Dark matter at colliders: A model-independent approach

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Assuming that cosmological dark matter consists of weakly interacting massive particles, we use the recent precise measurement of cosmological parameters to predict the guaranteed rates of production of such particles in association with photons at electron-positron colliders. Our approach is based on general physical principles such as detailed balancing and soft/collinear factorization. It leads to predictions that are valid across a broad range of models containing WIMPs, including supersymmetry, universal extra dimensions, and many others. We also discuss the discovery prospects for the predicted experimental signatures.

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I. INTRODUCTION

A variety of astrophysical and cosmological observations indicate that a substantial fraction (perhaps as much as 30%) of the energy density in the Universe is due to nonrelativistic, nonbaryonic, nonluminous matter. The microscopic nature of this "dark" matter is at present unknown. Perhaps the most attractive explanation is provided by the *"WIMP hypothesis"*: dark matter is assumed to consist of hypothetical stable particles with masses around the scale of electroweak symmetry breaking, in the 10 GeV–1 TeV range, whose interactions with other elementary particles are of the strength and range similar to the familiar weak interactions of the Standard Model. Such weakly interacting massive particles (WIMPs) naturally have a relic abundance of the correct order of magnitude to account for the observed dark matter, making them appealing from a theoretical point of view. Moreover, many extensions of the Standard Model contain particles which can be identified as WIMP dark matter candidates. Examples include supersymmetric models [1], models with "universal" extra dimensions [2], little Higgs theories [3], etc.

Particle accelerators of the next generation may have enough energy to produce WIMPs. Once produced, WIMPs escape the detector without interactions, leading to an apparent energy imbalance, or "missing energy" signature. In this paper, we will use the known cosmological abundance of WIMPs to predict the rate of such events at future colliders. In sharp contrast to all existing studies, we will do so without making *any* assumptions about the particle physics model responsible for the WIMP: our results are equally valid in all theories listed in the previous paragraph, as well as in any other model containing WIMPs. In our approach, all model uncertainties are reduced to a single continuous parameter with a transparent physical meaning, plus a small number of discrete choices such as the spin of the WIMP.

II. WIMP ANNIHILATION CROSS SECTIONS FROM COSMOLOGY

We assume that the observed dark matter is entirely due to a single WIMP particle, χ. This particle carries a new conserved quantum number which prevents it from decaying into lighter standard model (SM) particles, making it stable. At the same time, two WIMPs can annihilate into a pair of standard model particles [4]:

$$\chi + \chi \to X_i + \bar{X}_i, \tag{1}$$

where $X_i = l, q, g, \ldots$ can be any SM particle. We assume that (1) is the only process important for the determination of the χ relic abundance. The present dark matter density depends on the cross sections of reactions (1) in the limit when the colliding χ particles are nonrelativistic. If v is the relative velocity of two χ's, each cross section can be expanded as [5]

$$\sigma_i v = \sum_{J=0}^{\infty} \sigma_i^{(J)} v^{2J}, \tag{2}$$

where $\sigma^{(0)}$ only receives a contribution from s-wave annihilation, $\sigma^{(1)}$ receives contributions from s- and p-wave channels, etc., It is clear that at low v, the lowest nonvanishing term in Eq. (2) will dominate. We define

$$\sigma_{\rm an} = \sum_i \sigma_i^{(J_0)}, \tag{3}$$

where J_0 is the angular momentum of the dominant partial wave contributing to χ annihilation in a given model, and the sum only runs over the final states that give a nonvanishing contribution at this order in J. For most of this paper, we will restrict our analysis to two cases: $J_0 = 0$ and $J_0 = 1$. We will refer to WIMPs in each case as "s-annihilators" and "p-annihilators", respectively.

