Skyrme model predictions for the $27_{J=3/2}$ mass spectrum and the $27_{3/2} - \overline{10}$ mass splittings

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The $\mathbf{27}_{J=3/2}$-plet mass spectrum and the $\mathbf{27}_{3/2} - \overline{\mathbf{10}}$ mass splittings are computed in the framework of the minimal $SU(3)_f$ extended Skyrme model. As functions of the Skyrme charge e and the $SU(3)_f$ symmetry breaking parameters the predictions are presented in tabular form. The predicted mass splitting $\mathbf{27}_{3/2} - \overline{\mathbf{10}}$ is the smallest among all $SU(3)_f$ baryonic multiplets, confirming earlier findings.

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The discovery [1] of the exotic baryon Θ^+, with strangeness $+1$ and probable spin 1/2, recently supported by the observations of Θ^+ in various experiments [2–5], and the discovery [6] of the exotic isospin 3/2 baryon with strangeness -2, $\Xi^{--}_{3/2}$, have produced huge excitement in the high energy physics community.

The Θ^+-baryon mass was successfully predicted in the "model-independent" way for the first time in Ref. [7]. However, it was the prediction of the narrow width of Θ^+ in the chiral quark-soliton model of Ref. [8] that stimulated experimental searches. To estimate baryon multiplets ($\mathbf{8}, \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$, etc.) mass spectra, relevant mass differences, and other baryon properties, various authors employed different types of methods and models [9–33].

The main aim of this Brief Report is the application of the minimal $SU(3)_f$ extended Skyrme model [13] in an attempt to predict the $\mathbf{27}_{3/2} - \overline{\mathbf{10}}$ mass splitting and the $\mathbf{27}_{3/2}$-plet mass spectrum. The minimally extended Skyrme model uses only one free parameter, the Skyrme charge e, and only flavor symmetry breaking (SB) term proportional to λ_8 in the action $\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_{\rm Sk} + \mathcal{L}_{\rm WZ} + \mathcal{L}_{\rm SB}$, where $\mathcal{L}_\sigma$, $\mathcal{L}_{\rm Sk}$, $\mathcal{L}_{\rm WZ}$, and $\mathcal{L}_{\rm SB}$ denote the σ-model, Skyrme, Wess-Zumino and SB terms [34–40], respectively. For the profile function in $\mathcal{L}$ we use the arctan ansatz [41] which makes possible to evaluate relevant overlap integrals analytically. The classical soliton mass $\mathcal{M}_{\rm csol}$ receives too large a value producing an unrealistic baryonic mass spectrum. We are using it only to obtain the dimensionless size of the skyrmion x_0 by minimizing $\mathcal{M}_{\rm csol}(x_0)$. The dimensionless size of the skyrmion x_0 includes the dynamics of SB effects, which take place within the skyrmion. It follows that [29]:

$$x_0^2 = \frac{15}{8}\left[1 + \frac{6\beta'}{f_\pi^2} + \sqrt{\left(1 + \frac{6\beta'}{f_\pi^2}\right)^2 + \frac{30\delta'}{e^2 f_\pi^4}}\right]^{-1}, \qquad (1)$$

where the SB parameters $\hat{x}$, β', δ' are given by [13]:

$$\hat{x} = \frac{2m_K^2 f_K^2}{m_\pi^2 f_\pi^2} - 1, \qquad \beta' = \frac{f_K^2 - f_\pi^2}{4(1-\hat{x})}, \qquad \delta' = \frac{m_\pi^2 f_\pi^2}{4} = \frac{m_K^2 f_K^2}{2(1+\hat{x})}. \qquad (2)$$

The symmetry breaker $\hat{x}$ was constructed systematically from the QCD mass term in the case of $SU(3)_f$. The δ' term is required to split pseudoscalar meson masses, while the β' term is required to split pseudoscalar decay constants (for details, see Ref. [13]).

