Little Higgs model completed with a chiral fermionic sector

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The implementation of the little Higgs mechanism to solve the hierarchy problem provides an interesting guiding principle to build particle physics models beyond the electroweak scale. Most model building works, however, do not pay much attention to the fermionic sector. Through a case example, we illustrate how a complete and consistent fermionic sector of the TeV effective field theory may actually be largely dictated by the gauge structure of the model. The completed fermionic sector has a specific flavor physics structure, and many phenomenological constraints on the model can thus be obtained beyond gauge, Higgs, and top physics. We take a first look at some of the quark sector constraints.

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The hierarchy problem, or the very unnatural fine-tuning required to fix the electroweak scale due to the quadratic divergent quantum corrections to the Higgs boson mass, is a major theoretical shortcoming of the standard model (SM). The fine-tuning problem can be alleviated, only if there is new physics at the TeV scale that guarantees the cancellation of the quadratic divergence to an acceptable level, or totally changes our picture of SM physics. A guaranteed cancellation has to come from some mechanism protected by a symmetry. Candidates of the kind include supersymmetry and the recently proposed little Higgs mechanism [1,2].

With the little Higgs idea, the SM Higgs boson is identified as the pseudo-Nambu Goldstone boson(s) of some global symmetries. Two separate global symmetries, each to be broken by a Higgs vacuum expectation value, are to be arranged such that a 1-loop (SM) Higgs mass diagram is protected by the (residue) symmetries to be free from quadratic divergence. The idea was motivated by dimensional deconstruction [1], though the mechanism may not necessarily follow from the strong interaction dynamics behind (see Ref. [3]). Simple group theoretical constructions of little Higgs models have also been proposed [2,4]. In this article, we take the perspective of considering a little Higgs model as an effective field theory at the TeV scale and look into some plausible implications on quark physics.

A little Higgs model typically has an extended electroweak gauge symmetry, with extra fermions, including at least a toplike quark T, carrying nontrivial gauge charges. Such extra fermions have to be vectorlike at the SM level. Chiral states cannot have mass much above the electroweak scale and are hence extremely dangerous phenomenologically. Individual chiral fermions also ruin the gauge anomaly cancellation within the SM; adding more fermions to restore the consistency is far from a trivial business. The extra fermions vectorlike under the SM symmetry are, however, likely to be of a chiral nature before the breaking of the extra gauge symmetry. The T quark has to be connected to the SM t quark by the symmetries that enforce the quadratic divergence cancellation. For any such symmetry to be compatible with the SM symmetries, t_L and t_R cannot belong to multiplets of the same gauge charged; hence, it would likely be the same for T_L and T_R. Having fermions of a fundamentally chiral nature also naturally fixes their mass at or below the corresponding gauge symmetry breaking scale. From the lesson of the SM itself, one can see that the chiral fermionic spectrum embodies the beauty of the model and dictates many of the properties of the fermions. In fact, it has been argued that the spectrum can be derived from the gauge anomaly cancellation requirement [5]. The latter is the best one can do answering the question of why there is what there is in a particle physics model up to now. We present here such a chiral fermionic spectrum as a consistent completion of a little Higgs model and take a first look into the resulted implication on quark physics. We are working in a specific model; however, similar issues should be relevant to all little Higgs models though the exact fermionic completion and other details would be model dependent.

Our background little Higgs model is a model with $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry given in Ref. [4]. The model has a problem with the quartic Higgs coupling, which can be fixed in a $SU(3)_C \times SU(4)_L \times U(1)_X$ extension [4]. Here, we mainly stick to the $SU(3)_L$ version for simple illustration. Generalizing to the $SU(4)_L$ version is mostly straightforward. Moreover, an alternative little Higgs model with the same gauge symmetry is given in Ref. [6], to which most of the results here will likely apply. Our focus is to illustrate the basic features a little Higgs model with a complete and consistent fermionic content could have. In our opinion, the

