\Phi' + \mathcal{H}'\frac{\delta q}{q'}\right]_\pm = 0, \qquad \left[\frac{\delta q}{q'}\right]_\pm = 0, \tag{27}$$

for the scalar perturbations and

$$[h_{ij}]_\pm = 0, \qquad [h'_{ij}]_\pm = 0 \tag{28}$$

for the gravitational waves. Equations (27) and (28) are the basis for the applications below.

V. JUNCTION CONDITIONS FOR INFLATON DRIVEN METRIC FLUCTUATIONS

In order to present the method and the notation, we rederive the standard result for the scalar metric fluctuations driven by inflaton fluctuations, using the general formalism of junction conditions for the metric fluctuation outlined above (see also Refs. [20,29]).

A. Long-wavelength modes evolution

So for the time being let us assume that inflation is driven by a slow-rolling scalar field φ. We can introduce the slow-roll parameters by

$$\varepsilon = \frac{M_p^2}{16\pi}\left(\frac{V'}{V}\right)^2, \qquad \eta = \frac{M_p^2}{8\pi}\left(\frac{V''}{V}\right). \tag{29}$$

Solving for $\varepsilon = 1 - \mathcal{H}'/\mathcal{H}^2$ gives the expression of the parameter

$$\mathcal{H}_- = \frac{1+\varepsilon}{-\eta_-}, \tag{30}$$

where conformal time during inflation is η_-. It is convenient to introduce the two quantities defined by

$$\mathcal{R} = \Phi + \frac{2}{3}\frac{\Phi' + \mathcal{H}\Phi}{\mathcal{H}(1+w)}, \qquad \zeta = \Phi + \mathcal{H}\frac{\delta\rho}{\rho'}, \tag{31}$$

that correspond to the curvature perturbation in a flat slicing gauge and in a comoving gauge (see, e.g., Ref. [31] for the history of the introduction of these quantities). For a general perfect fluid, including the particular case of a single scalar field, the perturbation equations take the form [32]

$$\Delta\Phi = \frac{\kappa a^2}{2}[\delta\rho - 3\mathcal{H}\rho(1+w)v], \tag{32}$$

$$\Phi' + \mathcal{H}\Phi = -\frac{\kappa a^2}{2}\rho(1+w)v, \tag{33}$$

where $\delta\rho$ and v are, respectively, the density perturbation and the velocity perturbation in Newtonian gauge and with $\kappa \equiv 8\pi G$. It follows that the density fluctuation can be expressed as

$$\frac{\kappa a^2}{2}\delta\rho = \Delta\Phi + 3\mathcal{H}(\Phi' + \mathcal{H}\Phi), \tag{34}$$

and that the two quantities defined in Eq. (31) are related, using that $\mathcal{H}' - \mathcal{H}^2 = \mathcal{H}\rho'/\rho$, by

$$\mathcal{R} + \frac{1}{3}\frac{\Delta\Phi}{\mathcal{H}' - \mathcal{H}^2} = \zeta \tag{35}$$

so that they are equal for super-Hubble modes. The evolution equation for the gravitational potential, see Ref. [32], can be shown to imply

$$\mathcal{R}' = 0, \qquad \zeta' = 0, \tag{36}$$

for super-Hubble adiabatic modes. In fact, the solution evolution equation of the gravitational potential takes the general form

$$\Phi = \frac{\mathcal{H}}{a^2}\left[B + A\int(1+w)a^2 d\eta\right]. \qquad (37)$$

The coefficient A corresponds to the growing mode and B to a decaying mode. If the equations of state vary continuously from $w = -1 + 2\varepsilon/3$ during inflation to $w = 1/3$ during the radiation era, we obtain that the gravitational potential in the radiation dominated universe (RDU), after the decaying modes have become negligible,

$$\Phi \sim \frac{2}{3}\frac{1+\varepsilon}{\varepsilon}A, \qquad (38)$$

where A is fixed during inflation. Since $\mathcal{R} \simeq \Phi/\varepsilon \simeq A/\varepsilon$ during inflation and $\mathcal{R} \simeq 3\Phi/2$ during RDU, the solution of Eq. (37) is equivalent to the continuity of $\mathcal{R}$, Eq. (36). Note also that Eq. (37) implies that the variation of the gravitational potential when the scale factor changes behavior changes from $a \propto t^{p_1}$ to $a \propto t^{p_2}$ at $t = t_*$ is $\Phi(t \gg t_*)/\Phi(t < t_*) = (1+p_1)/(1+p_2)$ (see Ref. [20]).

