Measuring α in $B(t) \to \rho^{\pm}\pi^{\mp}$

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Defining a most economical parametrization of time-dependent $B \to \rho^{\pm}\pi^{\mp}$ decays, including a measurable phase $\alpha_{\rm eff}$ which equals the weak phase α in the limit of vanishing penguin amplitudes, we propose two ways for determining α in this processes. We explain the limitation of one method, assuming only that two relevant tree amplitudes factorize and that their relative strong phase δ_t is negligible. The other method, based on broken flavor SU(3), permits a determination of α in $B^0 \to \rho^{\pm}\pi^{\mp}$ in an overconstrained system using also rate measurements of $B^{0,+} \to K^*\pi$ and $B^{0,+} \to \rho K$. Current data are shown to restrict two ratios of penguin and tree amplitudes $r_{\pm}$ to a narrow range around 0.2 and to imply an upper bound $|\alpha_{\rm eff} - \alpha| < 15°$. Assuming that δ_t is much smaller than 90°, we find $\alpha = (93 \pm 16)°$ and $(102 \pm 20)°$ using *BABAR* and BELLE results for $B(t) \to \rho^{\pm}\pi^{\mp}$. Avoiding this assumption for completeness, we demonstrate the reduction of discrete ambiguities in α with increased statistics and show that SU(3) breaking effects are effectively second order in $r_{\pm}$.

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I. INTRODUCTION

A proposal made 14 years ago to measure the Cabibbo-Kobayashi-Maskawa (CKM) angle α in $B \to \pi\pi$ through an isospin analysis [1] was followed shortly afterwards by a suggestion to apply this technique also to the quasi two-body decays, $B \to \rho\pi$ [2,3]. The analysis requires a construction of two pentagons, for B and $\bar{B}$, the sides of which describe decay amplitudes into different-charge $\rho\pi$ final states. This is a challenging task, requiring precise measurements of decay rates and asymmetries in all five modes. In addition it also involves a large number of discrete ambiguities and in certain cases continuous ambiguities in α [3]. A simplification occurs when the decay amplitude of $B^0 \to \rho^0\pi^0$ is much smaller than the amplitudes of the other four processes, in which case the pentagons turn into approximate quadrangles. Recently the BELLE Collaboration reported evidence for $B^0 \to \rho^0\pi^0$ [4] at a level implying that this simplification may not occur in practice. This seems to indicate that a useful measurement of α using the full isospin analysis may be impractical even with super-B-factorylike luminosities [5].

A complementary and more promising way of learning α in these decays is based on performing also a time-dependent Dalitz plot analysis of $B^0 \to \pi^+\pi^-\pi^0$ [6]. One uses the interference between two ρ resonance bands to study the phase differences between distinct amplitudes contributing to the decay. This raises issues such as the precise shapes of the tails of the Breit-Wigner functions, and the effect of interference with other resonant and nonresonant contributions [7]. A complete implementation of this method requires higher statistics than available today.

An interesting and more modest question, which may already be studied now using time-dependent decay measurements of $B^0(\overline{B}^0) \to \rho^{\pm}\pi^{\mp}$ by the *BABAR* [8] and BELLE [9] Collaborations, is what can be learned about the weak phase α using this limited information alone. Since these processes involve more hadronic parameters than measurable quantities, further assumptions are required to answer the question in a model-independent manner. Flavor SU(3) [10,11], a symmetry less precise than isospin, provides a suitable framework for an answer. SU(3) symmetry relates $B^0 \to \rho^{\pm}\pi^{\mp}$ to processes of the type $B \to K^*\pi$ and $B \to \rho K$ [12]. Allowing for certain SU(3) breaking effects, which may be justified on theoretical grounds and tested experimentally, improves the quality of such an analysis. A recent application of broken flavor SU(3) to the considerably simpler case of measuring α in $B^0(t) \to \pi^+\pi^-$ was studied in [13].

In the present paper we extend the SU(3) analysis of $B^0 \to \pi^+\pi^-$ to study $B^0(t) \to \rho^{\pm}\pi^{\mp}$. We also suggest an alternative approach to measure α in $B \to \rho^{\pm}\pi^{\mp}$, which does not rely on flavor SU(3), but instead reduces the number of hadronic parameters by two when assuming that tree amplitudes factorize to a very good approximation.