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perspective could take little Higgs model construction and phenomenological studies to a new level. We skip discussion on the scalar and gauge boson sector, as well as the working of the little Higgs mechanism itself, for which readers are referred to the original reference [4]. Within the model, the T_L forms a $SU(3)_L$ triplet with the SM (t, b) doublet, while T_R is a singlet, the same as t_R. The first point to note here is that one cannot embed the other two families of SM quarks in the same way. Unlike $SU(2)_L$, $SU(3)_L$ multiplets are not free from triangle anomaly. In fact, the existence of a phenomenologically viable anomaly free embedding for the full content of the SM fermions together with T_L and T_R is not *a priori* obvious. Solving the problem more or less dictates the properties of the admissible complete model. Interestingly enough, a simple solution exists [7]. We present the spectrum in Table I and a similar result for the $SU(4)_L$ model in Table II. We emphasize again the results are not arbitrary choices of anomaly free spectra; they are essentially the minimal chiral spectra satisfying the requirement. Moreover, in the case of the one-family SM spectrum, the anomaly cancellation conditions tied the multiplets together so closely that no (simple) alternative is possible. It is expected to be the same case here. It is also of interest to note that the embedding of the three families of SM fermions is different, with gauge anomaly cancellation among them. In this sense, the consistent spectrum has to contain three families, which can be taken as a way to understand why there is a triplication of the anomaly free spectrum for the SM itself.

With the spectrum, we look into the possible couplings of the two $SU(3)_L$ Higgs multiplets Φ_i, as given in Ref. [4], to the fermions. One should note that the content and quantum numbers of Higgs multiplets are a central, non-negotiable, feature of the little Higgs model. The couplings are responsible for the SM Yukawa couplings, and the basic properties of the extra singlet quarks, as well as the leptons. Below we give details of the quark sector Yukawa couplings, assuming that the lowest order terms admitted by the gauge symmetry are all allowed. The discussion is to illustrate explicitly that the models at least do admit sensible Yukawa couplings for the SM quarks, with the extra, SM singlet quarks generally getting masses from the $SU(3)_L \times U(1)_X \to SU(2)_L \times U(1)_Y$ symmetry breaking. We will comment on the resultant mass matrices result in view of experimental data below.

TABLE I. Fermion spectrum for the $SU(3)_C \times SU(3)_L \times U(1)_X$ model with little Higgs. Here, we give the hypercharges of the electroweak states, with SM doublets put in $[\cdot]$'s. The other states are singlets. The embedding of the electric charge is given by $Q = \frac{1}{2}\lambda^3 - (1/2\sqrt{3})\lambda^8 + X$ (with $\mathrm{Tr}\{\lambda^a\lambda^b\} = 2\delta^{ab}$).

	$U(1)_Y$ states	
$(\mathbf{3}_C, \mathbf{3}_L, \frac{1}{3})$	$\frac{1}{6}[Q]$	$\frac{2}{3}(T)$
$2\ (\mathbf{3}_C, \bar{\mathbf{3}}_L, \mathbf{0})$	$2\ \frac{1}{6}[2Q]$	$2\ \frac{-1}{3}(D, S)$
$3\ (\mathbf{1}_C, \mathbf{3}_L, \frac{-1}{3})$	$3\ \frac{-1}{2}[3L]$	$3\ \mathbf{0}(3N)$
$4\ (\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{-2}{3})$	$4\ \frac{-2}{3}\ (\bar{u}, \bar{c}, \bar{t}, \bar{T})$	
$5\ (\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{1}{3})$	$5\ \frac{1}{3}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S})$	
$3\ (\mathbf{1}_C, \mathbf{1}_L, \mathbf{1})$	$3\ \mathbf{1}\ (e^+, \mu^+, \tau^+)$	

TABLE II. Fermion spectrum for a direct $SU(3)_C \times SU(4)_L \times U(1)_X$ extension of the model (see Ref. [4] for the little Higgs structure). Again, we give the hypercharges of the electroweak states, with SM doublets put in $[\cdot]$'s. The embedding of the electric charge is given by $Q = \frac{1}{2}\lambda^3 - (1/2\sqrt{3})\lambda^8 + -(1/2\sqrt{6})\lambda^{15} + X$.