B. Derivation by means of the junction conditions

Let us now do the same exercise but by means of the junction conditions and assume inflation suddenly ends with a transition to a radiation era. As long as the background dynamics is concerned, the matching conditions for the background quantities, that is the continuity of the scale factor and of the Hubble parameter, Eq. (26), imply that the transition happens at $\eta_+ = \eta_* = -(1-\varepsilon)\eta_-$ and we have

$$a_-(\eta_-) = C/(-\eta_-)^{1+\varepsilon}, \quad a_+(\eta_+) = C/[(1+\varepsilon)\eta_*^{1+\varepsilon}](\eta_+/\eta_*). \qquad (39)$$

Now, the transition is due to a sudden change in the equation of state and takes place on a constant density hypersurface [20]. The matching conditions (27) imply that

$$\left[\frac{\delta\rho}{\rho'}\right]_\pm = 0, \qquad [\Phi]_\pm = 0, \quad \left[\Phi' + (\mathcal{H}' - \mathcal{H}^2)\frac{\delta\rho}{\rho'}\right]_\pm = 0. \qquad (40)$$

The first two conditions and the continuity of $\mathcal{H}$ imply that

$$[\zeta]_\pm = 0, \qquad (41)$$

and thus that

$$\left[\mathcal{R} + \frac{1}{3}\frac{\Delta\Phi}{\mathcal{H}' - \mathcal{H}^2}\right]_\pm = 0. \qquad (42)$$

The first condition implies that

$$\left[\frac{\Delta\Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi)}{1+w}\right]_\pm = 0$$

which reduces to

$$\left[\frac{3\mathcal{H}(\Phi' + \mathcal{H}\Phi)}{1+w}\right]_\pm = 0 \qquad (43)$$

on super-Hubble scales. It follows that the matching conditions imply the continuity of ζ, Eq. (41) and the continuity of $\mathcal{R}$ on the super-Hubble scale. Note however that this conclusion relies strongly on the fact that $\delta\rho/\rho'$ is continuous, and thus on the choice of the matching surface (see Ref. [29] for further discussion on this issue). Note that matching on a constant φ hypersurface will have implied, from Eq. (27) that $[\mathcal{R}]_\pm = 0$, instead of Eq. (41). Interestingly, Eq. (35) shows that for supper-Hubble modes, it is equivalent to match on a constant field or constant density hypersurface.

The general solution for the gravitational potential, Eq. (37), implies that $\Phi = A_- + B_-(-\eta)$ and $\Phi = A_+ + B_+/\eta^3$ respectively during inflation and RDU. The continuity of Φ and the condition (36) imply that

$$A_+ = \frac{2}{3}\frac{1+\varepsilon}{\varepsilon}A_-, \qquad B_+ = \frac{1}{3}\frac{\varepsilon - 2}{\varepsilon}A_+, \qquad (44)$$

if we neglect the decaying mode during inflation. At a time still in the RDU but far enough from the transition, we get that

$$\Phi_+ \sim \frac{2}{3}\frac{1+\varepsilon}{\varepsilon}\Phi_-, \qquad (45)$$

which is the standard result, Eq. (38). Again, since $\mathcal{R}_- = \Phi_-/\varepsilon$ and $\mathcal{R}_+ = 3\Phi_+/2$, Eq. (37) is equivalent to the

