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Charged rotating black holes in five dimensional $U(1)^3$ gauged $\mathcal{N} = 2$ supergravity

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We obtain the general solution for nonextremal 3-charge dilatonic rotating black holes in the five dimensional $U(1)^3$ gauged $\mathcal{N} = 2$ supergravity coupled to two vector multiplets, in the case where the two rotation parameters are set equal. These solutions encompass all the previously-known extremal solutions, and, by setting the three charges equal, the recently-obtained nonextremal solutions of $\mathcal{N} = 2$ gauged five dimensional pure supergravity.

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Valuable tests of the AdS/CFT correspondence can be performed by taking charged black holes with nonzero cosmological constant as gravitational backgrounds [1,2]. The black-hole charges play the rôle of R-charges in the dual field theory [3]. Furthermore, one can gain insight into the phase structure of the strongly-coupled dual field theory by studying the thermodynamic stability of the black-hole solutions, and the analogue of the Hawking-Page transition [3–5]. The first examples of nonextremal charged black holes in five dimensions, as solutions of a gauged supergravity theory, were obtained in [6]. These, and some higher-dimensional generalizations obtained in [7], were all nonrotating.

Charged rotating black holes in four dimensional theories with a cosmological constant were obtained long ago [8], but until recently no analogous five dimensional charged rotating solutions were known, except in certain extremal supersymmetric (BPS) limiting cases [9,10]. In a recent paper [11], we constructed general solutions for charged rotating black holes in five dimensional gauged $\mathcal{N} = 2$ pure supergravity, in the case where the two angular momenta are taken to be equal. These nonextremal solutions encompass the extremal solutions of [9,10] as special cases. By instead setting the charge to zero, the solutions in [11] reduce to the rotating five dimensional Kerr-de Sitter black holes of [12], in the special case where the two rotation parameters are set equal.

In this letter we extend our previous results, by constructing a general class of nonextremal charged rotating black-hole solutions in the five dimensional $U(1)^3$ gauged theory of $\mathcal{N} = 2$ supergravity coupled to two vector multiplets. We obtain the general nonextremal solutions of this dilatonic theory, with three independent electric charges, subject to the specialisation that the two angular momenta in the orthogonal 4-space are set equal. These 3-charge solutions are important for probing fully the microscopic degrees of freedom associated with the 3 R-charges in the dual $\mathcal{N} = 4$ CFT on the boundary, without the loss of information that would be inherent if the three charges were set equal.

Our new 3-charge solutions are generalisations to the gauged theory of the 3-charge spinning black-hole solutions (with two rotation parameters set equal) of the corresponding five dimensional *ungauged* supergravity, obtained in [13]. They also, of course, specialize to our previous results in [11] if one sets the three electric charges equal, under which circumstance the two dilatonic scalars decouple and become constant.

The bosonic sector of the five dimensional $\mathcal{N} = 2$ gauged supergravity coupled to two vector multiplets is described by the Lagrangian

$$e^{-1}\mathcal{L} = R - \frac{1}{2}\partial\vec{\varphi}^2 - \frac{1}{4}\sum_{i=1}^{3} X_i^{-2}(F^i)^2 - \lambda\sum_{i=1}^{3} X_i^{-1} + \frac{1}{24}\epsilon_{ijk}\epsilon^{\mu\nu\rho\sigma\lambda}F^i_{\mu\nu}F^j_{\rho\sigma}A^k_\lambda, \tag{1}$$

where $\vec{\varphi} = (\varphi_1, \varphi_2)$, and

$$X_1 = e^{-(1/\sqrt{6})\varphi_1-(1/\sqrt{2})\varphi_2}, \qquad X_2 = e^{-(1/\sqrt{6})\varphi_1+(1/\sqrt{2})\varphi_2}, \qquad X_3 = e^{(2/\sqrt{6})\varphi_1}. \tag{2}$$

The gauge-coupling constant g is related to λ by $\lambda = -g^2$.

The solutions that we have obtained are as follows:

$$ds_5^2 = -\frac{Y-f_3}{R^2}dt^2 + \frac{r^2R}{Y}dr^2 + Rd\Omega_3^2 + \frac{f_1-R^3}{R^2}(\sin^2\theta d\phi + \cos^2\theta d\psi)^2 - \frac{2f_2}{R^2}dt(\sin^2\theta d\phi + \cos^2\theta d\psi), \tag{3}$$

$$A^i = \frac{\mu}{r^2H_i}[s_ic_idt + \ell(c_is_js_k - s_ic_jc_k)(\sin^2\theta d\phi + \cos^2\theta d\psi)], \tag{4}$$

$$X_i = \frac{R}{r^2 H_i}, \qquad i = 1, 2, 3 \tag{5}$$

where

$$\begin{aligned} R &\equiv r^2\Big(\prod_{i=1}^{3} H_i\Big)^{1/3}, \qquad H_i \equiv 1 + \frac{\mu s_i^2}{r^2}, \\ d\Omega_3^2 &= d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2, \end{aligned} \tag{6}$$

and s_i and c_i are shorthand notations for

$$s_i \equiv \sinh\delta_i, \qquad c_i \equiv \cosh\delta_i, \qquad i = 1, 2, 3. \tag{7}$$