Several earlier studies of α in $B \to \rho^{\pm}\pi^{\mp}$ have been performed. An application of flavor SU(3) to these processes has been carried out in [14,15]. We study a wider range of aspects, such as consequences of factorization of tree amplitudes, SU(3) breaking effects, bounds on ratios of penguin-to-tree amplitudes, and the nature of discrete ambiguities in values obtained for α. For completeness, we will present a proof of several bounds on two *unmeasurable quantities* $\alpha^{\pm}_{\rm eff} - \alpha$ [14,15] one of which will turn out to be stronger than bounds obtained earlier. We will then define a *measurable phase* $\alpha_{\rm eff}$ which provides an approximate measure for α.

An SU(3) relation between $B^0 \to \rho^{\pm}\pi^{\pm}$ and $B^0 \to K^{*\pm}\pi^{\mp}$ has also been discussed in [16]; however this work made no use of the interference between $B^0 - \overline{B}^0$ mixing and $B \to \rho\pi$ decay amplitudes which is a crucial input in our study. Implications of $B^0(t) \to \rho^\pm\pi^\mp$ for a global SU(3) fit to all charmless B decays into pairs of a vector and a pseudoscalar meson $B \to VP$ were studied recently in [17]. Our study will focus on $B \to \rho^\pm\pi^\mp$ and on their direct SU(3) counterparts. Our model-independent approach differs from other studies of $B(t) \to \rho^\pm\pi^\mp$ involving *a priori* calculations of decay amplitudes and strong phases based on QCD and factorization [18,19].

The paper is organized as follows: Section II provides definitions of decay amplitudes and expressions for time-dependent decay rates in terms of a minimal set of parameters describing these amplitudes. Section III defines and discusses the use of $\alpha_{\rm eff}$, a measurable that is equal to α in the absence of penguin amplitudes. Section IV considers a seemingly useful method of reducing the number of hadronic parameters by assuming approximate factorization of tree amplitudes, pointing out its intrinsic limitation. Section V draws SU(3) relations between $B \to \rho^\pm\pi^\mp$ and several processes of the type $B \to K^*\pi$ and $B \to \rho K$. In Sec. VI we summarize the current experimental measurements of relevant rates and asymmetries, deriving numerical bounds on ratios of penguin and tree amplitudes in $B^0 \to \rho^\pm\pi^\mp$ and on the shift $\alpha_{\rm eff} - \alpha$, obtaining a range of values for α. In Section VII we study the sensitivity to experimental errors of the flavor SU(3) method for determining α. This discussion involves certain discrete ambiguities, which will be discussed briefly in this section and will be dealt with in more detail in the appendix. We conclude with a summary in Sec. VIII.

II. AMPLITUDES AND TIME-DEPENDENT DECAY RATES IN $B \to \rho^\pm\pi^\mp$

We start by setting notations and conventions. B^0 decay amplitudes A_+ and A_- are denoted by the charge of the outgoing ρ, and corresponding $\overline{B}^0$ amplitudes into charge conjugate states are denoted by $\overline{A}_+$ and $\overline{A}_-$, respectively:

$$\begin{aligned} A_+ &\equiv A(B^0 \to \rho^+\pi^-), \qquad & A_- &\equiv A(B^0 \to \rho^-\pi^+), \\ \overline{A}_+ &\equiv A(\overline{B}^0 \to \rho^-\pi^+), \qquad & \overline{A}_- &\equiv A(\overline{B}^0 \to \rho^+\pi^-). \end{aligned} \tag{1}$$

Each of the four amplitudes can be expressed in terms of two terms, a "tree" and a "penguin" amplitude, carrying specific CKM factors. We adopt the c-convention, in which the top-quark has been integrated out in the $b \to d$ penguin transition and unitarity of the CKM matrix has been used to move a $V_{ub}^* V_{ud}$ term into the tree amplitude. Absorbing absolute magnitudes of CKM factors in tree (t) and penguin (p) amplitudes, we write

$$A_\pm = e^{i\gamma} t_\pm + p_\pm, \qquad \overline{A}_\pm = e^{-i\gamma} t_\pm + p_\pm. \tag{2}$$