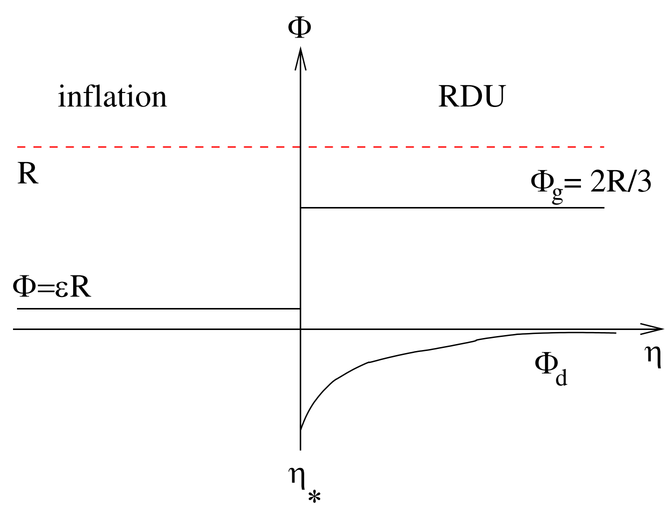

FIG. 2 (color online). The evolution of the gravitational potential Φ through the transition. Both $\mathcal{R}$ and Φ are continuous (dashed line) if the transition is a constant density hypersurface. Φ_g and Φ_d refer, respectively, to the growing and decaying modes.

continuity of $\mathcal{R}$. The behavior of the different quantities is depicted in Fig. 2. Note that the growing mode of the gravitational potential inherits a contribution from the gravitational potential during inflation (Φ_-) and a contribution from the density perturbation on the matching surface $[(2-\varepsilon)\Phi_-/3\varepsilon]$. In the modulated inflationary model, this second contribution will be modified, as we will see in the next section.

C. Initial power spectra and the consistency relation

The preceding argument simply shows that in the standard case it is equivalent to use the matching conditions or the continuity of ζ to relate the gravitational potential generated during inflation to the one in the radiation era. Indeed, one still needs to determine the gravitational potential generated during inflation that can be obtained from the quantification of the density fluctuations during inflation. This is part of the standard lore.

For the scalar modes, let us introduce the Mukhanov variables [32]:

$$u = z\mathcal{R} = z\left(\Phi + \frac{\mathcal{H}}{\varphi'}\delta\varphi\right), \qquad z = \frac{a\varphi'}{\mathcal{H}} \tag{46}$$

in terms of which the equation of evolution of the scalar modes is

$$u'' + (k^2 - z''/z)u = 0. \tag{47}$$

In terms of the slow-roll parameter $z''/z = (\nu^2 - 1/4)/\eta^2$ with $\nu = 3/2 + 3\varepsilon - \eta$ so that the general solution of Eq. (47) can be expressed in terms of Hankel functions. On super-Hubble scales, it reduces to

$$u_k \simeq \frac{1}{\sqrt{2k}}(-k\eta)^{1/2-\nu}. \tag{48}$$

We deduce that the curvature perturbation, $\mathcal{R}_k = u_k/z$, is given by

$$\mathcal{R}_k \simeq \frac{1}{\sqrt{2k^3}}\frac{H}{M_p}\frac{1}{\sqrt{\varepsilon}}(-k\eta)^{\eta-3\varepsilon} \tag{49}$$

before the transition. Defining the power spectrum of any field $X_{\mathbf{k}}$ as

$$\langle X_{\mathbf{k}} X_{\mathbf{k}'}\rangle = \frac{2\pi^2}{k^3}P_X(k)\delta(\mathbf{k}-\mathbf{k}'), \tag{50}$$

one obtains that

$$P_{\mathcal{R}}^{1/2} = \frac{1}{2\pi}\left(\frac{H}{M_p}\right)\frac{1}{\sqrt{\varepsilon}}(-k\eta_-)^{\eta-3\varepsilon} \simeq \frac{1}{\varepsilon}P_{\Phi_-}^{1/2} \simeq \frac{2}{3}P_{\Phi_+}^{1/2}, \tag{51}$$

where the last equality derives from Eq. (45).

Concerning the gravitational waves, one can follow the same routes but now the matching conditions Eq. (28) are trivial. Introducing the variable [32] $u_T = aM_p h$, the evolution of the tensor modes is dictated by the equation

$$u_T'' + (k^2 - a''/a)u_T = 0. \tag{52}$$

The general solution is given in terms of the Hankel function and the growing mode on super-Hubble scales is given by

$$h_+ \sim h_- \sim \frac{1}{\sqrt{2k^3}}\frac{H}{M_p}(-k\eta)^{-\varepsilon} \tag{53}$$

from which one deduces a consistency relation between the relative amplitude between scalar and tensor contributions and the slow-roll parameter

$$\frac{T}{S} = \varepsilon = -n_T/2, \tag{54}$$

where T and S are measuring the amplitude of the power spectra of, respectively, the tensor and scalar curvature modes and where n_T is the tensor modes spectral index.