The present cosmological abundance of WIMPs is mainly determined by the values of J_0 and $\sigma_{\rm an}$. It is these parameters that are strongly constrained by cosmological observations. In Fig. 1, we show the cosmological con-

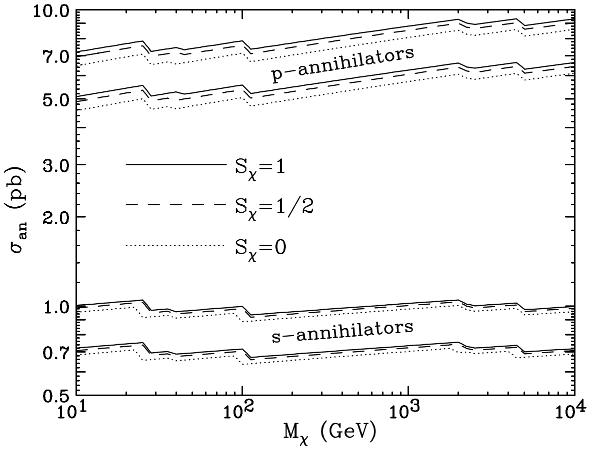

FIG. 1. Values of the quantity σ_{an} defined in (3) allowed at 2σ level as a function of the WIMP mass. The lower (upper) band is for models where s-wave (p-wave) annihilation dominates.

straint on σ_{an}, as a function of M_χ, for s- and p-annihilators and three values of the WIMP spin, $S_\chi = 0, 1/2, 1$. We have used the WMAP value of the present dark matter abundance, $\Omega_{dm}h^2 = 0.112 \pm 0.009$ [6]. The constraint on σ_{an} is surprisingly robust, as the logarithmic dependence on M_χ is offset by the change in the effective number of degrees of freedom at different freezeout temperatures.

III. DETAILED BALANCING AND WIMP PRODUCTION AT COLLIDERS

The cross sections of the reaction (1) and its inverse are related by the detailed balancing equation:

$$\frac{\sigma(\chi + \chi \to X_i + \bar{X}_i)}{\sigma(X_i + \bar{X}_i \to \chi + \chi)} = 2\frac{v_X^2(2S_X + 1)^2}{v_\chi^2(2S_\chi + 1)^2}, \tag{4}$$

where the cross sections are averaged over spins but not other quantum numbers such as color. For each SM particle X_i, we define the "annihilation fraction" κ_i as

$$\kappa_i = \frac{\sigma_i^{(J_0)}}{\sigma_{an}}. \tag{5}$$

Note that $\sum \kappa_i = 1$. Using Eqs. (4) and (5) we obtain the following expression for the production of nonrelativistic χ pairs in $X_i\bar{X}_i$ collisions

$$\sigma(X_i\bar{X}_i \to 2\chi) = 2^{2(J_0-1)}\kappa_i\sigma_{an}\frac{(2S_\chi + 1)^2}{(2S_X + 1)^2} \times\left(1 - \frac{4M_\chi^2}{s}\right)^{1/2+J_0}, \tag{6}$$

where we have assumed that the initial state particles are relativistic ($M_X \ll M_\chi$). This formula is only valid at center of mass energies slightly above the 2χ threshold, $v = 2v_\chi = 2\sqrt{1 - 4M_\chi^2/s} \ll 1$, and receives corrections of order v^2. Taking $X_i = q$ or g (or even W, Z) for a hadron collider or $X_i = e$ for an electron-positron machine, Eq. (6) provides a prediction of the WIMP production rate. The model-dependence of this prediction is contained in a small number of parameters with a clear physical meaning: the mass M_χ and the spin S_χ of the WIMP, the value of J_0, and the annihilation fraction κ_i for the given initial state. Crucially, the overall scale for this prediction, the quantity σ_{an}, is provided by cosmology, as shown in Fig. 1.

IV. TAGGING AND FACTORIZATION

Unfortunately, the 2χ production process is not measurable at colliders. Much like neutrinos, WIMPs cannot be directly observed due to the weakness of their interactions with matter. At least one detectable particle is required for the event to pass the triggers. In order to retain the model-independence of our analysis, we need to study processes in which two WIMPs are produced in association with a photon or a gluon radiated from the known initial state.