	$U(1)_Y$ states		
$(\mathbf{3}_C, \mathbf{4}_L, \frac{5}{12})$	$\frac{1}{6}[Q]$	$\frac{2}{3}(T)$	$\frac{2}{3}(T')$
$2\ (\mathbf{3}_C, \bar{\mathbf{4}}_L, \frac{-1}{12})$	$2\ \frac{1}{6}\ [2Q]$	$2\ \frac{-1}{3}\ (D, S)$	$2\ \frac{-1}{3}\ (D', S')$
$3\ (\boldsymbol{l}, \mathbf{4}_L, \frac{-1}{4})C$	$3\ \frac{-1}{2}\ [3L]$	$3\ \mathbf{0}(3N)$	$3\ \mathbf{0}(3N')$
$5\ (\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{-2}{3})$	$4\ \frac{-2}{3}\ (\bar{u}, \bar{c}, \bar{t}, \bar{T})$		$\frac{-2}{3}\ (\bar{T}')$
$7\ (\bar{\mathbf{3}}_C, \mathbf{1}_L, \frac{1}{3})$	$5\ \frac{1}{3}\ (\bar{d}, \bar{s}, \bar{b}, \bar{D}, \bar{S})$		$2\ \frac{1}{3}\ (\bar{D}', \bar{S}')$
$3\ (\mathbf{1}_C, \mathbf{1}_L, \mathbf{1})$	$3\ \mathbf{1}\ (e^+, \mu^+, \tau^+)$		

First comes the top sector. It is given in Ref. [4] as

$$\begin{aligned}\mathcal{L}_{\rm top} &= \lambda_1^t \bar{t}'_a \Phi_1 Q^a + \lambda_2^t \bar{T}'_a \Phi_2 Q^a \\ &= f(\lambda_1^t \bar{t}' + \lambda_2^t \bar{T}')T + \frac{i}{\sqrt{2}}(\lambda_1^t \bar{t}' - \lambda_2^t \bar{T}')h\begin{pmatrix} t \\ b \end{pmatrix} + \cdots \\ &= m_T \bar{T} T - i y_t \bar{t} h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots. \end{aligned} \tag{1}$$

Here Q^a denotes the triplet of t, b, and T quarks (these are chiral fermionic states; here and below, we suppress the L or R subscripts); and we suppress the color indices (a) after the first line. Both λ_1^t and λ_2^t are expected to be of order 1 to produce the phenomenological top mass from electroweak symmetry breaking. In fact, we have $m_T = \sqrt{(\lambda_1^t)^2 + (\lambda_2^t)^2} f$ and $y_t = \sqrt{2}\lambda_1^t\lambda_2^t / \sqrt{(\lambda_1^t)^2 + (\lambda_2^t)^2}$. A piece not explicitly given in the above top sector (or rather up sector) Yukawa couplings to the triplet Q is the term

$$\frac{-i}{\sqrt{2}} \frac{(\lambda_2^t)^2 - (\lambda_1^t)^2}{(\lambda_1^t)^2 + (\lambda_2^t)^2} \bar{T} h \begin{pmatrix} t \\ b \end{pmatrix}. \tag{2}$$

This term represents a deviation of up-sector quark physics from that of the SM, and hence, deserves attention [8]. A nonzero value of the term, in fact, also signifies deviations of m_T and y_t given above from what the notation suggests. There are actually more mixings of the SM quarks with the electroweak singlet T, as shown below.