VI. THE CASE OF MODULATED FLUCTUATIONS

Now we turn to the main subject of the paper, namely, the generation of modulated fluctuations. Contrary to the previous case of inflaton driven fluctuations, the transition between the inflationary era and radiation era does not take place on a constant density hypersurface but on a hypersurface of constant value of the bifurcation point. We derive in Sec. VI A the general expression of the curvature fluctuations in the radiation era and then apply it to the case of modulated fluctuation scenarios (Sec. VI B).

A. General results

Let us start by matching the inflationary stage to the radiation era on a general constant q hypersurface. From the junction conditions (28), we deduce that the gravitational potential in the radiation era is given by

$$\Phi_+(\eta) = \Phi_-(-\eta_*) + \frac{2-\varepsilon}{3}\frac{\mathcal{H}}{q'}\bigg|_*\delta q(-\eta_*) \times\left[1 - \left(\frac{\eta_*}{\eta}\right)^3\right], \tag{55}$$

where ε is given by Eq. (29) and depends on the details of the inflationary stage. Using that $\mathcal{R}_+ = 3\Phi_+/2$, we deduce that the curvature perturbation deep in the radiation era but long enough after the transition is given by

$$\mathcal{R}_+(\eta) = \frac{3}{2}\Phi_-(-\eta_*) + \left(1 - \frac{\varepsilon}{2}\right)\frac{H\delta q}{\dot{q}}(-\eta_*). \tag{56}$$

To go further, we need to specify (i) the nature of the inflationary period which will fix $\Phi_-(\eta_-)$ and ε and (ii) the nature of the transition which will fix q.

Let us note that the formula (56) contains the standard case discussed in Sec. V that is recovered by simply choosing $q = \varphi$, taking into account that Eq. (29) implies that $H/\dot{\varphi} = \sqrt{4\pi/\varepsilon}/M_p$. We now apply this general result to the case of modulated fluctuations.

B. Modulated fluctuations from slow-roll hybrid inflation

We consider the realistic scenario of a slow-roll hybrid inflationary stage in which the value of the bifurcation point is modulated by some light fields, as described in Sec. III. It follows that inflation ends when $\varphi = \varphi_c(\chi_a)$ so that the parameter q introduced in Sec. IV has to be chosen as $q(\varphi, \chi_a) = \varphi - \varphi_c(\chi_a)$. Let us stress that q is a function of all light fields including the inflaton. We deduce that the quantities needed to apply the matching conditions (27) and (28) are given by

$$\delta q = \delta\varphi - \sum_a \frac{d\varphi_c}{d\chi_a}\delta\chi_a \tag{57}$$

and by

$$\dot{q} = \dot{\varphi} \tag{58}$$

because $\dot{\chi}_a \simeq 0$. We can now apply the matching conditions between the inflationary solution and the radiation era solution, exactly as in Eq. (44), and making use of the background quantities defined in Eq. (39) we obtain the equivalent of Eq. (55):

$$\Phi_+(\eta) = \Phi_-(-\eta_*) + \frac{2-\varepsilon}{3}\frac{\mathcal{H}}{\varphi'}\bigg|_*\bigg[\delta\varphi(-\eta_*) - \sum_a \gamma_a \delta\chi(-\eta_*)\bigg]\bigg[1 - \left(\frac{\eta_*}{\eta}\right)^3\bigg], \tag{59}$$

where we have introduced the coefficients

$$\gamma_a \equiv \frac{d\varphi_c}{d\chi_a}(-\eta_*). \tag{60}$$

Using that $\Phi_- \sim \varepsilon\mathcal{R}_-$, we deduce that $\mathcal{R}_+ = 3\Phi_+/2$ is given, at a late enough time ($\eta \gg \eta_*$), by

$$\mathcal{R}_+ = \mathcal{R}_- - \left(1 - \frac{\varepsilon}{2}\right)\frac{H}{\dot{\varphi}}\bigg|_* \sum_a \gamma_a \delta\chi_a(-\eta_*). \tag{61}$$

Using Eq. (29) to express $H/\dot{\varphi}$ at the time of the transition, we end up with

$$\mathcal{R}_+ = \mathcal{R}_- - \sqrt{4\pi}\frac{1-\varepsilon/2}{\sqrt{\varepsilon}}\sum_a \gamma_a \frac{\delta\chi_a(-\eta_*)}{M_p}. \tag{62}$$

