Representation mixing and exotic baryons in the Skyrme model

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(Received 9 June 2004; revised manuscript received 9 July 2004; published 29 October 2004)

We study the effect of representation mixing in the SU(3) Skyrme model by diagonalizing exactly the representation-dependent part. It is observed that even without the next-to-leading-order symmetry breaking terms the low-lying baryon masses as well as the recently discovered Θ^+ and $\Xi_{\bar{10}}$ can be fairly well reproduced within 3% accuracy. It is also demonstrated that the mixing effect is not negligible in decay processes of $\{\bar{10}\}$. In particular the effect of mixing with $\{27\}$ is found to be quite large. These results are compatible with the second-order perturbation scheme. The decay widths are found to be sensitive to the mass values. The decay widths of $\{\bar{10}\}$ are estimated to be smaller than those of $\{10\}$ by an order of magnitude due to the destructive interference between operators, although the kinematic factors are comparable.

DOI: 10.1103/PhysRevD.70.074033 PACS numbers: 12.39.Dc, 13.30.Eg, 14.20.–c

The recent discoveries [1] of Θ^+ have generated lots of interesting developments in hadron spectroscopy, in particular, in understanding the exotic nature of the state. The state is exotic in the sense that the quantum numbers cannot be explained as a system of three quarks, as the smallest number of quarks consistent with Θ^+ is five, or that it cannot be classified into conventional classifications $\{8\}$ and $\{10\}$. The lowest multiplet consistent with Θ^+ is $\{\bar{10}\}$ in the scheme of flavor SU(3) symmetry.

The chiral soliton model proposed by Skyrme [2] has been explored theoretically and phenomenologically with many interesting successes [3–6] in describing the properties of low-lying hadrons. The importance of the higher multiplets beyond octet and decuplet has been noticed in the chiral soliton model in treating the symmetry breaking part as perturbations. The symmetry breaking part is not diagonal in the SU(3) multiplet space so that in higher-order perturbation [6–8] or in diagonalizing the Hamiltonian [9,10] the mass eigenstate should be mixed with higher representations. For example, the nucleon is dominantly described by $\{8\}$ but with nonvanishing mixing amplitudes of $\{\bar{10}\}, \{27\}, \cdots$ and the Δ also has nonvanishing mixing amplitudes of $\{27\}, \{35\}, \cdots$.

The prediction [7,11] of Θ^+ as the lowest state among the higher multiplet $\{\bar{10}\}$ has now been confirmed. One of the characteristics of Θ^+ as an isospin singlet and hypercharge 2 state with respect to representation mixing is that it has no corresponding state in the $\{8\}$ and $\{27\}$, i.e., no representation mixing is possible. On the other hand more massive states in the same multiplet $\{\bar{10}\}$ have non-negligible mixing with other representations, and the masses and decay widths are supposed to depend on the mixing. The effect of mixing in second-order perturbation has been extensively discussed recently [12–14], in which the effect of mixing is found to be non-negligible but depends much on the parameters of the underlying effective theory. Similar observations have been made in the exact diagonalization method [15,16] for the exotic baryon masses. In this short paper we discuss the mixing effect on the decay process further, using the exact diagonalization method keeping only the chiral symmetry breaking term that is of leading-order in N_c.

The effective action for the pseudoscalar mesons, which realizes the global $SU(3)_L \times SU(3)_R$ in the Goldstone mode, can be written in general as

$$S_{\rm eff} = S_2 + S_{HOD} + S_{SB} + S_{WZ} \tag{1}$$

where S_2 and S_{HOD} are the leading kinetic term and the higher-order derivative terms including the Skyrme term. S_{WZ} is the Wess-Zumino action and S_{SB} is an explicit symmetry breaking term depending on the meson masses. The effective Hamiltonian after quantizing the "degenerate rotational mode" of the SU(2) soliton of hedge-hog ansatz [2] embedded into SU(3) is known to have the following form for $N_c = 3$ and $B = 1$:

$$H = M_{cl} + \frac{1}{2}\left(\frac{1}{I_1} - \frac{1}{I_2}\right) C_2[\mathrm{SU}(2)_R] - \frac{3}{8I_2} + \frac{1}{2I_2} C_2[\mathrm{SU}(3)_L] - \alpha[1 - D^8_{88}(A)], \tag{2}$$