Note that in the expressions (4) for the vector potentials A^i, the triplet indices (i, j, k) are all unequal: $(i \neq j \neq k \neq i)$. The functions (f_1, f_2, f_3, Y) are given by

$$\begin{aligned} f_1 &= R^3 + \mu\ell^2 r^2 + \mu^2\ell^2\Big[2\Big(\prod_i c_i - \prod_i s_i\Big)\prod_j s_j - \sum_{i<j} s_i^2 s_j^2\Big], \quad f_2 = \gamma\ell\lambda R^3 + \mu\ell\Big(\prod_i c_i - \prod_i s_i\Big) r^2 + \mu^2\ell\prod_i s_i, \\ f_3 &= \gamma^2\ell^2\lambda^2 R^3 + \mu\ell^2\lambda\Big[2\gamma\Big(\prod_i c_i - \prod_i s_i\Big) - \Sigma\Big] r^2 + \mu\ell^2 - \lambda\Sigma\mu^2\ell^2\Big[2\Big(\prod_i c_i - \prod_i s_i\Big)\prod_j s_j - \sum_{i<j} s_i^2 s_j^2\Big] \\ &\quad + 2\lambda\gamma\mu^2\ell^2\prod_i s_i, \\ Y &= f_3 - \lambda\Sigma R^3 + r^4 - \mu r^2, \end{aligned} \tag{8}$$

where

$$\Sigma \equiv 1 + \gamma^2\ell^2\lambda. \tag{9}$$

It is helpful to note that $\sqrt{-g}$ takes a simple form, namely $\sqrt{-g} = rR\sin\theta\cos\theta$.

We arrived at the above solution by making conjectures for the expressions for the metric, vector potentials and dilatonic scalars that reduced to previously-known cases under appropriate limits. In particular, we were guided by the results for the ungauged case in [13], and the results for the nondilatonic gauged case (i.e., with three equal charges) in [11]. Verifying that the conjectured configuration solves the equations of motion following from (1) is then a straightforward mechanical exercise, which is most easily accomplished with the aid of a computer. (We used Mathematica for this purpose.) There are six free parameters in the solution, namely $(\mu, \delta_1, \delta_2, \delta_3, \ell, \gamma)$. The constant μ, together with the three "nonextremality parameters" δ_i, characterize the mass and the three electric charges associated with the three vector potentials A^i. The parameter ℓ characterizes the rotation of the black hole. One can define "physical" mass, charge and angular momentum parameters M, Q_i and J, according to

$$\begin{aligned} M &= \frac{1}{2}\mu\sum_i (s_i^2 + c_i^2), \qquad Q_i = \mu s_i c_i, \\ J &= \mu\ell\Big(\prod_i c_i - \prod_i s_i\Big). \end{aligned} \tag{10}$$

The sixth constant γ is trivial.

In order to make the global structure of the metrics more apparent, it is convenient to rewrite the metric (3) in terms of left-invariant 1-forms σ_i on S^3. Defining

$$\begin{aligned} \sigma_1 &= \cos\tilde\psi d\tilde\theta + \sin\tilde\psi\sin\tilde\theta d\tilde\phi, \\ \sigma_2 &= -\sin\tilde\psi d\tilde\theta + \cos\tilde\psi\sin\tilde\theta d\tilde\phi, \\ \sigma_3 &= d\tilde\psi + \cos\tilde\theta d\tilde\phi, \end{aligned} \tag{11}$$

where

$$\psi - \phi = \tilde\phi, \qquad \psi + \phi = \tilde\psi, \qquad \theta = \frac{1}{2}\tilde\theta, \tag{12}$$

we find that (3) can be rewritten as

$$\begin{aligned} ds_5^2 &= -\frac{RY}{f_1}dt^2 + \frac{r^2 R}{Y}dr^2 + \frac{1}{4}R(\sigma_1^2 + \sigma_2^2) \\ &\quad + \frac{f_1}{4R^2}\Big(\sigma_3 - \frac{2f_2}{f_1}dt\Big)^2, \end{aligned} \tag{13}$$

while the vector potentials in (4) become

$$A^i = \frac{\mu}{r^2 H_i}\Big[s_i c_i dt + \frac{1}{2}\ell(c_i s_j s_k - s_i c_j c_k)\sigma_3\Big]. \tag{14}$$

I. REDUCTIONS TO PREVIOUSLY-KNOWN SOLUTIONS

Various limits of our new solutions reduce to previously-known cases. These include the nonextremal 3-charge spinning black-hole solutions of the ungauged theory in [13] (specialized to the case of equal angular momenta); the nonextremal charged rotating solutions of the pure $\mathcal{N} = 2$ gauged theory found recently in [11]; the BPS 3-charge rotating solutions of Klemm and Sabra [14]; and the BPS 3-charge rotating solutions of Gutowski and Reall [15]. In detail, these various cases arise as follows:

(i) The ungauged limit (i.e., $\lambda = 0$) leads to the special case of the solutions of [13] where one sets the two angular momenta parameters equal, i.e., $\ell_1 =$

$\ell_2 = \ell$. This limit is obtained from our solution by setting $\lambda = 0$, replacing μ by $2m$, and by redefining $r^2 \to r^2 + \ell^2$.