We have two more SM quark doublets residing in the $\bar{3}_L$ representations Q'_j ($j = 1$ and 2). There are admissible dimension five terms

$$\mathcal{L}_{Q'} = \frac{1}{M}\lambda^u_{\alpha j}\bar{u}'_\alpha \Phi_1\Phi_2 Q'_j = \frac{-i\sqrt{2}f}{M}\lambda^u_{\alpha j}\bar{u}'_\alpha h \begin{pmatrix} u_j \\ d_j \end{pmatrix} + \cdots, \tag{3}$$

where color indices are suppressed (same below), with M being a background mass scale factor and $\lambda^u_{\alpha j}$ a 4×2 matrix of couplings with obvious indexing for the quark states. Note that in the generic case, all four quark singlets have to be allowed to couple to $\Phi_1\Phi_2 Q'_j$. However, we are still left with an SU(2) flavor degree of freedom of basis choice among the singlets besides $\bar{t}$ and $\bar{T}$ and another SU(2) flavor basis choice among the two Q'_j's. Hence, we can give an optimal parametrization of the up-sector mass matrix as

$$\mathcal{M}^u = \begin{pmatrix} m'_u & 0 & 0 & 0 \\ 0 & m'_c & 0 & 0 \\ m_{tu} & m_{tc} & m'_t & 0 \\ m_{Tu} & m_{Tc} & m_{Tt} & m_T \end{pmatrix}. \tag{4}$$

The m_{Tt} is from the coupling as given by expression (2), which we discussed. The other mass mixing terms are all from the dimension five terms involving the Q'_j's.

Next, we look at the down-quark sector. The bottom quark has to get its Yukawa coupling from the dimension five $\bar{b}\Phi_i^\dagger\Phi_j^\dagger Q$ term, which may naturally give the desired suppression in its mass. For the first two families, however, the $1_L\Phi_i^\dagger\bar{3}_L$ terms are naively allowed for all the down-type quarks. Putting all that together, we have the following dimension four and five terms responsible for the quark masses:

$$\begin{aligned}\mathcal{L}_{\text{down}} &= \lambda^{d1}_{\beta j}\bar{d}'_\beta\Phi_1^\dagger Q'_j + \lambda^{d2}_{\beta j}\bar{d}'_\beta\Phi_2^\dagger Q'_j + \frac{1}{M}\lambda^b_\beta\bar{d}'_\beta\Phi_1^\dagger\Phi_2^\dagger Q \\ &= f(\lambda^{d1}_{\beta j}\bar{d}'_\beta + \lambda^{d2}_{\beta j}\bar{d}'_\beta)D_j - \frac{i}{\sqrt{2}}(\lambda^{d1}_{\beta j}\bar{d}'_\beta \\ &\quad - \lambda^{d2}_{\beta j}\bar{d}'_\beta)h^\dagger\begin{pmatrix} u_j \\ d_j\end{pmatrix} + \frac{i\sqrt{2}f}{M}\lambda^b_\beta\bar{d}'_\beta h^\dagger\begin{pmatrix} t \\ b\end{pmatrix} + \cdots.\end{aligned} \tag{5}$$

We note that the β index goes from 1 to 5. The basic notation should be obvious. The first term can be used to extract the two states within the SU(5) flavor space of the five quark singlets as $\bar{D}_j$'s, the singlets that couple directly to the D_j's. However, one cannot then avoid having the couplings of the $\bar{D}_j$'s in the latter two terms simultaneous without extra assumption. The mass matrix for the down-sector quarks may then be written in the $3 + 2$ block form

$$\mathcal{M}^d = \begin{pmatrix} m^d & 0 \\ m^{Dd} & m^D \end{pmatrix}, \tag{6}$$

where we are leaving m^d and m^{Dd} as generic matrices to stick to the left-handed basis of the SM doublets as in $\mathcal{M}^u$ above and to accommodate the required nontrivial Cabibbo-Kobayashi-Maskawa (CKM) mixings of the SM quarks.