where $C_2[\mathrm{SU}(2)_R]$ and $C_2[\mathrm{SU}(3)_L]$ are the corresponding Casimir operators $(C_2(\mathrm{SU}(2)_R) = J(J+1), C_2(\mathrm{SU}(3)_L) = \frac{1}{3}[p^2 + q^2 + 3(p+q) + pq]$. In this frame work we are left with four parameters, M_{cl}, I_1, I_2 and α, which should be in principle determined unambiguously from the effective action. In this work however we take them as a set of free parameters for the phenomenological study.

In Eq. (2), the SU(3) symmetric limit can be achieved when the last term vanishes. The mass spectrum of the baryon can be determined by treating the symmetry breaking term in a perturbative way. One can also include

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additional terms of next-to-leading-order in chiral symmetry breaking to reproduce the low-lying baryon spectrum well in the first-order perturbation calculation [11]. In this work when the Hamiltonian is to be diagonalized, we do not include these terms which are of next-to leading-order in the $1/N_c$ expansion to make the analysis free from possible ambiguities due to the extra parameters in the effective theory. For the diagonalization the Hamiltonian can be divided into two parts, representation-independent (H_0) and dependent (H_R) parts:

$$H_0 = M_{cl} + \frac{1}{2}\left(\frac{1}{I_1} - \frac{1}{I_2}\right)C_2[\mathrm{SU}(2)_R] - \frac{3}{8I_2}, \quad (3)$$

$$H_R = \frac{1}{2I_2}C_2[\mathrm{SU}(3)_L] - \alpha[1 - D^8_{88}(A)]. \quad (4)$$

Minimal extensions beyond octet and decuplet can be guided by considering the quark content of the baryons. Three-quark system leads up to decuplet. With an additional quark-antiquark pair for a penta-quark system, $qqq\bar{q}q$, the possible representation can be extended up to $\{\bar{10}\}$, $\{27\}$,and $\{35\}$. With the constraint $Y_R = 1$ for $B = 1$ baryon, the state vectors [10] for spin 1/2 baryons and spin 3/2 baryons can be written as

$$\left|B\left(J = \frac{1}{2}\right)\right\rangle = C^a_8|\Psi^8_{\mu,\nu}\rangle + C^a_{\bar{10}}|\Psi^{\bar{10}}_{\mu,\nu}\rangle + C^a_{27}|\Psi^{27}_{\mu,\nu}\rangle, \quad (5)$$

$$\left|B\left(J = \frac{3}{2}\right)\right\rangle = C^b_{10}|\Psi^{10}_{\mu,\nu}\rangle + C^b_{27}|\Psi^{27}_{\mu,\nu}\rangle + C^b_{35}|\Psi^{35}_{\mu,\nu}\rangle, \quad (6)$$

where $a(b)$ refer to a baryon with flavor part $\mu = (Y, I, I_3)$ and spin part $\nu = (Y_R, J, J_3)$ with spin $J = 1/2(3/2)$. By diagonalizing the Hamiltonian H_R in the form of 3×3 matrix for each baryon state, we can calculate the corresponding mass as an eigenvalue of the Hamiltonian.

The eigenvalues and the mixing amplitudes in Eqs. (5) and (6) are of course functions of four parameters, M_{cl}, I_1, I_2 and α. We fix the parameters by a best fit to the masses of the low-lying octet and decuplet states. The best fit to the mass differences can be obtained with the central value of $I_2 = 2.91 \times 10^{-3}$ MeV^{-1} and $\alpha = -750$ MeV. Then the mass fit gives $M_{cl} = 773$ MeV and $I_1 = 6.32 \times 10^{-3}$ MeV^{-1}. It is interesting to note that these values are comparable to those used in the perturbation scheme [12,13].

