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Static quark-antiquark free energy and the running coupling at finite temperature

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We analyze the free energy of a static quark-antiquark pair in quenched QCD at short and large distances. From this we deduce running couplings, g^2 (r, T), and determine the length scale that separates at high temperature the short distance perturbative regime from the large distance nonperturbative regime in the QCD plasma phase. Ambiguities in the definition of a coupling beyond the perturbative regime are discussed in their relation to phenomenological considerations on heavy quark bound states in the quark gluon plasma. Our analysis suggests that it is more appropriate to characterize the nonperturbative properties of the QCD plasma phase close to T_c in terms remnants of the confinement part of the QCD force rather than a strong Coulombic force.

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I. INTRODUCTION

On quite general grounds it is expected that fundamental forces between quarks and gluons get modified at finite temperature. More precisely, we expect that forces between static quarks, i.e., static test charges in a thermal medium, change because the gluons, which mediate the interaction between the static quarks, also interact with the constituents (quarks and gluons) of the thermal bath. In particular, above the deconfinement temperature T_c the potential is expected to be exponentially screened at large distances ($r \gg 1/T$) [1]. In leading order perturbation theory this happens due to the generation of a chromoelectric (Debye) mass of order gT (with g being the gauge coupling). Beyond leading order, however, chromoelectric and chromomagnetic screening effects cannot be separated unambiguously. It is this nonperturbative large distance physics, which plays a central role in our attempts to understand the bulk properties of the QCD plasma phase, e.g., the equation of state and the apparent deviations from ideal gas behavior found in numerical calculations [2]. On the other hand it is the short and intermediate distance regime which is most important in the discussion of signals which are considered today as being suitable to gain information on properties of hot and dense matter generated experimentally in heavy ion collisions. In this paper we will quantify the temperature dependence of the length scale which separates these different regimes and analyze in detail the properties of the QCD coupling constant at short and large distances.

Although a detailed understanding of screening phenomena at large distances is still missing, it is evident that in this regime the temperature is the dominant scale and consequently will control the running of the QCD coupling, i.e., $g \simeq g(T)$ for $(rT \gg 1, T \gg T_c)$[1]. However, at short distances, $r \cdot \max(T, T_c) \ll 1$, hard processes dominate the physics of the quark gluon plasma even at high temperature and it is expected that a scale appropriate for this short distance regime will control the running of the QCD coupling, i.e., $g \simeq g(r)$. The interplay between short and large distance length scales plays a crucial role for a quantitative understanding of hard as well as soft processes in dense matter. It will, for instance, determine the range of applicability of perturbative calculations for thermal dilepton rates or the production of jets as well as the analysis of processes that can lead to thermalization of the dense matter produced in heavy ion collisions. Moreover, the short and intermediate distance regime also is most relevant for the discussion of in-medium modifications of heavy quark bound states which are sensitive to thermal modifications of the heavy quark potential as well as the role of quasiparticle excitations in the quark gluon plasma. In all these cases it is not immediately evident that temperature is the relevant scale that controls the running of the QCD coupling at energies currently relevant in heavy ion physics where temperatures may be reached which are only moderately larger than the phase transition temperature T_c. An analysis of this question becomes of particular interest in view of the recently suggested scenario for the existence of a large number of Coulombic bound states in the QCD plasma phase close to T_c [3,4].

It is the purpose of this paper, to firmly establish that also at finite temperature the QCD coupling indeed runs as function of the length scale r and agrees with the zero temperature running coupling at sufficiently short dis-

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[1]We use the deconfinement temperature T_c as a characteristic energy scale rather than a more conventionally used Λ-parameter.

tances. In fact, we will show that in the entire regime of distances for which at zero temperature the heavy quark potential is considered to be described well by QCD perturbation theory [5–7] ($r \lesssim 0.1$ fm) the QCD coupling remains unaffected by temperature effects up to $T \simeq 3T_c$. Furthermore, we will analyze the interplay between the two relevant scales controlling the behavior of heavy quark free energies, $F(r, T)$, and we will quantify the distance scale below which processes in the QCD plasma phase are still dominated by properties of the QCD vacuum and above which screening dominates the physics in the plasma phase.

We will start in Section II with a discussion of heavy quark free energies and their relation to the QCD coupling constant and give some details on our simulation parameters in Section III. In Section IV we discuss the calculation of running couplings at finite temperature and short distances from color singlet free energies and relate these results to properties of the running coupling at large distances in Section V. Section VI contains our conclusions.

II. FREE ENERGIES AND RUNNING COUPLINGS

A. Free energy of a static quark-antiquark pair in lattice QCD

Our main concern in this study is the determination of a running coupling from correlation functions of Polyakov loops. In particular we want to make contact to calculations performed at zero temperature. Here the running of the QCD coupling has been determined successfully in lattice calculations from the short distance properties of the heavy quark potential [8–10] and contact could be made with perturbative results obtained in 2-loop calculations [5,6]. We thus want to analyze a finite temperature observable which naturally is related to the zero temperature heavy quark potential and has a well defined (perturbative) interpretation that allows us to make contact with perturbative definitions of a running coupling. For this reason we will analyze properties of heavy quark free energies in the singlet channel.