In this paper, we concentrate on the case of $e^+e^- \to 2\chi + \gamma$. For general kinematics, there is no model-independent relation between the rate of this process and that of $e^+e^- \to 2\chi$ predicted by Eq. (6). However, if the emitted photon is either *soft* or *collinear* with the incoming electron or positron, soft/collinear factorization theorems provide such a relation. Emission of collinear photons is described by

$$\frac{d\sigma(e^+e^- \to 2\chi + \gamma)}{dx\,d\cos\theta} \approx \mathcal{F}(x, \cos\theta)\hat{\sigma}(e^+e^- \to 2\chi), \tag{7}$$

where $x = 2E_\gamma/\sqrt{s}$ (E_γ is the photon energy), θ is the angle between the photon and the incoming electron beam, $\mathcal{F}$ denotes the collinear factor:

$$\mathcal{F}(x, \cos\theta) = \frac{\alpha}{\pi}\frac{1 + (1 - x)^2}{x}\frac{1}{\sin^2\theta}, \tag{8}$$

and $\hat{\sigma}$ is the WIMP pair-production cross section evaluated at the reduced center of mass energy, $\hat{s} = (1 - x)s$. Note that upon integration over θ, the above equation reproduces the familiar Weizsacker-Williams distribution function. The factor $\mathcal{F}$ is universal: it does not depend on the nature of the neutral particles produced in association with the photon. Emission of soft photons is described by the leading piece of Eq. (7) in the $x \to 0$ limit.

Combining Eqs. (6) and (7), we find

$$\frac{d\sigma}{dx\,d\cos\theta} \approx \frac{\alpha\kappa_e\sigma_{an}}{16\pi}\frac{1 + (1 - x)^2}{x}\frac{1}{\sin^2\theta}2^{2J_0}(2S_\chi + 1)^2 \times\left(1 - \frac{4M_\chi^2}{(1 - x)s}\right)^{1/2+J_0}. \tag{9}$$

This formula, applicable for collinear photons ($\theta \to 0$ or $\theta \to \pi$), is the main result of this paper.

V. VALIDITY OF COLLINEAR APPROXIMATION

Equation (9) predicts the rate of events with a single collinear photon and missing energy due to WIMP production in e^+e^- collisions. However, very collinear photons cannot be detected in an experiment, due to an incomplete electromagnetic calorimeter coverage around the beam pipe as well as the lower cut on the $p_{T,\gamma} \equiv E_\gamma \sin\theta$ that has to be imposed to reject backgrounds such as $e^+e^- \to e^+e^-\gamma$ where electron and positron are too forward to be detected. Do the predictions made using the collinear factorization approach have any value in realistic circumstances? To address this question, we have compared the event rates obtained by integrating Eq. (9) with realistic cuts with those obtained in explicit models containing WIMPs without making any approximations. For this comparison, we have chosen $\sqrt{s} = 500$ GeV, assumed the electromagnetic calorimeter acceptance $\sin\theta > 0.1$ [7], and required $p_{T,\gamma} > 7.5$ GeV corresponding to the mask calorimeter acceptance of 1 °. The results of the comparison are shown in Fig. 2. The red (dark-gray) histograms show the photon spectra from the reaction $e^+e^- \to \chi_1^0\chi_1^0\gamma$ within the minimal supersymmetric standard model (MSSM) [8] with the parameters suitably chosen to provide the correct neutralino relic density [9]. (Explicitly, for $M_\chi = (100, 150, 200, 225)$ GeV, the MSSM parameters take the following values at the weak scale: $M_1 = (115, 168, 217, 242)$ GeV, $\mu = (185, 225, 275, 300)$ GeV, and $m_{\tilde{\ell}_R} = (115, 177, 237, 268)$ GeV; for all four points, $M_2 = 2M_1$, $\tan\beta = 10$, and all the mass parameters not listed above are fixed at one TeV.) The green (light-gray) lines on the same figure show the spectra predicted by Eq. (9) for a "generic" p-annihilator of the corresponding mass and κ_e. We conclude that our approach works quite well. The photon spectrum near the endpoint is correctly reproduced for all M_χ. Equation (9) fails for lower values of E_γ; this effect is especially noticeable for low M_χ. This is due not to the failure of collinear approximation, but rather to the fact that the relative motion of the produced χ particles becomes relativistic in this regime, and the higher-order terms in the v^2 expansion of Eq. (2), not captured by $\sigma_{\rm an}$, are important. Model-independent WIMP searches at e^+e^- colliders, which we discuss below, should take this limitation into account by concentrating on the photons near the endpoint of the spectrum. Note that we did not have to impose an additional cut to eliminate central photons: collinear emission naturally dominates the signal.