The masses in the best fit are given by

$$\begin{aligned} &M(N) = 939, && M(\Lambda) = 1108, && M(\Sigma) = 1226,\\ &M(\Xi) = 1345, && M(\Delta) = 1231, && M(\Sigma_{10}) = 1385,\\ &M(\Xi_{10}) = 1506, && M(\Omega) = 1638, && M(\Theta^+) = 1570,\\ &M(N_{\bar{10}}) = 1705, && M(\Sigma_{\bar{10}}) = 1811, && M(\Xi_{\bar{10}}) = 1818. \end{aligned} \quad (7)$$

One can see that the masses for the low-lying octet and decuplet are reasonably well reproduced in the exact diagonalization method, with results that are comparable also to those obtained in the perturbation scheme (either in the first-order [17] or in the second-order perturbation [12]). It is found that the estimated masses of Θ^+ and $\Xi_{\bar{10}}$ are consistent with the experimental values within 3% accuracy. The mixing amplitudes for the corresponding states can be read out from the normalized mass eigenstates. For example, the mixing amplitudes for N, Δ and $N_{\bar{10}}$ are given by

$$\begin{aligned} &C^N_8 = 0.953, && C^N_{\bar{10}} = 0.234, && C^N_{27} = 0.191,\\ &C^\Delta_{10} = 0.877, && C^\Delta_{27} = 0.464, && C^\Delta_{35} = 0.125,\\ &C^{N_{\bar{10}}}_8 = -0.234, && C^{N_{\bar{10}}}_{\bar{10}} = 0.970, && C^{N_{\bar{10}}}_{27} = 0.024, \end{aligned} \quad (8)$$

which are comparable to those in [12,14,17]. For $\{\bar{10}\}$, it should be noted that the equal spacing rule in the first-order perturbation is not literally respected due to the effects of the mixing in the second-order perturbation [12]. It is observed that there are no appreciable differences in the mixing amplitudes between the exact diagonalization scheme and second-order perturbation scheme, which is consistent with the higher-order perturbative calculations [8].

Given the wave function in the representation space, Eqs. (5) and (6), the decay width of a baryon B into a low-lying B' and meson φ can be obtained by evaluating the matrix element of the baryon decay operators. The Yukawa coupling in general as well as the decay operator, in particular, which is basically a meson-baryon-baryon ($\varphi BB'$) coupling, has been discussed by many authors [18] in the context of the chiral soliton model. In this work, we choose an operator based on the suggestion of Adkins *et al.* [4] in relation to the axial current coupling and developed further by Blotz *et al.* [19], which has the form [11,12]:

$$\hat{O}^{(8)}_\varphi = 3\left[G_0 D^{(8)}_{\varphi i} - G_1 d_{ibc} D^{(8)}_{\varphi b}\hat{S}_c - G_2\frac{1}{\sqrt{3}}D^{(8)}_{\varphi 8}\hat{S}_i\right] \times p_{\varphi i}, \quad (9)$$

where $i = 1, 2, 3$ and $b, c = 4, ..., 7$. The decay amplitude and the decay width are given by

$$M_{B\to B'+\varphi} = \langle B'|\hat{O}^{(8)}_\varphi|B\rangle, \qquad \Gamma_{B\to B'\phi} = K \cdot \bar{A}^2 \quad (10)$$

where $\bar{A}^2 = |M|^2/3p^2$ and K is a kinematic factor:

$$K = \frac{p^3}{8\pi M_B M_{B'}}\frac{\overline{M}_{B'}}{\overline{M}_B}. \quad (11)$$

Here M_B's (m) are the corresponding masses of the baryons (mesons), $\overline{M}$'s are the mean masses of the multiplet. We take $\overline{M}_8 = 1154.5$ MeV, $\overline{M}_{10} = 1436$ MeV, $\overline{M}_{\bar{10}} = 1726$ MeV.