It is clear from the above that the quantum number assignment scheme does admit a heavy, $SU(3)_L \times U(1)_X \to SU(2)_L \times U(1)_Y$ symmetry breaking scale, and masses to the extra singlet quarks, together with Yukawa couplings for all the SM quarks. There are also mass mixings between the two classes, as parametrized above by $m^{Tu} = (m_{Tu}m_{Tc}m_{Tt})$ and m^{Dd}. At least in the limit that these mixings are vanishingly small, one will obtain SM quark physics with the heavy quarks decoupled. This result is, of course, what is to be expected naively. We have only to emphasize that there does exist a large enough number of independent couplings, in the Lagrangian parts, as given by Eqs. (1), (4), and (6) to make the scenario admissible. It should also be noted that the couplings as given here do not automatically produce the full hierarchical quark mass pattern.

The physics of the type of extra vectorlike quarks have been studied under various scenarios before [9]. We will take a step in the direction to get an idea on some of the constraints on the model. This is mainly an attempt to illustrate the strong phenomenological implications the fermion sector of a little Higgs model may have, as well as to outline the particular type of phenomenological constraints to be expected for the current model under discussion. We leave a detailed phenomenological study along the line to future publications.

The first set of stringent constraint on a SM extension with vectorlike quarks mixing with the SM ones is the precision results on partial widths of the Z decay. The heavy quarks have to be above the decay threshold; but the modified electroweak nature of the SM quarks changes the partial widths of the latter. The effective coupling of an electroweak state f to the Z^0 boson is proportional to $T^3_f - \mathcal{Q}_f\sin^2\theta_W$ from which one can easily work out the mass eigenstate couplings. In our case, the extra quarks are all electroweak singlets. The first order mixings are among the L-handed states. Since these are mixings among states of different electroweak character, nonuniversal flavor diagonal couplings as well as flavor changing neutral current (FCNC) couplings are induced. We introduce diagonalizing matrices for $\mathcal{M}^{u\dagger}\mathcal{M}^u$ and $\mathcal{M}^{d\dagger}\mathcal{M}^d$ in $3 + 1$ and $3 + 2$ block form as

$$U^f_L = \begin{pmatrix} K^f & R^f \\ S^f & T^f \end{pmatrix}, \tag{7}$$

for $f = u$ and d. Obviously, admissible mixings between the SM L-handed quarks and the heavy L-handed singlet quarks have to be very small. A block perturbative analysis then yields the solution:

$$R^d \simeq m^{Dd\dagger}(m^{D\dagger})^{-1}T^d, \qquad S^d \simeq -(m^D)^{-1}m^{Dd}K^d, \tag{8}$$

and

$$R^u \simeq \frac{1}{m_T} m^{Tu\dagger}, \qquad S^u \simeq \frac{-1}{m_T} m^{Tu} K^u. \tag{9}$$

Here, the K^f and T^f matrices are essentially the unitary matrices that diagonalize the corresponding diagonal blocks (T^u is just the unit element 1).

Couplings of L-handed SM quark mass eigenstates to the Z^0 boson are modified to

$$\begin{aligned} g_L(u) &= \tfrac{1}{2}[1 - |S_u|^2] - \tfrac{2}{3}\sin^2\theta_W, \\ g_L(c) &= \tfrac{1}{2}[1 - |S_c|^2] - \tfrac{2}{3}\sin^2\theta_W, \end{aligned} \tag{10}$$

for the u and c quarks, where the $(S_u S_c S_t)$ denote the 1×3 matrix S^u, and for the down sector

$$\begin{aligned} &g_L(q) = -\tfrac{1}{2}[1 - |(S^d)_{1q}|^2 - |(S^d)_{2q}|^2] + \tfrac{1}{3}\sin^2\theta_W \\ &(\text{for } q = d, s, \text{ and } b). \end{aligned} \tag{11}$$

Similarly, the induced FCNC couplings are given by expression of the form

$$g_L(\bar{u}c) = \tfrac{1}{2}[-S_u^* S_c], \tag{12}$$

for example.