Assuming for simplicity that there is only one light scalar field and taking into account that $\mathcal{R}_-$ and $\delta\chi_a$ are not correlated, see the discussion in Sec. II, we conclude that

$$P_{\mathcal{R}_+} = P_{\mathcal{R}_-} + 4\pi\frac{1-\varepsilon}{\varepsilon}\gamma^2\frac{P_\chi}{M_p^2}. \tag{63}$$

We now need to determine the one of χ_a that can be considered as a test field. Let us recall the evolution of any light field χ satisfies, in Fourier space, an equation of the form

$$(a\delta\chi)'' + \bigg[k^2 + m^2a^2 - \frac{a''}{a}\bigg](a\delta\chi) = 0. \tag{64}$$

Assuming that the background spacetime is described by slow-roll inflation, the bracket expression reduces to $(\nu^2 - 1/4)/\eta^2$ with $\nu \sim 3/2 - m^2/3H^2 + \varepsilon$ if one uses Eq. (30). The solution leads, on super-Hubble scales, to

$$\delta\chi_k \sim \frac{H}{\sqrt{2k^3}}(-k\eta)^{3/2-\nu}. \tag{65}$$

The power spectrum of any light field, as defined in Eq. (50), is thus given by

$$P_\chi(k) = \left(\frac{H}{2\pi}\right)^2(-k\eta)^{2m^2/3H^2-2\varepsilon}. \tag{66}$$

It follows that the power spectrum of the curvature perturbation deep in the radiation era is given by [33]

$$P_{\mathcal{R}_+} = \frac{1}{4\pi^2}\left(\frac{H}{M_p}\right)^2\frac{1}{\varepsilon}(k\eta_*)^{-2\varepsilon}[(k\eta_*)^{2\eta-4\varepsilon} + 4\pi\gamma^2(1-\varepsilon)(k\eta_*)^{2m^2/3H^2}]. \tag{67}$$

Contrary to the standard case described in Sec. V, $\mathcal{R}$ is not conserved through the transition, his jump being given by Eq. (62). This is due to the fact that the perturbation of the light fields, which were isocurvature perturbations during inflation, are transferred to the adiabatic mode at the beginning of the radiation era. Note also that φ' has been expressed in terms of the slow-roll parameters so that $\delta\varphi$ and Φ_- combine to give $\mathcal{R}_+$. Also, the standard inflationary case described in Sec. V is recovered when $\gamma = 0$, that is when the value of the bifurcation point does not depend on any light field and when the end of inflation takes place on a constant φ hypersurface.

C. Spectral index and the consistency relation of modulated fluctuations

An important qualitative result we obtain here is that the modulated fluctuations from the hybrid inflation are inevitably accompanied by the usual inflaton fluctuations. Their relative amplitude is given by the factor $4\pi\gamma^2(1-\epsilon)$ and their spectra are not generically the same. While the index of the inflaton driven fluctuations is $n_S - 1 = 2\eta - 6\epsilon$, that of the modulated fluctuations $n_S - 1 = 2m^2/3H^2 - 2\varepsilon$. The observed scalar spectrum is therefore the sum of two power laws and the scalar spectral index can run between these two limiting values.

Concerning gravitational waves, the super-Hubble solution in RDU takes the form $h_+ = A_+ + B_+/\eta$ so that the matching conditions (28) imply

$$h_+ \sim \frac{1}{\sqrt{2k^3}}\frac{H}{M_p}(k\eta_*)^{-\varepsilon} \tag{68}$$

on super-Hubble scales at any time $\eta \gg \eta_*$. It follows that deep in the radiation era,

$$P_h(k) = \frac{1}{4\pi^2}\left(\frac{H}{M_p}\right)^2 (k\eta_*)^{-2\varepsilon}, \quad (69)$$

as in the standard inflationary case.

In the standard case described in Sec. V, the amplitude of the gravitational waves was set by the energy scale of inflation, $(H/M_p)^{1/2} \sim V^{1/4}/M_p$ and their relative amplitude compared to the scalar modes was controlled by the slow-roll parameter ε via the consistency relation (54). This implies that the detection, or limit on the amplitude, of the gravitational waves set a constraint on the energy scale at which inflation took place.