Correlation functions of Polyakov loops define the free energy of a heavy quark-antiquark pair. One generally considers the so-called color averaged free energy

$$e^{-F(r,T)/T+C} = \frac{1}{9}\langle \mathrm{Tr}L(\mathbf{x})\mathrm{Tr}L^{\dagger}(\mathbf{0})\rangle, \tag{1}$$

where the Polyakov loop $L(\mathbf{x})$ is defined on lattices with temporal extent N_τ in terms of temporal link variables $U_0(\mathbf{x}, \tau) \in \mathrm{SU}(3)$,

$$L(\mathbf{x}) = \prod_{\tau=1}^{N_\tau} U_0(\mathbf{x}, \tau). \tag{2}$$

Furthermore, $r = |\mathbf{x}|$ and C is a suitably chosen renormalization constant, which can be determined from a matching of finite temperature free energies to the zero temperature heavy quark potential [11,12]. Its precise value is of no importance for us here as we will be interested in the r-dependence of the free energy to determine running couplings. We plan to be more specific on the renormalization process in a forthcoming publication.

The color averaged free energy can be considered as a thermal average over contributions corresponding to quark-antiquark sources in color singlet and octet states, respectively, [13,14]

$$e^{-F(r,T)/T} = \frac{1}{9}e^{-F_1(r,T)/T} + \frac{8}{9}e^{-F_8(r,T)/T}, \tag{3}$$

where

$$e^{-F_1(r,T)/T+C} = \frac{1}{3}\mathrm{Tr}\langle L(\mathbf{x})L^{\dagger}(\mathbf{0})\rangle, \tag{4}$$

$$e^{-F_8(r,T)/T+C} = \frac{1}{8}\langle \mathrm{Tr}L(\mathbf{x})\mathrm{Tr}L^{\dagger}(\mathbf{0})\rangle - \frac{1}{24}\mathrm{Tr}\langle L(\mathbf{x})L^{\dagger}(\mathbf{0})\rangle. \tag{5}$$

While the color averaged free energy is defined in terms of a gauge invariant Polyakov loop correlation function, the singlet and octet correlation functions are given in terms of a gauge dependent correlator, $\mathrm{Tr}L(\mathbf{x})L^{\dagger}(\mathbf{0})$, and thus have to be evaluated in a fixed gauge. Nonetheless, $\mathrm{Tr}L(\mathbf{x})L^{\dagger}(\mathbf{0})$ is related to an appropriately chosen gauge invariant correlator and thus has a proper gauge invariant interpretation. In fact, when evaluated in Coulomb gauge the singlet correlation function constructed from the Polyakov loops defined in Eq. (2) may be viewed as resulting from gauge fixing nonlocal but gauge invariant operators, i.e., *dressed* Polyakov loops, where the static quark and antiquark sources are surrounded by gluon clouds [15,16]. We also note that in a recent paper [17] it has been argued that in the context of the transfer matrix description of Polyakov loop correlation functions the operators introduced above for the color averaged and color octet free energies receive contributions only from color singlet eigenstates. In the zero temperature limit also the octet operator thus could project onto the lowest lying singlet eigenstate, if the corresponding matrix element is nonzero.

For our purpose of defining a running coupling at finite temperature the singlet free energy is most appropriate as it has at short ($r \cdot \max(T, T_c) \ll 1$) as well as large ($rT \gg 1$, $T \gg T_c$) distances and temperatures a simple asymptotic behavior which is dominated by one gluon exchange[2], i.e.,

[2]Although this is usually called the leading order perturbative result in the high temperature phase the screened potential already involves summation of an infinite set of ladder diagrams which leads to the screening mass $\mu(T) = g(T)T$. We note that this leading order perturbative result is gauge invariant.

$$F_1(r,T) = \begin{cases} -\frac{g^2(r)}{3\pi r}, & r \cdot \max(T, T_c) \ll 1 \\ -\frac{g^2(T)}{3\pi r} \mathrm{e}^{-g(T)rT}, & (rT \gg 1, T \gg T_c) \end{cases}. \quad (6)$$

Here we have already anticipated the running of the couplings with the dominant scales in both limiting regimes, although their running, of course, only arises in higher order perturbative calculations. We also have suppressed any additative constants which result from the renormalization of the free energy [11] and, in particular, at high temperature will dominate the free energy in the large distance limit.

While the relation between the singlet free energy and a running coupling is straightforward the definition of a running coupling with the help of the color averaged free energy is problematic. The exact cancellation of leading order perturbative terms in high temperature perturbation theory, $F(r,T)/T \sim [F_1(r,T)/T]^2$, which generically does not occur at finite distances and temperatures, makes it difficult to define a running coupling which easily could be motivated by perturbation theory. While the above quadratic relation between F (r, T) and F_1 (r, T) holds at large distances it has been demonstrated by us recently that even at high temperature the short distance part of the color averaged free energy is dominated by the singlet contribution and $F(r,T) \sim F_1(r,T)$ holds at short distances [11]. In fact, for this reason the determination of a screening mass from the exponential fall of F (r, T) at large distances, i.e., $F(r,T) \sim \exp\{-\mu(T)r\}$, turned out to be quite difficult and strongly dependent on the asymptotic form used in fits of the large distance behavior of $F(r,T)$ [18]. The subleading powerlike corrections were found to be strongly temperature dependent and turned out to be difficult to control.