To obtain the $\mathbf{27}_{3/2} - \overline{\mathbf{10}}$ mass splittings and the $\mathbf{27}$ 3/2 mass spectrum, the following definitions of the mass formulas are used:

$$\mathrm{M}_B^{\overline{\mathbf{10}}}(x_0) = \mathcal{M}^{\mathbf{8}} + \frac{3}{2\lambda_s(x_0)} - \frac{\gamma(x_0)}{2}\delta_B^{\overline{\mathbf{10}}}, \qquad (3)$$

$$\mathrm{M}_B^{\mathbf{27}}(x_0) = \mathcal{M}^{\mathbf{8}} + \frac{3}{2\lambda_c(x_0)} + \frac{1}{\lambda_s(x_0)} - \frac{\gamma(x_0)}{2}\delta_B^{\mathbf{27}}. \qquad (4)$$

Here the experimental octet mean mass $\mathcal{M}^{\mathbf{8}} = \frac{1}{8} \times \sum_{B=1}^{\mathbf{8}} \mathrm{M}_B^{\mathbf{8}} = 1151$ MeV was used instead of $\mathcal{M}^{\mathbf{8}} = \mathcal{M}_{\rm csol}(x_0) + \frac{3}{2\lambda_c(x_0)}$. From experiment it also follows that the decuplet mean mass $\mathcal{M}^{\mathbf{10}} = \frac{1}{10}\sum_{B=1}^{10} \mathrm{M}_B^{\mathbf{10}} = 1382$ MeV [42]. The splitting constants $\delta_B^{\overline{\mathbf{10}}}$ and $\delta_B^{\mathbf{27}}$ are given in [28] and in Table I of Ref. [26], respectively. The moment of inertia λ_c for rotation in coordinate space, the moment of inertia λ_s for flavor rotations in the direction of the strange degrees of freedom (except for the eighth direction), and the symmetry breaking quantity γ, [the coefficient in the SB piece $\mathcal{L}_{\rm SB} = -\frac{1}{2}\gamma(1 - D_{88})$ of a total collective Lagrangian $\mathcal{L}$], are given in Ref. [29].

From (3) and (4) the $\mathbf{27}_{3/2} - \overline{\mathbf{10}}$ mean mass splitting $\Delta_{\mathbf{27}}^{\overline{\mathbf{10}}}$ is given by

$$\Delta_{\mathbf{27}}^{\overline{\mathbf{10}}} \equiv \mathcal{M}_{3/2}^{\mathbf{27}} - \mathcal{M}^{\overline{\mathbf{10}}} = = \frac{1}{2}\left[\frac{3}{\lambda_c(x_0)} - \frac{1}{\lambda_s(x_0)}\right] \equiv \Delta_{\mathbf{8}}^{\mathbf{10}} - \frac{1}{3}\Delta_{\mathbf{8}}^{\overline{\mathbf{10}}}, \qquad (5)$$

where $\Delta^{\overline{10}}_{27}$ is also expressed in terms of the decuplet-octet Δ^{10}_{8} and the antidecuplet-octet $\Delta^{\overline{10}}_{8}$ mean mass splittings [29]. In the computations of the mean masses $\mathcal{M}^{\overline{10}}$ and $\mathcal{M}^{27}_{3/2}$ the sum of D_{88} diagonal elements over all components of irreducible representations cancels out because of the properties of the SU(3) Clebsh-Gordan coefficients.

The mass splittings between the same quark flavor content baryons of $\mathbf{27}_{3/2}$ and $\overline{\mathbf{10}}$-plets are:

$$\begin{aligned}
\delta_1 &= \mathrm{M}^{27}_{3/2}(\Theta_1) - \mathrm{M}^{\overline{10}}(\Theta^+) = \Delta^{\overline{10}}_{27} + \frac{3}{56}\gamma(x_0),\\
\delta_2 &= \mathrm{M}^{27}_{3/2}(N^*_{\frac{3}{2}}) - \mathrm{M}^{\overline{10}}(N^*) = \Delta^{\overline{10}}_{27} + \frac{1}{224}\gamma(x_0),\\
\delta_3 &= \mathrm{M}^{27}_{3/2}(N^*_{\frac{1}{2}}) - \mathrm{M}^{\overline{10}}(N^*) = \Delta^{\overline{10}}_{27} + \frac{5}{112}\gamma(x_0),\\
\delta_4 &= \mathrm{M}^{27}_{3/2}(\Sigma_2) - \mathrm{M}^{\overline{10}}(\Sigma) = \Delta^{\overline{10}}_{27} - \frac{5}{112}\gamma(x_0),\\
\delta_5 &= \mathrm{M}^{27}_{3/2}(\Sigma_1) - \mathrm{M}^{\overline{10}}(\Sigma) = \Delta^{\overline{10}}_{27} + \frac{1}{112}\gamma(x_0),\\
\delta_6 &= \mathrm{M}^{27}_{3/2}(\Lambda^*) - \mathrm{M}^{\overline{10}}(\Sigma) = \Delta^{\overline{10}}_{27} + \frac{1}{28}\gamma(x_0),\\
\delta_7 &= \mathrm{M}^{27}_{3/2}(\Xi^*_{\frac{3}{2}}) - \mathrm{M}^{\overline{10}}(\Xi_{\frac{3}{2}}) = \Delta^{\overline{10}}_{27} - \frac{3}{112}\gamma(x_0),\\
\delta_8 &= \mathrm{M}^{27}_{3/2}(\Xi^*_{\frac{1}{2}}) - \mathrm{M}^{\overline{10}}(\Xi_{\frac{3}{2}}) = \Delta^{\overline{10}}_{27} + \frac{3}{224}\gamma(x_0).
\end{aligned} \tag{6}$$

The Ξ isoquartet and isodoublet from the **27**, spin 3/2, we mark as $\Xi^*_{3/2}$ and $\Xi^*_{1/2}$, to distinguish them from the Ξ isoquartet and isodoublet from the $\overline{\mathbf{10}}$, spin 1/2. We also mark the **27**-plet isosinglet as Λ^*.

Considering the SB parameters (2), at this point we introduce three different dynamical assumptions based on the SB part of the Lagrangian producing three fits which will be used further in our numerical analysis:

$$\begin{aligned}
&\text{(i)}\quad m_\pi = m_K = 0, \quad f_\pi = f_K = 93\ \text{MeV}\\
&\qquad \longrightarrow \hat{x} = 1, \quad \beta' = \delta' = 0;\\
&\text{(ii)}\quad m_\pi = 138, \quad m_K = 495, \quad f_\pi = f_K = 93\ \text{MeV}\\
&\qquad \longrightarrow \hat{x} = 24.73, \quad \beta' = 0,\ \delta' = 4.12\times 10^7, \quad \text{MeV}^4;\\
&\text{(iii)}\quad m_\pi = 138, \ m_K = 495, \quad f_\pi = 93, \ f_K = 113\ \text{MeV}\\
&\qquad \longrightarrow \hat{x} = 36.97, \quad \beta' = -28.6\ \text{MeV}^2,\\
&\qquad\quad \delta' = 4.12\times 10^7, \quad \text{MeV}^4.
\end{aligned} \tag{7}$$

Switching off $SU(3)_f$ symmetry breaking, which corresponds to case (i), the absolute masses of each member of the multiplet become equal for each fixed e. In the chiral limit,

$$x_0 = \frac{\sqrt{15}}{4} \longrightarrow \Delta^{\overline{10}}_{27} = \delta_{1,\ldots,8} = \frac{52e^3 f_\pi}{285\sqrt{30}\pi^2}. \tag{8}$$

For example, from (4) and (8) one would have $\mathrm{M}^{27}_{3/2} =$ 1898 MeV and $\Delta^{\overline{10}}_{27} = 32.6$ MeV, for $e = 4.7$.

The mass splittings (5) and (6) as functions of two different dynamical assumptions, (ii, iii), and the Skyrme charge e are given in Table I. We have chosen four values of the Skyrme charge $e = 3.4, 4.2, 4.4, 4.7$ because in the minimal approach they give the best fit for the nucleon axial coupling constant $g_A = 1.25$ [43], the mass splitting $(\Delta^{10}_{8})_{\text{exp}} = 231$ MeV, and the pentaquark masses $\mathrm{M}^{\text{exp}}_{\Theta^+} = 1540$ MeV and $\mathrm{M}^{\text{exp}}_{\Xi^{--}_{3/2}} = 1861$ MeV, respectively.