VI. EXPERIMENTAL SEARCHES FOR WIMPS

The main irreducible background to the search for anomalous γ + missing E events is provided by the Standard Model reaction $e^+e^- \to \nu\bar{\nu}\gamma$. At the energies well above the Z peak, this reaction is dominated by the t-channel W exchange contribution, and has a rather large

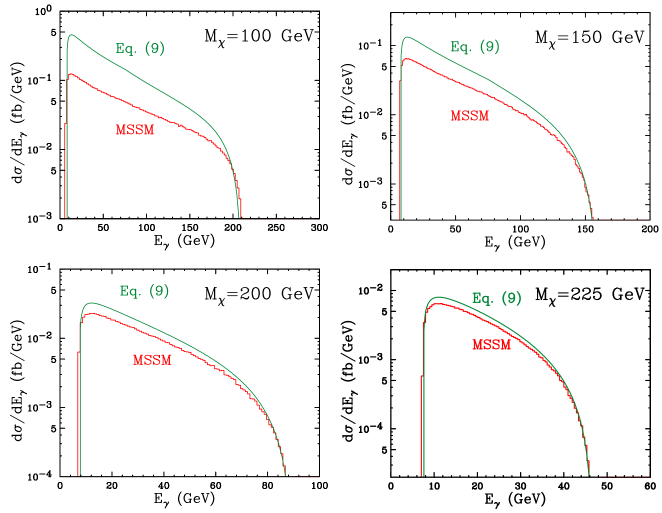

FIG. 2 (color online). Comparison between the photon spectra from the process $e^+e^- \to 2\chi_1^0 + \gamma$ in the explicit supersymmetric models defined in the text (red/dark-gray) and the spectra predicted by Eq. (9) for a p-annihilator of the corresponding mass and κ_e (green/light-gray).

cross section. Nevertheless, the enhancement of the rate predicted by Eq. (9) may well be observable. In Fig. 3, we show the reach of a 500 GeV linear collider (LC) with an integrated luminosity of 500 fb^{-1} to p-annihilator WIMPs in terms of the values of κ_e that can be probed at three and five σ level, as a function of the WIMP mass M_χ. (For comparison, a typical value of the κ_e parameter in the bulk of mSUGRA parameter space is between 0.2 and 0.3.) The kinematic acceptance cuts imposed on the

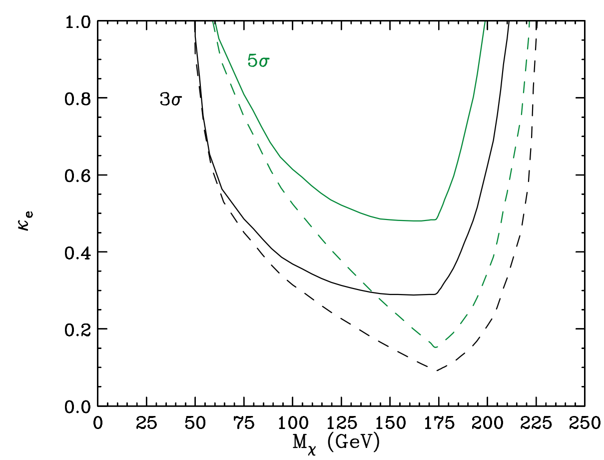

FIG. 3 (color online). The reach of a 500 GeV unpolarized electron-positron collider with an integrated luminosity of 500 fb − 1 for the discovery of p-annihilator WIMPs, as a function of the WIMP mass M_χ and the e^+e^- annihilation fraction κ_e. The three σ (black) and five σ (green/light-gray) contours are shown. The dashed lines include only statistical uncertainty, whereas the solid lines include a systematic uncertainty of 0.3% [7].