B. Running couplings

The perturbative short and large distance relations for the singlet free energy will be used to define a running coupling at finite temperature. In general, the definition of a running coupling in QCD is not unique beyond the validity range of 2-loop perturbation theory; aside from the scheme dependence of higher order coefficients in the QCD β-functions it will strongly depend on nonperturbative contributions to the observable used for its definition. This is quite apparent when defining the coupling in QCD either in terms of the free energy ($T = 0$: potential)

$$\alpha_{\mathrm{V}}(r,T) = -\frac{3r}{4} F_1(r,T), \quad (7)$$

or its derivative ($T = 0$: force)

$$\alpha_{\mathrm{qq}}(r,T) = \frac{3r^2}{4} \frac{\mathrm{d}F_1(r,T)}{\mathrm{d}r}. \quad (8)$$

At low temperature the former necessarily has to change sign at some intermediate distance due to the dominance of the linearly rising confinement part in the potential [19]. The latter, however, stays positive as $F_1(r,T)$ increases monotonically with r. In fact, for a linear confining potential α_{V} defined through Eq. (7) will become negative and drop quadratically while α_{qq} defined through Eq. (8) will rise quadratically at large distances. The latter gives the possibility of smoothly matching the increasing coupling in the perturbative regime to the nonperturbative increase. This and the poor convergence of the perturbative expansion for $\alpha_{\mathrm{V}}(r)$ have been reasons for analyzing in lattice calculations running couplings defined through Eq. (8) (qq-scheme) rather than Eq. (7) (V-scheme). We will in the following consider both definitions as this will also help to distinguish the short and large distance regimes at finite temperature.

In the perturbative regime different definitions of the running coupling are uniquely related through the QCD β-function

$$\alpha_{\mathrm{qq}}(r,T) = \alpha_{\mathrm{V}}(r,T) - r\frac{\mathrm{d}\alpha_{\mathrm{V}}(r,T)}{\mathrm{d}r} \equiv \alpha_{\mathrm{V}}(r,T) + \frac{g(r,T)}{2\pi}\beta(g), \quad (9)$$

with $\beta(g) = -b_0 g^3 - b_1 g^5 - b_2 g^7 + \mathcal{O}(g^9)$ and universal coefficients $b_0 = 11/16\pi^2$, $b_1 = 102/(16\pi^2)^2$. In higher orders the coefficients of the β-function are scheme dependent. At zero temperature the heavy quark potential has been calculated in 2-loop perturbation theory [5–7]. From these calculations the 3-loop coefficient b_2 in the V-scheme [6] and the qq-scheme [9] could be extracted and allowed for a detailed comparison of running couplings determined in perturbative and nonperturbative lattice calculations [9,10]. Good agreement has been found at distances $r \lesssim 0.1$ fm, which also has been estimated to be the range of validity of the perturbative calculations. At larger distances, however, any perturbatively motivated definition of the running coupling will also become sensitive to nonperturbative effects and may lead to quite different results.

We will extent here the zero temperature studies of the heavy quark potential and the force between static charges to finite temperature. In this case the appropriate observable is the heavy quark free energy and its derivative. At short distances we follow the approach used also at $T = 0$ and introduce a running coupling by analyzing the r-dependence of the force between static quark-antiquark sources, Eq. (8), and will compare it with the definition of a coupling in terms of the potential, Eq. (7). At large distances we will determine a T-dependent running coupling directly from fits of $F_1(r,T)$ which are motivated by the perturbative large distance form given in Eq. (6). We will be more specific on this in Section V.

III. SIMULATION PARAMETERS

We will analyze in the following properties of heavy quark-antiquark pairs in a thermal heat bath of gluons, i.e., we consider correlation functions of Polyakov loops in the SU(3) gauge theory (quenched QCD) at finite temperature calculated on Euclidean lattice of size $N_\sigma^3 \times N_\tau$. All our calculations have been performed on lattices with spatial extent $N_\sigma = 32$ and $N_\tau = 4$, 8, and 16 using the tree-level Symanzik-improved gauge action [20]. It has been verified by us earlier [11] that this choice of action and lattice parameters is sufficient to suppress finite volume effects and finite size effects such as the breaking of rotational symmetry in the analysis of correlation functions at short and intermediate distances. Our simulations have been performed in the temperature range $T_c < T \leq 12T_c$. The simulation parameters are summarized in Table I. The temperature scale for the Symanzik-improved action was obtained earlier from calculations of the string tension at zero temperature and a determination of the critical coupling for the deconfinement transition on lattices with temporal extent $N_\tau = 4$ and 6 [21].

The calculation of singlet free energies has been performed in Coulomb gauge, which on the lattice is realized by maximizing $\mathrm{Tr}\sum_{\mu=1}^{3} U_\mu(\mathbf{x}, \tau)$ in each time slice, i.e., for fixed τ. A residual gauge degree of freedom is fixed by demanding $\sum_{\mathbf{x}} U_0(\mathbf{x}, \tau)$ to be independent of τ. Typically we have analyzed the correlation functions on 100–500 independent gauge field configurations.