While dependence on the weak phase γ is displayed explicitly, strong phases are implicit in the definitions of complex amplitudes. We define three strong phase differences,

$$\delta_\pm = \arg(p_\pm/t_\pm), \qquad \delta_t = \arg(t_-/t_+). \tag{3}$$

For convenience we also define ratios of penguin and tree amplitudes in the two processes and a ratio of the two tree amplitudes,

$$r_\pm \equiv \left| \frac{p_\pm}{t_\pm} \right|, \qquad r_t \equiv \left| \frac{t_-}{t_+} \right|. \tag{4}$$

Counting parameters, we find a total of eight, consisting of seven hadronic quantities $|t_\pm|$, $|p_\pm|$, $\delta_\pm$, δ_t and the weak phase γ or α. We will assume β to be given [15,20] and use $\gamma = \pi - \beta - \alpha$. In general, the amplitudes $t_+(p_+)$ and $t_-(p_-)$ have different dynamical origins and are expected to involve different magnitudes and different strong phases. Amplitudes with subscripts $+(-)$ in Eq. (2) describe transitions in which the final-state meson incorporating the spectator quark is a π (ρ) (cf. Fig. 1). This characterization was shown to be useful in the context of a SU(3) analysis of charmless B decays into a vector and a pseudoscalar meson $B \to VP$ [12,17] where $t_+(p_+)$ and $t_-(p_-)$ represent SU(3) amplitudes (denoted $t_P(p_P)$ and $t_V(p_V)$ in [12,17]). This broader framework will be used in our discussion below.

Let us now consider measurables in time-dependent rates. Neglecting the width difference in the B^0 system, and neglecting tiny effects of CP violation in $B^0 - \overline{B}^0$ mixing, time-dependent decay rates for initially B^0 decaying into $\rho^\pm\pi^\mp$ are given by [21]

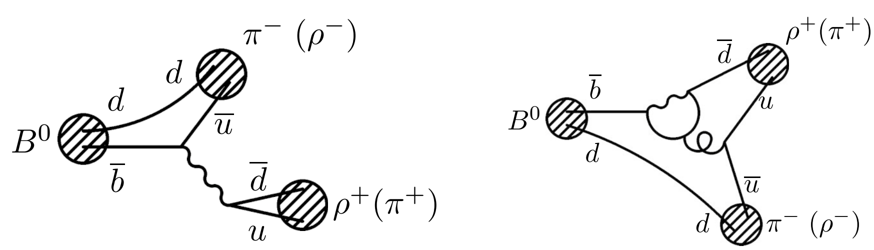

FIG. 1. The tree (left) and penguin (right) diagrams for the $B^0 \to \rho^+\pi^-$ ($B^0 \to \rho^-\pi^+$) decays.

$$\Gamma[B^0(t)\to\rho^\pm\pi^\mp] = e^{-\Gamma t}\frac{1}{2}(|A_\pm|^2+|\overline{A}_\mp|^2)\times[1+(C\pm\Delta C)\cos\Delta mt - (S\pm\Delta S)\sin\Delta mt], \tag{5}$$

where

$$C\pm\Delta C \equiv \frac{|A_\pm|^2-|\overline{A}_\mp|^2}{|A_\pm|^2+|\overline{A}_\mp|^2}, \qquad S\pm\Delta S\equiv\frac{2\mathrm{Im}(e^{-2i\beta}\overline{A}_\mp A^*_\pm)}{|A_\pm|^2+|\overline{A}_\mp|^2}. \tag{6}$$

Here Γ and Δm are the average B^0 width and mass difference, respectively. For initially $\overline{B}^0$ decays, the $\cos\Delta mt$ and $\sin\Delta mt$ in (5) have opposite signs.

Counting the number of independent measurables, we find a total of six, consisting of two CP violating quantities, S and C, two CP conserving measurables, ΔC and ΔS, and two rates, $\langle\Gamma_\pm\rangle\equiv\frac{1}{2}(|A_\pm|^2+|\overline{A}_\mp|^2)$. These two rates are related to the CP conserving charge averaged $\rho^\pm\pi^\mp$ combined decay rate $\Gamma^{\rho\pi}$ and the overall CP violating asymmetry $\mathcal{A}^{\rho\pi}_{\rm CP}$

$$\Gamma^{\rho\pi}\equiv\langle\Gamma_+\rangle+\langle\Gamma_-\rangle, \qquad \mathcal{A}^{\rho\pi}_{\rm CP}\equiv\frac{\langle\Gamma_+\rangle-\langle\Gamma_-\rangle}{\langle\Gamma_+\rangle+\langle\Gamma_-\rangle}, \tag{7}$$

implying

$$\langle\Gamma_\pm\rangle = \frac{1}{2}\Gamma^{\rho\pi}(1\pm\mathcal{A}^{\rho\pi}_{\rm CP}). \tag{8}$$