photon are $\sin\theta > 0.1$, $p_{T,\gamma} > 7.5$ GeV. Moreover, the accepted photons have to satisfy

$$\frac{\sqrt{s}}{2}\left(1 - \frac{8M_\chi^2}{s}\right) \le E_\gamma \le \frac{\sqrt{s}}{2}\left(1 - \frac{4M_\chi^2}{s}\right). \quad (10)$$

The lower cut ensures that the relative motion of the produced WIMPs is nonrelativistic, $v_\chi^2 < 1/2$ in the center of mass frame of the WIMP pair. The upper cut corresponds to the endpoint of the photon spectrum for a given M_χ, and serves to improve the signal/background (S/B) ratio. Note that the cuts in Eq. (10), and therefore the data set used to test the WIMP hypothesis, depend on the assumed WIMP mass. The values of the signal and background cross sections with the cuts specified above for a few representative values of M_χ are given in Table I. The table also lists the signal cross section values for the case of s-annihilators. It is clear that the sensitivity of the proposed search in this case is at best marginal.

The reach of the LC can be further increased by polarizing the beams: while the background is dominated by the W exchange diagrams which only appear for left-handed electrons, there is no reason to expect that the WIMP couplings have the same asymmetry. For polarized beams, the signal cannot be fully characterized by the spin-averaged annihilation fraction κ_e introduced in Eq. (5); instead, four independent annihilation fractions are needed, corresponding to the four possible e^+e^- helicity configurations. To apply Eq. (9) to this case, we make a replacement

$$\begin{aligned}\kappa_e \rightarrow &\frac{1}{4}(1 + P_-)[(1 + P_+)\kappa(e_-^R e_+^L) + (1 - P_+)\kappa(e_-^R e_+^R)] \\ &+ \frac{1}{4}(1 - P_-)[(1 + P_+)\kappa(e_-^L e_+^L) \\ &+ (1 - P_+)\kappa(e_-^L e_+^R)], \quad (11)\end{aligned}$$

where $P_\pm$ are the polarizations of the positron and electron beams ($P = 0$ corresponds to unpolarized beams, $P_- = 1$ to pure right-handed electron beam, and $P_+ = 1$ to pure left-handed positron beam). Ignoring the Z exchange contribution, the background cross section scales as $(1 - P_-)(1 - P_+)$.

TABLE I. Signal and background cross sections at $\sqrt{s} = 500$ GeV with no polarization and the cuts specified in the text, for a few representative values of M_χ. The signal cross sections are listed for p- and s-annihilators with $\kappa_e = 1$, and scale linearly with κ_e.

M_χ, GeV	75	100	125	150	175	200	225
$\sigma_{\rm bg}$, fb	36	83	202	590	2030	1800	1200
$\sigma_{\rm sig}$ (p-ann.), fb	1.8	3.9	8.4	21	64	27	4.9
$\sigma_{\rm sig}$ (s-ann.), fb	0.4	0.9	1.9	4.5	13	8.7	3.6

VII. DISCUSSION

All existing studies discussing the prospects for discovery of WIMPs at colliders do so within a specific model, fixing the model parameters to reflect the observed dark matter abundance. Their interpretation is hindered by the large number of theoretical assumptions made about the details of particle physics at the TeV scale. For example, supersymmetric models often lead to high rates of events with missing E_T at the LHC due to production of strongly-coupled gluinos and squarks, whose decay chains necessarily involve neutralinos. It is possible, and indeed likely, that such high rates will be observed. However, this is by no means guaranteed by the WIMP hypothesis itself. In contrast, the rates predicted here are guaranteed (up to the unknown annihilation fraction) once the WIMP hypothesis is accepted. Therefore, the signatures we have discussed provide a unique opportunity to directly test the WIMP hypothesis at high-energy colliders. We believe that this direction is well worth pursuing.

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