As we are interested in the short distance behavior of the heavy quark free energy it is important to correct for the violation of rotational symmetry which is most pronounced in this region. Following [10] we have replaced $F(r, T)$ by $F(r_I, T)$ (similarly for F_1 and F_8) where r_I relates the separation between the static quark and antiquark sources to the Fourier transform of the tree-level lattice gluon propagator, $D_{\mu\nu}$, i.e.,

$$r_I^{-1} = 4\pi \int_{-\pi}^{\pi} \frac{d^3k}{(2\pi)^3} \exp(i\vec{k} \cdot \vec{r}) D_{00}(k). \tag{10}$$

For the Symanzik-improved action the timelike component of $D_{\mu\nu}$ is given by

$$D_{00}^{-1}(k) = 4\sum_{i=1}^{3}\left(\sin^2\frac{k_i}{2} + \frac{1}{3}\sin^4\frac{k_i}{2}\right). \tag{11}$$

This procedure removes most of the lattice artifacts as is evident from the smooth short distance behavior of the free energies shown in Fig. 1. Therefore in what follows we will always show free energies plotted versus r_I, but will suppress the subscript I in our formulas.

TABLE I. Parameters of the simulations on $32^3 \times N_\tau$ lattices using the tree-level Symanzik-improved action.

	β		
T/T_c	$N_\tau = 4$	$N_\tau = 8$	$N_\tau = 16$
1.03(1)	4.090	⋯	⋯
1.05(2)	4.100	4.5592	⋯
1.10(1)	4.127	⋯	⋯
1.15(1)	4.154	⋯	⋯
1.20(3)	4.179	4.6605	⋯
1.24(1)	4.200	⋯	⋯
1.30(1)	4.229	4.7246	⋯
1.50(3)	4.321	4.8393	5.4261
1.60(2)	4.365	4.8921	⋯
1.68(2)	4.400	⋯	⋯
2.21(5)	4.600	⋯	⋯
3.0(1)	4.839	5.4261	⋯
6.0(3)	⋯	6.0434	⋯
9.0(3)	⋯	6.3910	⋯
12.0(5)	⋯	6.6450	⋯

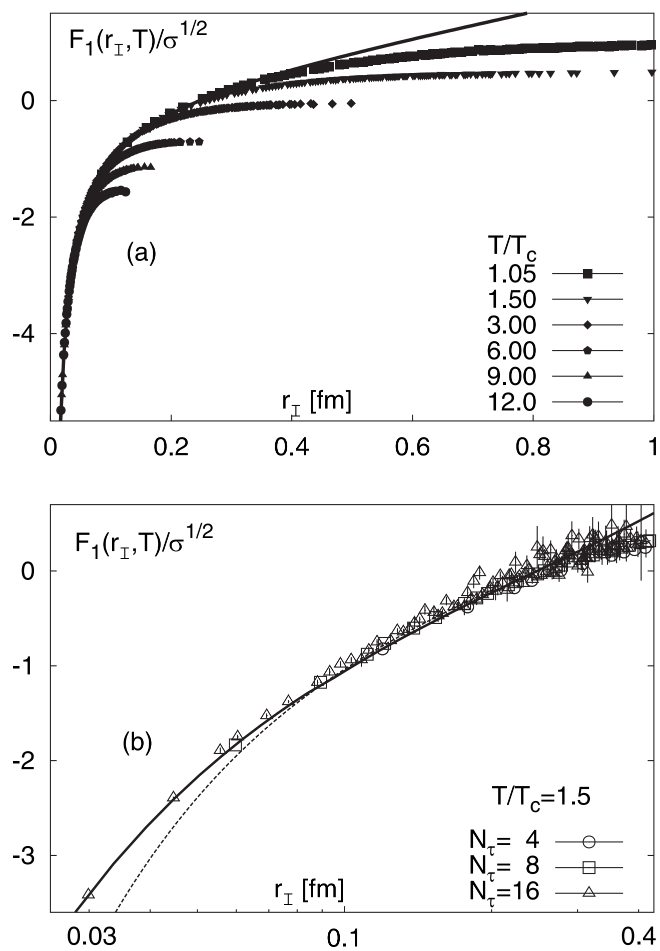

FIG. 1. Heavy quark free energies in the singlet channel calculated for several values of the temperature above T_c (a) and the short distance part of the singlet free energy at $T/T_c = 1.5$ (b). The dashed line shows $V_{\mathrm{string}}(r)$ defined in Eq. (13) and the solid line is the perturbative result of Ref. [10] for $V_{q\bar{q}}(r)$. This has been matched smoothly to $V_{\mathrm{string}}(r)$ at $r \simeq 0.1$ fm. Physical units have been obtained by using $\sqrt{\sigma} = 420$ MeV.

IV. RUNNING COUPLING CONSTANT AT SHORT DISTANCES

In Fig. 1 we show our results for the singlet free energies for several temperatures above T_c. The free energies have been calculated in Coulomb gauge and have been renormalized by matching the short distance part to the zero temperature heavy quark potential of Ref. [10].