Of particular interest are the two direct CP asymmetries between $B^0(\overline{B}^0)\to\rho^+\pi^-$ and $\overline{B}^0(B^0)\to\rho^-\pi^+$ decay rates,

$$\mathcal{A}^+_{\rm CP}\equiv\frac{|\overline{A}_+|^2-|A_+|^2}{|\overline{A}_+|^2+|A_+|^2}, \qquad \mathcal{A}^-_{\rm CP}\equiv\frac{|\overline{A}_-|^2-|A_-|^2}{|\overline{A}_-|^2+|A_-|^2}. \tag{9}$$

These may be expressed in terms of three of the above measurables, C, ΔC and $\mathcal{A}^{\rho\pi}_{\rm CP}$,

$$\mathcal{A}^+_{\rm CP} = -\frac{\mathcal{A}^{\rho\pi}_{\rm CP}(1+\Delta C)+C}{1+\mathcal{A}^{\rho\pi}_{\rm CP}C+\Delta C}, \qquad \mathcal{A}^-_{\rm CP} = \frac{\mathcal{A}^{\rho\pi}_{\rm CP}(1-\Delta C)-C}{1-\mathcal{A}^{\rho\pi}_{\rm CP}C-\Delta C}. \tag{10}$$

The above observables, of which six are independent, can be expressed in terms of the eight parameters describing $B\to\rho^\pm\pi^\mp$ in (2) and (3). This leads to rather lengthy expressions, which we do not fully display. Here we give the example of the overall CP asymmetry $\mathcal{A}^{\rho\pi}_{\rm CP}$ which does not depend on δ_t,

$$\mathcal{A}^{\rho\pi}_{\rm CP} = \frac{2\sin(\beta+\alpha)(r_+\sin\delta_+ - r_t^2 r_-\sin\delta_-)}{1+r_+^2+r_t^2(1+r_-^2)-2\cos(\beta+\alpha)(r_+\cos\delta_+ + r_t^2 r_-\cos\delta_-)}. \tag{11}$$

As demonstrated by this example which contains ratios of amplitudes rather than the amplitudes themselves, it is useful to consider ratios of rates, thus trading two CP conserving measurables for two parameters $|t_+|$ and $|t_-|$. Our study is simplified by a judicial choice of the remaining four observables, such that they depend only on the six parameters $|r_\pm|$, $\delta_\pm$, δ_t, and α without depending on r_t.

We now display a convenient (but not unique) choice for this minimal set of observables. Two of the observables are naturally the direct asymmetries $\mathcal{A}^\pm_{\rm CP}$ which depend neither on r_t nor on δ_t,

$$\mathcal{A}^\pm_{\rm CP} = -\frac{2r_\pm\sin\delta_\pm\sin(\beta+\alpha)}{1+r_\pm^2-2r_\pm\cos\delta_\pm\cos(\beta+\alpha)}. \tag{12}$$

Instead of S and ΔS we define:

$$\overline{S} = \frac{1}{2\sqrt{(1+\Delta C)^2-C^2}}\left[(S+\Delta S)\left(\frac{1+\mathcal{A}^{\rho\pi}_{\rm CP}}{1-\mathcal{A}^{\rho\pi}_{\rm CP}}\right)^{1/2} + (S-\Delta S)\left(\frac{1-\mathcal{A}^{\rho\pi}_{\rm CP}}{1+\mathcal{A}^{\rho\pi}_{\rm CP}}\right)^{1/2}\right], \tag{13}$$

$$\Delta\overline{S} = \frac{1}{2\sqrt{(1+\Delta C)^2-C^2}}\left[(S+\Delta S)\left(\frac{1+\mathcal{A}^{\rho\pi}_{\rm CP}}{1-\mathcal{A}^{\rho\pi}_{\rm CP}}\right)^{1/2} - (S-\Delta S)\left(\frac{1-\mathcal{A}^{\rho\pi}_{\rm CP}}{1+\mathcal{A}^{\rho\pi}_{\rm CP}}\right)^{1/2}\right]. \tag{14}$$