(ii) The equal-charge limit, i.e., setting $\delta_1 = \delta_2 = \delta_3 \equiv \delta$, reduces to the solution found in [11]. Note that the parameters β and J in [11] are related to γ and ℓ of the present paper by

$$\beta = \gamma e^{\delta}, \qquad J = \ell e^{-\delta}, \tag{15}$$

while the parameters M and Q in [11] can be read off from (10). The radial variable in [11] is given by sending $(r^2 + \mu \sinh^2\delta) \to r^2$.

(iii) The Klemm-Sabra solution [14], which has closed timelike curves, is a BPS limit of our solution, obtained by taking

$$\begin{aligned} &\mu \to 0, \qquad \delta_i \to -\infty, \\ &Q_i = \frac{1}{4}\mu e^{-2\delta_i}, \qquad \ell = \alpha\sqrt{\mu}, \end{aligned} \tag{16}$$

where the three charges Q_i and the constant α are kept finite and nonzero. The black-hole mass and angular momentum, defined in (10), are then given by

$$M = -(Q_1 + Q_2 + Q_3), \qquad J = 2\alpha\sqrt{Q_1 Q_2 Q_3}. \tag{17}$$

Note that since $\ell \to 0$ and the γ parameter appears only in a product with ℓ, the solution does not depend on γ.

(iv) The Gutowski-Reall solution [15] is a regular BPS limit of our solution, obtained by taking

$$\begin{aligned} &\mu \to 0, \qquad \delta_i \to +\infty, \qquad Q_i = \frac{1}{4}\mu e^{2\delta_i}, \\ &\gamma = \sqrt{\mu}, \qquad \ell = \frac{1}{\sqrt{-\lambda\mu}}, \end{aligned} \tag{18}$$

where the charges Q_i are kept finite and nonzero. Note that γ goes to zero, while ℓ goes to infinity, when the limit is taken. (The solution remains finite, however.) The black-hole mass and the angular momentum, following from (10), are:

$$\begin{aligned} M &= +(Q_1 + Q_2 + Q_3), \\ J &= \frac{Q_1Q_2 + Q_1Q_3 + Q_2Q_3}{2\sqrt{-\lambda}\sqrt{Q_1Q_2Q_3}}. \end{aligned} \tag{19}$$

As in the solutions corresponding to setting all three charges equal, obtained in [11], we can obtain more general BPS limits from our general solutions, because of the additional free parameter γ.

II. FURTHER REMARKS

The metric (13) has horizons at values of the radial coordinate where RYf_1^{-1} vanishes. In order to avoid naked singularities, the outer horizon at $r = r_H$ should lie outside the curvature singularity at $R = 0$, and thus we require that it occur at the largest positive root of $Y(r_H) = 0$. In order to avoid having closed timelike curves (CTCs), f_1 should be positive for all $r > r_H$. A detailed analysis of the restrictions on the parameters in order to obtain solutions free of naked singularities or closed CTCs is quite involved, and we shall not present it here. It is analogous to the one given in [11] for the case where all three charges are equal. Clearly, there exist appropriate ranges of the parameters for which such "regular" black holes arise.

On the horizon, the Killing vector

$$l = \frac{\partial}{\partial t} - \frac{2f_2(r_H)}{f_1(r_H)}\frac{\partial}{\partial\tilde{\psi}} \tag{20}$$

becomes null, and thus $r = r_H$ corresponds to a Killing horizon. The surface gravity κ, which is constant over the horizon, can be calculated from

$$\kappa^2 = (\partial_\mu K)(\partial^\mu K)|_{r=r_H}, \tag{21}$$

where $K = \sqrt{-l^\mu l_\mu}$, implying that

$$\kappa = \left|\frac{Y'(r_H)}{4r_H^2 f_1(r_H)}\right|. \tag{22}$$

The area of the Killing horizon is given by

$$A = 2\pi^2\sqrt{f_1(r_H)}. \tag{23}$$

The Hawking temperature and entropy are therefore given by

$$T = \left|\frac{Y'(r_H)}{8\pi r_H^2 f_1(r_H)}\right|, \qquad S = \frac{1}{2}\pi^2\sqrt{f_1(r_H)}. \tag{24}$$

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