Applying the above results against the experimental data, we can get an idea of the constraints on the very nontrivial flavor structure of the model. In particular all the $g_L(q)$ results decrease as a result of the S^u and S^d mixings. Current data [10] allow roughly only a decrease of the total hadronic width by 0.115%. The magnitude of even a single dominant element of the S^u and S^d matrices would then be bounded roughly by 0.014, which reflexes the allowable order of magnitude for a mass ratio of the form $\frac{\text{mixing mass}}{\text{heavy mass}}$, such as $\frac{m_{Tc}}{m_T}$. The up sector is particularly interesting, as the kind of mixings most probably exist for any little Higgs model. From Eq. (4), we can see that the required suppression here is actually not too bad, for a mass term like m_{Tc} would naturally be at scale $\sim \frac{f}{M} m_t$ or below (if the coupling is suppressed). Mixings <0.014 also imply that FCNC couplings induced are largely safe. For example, $g_L(\bar{u}c)$ contributions to D-meson mixing requires only

$$|S_u^* S_c| \lesssim 2\frac{\cos\theta_W M_Z}{g_2} \frac{1}{f_D}\left(\frac{3\Delta m_D}{2m_D B_D}\right)^{1/2} \simeq 0.001, \tag{13}$$

which is not stronger.

A little Higgs model other than the example case here may not contain extra vectorlike down-type quarks. So, similar constraints from the down sector are less generic. Otherwise, the constraints are no weaker. For example, we have $|(S^{d\dagger}S^d)_{ds}|$ roughly bounded by 3×10^{-4} from kaon physics, without taking into consideration CP phase dependent bounds. On the whole, the <0.014 bound on light heavy quark mixings is the major guideline to be taken.

Readers will realize that our discussion of the phenomenological constraints above is quite generic. They are performed on parameters within the quark mass mixing matrices rather than explicit model parameters. Of course the mass mixing parameters come from the Lagrangian parts illustrated. It should be obvious that the explicit connection is difficult to be addressed analytically though. There is ambiguity in the implementation of CKM mixings onto the quark mass matrices $\mathcal{M}^u$ and $\mathcal{M}^d$, for example. Our goal here is simply to illustrate the kind of strong phenomenological implications the nontrivial flavor structure, dictated by the fermionic spectrum, has. A few comments on their role and their relation to the other electroweak constraints are in order.

Short of a detailed numerical study, we can still take a look into the likely implications of the constraints above on the various parameters of our model at hand. The λ_i^t's have to be order 1. Mixing among the physical t and T quark will be naturally of order m_t/m_T. The lighter two quarks of the sector have their masses from the dimension five term of Eq. (4), and hence suppressed by an f/M factor. Further suppressions from the $\lambda_{\alpha j}^u$ couplings are needed to get the right masses for the u and c quarks. Natural values for the mixings S_u and S_c are expected to be m_u/m_T and m_c/m_T. Hence, they look fine. In summary, the small mixings are "natural" if the singlet states are heavy and the mixing mass terms are of the same order as the light masses. This is the case for the up sector. The story for the down sector, however, is more complicated.