Note that $\overline{S}$ and $\Delta\overline{S}$ are CP violating and CP conserving, respectively, in complete analogy to S, ΔS. They are free of r_t, and their dependence on other hadronic parameters is given by

$$\begin{aligned}\overline{S} = \frac{1}{\sqrt{}}\{&[\sin2\alpha-(r_+\cos\delta_+ + r_-\cos\delta_-)\sin(\alpha-\beta) \\ &- r_+r_-\sin2\beta\cos(\delta_+-\delta_-)]\cos\delta_t \\ &- [(r_+\sin\delta_+ - r_-\sin\delta_-)\sin(\alpha-\beta) \\ &+ r_+r_-\sin2\beta\sin(\delta_+-\delta_-)]\sin\delta_t\},\end{aligned} \tag{15}$$

$$\begin{aligned}\Delta\overline{S} = \frac{1}{\sqrt{}}\{&[\cos2\alpha-(r_+\cos\delta_+ + r_-\cos\delta_-)\cos(\alpha-\beta) \\ &+ r_+r_-\cos2\beta\cos(\delta_+-\delta_-)]\sin\delta_t \\ &+ [(r_+\sin\delta_+ - r_-\sin\delta_-)\cos(\alpha-\beta) \\ &- r_+r_-\cos2\beta\sin(\delta_+-\delta_-)]\cos\delta_t\},\end{aligned} \tag{16}$$

where

$$\sqrt{} \equiv \sqrt{[1 - 2r_+ \cos(\beta + \alpha + \delta_+) + r_+^2][1 - 2r_- \cos(\beta + \alpha + \delta_-) + r_-^2]}. \quad (17)$$

In our discussion in Sec. VII of determining α in a broken SU(3) analysis we will use this most economical parametrization of time-dependent measurements in $B^0(t) \to \rho^\pm \pi^\mp$, given by the four measurables, $A_{CP}^\pm$, $\overline{S}$ and $\Delta\overline{S}$ in Eqs. (12), (15), and (16) in terms of the six parameters, $r_\pm$, $\delta_\pm$, δ_t, and α.

III. THE USE OF $\alpha_{\rm eff}$

We follow the simpler case of $B^0(t) \to \pi^+\pi^-$, where a contribution of a penguin amplitude modifies the value of α to $\alpha_{\rm eff} = \frac{1}{2}\arg[e^{-2i\beta}A(\overline{B}^0 \to \pi^+\pi^-)A^*(B^0 \to \pi^+\pi^-)]$, measured from the two coefficients of the $\sin\Delta mt$ and $\cos\Delta mt$ terms [22]. The isospin analysis [1] provides a way of determining the shift $\alpha_{\rm eff} - \alpha$. In $B \to \rho^\pm\pi^\mp$ we now define two corresponding quantities [8,14,15],

$$\alpha_{\rm eff}^\pm \equiv \frac{1}{2}\arg(e^{-2i\beta}\overline{A}_\pm A_\pm^*). \quad (18)$$

Note that these phases do not occur in the time-dependent rates (5) and are unmeasurable in $B \to \rho^\pm\pi^\mp$ alone. Instead, the observables $S \pm \Delta S$ (6) involve two other related phases which can be measured in these decays,

$$2\alpha_{\rm eff}^\pm \pm \hat{\delta} \equiv \arg(e^{-2i\beta}\overline{A}_\pm A_\mp^*) = \arcsin\left(\frac{S \mp \Delta S}{\sqrt{1-(C \mp \Delta C)^2}}\right), \quad (19)$$

where

$$\hat{\delta} \equiv \arg(A_-^* A_+), \quad (20)$$

is an unknown relative phase between the two decay amplitudes. Consequently, although $\alpha_{\rm eff}^+$ and $\alpha_{\rm eff}^-$ cannot be measured separately, their algebraic average is measurable. We therefore define:

$$\alpha_{\rm eff} \equiv \frac{1}{2}(\alpha_{\rm eff}^+ + \alpha_{\rm eff}^-) = \frac{1}{4}\left[\arcsin\left(\frac{S+\Delta S}{\sqrt{1-(C+\Delta C)^2}}\right) + \arcsin\left(\frac{S-\Delta S}{\sqrt{1-(C-\Delta C)^2}}\right)\right]. \quad (21)$$