The decay amplitudes of the baryons can be calculated in a straightforward way and result in lengthy formulas. For example, the amplitudes squared for $\Delta \rightarrow N + \pi$ and $\Theta^+ \rightarrow N + K$ are given by

$$\begin{aligned}\bar{A}^2(\Delta \rightarrow N + \pi) = \frac{3}{5}\Big[& G_{10} c_8^N a_{10}^{\Delta} + \frac{\sqrt{30}}{9} G_{27} c_8^N a_{27}^{\Delta} + \frac{5\sqrt{6}}{18} F_{35} c_{\bar{10}}^N a_{27}^{\Delta} \\ & + \frac{1}{3\sqrt{6}} G'_{27} c_{27}^N a_{10}^{\Delta} + \frac{\sqrt{5}}{7} G_{27} c_{27}^N a_{27}^{\Delta} + \frac{25}{18}\sqrt{\frac{3}{7}} F_{35} c_{27}^N a_{35}^{\Delta} \Big]^2, \\ \bar{A}^2(\Theta^+ \rightarrow N + K) = \frac{3}{5}\Big[& G_{\bar{10}} c_8^N d_{\bar{10}}^{\Theta^+} + \frac{\sqrt{5}}{4} H_{\bar{10}} c_{\bar{10}}^N d_{\bar{10}}^{\Theta^+} - \frac{7}{4\sqrt{6}} H'_{27} c_{27}^N d_{\bar{10}}^{\Theta^+} \Big]^2,\end{aligned} \tag{12}$$

where $G_{10} = G_0 + \frac{1}{2}G_1$, $G_{27} = G_0 - \frac{1}{2}G_1$, $G'_{27} = G_0 - 2G_1$, $F_{35} = G_0 + G_1$, $G_{\bar{10}} = G_0 - G_1 - \frac{1}{2}G_2$, $H_{\bar{10}} = G_0 - \frac{5}{2}G_1 + \frac{1}{2}G_2$, $H'_{27} = G_0 + \frac{11}{14}G_1 + \frac{3}{14}G_2$. Introducing a parameter ρ [13] as $G_1 = \rho G_0$, we take G_0 and ρ as parameters in this phenomenological analysis. We find a ρ and G_0 that are consistent with the overall fit to the experimental values of the widths of the decuplet. The overall fit is obtained with $G_0 = 17.5$ and $\rho = .5$. The decay width is found to be quite sensitive to the masses of the particles involved in the decay process. This is because the kinetic part is very sensitive to the masses. We calculate the possible range of the calculated widths by allowing 3% variations of the masses. As shown in the parentheses in Table I, the kinetic terms K and therefore the decay widths are changing in a relatively large range even with 3% variation with masses. On the other hand by allowing $\pm 3\%$ variation in masses, reasonably well reproduced in this model, one can explain the experimental values of $\{10\}$ decay widths within the right range. Now given the set of parameters determined by the low-lying baryons, one can make the prediction for the decay widths of exotic $\{\bar{10}\}$ baryons. In this work we adopt the parametrization for G_2 as in [12], $G_2 = (\frac{9F/D-5}{3F/D+5}) \times (\rho + 2)G_0$. The estimated decay widths are given in Table II.

Compared to the decuplet, the decay amplitudes for the antidecuplet are found to be much smaller by an order of magnitude whereas the kinetic terms are comparable to each other. It has been understood that this is mainly due to the destructive interference between the operators [11]. In the fourth column, the amplitudes with $G_1 = G_2 = 0$ are shown, which clearly shows that the effect of interferences are substantially large. To see the effect of representation mixing particularly with $\{27\}$, the results without $\{27\}$ mixing are shown in the fifth column. The overall tendency is that the nonvanishing mixing with $\{27\}$ reduces the amplitudes [13]. However, for the processes $\Xi_{\bar{10}} \rightarrow \Xi + \pi$ and $\Sigma_{\bar{10}} \rightarrow N + K$, the mixing enhances the decay amplitudes [13], whereas $\Xi_{\bar{10}} \rightarrow \Sigma + K$ and $\Sigma_{\bar{10}} \rightarrow \Lambda + \eta$ are found to be insensitive to $\{27\}$ mixing. The values in parenthesis are those with $\pm 3\%$ variations of the baryon masses. According to our calculated masses, the process $N_{\bar{10}} \rightarrow \Sigma + K$ and $\Sigma_{\bar{10}} \rightarrow \Xi + K$ are beyond the threshold in the best fit.