The mixings of the heavy D quarks with the SM quarks are expected to be more alarming. This is especially true in the case of the bottom quark. To appreciate that better, let us recall the basic admissible couplings as given in Eq. (6). The bottom quark mass eigenvalue is to come mainly from the dimension five term $\frac{1}{M}\lambda_\beta^b \bar{d}'_\beta \Phi_1^\dagger \Phi_2^\dagger Q$, while those of the strange and down quarks the dimension four terms $\lambda_{\beta j}^{d1} \bar{d}'_\beta \Phi_1^\dagger Q'_j$ and $\lambda_{\beta j}^{d2} \bar{d}'_\beta \Phi_2^\dagger Q'_j$. The latter are also the source of the heavy D quark masses and their mixings with d and s. The SM quark mass hierarchy then dictates, naively, very small values for the couplings $\lambda_{\beta j}^{d1}$ and $\lambda_{\beta j}^{d2}$. This is a big contrast to the up-sector situation where an f/M factor helps to suppress the lighter quark masses, with independent Yukawa terms responsible for the heavier quark masses. Here, the f/M factor suppresses the b mass, relative to the t, but enhances the d and s masses relative to the b itself. We have then a dilemma. We need small couplings to get the right d and s masses. We need, however, relative big masses for the extra singlet D states. The latter being light enough to add extra channels to the hadronic Z width is far too dangerous. For instance, the effective s quark Yukawa coupling has to be $\sim 10^{-3}$, while an effective D quark Yukawa of the same order would give a D mass of $\sim 10^{-3} f$—namely at the GeV order. There is a possible way out though. Consider phenomenologically admissible effective Yukawa couplings for the D quarks. Couplings of the required magnitude may be restricted to involve the $\bar{D}$ states among the right-handed singlets. One will have to tune the couplings among $\lambda^{d1}_{\bar{D}j}$ and $\lambda^{d2}_{\bar{D}j}$ [cf. Eq. (6)] to get small enough values of mixing masses with d and s—essentially, the magnitude of $(\lambda^{d1}_{\bar{D}j} - \lambda^{d2}_{\bar{D}j})v/f$. Finally, one has to take simply small λ^d-type couplings for the $\bar{d}'$ states orthogonal to the $\bar{D}$ states to get the right d and s masses. All in all, we see that the model most probably does have parameter space regions that can pass the flavor sector constraints discussed here. It does, however, impose a strong requirement on the couplings, especially that of the down sector. Relatively light singlet D quarks are preferred. A more detailed analysis, together with numerical studies on the flavor physics of the model, should be performed.

The stringent fermion sector constraints do not stand alone phenomenologically. Contributions to FCNCs from quark mixings have to be considered together with the corresponding contributions from the heavy gauge boson (Z' and W') exchanges. The two types of FCNC contributions typically come into the same processes. A realistic analysis has to combine the two parts together. There are also other precision electroweak constraints largely complementary to the FCNC ones. For instance, Ref. [4] claims a lower bound on f of about 1.5 TeV as a result.

We have a specific model here that contains vectorlike quarks of both the up and down types, which generally mix with the SM quarks. There are very stringent constraints on the admissible mixings. What we want to emphasize, however, is that any realistic little Higgs model completed with a consistent fermionic spectrum is likely to contain extra quarks. There has to be certainly an extra toplike quark which mixes not only with the top, but most probably with the up and charm too. If a consistent chiral spectrum can be found, anomaly cancellation requirements would likely dictate the existence of other SM singlet fermions. At least with the example(s) at hand, there are also extra down-sector quarks and leptons. This has a strong implication on the flavor physics structure from which interesting constraints can be obtained besides the constraints on the gauge and Higgs sectors. More detailed analyses of the interplay of all the constraints for a realistic model should be taken seriously. In particular, it will be interesting to check if the more realistic $SU(4)_L \times U(1)_X$ model may satisfy the FCNC constraints more naturally.

We illustrate above, with the case example, how a complete and consistent fermionic sector of the TeV effective field theory may actually be largely dictated by the gauge structure of the model. While the specific solution spectrum construction strategy can hardly be generalized to little Higgs models with an extended electroweak gauge symmetry beyond the $SU(N) \times U(1)$ type, the paramount importance of the gauge anomaly cancellation constraints and the plausible implication of a solution fermionic spectrum are generic. The latter may be contrary to the impression one may get from the literature on little Higgs, as the author seems to be the only one drawing attention to the issue so far. Especially because of that, we want to emphasize and elaborate further on the point here.