Assuming equal spacing for antidecuplets, from the recent experimental data ($\mathrm{M}^{\text{exp}}_{\Theta^+} = 1540$ MeV and $\mathrm{M}^{\text{exp}}_{\Xi^{--}_{3/2}} = 1861$ MeV), in Ref. [29] we have found the following masses of antidecuplets $\mathrm{M}_{N^*} = 1647$ MeV, $\mathrm{M}_{\Sigma_{\overline{10}}} = 1754$ MeV, the mean mass $\mathcal{M}^{\overline{10}} = \frac{1}{10}\times\sum_{B=1}^{10}\mathrm{M}^{\overline{10}}_{B} = 1754$ MeV and the mass difference $\Delta^{\overline{10}}_{8} =$ 603 MeV. Taking 603 MeV, bonafide, as an "experimental" estimate for $\Delta^{\overline{10}}_{8}$, together with $(\Delta^{10}_{8})_{\text{exp}} = 231$ MeV, via Eq. (5), we estimate $\Delta^{\overline{10}}_{27} = 30$ MeV. It turns out from Table I that only $e \simeq 3.2$, in the most realistic case (iii), could account for the small value of $\Delta^{\overline{10}}_{27}$. However, $e =$ 3.2 gives too small values for Δ^{10}_{8} and $\Delta^{\overline{10}}_{8}$.

Using 1754 MeV for the $\overline{\mathbf{10}}$-plet mean mass and the predicted range for the mean mass splitting $30 \le \Delta^{\overline{10}}_{27} \le$ 95 MeV, we find the range for the $\mathbf{27}_{3/2}$-plet mean mass as $1784 \le \mathcal{M}^{27}_{3/2} \le 1849$ MeV, which is approximately placed into the center of the $\mathbf{27}_{3/2}$-plet mass spectrum displayed in Fig. 4 of Ref. [15] (for A and B fits), and in Fig. 4 of Ref. [28]. Careful inspection of the results for the $\mathbf{27}_{3/2}$-plet mass spectrum from Fig. 4 of Ref. [15] shows approximate agreement with our results, $\delta_1, \ldots, \delta_8$, for $4.2 \le e \le 4.7$ fit (iii), presented in Table I.

Comparing the pure Skyrme model prediction of Ref. [15] (fits A and B in Fig. 4) with our results from Table II, we have found that our case (iii) with $4.3 \le e \le$ 4.7 supports fit B, and for $4.4 \le e \le 4.6$ agrees nicely with fit A. Both fits A and B from [15] lie between $4.0 \le e \le 4.6$ for case (ii). Case (iii) with $4.2 \le e \le 4.7$ also supports the results presented in Table I of Ref. [26]. From Table II we conclude that the best fit for the $\mathbf{27}_{3/2}$ baryon mass spectrum, as a function of e and for $f_K \neq f_\pi$, would lie between $e \simeq 4.2$ and $e \simeq 4.7$, just like that for the octet, decuplet, and antidecuplet mass spectra [29]. In Table II the masses of Λ^* and $\Xi^*_{3/2}$ are equal owing to the absence of anomalous moments of inertia [7,12] in the model used. Note, however, that the anomalous moments of inertia contributions are estimated to be at best $\sim 1\%$ for the $\Xi^*_{3/2}$ mass [26,28], for example.

Next we comment on possible effects coming from the mixing between exotic rotational excitations and vibrational (or radial) excitations [31,32] in the minimal $SU(3)_f$ extended Skyrme model. Let us note that, in

TABLE I. The $\mathbf{27}_{3/2} - \overline{\mathbf{10}}$ mass splittings (MeV) as functions of the Skyrme charge e and for fits (ii), (iii).

Fit			(ii)					(iii)		
e	3.2	3.4	4.2	4.4	4.7	3.2	3.4	4.2	4.4	4.7
$\Delta_{\mathbf{8}}^{\overline{\mathbf{10}}}$	110	129	229	260	312	109	128	227	257	309
$\Delta_{\mathbf{8}}^{\overline{\mathbf{10}}}$	302	354	621	704	843	233	273	474	536	641
$\Delta_{\mathbf{27}}^{\overline{\mathbf{10}}}$	9	11	22	25	31	31	37	69	79	95
δ_1	99	89	67	66	65	179	165	146	148	154
δ_2	17	18	26	29	34	44	48	75	84	100
δ_3	84	76	60	59	59	154	144	133	136	144
δ_4	−66	−53	−16	−8	3	−91	−69	4	21	46
δ_5	24	24	30	32	37	56	59	82	90	105
δ_6	69	63	52	52	54	130	123	120	125	134
δ_7	−36	−27	−1	5	14	−42	−27	30	44	66
δ_8	32	31	33	36	40	68	69	88	96	110