In this work, we discussed the effect of representation mixing obtained in SU(3) Skyrme model by diagonalizing the representation-dependent part in the Hamiltonian resulting from quantizing the rotational mode. It is shown that even without the next-to-leading-order (in N_C) symmetry breaking terms the low-lying baryon masses can be fairly well reproduced by allowing the mixing with higher representation. One of the major differences in the mass results obtained in the exact diagonalization method compared to the first-order estimation [11] is that there is a deviation from the equal spacing rule with hypercharge in the $\{\bar{10}\}$ multiplet. It is due to the non-negligible mixing with other representations [7]. It is also observed that the mixing effect is not negligible in the decay widths. The effect of mixing with $\{27\}$ is found to be particularly large. These results are consistent with the second-order

TABLE I. $\{10\} \rightarrow \{8\} + \varphi$

Decay	K[a]	$\bar{A}^2$	Γ[a]	$\Gamma^{\text{Exp.}}$
$\Delta \rightarrow N + \pi$	0.33 (0.13–0.64)	367	121 (46–233)	115–125
$\Sigma_{10} \rightarrow \Lambda + \pi$	0.17 (0.04–0.44)	177	31 (8–79)	34.7
$\Sigma_{10} \rightarrow \Sigma + \pi$	0.001 (<0.18)	43	0.70 (<7.9)	4.73
$\Xi_{10} \rightarrow \Xi + \pi$	0.01 (<0.22)	135	1.2 (<30)	9.9

[a]Values in the parentheses are obtained with $\pm 3\%$ mass variations.

TABLE II. $\overline{\{10\}} \to \{8\} + \varphi$

Decay	K[a]	$\bar{A}^2_{(\text{best fit})}$	$\bar{A}^2_{(G_1=0,G_2=0)}$	$\bar{A}^2_{(\text{without}\{27\})}$	Γ[a]
$\Theta^+ \to N + K$	0.52 (0.15–1.04)	7.60	165	29.00	4 (1.2–7.9)
$\Xi_{\overline{10}} \to \Xi + \pi$	0.66 (0.42–1.3)	39	125	16	26 (16–50)
$\Xi_{\overline{10}} \to \Sigma + K$	0.23 (0.06–0.86)	17	41	17	4 (0.97–14)
$N_{\overline{10}} \to N + \pi$	3.3 (2.5–4.2)	0.61	15.2	14	2 (1.5–2.6)
$N_{\overline{10}} \to N + \eta$	1.1 (0.56–1.8)	6.6	24	14	7.3 (3.6–12)
$N_{\overline{10}} \to \Lambda + K$	0.28 (0.01–0.67)	3.6	31	0.7	1.0 (0.03–2.40)
$N_{\overline{10}} \to \Sigma + K$	⋯ (<0.27)	0.40	45	2.3	⋯ (<0.11)
$\Sigma_{\overline{10}} \to N + K$	2.4 (1.6–3.3)	0.50	22	0.02	1.2 (0.81–1.7)
$\Sigma_{\overline{10}} \to \Sigma + \pi$	1.3 (1.0–2.2)	1.3	13	12	1.7 (1.3–2.9)
$\Sigma_{\overline{10}} \to \Sigma + \eta$	0.06 (<0.57)	18	46	34	1.0 (<10)
$\Sigma_{\overline{10}} \to \Lambda + \eta$	0.57 (0.13–1.1)	1.1	22	1.1	0.61 (0.14–1.2)
$\Sigma_{\overline{10}} \to \Xi + K$	⋯ (<0.20)	33	70	91	⋯ (<6.60)

[a]Values in the parentheses are obtained with ±3% mass variations.

perturbation scheme, where higher-order corrections are found to be relatively large [13]. Although the decay width estimations in this work are based on a specific form of the decay operator [4,19], the observation that the results of the exact diagonalization method and the second-order perturbation scheme are consistent with each other demonstrates that the higher-order corrections beyond the second-order might not be important in numerical estimations. However, it should be noted that the exact diagonalization can make more sense only when the Hamiltonian to be diagonalized is as complete as possible at least for the symmetry breaking part.

We would like to thank Mannque Rho for useful discussions. We benefited also from discussions with Hyun-Chul Kim, Vladmir Kopeliovich, Ghill-Seok Yang, and Kwang-Yun Youm. H. K. L. is grateful also to the Killam Trust for its partial financial support.

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