In particular, let us take a look at a $SU(5)/SO(5)$ little Higgs model[11], which is arguably the most popular one around. In this case, the electroweak symmetry is to be extended with an extra SU(2) or $SU(2) \times U(1)$ factor. The only extra fermionic state explicitly discussed in the paper is the heavy top quark, with its full gauge quantum numbers not explicitly stated. Naively, one may be led to the simple choice of a vectorlike singlet under the extended electroweak symmetry. The choice looks like there is no need for any further fermionic states and the model is completed. This is actually not what the original authors had in mind, as clearly indicated by the sentence we quote: "We do not concern ourselves with the cancellation of the $G_1 \times G_2$ anomalies in this low energy effective theory, since there may be additional fermions at the cutoff which cancel the anomalies involving the broken subgroup" [11]. So, the gauge anomaly issue is recognized, but pushed aside instead of solved.

In our opinion, the simple vectorlike singlet choice is not really quite feasible, nor desirable. Moreover, the idea of pushing any additional fermions to the cutoff scale may not be in much better shape either.

The authors of Ref. [11] did have the anomalies issue in mind. Only fermions chiral with respect to the full gauge symmetry contribute to the anomalies. Mass for a chiral fermion is by definition ruled out by gauge invariance, while a vectorlike pair has admissible Dirac mass naturally at the cutoff scale. If the heavy top is vectorlike, before any gauge symmetry breaking, its mass would likely be at the cutoff M. It is the chiral states which only match into a vectorlike pair of the broken gauge symmetry that should be expected to have mass at the scale f or below. Chiral fermions, rather than vectorlike ones, are what is more relevant to low energy physics. It is possible to make the extra fermions heavy by the adoption of nonperturbatively large Yukawa couplings, provided that they do form vectorlike pairs with respect to the SM gauge group. It is however not very appealing to have mass terms forbidden by some gauge symmetry to be larger than the gauge invariant mass terms.

The quantum numbers of a full gauge multiplet containing any SM multiplet or the heavy top states are dictated by the symmetry embedding of a gauge group into the parent (global) SU(5). While the version of the model with only an extra SU(2) gauge symmetry is formally consistent having only the vectorlike T singlet, so long as one does not give up the idea of a possible SU(5) description of the fermion sector, the existence of extra fermionic states charged under the extra gauge symmetry is not a matter of arbitrary choice. It is a model consistency issue to be looked into carefully. For a generic model, the existence of such a consistent fermionic spectrum is not *a priori* guaranteed. It is not just about adding states to cancel the anomalies. One has also to make sure that the resulted spectrum, when split into SM multiplets, gives no other chiral electroweak states beyond that of the three SM families that will run into conflict with phenomenology. And it will be of great interest to see if there are other fermions beyond the heavy top to enrich the prediction of such a model at the TeV scale.

A little Higgs model is supposed to describe a TeV scale effective theory. A so-called UV completion model of strong dynamics is expected to be behind the cutoff. Independent of any little Higgs model, it does not sound likely at all that one with the minimal fermion spectrum of the SM parts plus only one extra, vectorlike, top quark would arise. We certainly hope that a more interesting spectrum would be obtained.

After all, the SM fermionic spectrum is fully chiral, and (for a single family) essentially dictated by the gauge anomaly cancellation conditions. This gives an explanation for why the spectrum is what it is, as well as the light masses of the resulted Dirac fermions. The only state within the SM that can have a gauge invariant mass term before electroweak symmetry breaking is the Higgs. The latter is then the only possible source of the electroweak scale. It is exactly the stabilization puzzle of this scale that the little Higgs idea aims at resolving. An all around appealing little Higgs model, in our opinion, should be one which maintains all these nice features of the SM and, hopefully, provides some insight on the origin of the three SM families. We illustrate a case example with some partial success in that direction. It should be very interesting to see if any other little Higgs model can be similarly completed with a chiral fermionic sector. Successful fermionic completion makes a little Higgs model a more compelling candidate theory beyond the SM. The kind of flavor physics constraints outlined here above then will likely play an important role in the experimental checking of the model.

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