The modulated fluctuations lead to another effect. The consistency relation (54) becomes

$$\frac{T}{S} = \frac{\varepsilon}{1 + 4\pi(1-\varepsilon)\gamma^2}, \quad (70)$$

and the contribution of the tensor modes is always smaller in the modulated fluctuations case than in the standard inflationary case. This can be easily understood because the modulated fluctuations are of a scalar type only and the gravitational waves are completely insensitive to the properties of the transition, as can be seen from the matching relation Eq. (28).

The situation where $\gamma \sim \mathcal{O}(1)$ is particularly interesting since T/S remains of order ε but a deviation from the standard consistency relation (54) of order ε appears. In this regime, the scalar power spectrum is the sum of two power laws of comparable amplitude, opening the possibility to have a break at an observable wavelength. This, in particular, the case in the explicit model presented in Sec. III, from Eq. (20) we get $\gamma^2 = \sum_{a=1}^{3} \gamma_a^2 = 2$.

When $\gamma \geq 1$ that is when most of the scalar perturbations are inherited from the modulation of the transition hypersurface the ratio T/S is then much smaller that ε and we get a mechanism that damps the gravitational waves contribution, whatever the energy scale of inflation. On one hand, it allows the energy scale of inflation to be higher than in the standard case and still have undetectable gravitational waves (see also Ref. [34] for a scenario that can boost the gravitational waves). In that case, gravitational waves then appear to contribute nonsignificantly, e.g., to the temperature anisotropies and their detectability, see, e.g., Refs. [35,36], and strong constraints on ε will be difficult to set.

VII. DISCUSSION AND CONCLUSION

Modulated curvature fluctuations in hybrid models can be put on firm ground whether it is from a model building perspective or for the computation of its phenomenological consequences. That such models are generic might be interesting at different levels. In such classes of models indeed the metric fluctuations are not necessarily associated with field fluctuations in the slow-roll direction. It allows one to break the relations between the slow-roll parameters and the shape of the power spectrum. Analogous conclusions were reached in Ref. [37] where a model that decouples the spectral index from the inflationary stage is presented. Moreover some of the difficulties encountered in the usual hybrid models might be circumvent. Indeed the production of topological defects at an energy scale comparable to that of the adiabatic fluctuations, no contribution of which have been detected, is natural in such models forcing a tuning of the parameters subsequently causing the suppression of "good" scalar fluctuations (see Ref. [24]). Here we avoid this problem. Actually what we have obtained here is a kind of decoupling of the background evolution sector from the generation of fluctuations.

Are thus models arbitrary constructions or could they be physically motivated? Let us recall that light fields, such as moduli, are generic in, e.g., string theory. They may have different effects related to the theory of structure formation and of interest for cosmology. They may induce a modulation of the coupling constants that can lead (i) to a spacetime modulation of the constants of nature, as, e.g., the fine structure constant (see Ref. [38] for the effect of a fluctuating light dilaton, Ref. [39] for an example of signature on the cosmic microwave background, and Ref. [40] for a review), (ii) to the generation of non-Gaussianity, and (iii) to the mechanism of modulated fluctuations described in this article.

The mechanism presented in the article shows that modulated inflationary models correspond to a large class of models that naturally emerges from supergravity where the couplings depending on light moduli fields can modulate the transition to the radiation era. In such models we have more specifically shown that gravitational waves were not affected by the modulation while scalar modes receive an extra contribution. The primordial power spectra however follow a modified consistency relation, Eq. (70), which depends on a new parameter, γ, that characterizes the dependence of the bifurcation point on the light fields. Moreover the fact that the contribution of the gravitational waves is always smaller in this context than in the standard inflationary case has two consequences: (i) it allows one to have a higher energy scale for inflation and still have undetectable tensor modes but (ii) makes the possibility of their detection via B-polarization measurements more difficult. An interesting situation arises when $\gamma \sim 1$. In that case, the tensor modes still have a relative amplitude comparable to the standard case but (i) we have a deviation, Eq. (70), from the standard, Eq. (54), that can be hoped to be measured and (ii) the scalar power spectrum is the sum of two power laws, opening the door for a possible break in the range of wavelengths of interest for cosmology.