The two shifts $\alpha_{\rm eff}^\pm - \alpha$ are expected to increase with the magnitudes of the corresponding penguin amplitudes, $|p_\pm|$. A relation between $\alpha_{\rm eff}^\pm - \alpha, |p_\pm|, \gamma$ and corresponding charge averaged rates and CP asymmetries is readily obtained using Eqs. (2) (a similar relation in the different t-convention was shown to hold in $B \to \pi^+\pi^-$ [23]),

$$4|p_\pm|^2\sin^2\gamma = (|A_\pm|^2 + |\overline{A}_\pm|^2)\left[1 - \sqrt{1-(\mathcal{A}_{CP}^\pm)^2}\cos 2(\alpha_{\rm eff}^\pm - \alpha)\right]. \quad (22)$$

The left-hand side of (22) can be bounded using flavor SU(3) as shown in Sec. V. This implies lower bounds on $\cos 2(\alpha_{\rm eff}^\pm - \alpha)$ or upper bounds on $|\alpha_{\rm eff}^\pm - \alpha|$ (see Eqs. (36) and (39) below). Using (21) will then provide an upper bound on $|\alpha_{\rm eff} - \alpha|$.

IV. ASSUMING FACTORIZATION OF TREE AMPLITUDES

Since the number of parameters in $B^0(t) \to \rho^\pm\pi^\mp$ exceeds the number of measurables by two, a certain input is required in order to determine α from these measurements. This input is provided by an assumption that the two tree amplitudes $t_\pm$ factorize and that their relative strong phase vanishes in this approximation. Given that factorization was shown to hold to leading order in $1/m_b$ and $\alpha_s(m_b)$ in a heavy quark QCD expansion [24,25], we will proceed under this assumption. Thus, neglecting for a moment a ratio of two form factors contributing to t_- and t_+ [19], we take $r_t \equiv |t_-|/|t_+|$ to be given by the ratio of corresponding decay constants,

$$r_t \simeq \frac{f_\pi}{f_\rho} = 0.63, \quad (23)$$

where $f_\pi = 130.7$ MeV, $f_\rho = 208$ MeV. We note that a value $r_t = 0.68$ was obtained in a global SU(3) fit to all $B \to VP$ decays [17], supporting both factorization of tree amplitudes and the assumption that $B \to \pi$ and $B \to \rho$ form factors do not differ much from one another. The absolute value of $|t_+|$ obtained in the fit of Ref. [17] agrees with $|t_+/t| \simeq f_\rho/f_\pi$, where t, the tree amplitude in $B^0 \to \pi^+\pi^-$, is obtained from a global SU(3) fit to B decays to two charmless pseudoscalars [26]. This also supports factorization of tree amplitudes and an assumption that the B to π form factor varies only slightly with q^2.

Factorization of tree amplitudes also implies that to a good approximation $\delta_t \approx 0$. A very small phase $\delta_t = (1 \pm 3)°$ supporting this assumption was calculated in [19]. (Somewhat larger values around $-20°$ were obtained in the global fit [17].) Taking r_t to be given by (23) and assuming $\delta_t \approx 0$ reduces by two the number of parameters describing $B \to \rho^\pm\pi^\mp$, to become equal to the number of observables. Although this situation seems perfectly suitable for a direct determination of α, we wish to point out its limitation.

As noted above, the four observables $\mathcal{A}_{CP}^\pm$, $\overline{S}$ and $\Delta\overline{S}$ depend on six parameters, $r_\pm$, $\delta_\pm$, δ_t and α, one of which is assumed here to vanish approximately, $\delta_t \approx 0$. The overall CP asymmetry $\mathcal{A}_{\rm CP}^{\rho\pi}$, given explicitly in (11), provides a fifth measurable, depending also on r_t, which is assumed to be given by (23). While in principle this permits a determination of α, this can be seen to rely on terms quadratic in $r_{\pm}$. Expanding Eqs. (11) and (12) up to terms linear in $r_{\pm}$, we find

$$\mathcal{A}_{\rm CP}^{\rho\pi} = -\mathcal{A}_{\rm CP}^{+} + r_t^2 \mathcal{A}_{\rm CP}^{-} + \mathcal{O}(r_{\pm}^2). \qquad (24)$$

That is, at this order the three observables are not independent when r_t is given. As we will show in Sec. VI, one expects $r_{\pm}$ to be small, $r_{\pm} \sim 0.2$, implying that a determination of α using these assumptions will be very difficult. One may turn things around, however, by using the linear relation (24) to determine r_t and thereby test factorization.