the case of the **27**-plet, states with $Y = \pm 2$ and $Y = +1$, $I = 3/2$ do not mix with either **8**, **10**, or $\overline{\mathbf{10}}$, or with their vibrational excitations. They will have vibrational excitations themselves, but, as results of Ref. [32] indicate, such vibrations are expected to have minor influence on "base" states. Therefore, for these states our predictions are correct within the approximations made, i.e., by neglecting $1/N_c$ corrections to $\mathcal{L}_{\text{SB}}$. All other states will be subject to mixing. However, their masses, given in Table II, represent the predictions under no mixing assumption. Considering the question of the decay width calculations, the Skyrme model is too crude to give reliable predictions for the widths [26,28]. Here the $1/N_c$ corrections, missing in the present approach, are of primary importance.

For the simplest version of the total Lagrangian, the results given in Tables I and II do agree well with the other Skyrme model based estimates [7,8,14–16,26,28]. In particular, our approach is similar to [15,16].

As has been discussed in [43], although the symmetry breaking effects are generally very important, the main effect comes from the D_{88} term confirming the results of [15,16,28,32]. In our approach, in the language of [32], the reduction of the influence of the so-called $\bar{s}s$ cloud was taken into account by inclusion of the SB term $(1 - D_{88})$ in mass formulae (3) and (4).

It is clear from Table I that for fixed e the difference between fits (ii) and (iii) is crucial for the correct description of the mass splittings (5) and (6). For small mass splittings the contribution of the term proportional to $(f_K^2 - f_\pi^2)$ in the Lagrangian $\mathcal{L}$ plays a major role.

The $\mathbf{27}_{3/2} - \overline{\mathbf{10}}$ mass splittings are the quantities whose measured values, together with measurements of the decay modes branching ratios, would determine the spins, 3/2 or 1/2, of observed objects, like $\Xi_{3/2}^{--}$, thus placing it into the right $\text{SU}(3)_f$ representation. We do expect that experimental analysis, considering other members of the $\overline{\mathbf{10}}$ and $\mathbf{27}_{3/2}$-plets, should also be performed.

We hope that the present calculation, taken together with the analogous calculation in [7,13,15,16,26–29] will contribute to the understanding of the overall picture of the baryonic mass spectrum and mass splittings in the Skyrme model, as well as to further computations of other nonperturbative, dimension-6 operator matrix elements between different baryon states [43,44].

TABLE II. The $\mathbf{27}_{3/2}$ mass spectrum (MeV) as functions of the Skyrme charge e and for fits (ii), (iii).

Fit			(ii)					(iii)		
e	3.2	3.4	4.2	4.4	4.7	3.2	3.4	4.2	4.4	4.7
Θ_1	1343	1413	1734	1827	1980	1219	1290	1590	1674	1808
$N_{3/2}^*$	1365	1433	1745	1837	1988	1256	1322	1610	1691	1823
Σ_2	1388	1452	1756	1847	1997	1293	1354	1629	1708	1838
$N_{1/2}^*$	1433	1491	1779	1867	2014	1367	1418	1668	1743	1867
Σ_1	1478	1529	1802	1887	2031	1440	1482	1706	1778	1897
Λ^*	1522	1568	1824	1908	2048	1514	1546	1745	1812	1926
$\Xi_{3/2}^*$	1522	1568	1824	1908	2048	1514	1546	1745	1812	1926
$\Xi_{1/2}^*$	1590	1626	1858	1938	2073	1624	1642	1803	1864	1970
Ω_1	1657	1684	1892	1968	2099	1735	1738	1861	1916	2014

Since the splittings (5) and (6) represent the smallest splittings among splittings between the $SU(3)_f$ multiplets **8**, **10**, $\overline{\mathbf{10}}$, **27**, **35** and $\overline{\mathbf{35}}$, we would urge our colleagues to continue experimental analysis of pentaquark spectral and decay modes and find the pentaquark members of the $\mathbf{27}_{3/2}$-plet which would mix with or lie just above the pentaquark family of the $\overline{\mathbf{10}}$-plet.

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