These results should be put in parallel to other extensions of standard inflation where the consistency relation can be modified. Generically in multifield inflation [41], Eq. (54) becomes $T/S = \varepsilon \sin^2\Theta$ where Θ is the isocurvature-adiabatic correlation angle. As in our case, $T/S < \varepsilon$. On the other hand in the curvaton scenario [8], the density perturbations arise from the fluctuation of a light field and primordial perturbations are entirely from an isocurvature mode and the consistency relation becomes $T/S \ll \varepsilon$ (see, e.g., Ref. [42]). This is analogous to a limit in which $\gamma \gg 1$.

This class of models also opens a new phenomenological avenue. Indeed nothing prevents the fields χ_a from developing non-Gaussianity prior to the transition as in the mechanism of Ref. [7]. In modulated inflation the transfer of modes is not due to a bent of the trajectory but to the modulation. The outcome of such a mechanism could be the superposition of Gaussian perturbations and non-Gaussian ones of the same variance. Let us stress that no non-Gaussianity is generated by the transition itself so that if fluctuations of the light field are initially Gaussian so will be the modulated fluctuations.

The last remarks are about the nature of modulated scalar fluctuations. First, note that this mechanism offers an example of a situation in which the adiabatic mode is not conserved, an issue raised in Ref. [43]. Second, before the end of inflation, the moduli field fluctuations are subdominant gravitationally and have a character of isocurvature fluctuations. When the equation of state is changed the moduli field remains to be gravitationally subdominant (contrary to the curvaton scenario where dominance of the curvaton is additionally assumed). However, the moduli field fluctuations vary the time hypersurface where equation of state is altered. At this moment their isocurvature fluctuations are transferred into adiabatic, scalar metric fluctuations (viz. the coupling which control the change of the equation of state). We have cosmological examples where initially isocurvature inhomogeneities are transferred into adiabatic fluctuations—examples include the isocurvature cold dark matter scenario or the curvaton scenario—in both cases the carrier of isocurvature fluctuations becomes gravitationally dominant. What is qualitatively new in the modulated fluctuations is the fact that the carrier of fluctuations remains always gravitationally subdominant; however, its isocurvature fluctuations are transformed into the adiabatic one after it modulates the jump in the equation of state.

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[1] V. F. Mukhanov and G. V. Chibisov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 549 (1981) [JETP Lett. **33**, 532 (1981)]; S.W. Hawking, Phys. Lett. **115B**, 295 (1982); A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982); A. H. Guth and S.Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).

[2] L. A. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Lett. **157B**, 361 (1985).

[3] A. Linde, Phys. Lett. B **259**, 38 (1991); Phys. Rev. D **49**, 748 (1994).

[4] L. Kofman, Phys. Lett. B **173**, 400 (1986); D. Polarski and A. A. Starobinsky, Phys. Rev. D **50**, 6123 (1994); D. Langlois, Phys. Rev. D **59**, 123512 (1999); C. Gordon *et al.*, Phys. Rev. D **63**, 023506 (2001).

[5] L. A. Kofman and A. D. Linde, Nucl. Phys. **B282**, 555 (1987); L. A. Kofman and D. Yu. Pogosian, Phys. Lett. B **214**, 508 (1988); D. S. Salopek, J. R. Bond, and J. M. Bardeen, Phys. Rev. D **40**, 1753 (1989).

[6] A. D. Linde and V. Mukhanov, Phys. Rev. D **56**, 535 (1997).

[7] F. Bernardeau and J.-P. Uzan, Phys. Rev. D **66**, 103506 (2002); **67**, 121301(R) (2003); F. Bernardeau, T. Brunier, and J.-P. Uzan, Phys. Rev. D **69**, 063520 (2004); F. Bernardeau and J.-P. Uzan, Phys. Rev. D **70**, 043533 (2004).

[8] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002).

[9] L. Kofman, in "Proceedings of the Cosmo02 Meeting, Chicago, 2002," http://pancake.uchicago.edu/~cosmo02/

[10] G. Dvali, A. Gruzinov, and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004).

[11] L. Kofman, astro-ph/0303614.

[12] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vol. 1.

[13] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); Phys. Rev. D **56**, 3258 (1997).