Figure 1(b) shows the short distance part of the singlet free energy calculated at $T = 1.5T_c$ on lattices with temporal extent $N_\tau = 4$, 8, and 16. This corresponds to lattice spacings ranging from $a \simeq 0.12$ fm down to $a \simeq 0.03$ fm. Apparently the short distance part of the singlet free energy agrees quite well with the zero temperature heavy quark potential including a perturbatively calculated Coulomb term [9]

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_V(r)}{r}, \tag{12}$$

and shows no significant cutoff dependence. On the other hand, deviations from a confinement potential with a constant Coulomb like term that arises from universal string fluctuations

$$V_{\mathrm{string}}(r) = -\frac{\pi}{12r} + \sigma r, \tag{13}$$

are clearly visible at these short distances. This already indicates that the short distance behavior of the singlet free energy is consistent with a running coupling that is controlled by the quark-antiquark separation, r, and shows no or only little temperature dependence. We also note that after having renormalized the free energy through a matching at short distances the large distance behavior is completely fixed. Above T_c the singlet free energy approaches a temperature dependent constant value at large distances which changes sign for $T \simeq 3T_c$.

To analyze the T and r-dependence of the coupling we first follow the approach used at $T = 0$ [8,9] and define $\alpha_{qq}(r, T)$ through Eq. (8). The derivatives of the singlet free energy with respect to the distance, $\mathrm{d}F_1(r, T)/\mathrm{d}r$, are obtained from a finite difference approximation using results at neighboring distances. We compare our finite temperature results to the high statistics calculation performed at zero temperature [10] in Fig. 2. In this figure the results obtained from the numerical calculation of the heavy quark potential at $T = 0$ and distances $r \gtrsim 0.1$ fm are summarized by a fat black line. At shorter distances a thin line represents the result of a perturbative calculation of the force [9,10] which is based on the 2-loop calculation of the heavy quark potential [5,6]. This perturbative result is smoothly matched to the lattice data at $r \simeq 0.1$ fm. Also shown in the figure as a dashed line is the effective coupling extracted from the confinement potential V_{string} using Eq. (8). It agrees quite well with the lattice data for $r \gtrsim 0.1$ fm but shows strong

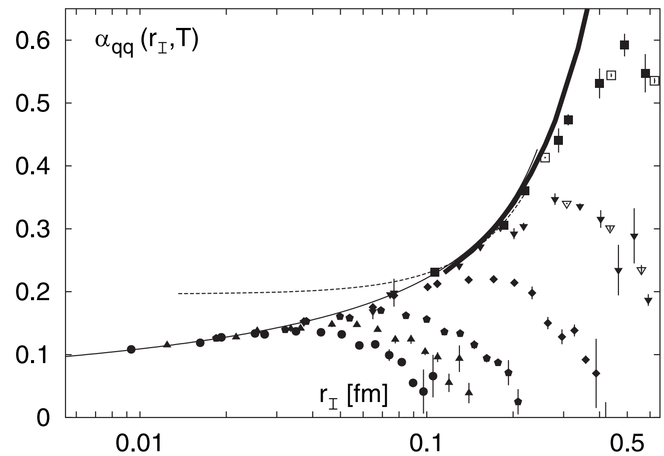

FIG. 2. The running coupling in the qq-scheme determined on lattices of size $32^3 \times N_\tau$ with $N_\tau = 4$ (open symbols) and 8 (filled symbols) from derivatives the short distance part of the singlet free energy ($T = 0$: from the force) at different temperatures. The relation of different symbols to the values of the temperature are as in Fig. 1(a). The various lines are explained in the text.

deviations from the perturbative as well as lattice calculation at shorter distances.

Our numerical results on α_{qq} at distances smaller than 0.1 fm cover also distances substantially smaller than those analyzed so far at $T = 0$. They clearly show the running of the coupling with the dominant length scale r also in the QCD plasma phase. For temperatures below $3T_c$ we find that α_{qq} agrees with the zero temperature perturbative result in its entire regime of validity, i.e., for $r \lesssim 0.1$ fm. At these temperatures thermal effects only become visible at larger distances and lead, as expected, to a decrease of the coupling relative to its zero temperature value; for larger temperatures thermal effects influence also the short distance behavior at distances $r \lesssim 0.1$ fm.

At distances larger than $r \simeq 0.1$ fm nonperturbative effects clearly dominate the properties of α_{qq}. It thus is to be expected that the properties of a running coupling will strongly depend on the physical observable used to define it. To quantify this we analyze directly the short distance behavior of the renormalized singlet free energies and define $\alpha_V(r, T)$ through Eq. (7). At $T = 0$ the singlet free energy simply is the heavy quark potential which for distances $r \gtrsim 0.1$ fm is quite well described by $V_{\mathrm{string}}(r)$. In the case of the string potential, V_{string}, one would obtain with the perturbative ansatz given in Eq. (7) for the running coupling $\alpha_V(r, 0) = \pi/16 - 0.75\sigma r^2$, which changes sign at $r \simeq 0.25$ fm. We expect to find a similar behavior also when using $F_1(r, T)$ at temperatures close to T_c. As can be seen from Fig. 3 such a behavior is indeed found for $T_c \leq T \lesssim 3T_c$. This again reflects the importance of remnants of the confining force in the QCD plasma phase. For larger temperatures α_V stays

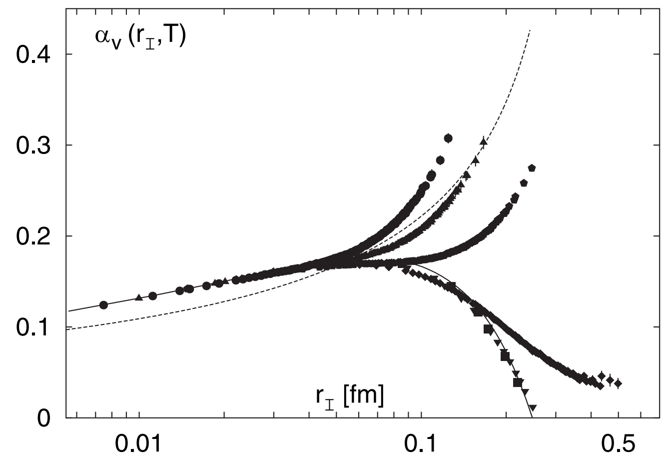

FIG. 3. The running coupling in the V-scheme determined on lattices of size $32^3 \times N_\tau$ with $N_\tau = 4$ (open symbols) and 8 (filled symbols) from the short distance part of the singlet free energy ($T = 0$: from the potential) at different temperatures. The relation of different symbols to the values of the temperature are as in Fig. 1(a). The dashed (solid) lines for $\alpha_{qq}(\alpha_V)$ represent the zero temperature results of Ref. [10] which are continued to shorter distances using our new results.

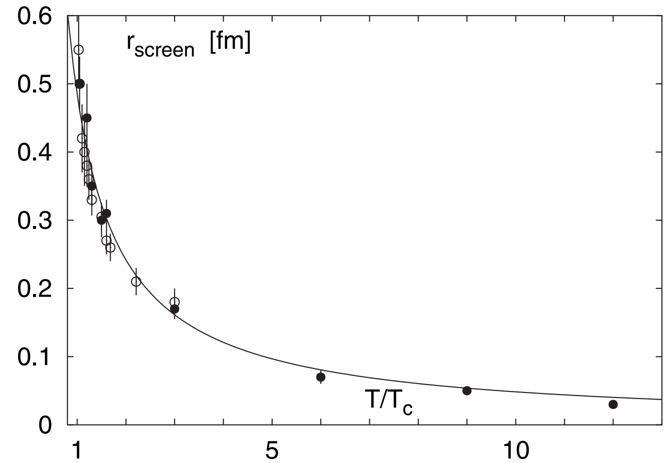

FIG. 4. The location of the maximum in $\alpha_{qq}(r, T)$ at fixed temperature versus temperature in units of the deconfinement temperature. Open (filled) symbols correspond to calculations on lattices with temporal extent $N_\tau = 4$ and 8, respectively. The solid line corresponds to $r_{\mathrm{screen}} = 0.48\ \mathrm{fm} \cdot T_c/T$.

positive reflecting the fact that $F_1(r, T)$ approaches a negative constant at large distances. We note that at short distance $\alpha_V(r, T) > \alpha_{qq}(r, T)$ as expected from the perturbative relation, Eq. (9), between both definitions of the running coupling. At distances $r \gtrsim 0.1$ fm the couplings, however, merely reflect the nonperturbative properties of the observable used to define them.

Let us return to an analysis of the properties of the running coupling defined in the qq-scheme. At temperatures close to T_c the coupling $\alpha_V(r, T)$ stays close to the zero temperature value up to distances $r \simeq 0.3$ fm. At these distances the strong r-dependence of $\alpha_{qq}(r, T)$ mimics the linear rising part of the zero temperature confinement potential. Remnants of the confining force thus survive the deconfinement transition and play an important role in quark-antiquark interactions at intermediate distances up to $r \simeq 0.3$ fm. At distances larger than $\sim$0.5 fm these are, however, rapidly screened also at temperatures close to T_c. We use the maxima in $\alpha_{qq}(r, T)$ to define a length scale r_{screen} which separates the short distance regime from the large distance regime. This is shown in Fig. 4. In the entire temperature interval analyzed by us we find that r_{screen} is inversely proportional to the temperature, i.e., we find $r_{\mathrm{screen}} = (0.48 \pm 0.01)\ \mathrm{fm} \cdot T_c/T$.

The fact that $\alpha_{qq}(r, T)$ can become large at some distance also in the deconfined phase of QCD has recently been exploited to discuss the scenario of a strongly interacting fluid of quasiparticles describing the thermodynamics above but close to T_c [3,4]. Our analysis of the running coupling shows that up to a certain distance scale confining features of the heavy quark potential indeed survive in the plasma phase and thus may support such a scenario. We note, however, that this effect is not related to an unexpectedly large coupling arising from thermal effects but on the contrary to the survival of vacuum physics below a certain characteristic length scale. In particular, there is no evidence for a larger coupling in the Coulomb part of the heavy quark potential arising from thermal effects. From this point of view it may be questionable whether the plasma phase really can support the existence of well localized colored bound states as it is advocated in [3].