[14] G. N. Felder and L. Kofman, Phys. Rev. D **63**, 103503 (2001); R. Micha and I. I. Tkachev, Phys. Rev. Lett. **90**, 121301 (2003).

[15] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde, and I. Tkachev, Phys. Rev. Lett. **87**, 011601 (2001).

[16] G. N. Felder, L. Kofman, and A. D. Linde, Phys. Rev. D **64**, 123517 (2001).

[17] S. Matarrese and A. Riotto, J. Cosmol. Astropart. Phys. 08 (2003) 007.

[18] F. Vernizzi, Phys. Rev. D **69**, 083526 (2004).
[19] J. Baacke, K. Heitmann, and C. Patzold, Phys. Rev. D **58**, 125013 (1998); P. B. Greene and L. Kofman, Phys. Lett. B **448**, 6 (1999); G. F. Giudice, M. Peloso, A. Riotto, and I. Tkachev, J. High Energy Phys. 08 (1999) 014.
[20] N. Deruelle and V. F. Mukhanov, Phys. Rev. D **52**, 5549 (1995).
[21] A. D. Linde and A. Riotto, Phys. Rev. D **56**, 1841 (1997).
[22] P. Binetruy and G. Dvali, Phys. Lett. B **388**, 241 (1996).
[23] D. H. Lyth and A. Riotto, Phys. Rep. **314**, 1 (1999).
[24] R. Kallosh and A. Linde, J. Cosmol. Astropart. Phys. 10 (2003) 008.
[25] R. Kallosh, L. Kofman, A. D. Linde, and A. Van Proeyen, Classical Quantum Gravity **17**, 4269 (2000).
[26] E. Witten, Phys. Lett. **155B**, 151 (1985).
[27] J. P. Hsu, R. Kallosh, and S. Prokushkin, J. Cosmol. Astropart. Phys. 12 (2003) 009.
[28] G. Darmois, Mémorial des Sciences Mathématiques **XXV** 1 (1927); K. Lanczos, Ann. Phys. (Leipzig) **74**, 518 (1924); N. Sen, Ann. Phys. (Leipzig) **73**, 365 (1924); W. Israel, Nuovo Cimento B **44**, 1 (1966).
[29] J. Martin and D. Schwarz, Phys. Rev. D **57**, 3302 (1998).
[30] N. Deruelle, D. Langlois, and J.-P. Uzan, Phys. Rev. D **56**, 7608 (1997).
[31] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cambridge, 2000).
[32] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[33] We assume that the slow-roll parameters ε and η are constant during inflation, which is the case at first order in the slow-roll parameters. If ε had varied between the time of horizon crossing of the modes of cosmological interest and the time of the transition then Eq. (67) will be $P_{\mathcal{R}_+} = (1/4\pi^2)(H/M_p)^2(1/\varepsilon)(k\eta_*)^{-2\varepsilon}[(k\eta_*)^{2\eta-4\varepsilon} + 4\pi\gamma^2(1-\varepsilon_*)(\varepsilon/\varepsilon_*)(k\eta_*)^{2m^2/3H^2}]$. We will get the same phenomenology but with γ^2 replaced by $\gamma^2\varepsilon/\varepsilon_*$.
[34] L. Pilo, A. Riotto, and A. Zaffaroni, Phys. Rev. Lett. **92**, 201303 (2004).
[35] L. Knox and Y.-S. Song, Phys. Rev. Lett. **89**, 011303 (2002).
[36] L. Knox and Y.-S. Song, Phys. Rev. D **68**, 043518 (2003); M. Kesden, A. Cooray, and M. Kamionkowski, Phys. Rev. Lett. **89**, 011304 (2002).
[37] C. Armendáriz-Picón, J. Cosmol. Astropart. Phys. 03 (2004) 009.
[38] T. Damour, F. Piazza, and G. Veneziano, Phys. Rev. D **66**, 046007 (2002).
[39] K. Sigurdson, A. Kurylov, and M. Kamionkowski, Phys. Rev. D **68**, 103509 (2003).
[40] J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).
[41] D. Wands, N. Bartolo, and A. Riotto, Phys. Rev. D **66**, 043520 (2002).
[42] D. Langlois and F. Vernizzi, astro-ph/0403258 [Phys. Rev. D (to be published)].
[43] S. Weinberg, Phys. Rev. D **67**, 123504 (2003).