V. T DEPENDENCE OF THE COUPLING AT LARGE DISTANCES

We now turn to an analysis of the large distance structure of heavy quark free energies and discuss the determination of a temperature dependent coupling from it. As inferred from the r-dependence of the coupling the crossover from the short to large distance regime sets in rather abruptly. In particular, the screening of the heavy quark free energies leads to an exponential suppression of α_{qq}. In order to extract the T-dependence of the QCD coupling conventionally used to describe the large distance properties of QCD at high temperature this screening effect should be eliminated. In Fig. 5 we show $r[F_1(\infty, T) - F_1(r, T)]$ on a logarithmic scale. Aside from deviations at short and intermediate distances this is seen to decay exponentially. We fit the large distance part of $F_1(r, T)$ with an ansatz motivated by the Debye screened perturbative result

$$\frac{F_{\mathrm{fit}}(r, T)}{T} = -\frac{4\alpha(T)}{3rT} \exp\{-\sqrt{4\pi\tilde{\alpha}(T)}rT\} + b(T), \quad (14)$$

where $\alpha(T)$ and $\tilde{\alpha}(T)$ are used as two independent fit

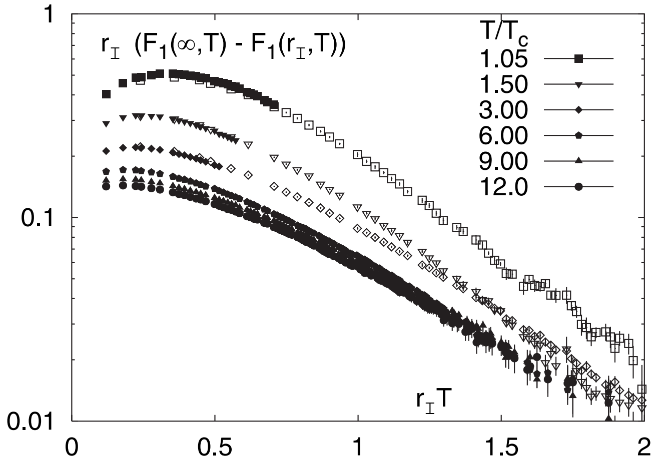

FIG. 5. The singlet free energy versus rT for several temperatures above T_c obtained from calculations on lattices with temporal extent $N_\tau = 4$ (open symbols) and $N_\tau = 8$ (filled symbols).

parameters, which at large temperature, i.e., in the perturbative limit, are expected to coincide,

$$\lim_{T\to\infty} \alpha(T)/\tilde{\alpha}(T) = 1. \qquad (15)$$

Both fit parameters as well as their ratio are shown in Fig. 6. Within statistical errors the above limit indeed is reached for temperatures $T/T_c \simeq 6$. At smaller temperatures, however, deviations from unity are large and reach $\alpha(T)/\tilde{\alpha}(T) \simeq 5$ close to T_c. Similar to the ambiguities that exist for the definition of a running coupling at large distances this suggests that for not too large temperatures, $T_c \leq T \lesssim 2T_c$, any definition of a temperature dependent running coupling will strongly depend on the physical process used to define $\alpha(T)$ or equivalently $g(T)$. Nonetheless for $T \gtrsim 6T_c$ the temperature dependence seems to follow the logarithmic behavior expected from perturbation theory. This is shown in the figure by the solid lines with the dotted error band which represent an appropriately rescaled perturbative running coupling, $\alpha(T) = 2.095(82)\alpha_{\rm pert}(T)$ where we have used for $\alpha_{\rm pert}$ the 2-loop perturbative running coupling

$$g^{-2}(T) = \frac{11}{8\pi^2} \ln\left(\frac{2\pi T}{\Lambda_{\overline{\rm MS}}}\right) + \frac{51}{88\pi^2} \ln\left[2 \ln\left(\frac{2\pi T}{\Lambda_{\overline{\rm MS}}}\right)\right], \quad (16)$$

and relate $\Lambda_{\overline{\rm MS}}$ to the critical temperature for the deconfinement transition, $T_c/\Lambda_{\overline{\rm MS}} \simeq 1.14(4)$ [8,21]. The rescaling factor has been determined from a common fit of the data for $\alpha(T)$ and $\tilde{\alpha}(T)$ at temperatures $T \geq 6T_c$. At least in this temperature regime all higher order perturbative as well as nonperturbative effects seem to be well described by a rescaling of the coupling with constant factor. A similar observation has been made for screening masses determined in an SU(2) gauge theory at high temperature [22].

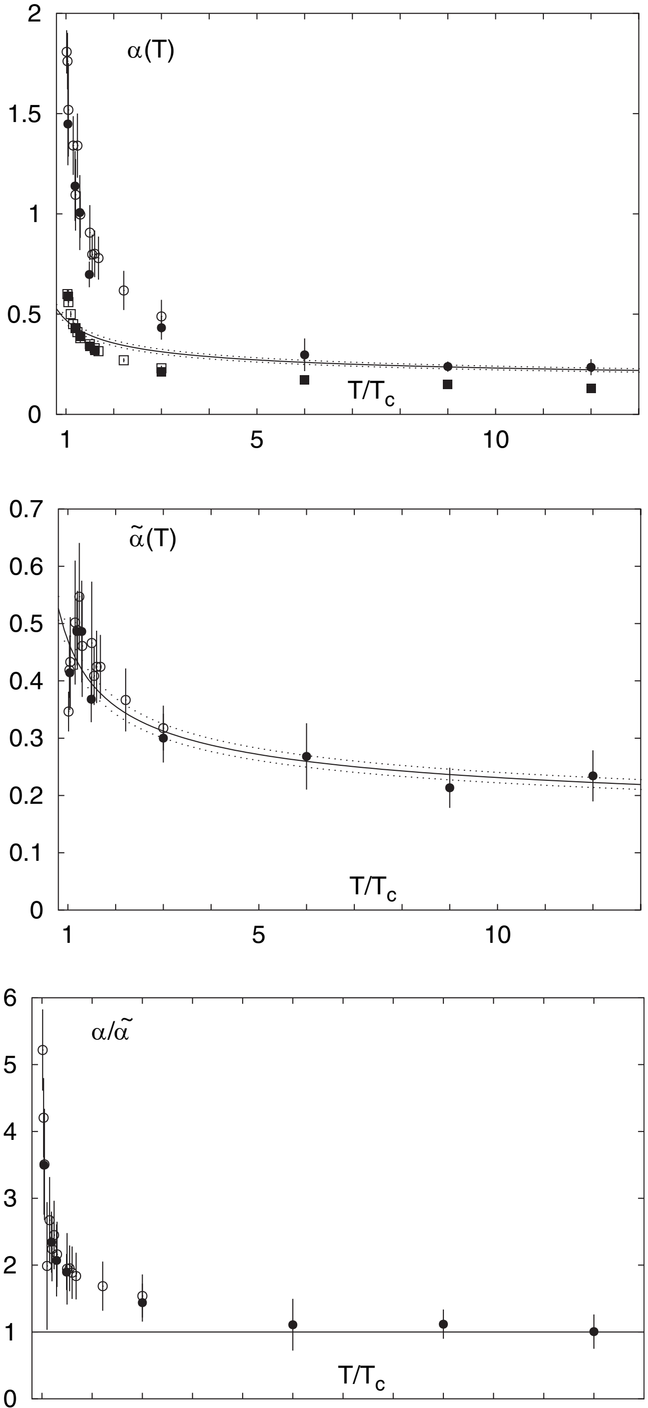

FIG. 6. The temperature dependent running coupling determined from the large distance behavior of the singlet free energy on lattices with temporal extent $N_\tau = 4$ (open symbols) and $N_\tau = 8$ (filled symbols). The upper figure shows $\alpha(T) \equiv g^2(T)/4\pi$ (dots) and the value $\alpha_{qq}(r_{\rm screen}, T)$ (squares) determined from the short distance behavior of the singlet free energy (see Fig. 3). The figure in the middle shows $\tilde{\alpha}(T) \equiv \tilde{g}^2(T)/4\pi$ and characterizes the temperature dependence of the screening mass. The lower figure gives the ratio of both fit parameters. The solid lines with the dotted error band are discussed in the text.

Also shown in the upper part of Fig. 6 is the maximal value of the running coupling determined from the short distance behavior of the singlet free energy, $\alpha_{qq}(r_{screen}, T)$. As can be seen, this coupling is significantly smaller than $\alpha(T)$ determined as coupling strength of the Debye screened Coulomb potential which describes the long distance part of the singlet free energy. We stress again that $\alpha(T)$ found here for $T \lesssim 3T_c$ at large distances is not appropriate to characterize the Coulombic part of $F_1(r, T)$ at short distances. As discussed in the previous section this is still controlled by an almost temperature independent coupling $\alpha_V(r, T)$.

VI. SUMMARY

We have performed a detailed study of the singlet free energy of a static $q\bar{q}$-pair in quenched QCD (SU(3) gauge theory) at short and large distances. We have shown that at sufficiently short distances the free energy agrees well with the zero temperature heavy quark potential and thus also leads to a temperature independent running coupling. The range of this short distance regime is temperature dependent and reduces from $r \simeq 0.5$ fm at $T \simeq T_c$ to $r \simeq 0.03$ fm at $T \simeq 12T_c$. At high enough temperatures, $T \gtrsim 6T_c$, the large distance behavior of the free energy is qualitatively similar to what is expected in perturbation theory and allows for a consistent definition of a temperature dependent running coupling characterizing large distance properties of QCD thermodynamics. However, at temperatures close to T_c the definition of a temperature dependent running coupling is not unique.

Our analysis suggests that it is more appropriate to characterize the nonperturbative properties of the QCD plasma phase close to T_c in terms remnants of the confinement part of the QCD force rather than a strongly coupled Coulombic force.

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