V. CONSTRAINTS FROM FLAVOR SU(3)

Another way of adding an input into the analysis of $B \to \rho^{\pm}\pi^{\mp}$ is provided by assuming flavor SU(3), as we show now. In order to improve the precision of our analysis, we introduce SU(3) breaking corrections in tree amplitudes. These amplitudes, which can be shown to factorize to leading order in $1/m_b$ and $\alpha_s(m_b)$ [24,25], will be assumed to involve SU(3) breaking factors given by ratios of meson decay constants. Penguin amplitudes, for which factorization is not expected to hold [25,27], will be assumed by default to obey exact SU(3). The effects of SU(3) breaking in penguin amplitudes will be discussed further in Sec. VII.

Strangeness changing amplitudes describing $B \to K^*\pi$ and $B \to \rho K$ will be denoted by primed quantities. The SU(3) counterparts of t_+, t_- and $p_{\pm}$, (2), are given by [12,17]

$$t'_+ = \frac{f_{K^*}}{f_\rho}\frac{V_{ub}^* V_{us}}{V_{ub}^* V_{ud}} t_+ = \frac{f_{K^*}}{f_\rho}\bar{\lambda} t_+,$$
$$t'_- = \frac{f_K}{f_\pi}\frac{V_{ub}^* V_{us}}{V_{ub}^* V_{ud}} t_- = \frac{f_K}{f_\pi}\bar{\lambda} t_-,$$
$$p'_{\pm} = \frac{V_{cb}^* V_{cs}}{V_{cb}^* V_{cd}} p_{\pm} = -\bar{\lambda}^{-1} p_{\pm}, \qquad (25)$$

where

$$\bar{\lambda} \equiv \frac{\lambda}{1-\lambda^2/2} = 0.230, \quad \frac{f_{K^*}}{f_\rho} = 1.04, \quad \frac{f_K}{f_\pi} = 1.22. \qquad (26)$$

SU(3) amplitudes represented by exchange and annihilation contributions (contributing to $\Delta S = 0$ and $\Delta S = 1$ decays, respectively) are $1/m_b$ suppressed relative to tree and penguin amplitudes [25] and will be neglected. We also neglect very small color-suppressed electroweak penguin contributions. These approximations and the SU(3) breaking factors in (25) can be tested in $B^0 \to K^{*+}K^-$, $B^+ \to K^{*+}\bar{K}^0$ and in other $B \to VP$ decays [17]. Other tests, relating CP asymmetries in $B \to \rho^{\pm}\pi^{\mp}$ and in strangeness changing decays, will be discussed in Sec. VII. Under these assumptions one finds the following expressions for strangeness changing decay amplitudes [12,17]

$$A(B^+ \to K^{*0}\pi^+) = -\bar{\lambda}^{-1} p_+,$$
$$A(B^+ \to \rho^+ K^0) = -\bar{\lambda}^{-1} p_-, \qquad (27)$$

and

$$A(B^0 \to K^{*+}\pi^-) = \frac{f_{K^*}}{f_\rho}\bar{\lambda} t_+ e^{i\gamma} - \bar{\lambda}^{-1} p_+,$$
$$A(B^0 \to \rho^- K^+) = \frac{f_K}{f_\pi}\bar{\lambda} t_- e^{i\gamma} - \bar{\lambda}^{-1} p_-. \qquad (28)$$

Denoting charge averaged decay rates by $\bar{\Gamma}(B \to f) \equiv [\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})]/2$, we now define the following ratios of charge averaged rates,

$$\mathcal{R}_+^0 \equiv \frac{\bar{\lambda}^2 \bar{\Gamma}(B^0 \to K^{*+}\pi^-)}{\bar{\Gamma}(B^0 \to \rho^+\pi^-)},$$
$$\mathcal{R}_+^+ \equiv \frac{\bar{\lambda}^2 \bar{\Gamma}(B^+ \to K^{*0}\pi^+)}{\bar{\Gamma}(B^0 \to \rho^+\pi^-)}, \qquad (29)$$

$$\mathcal{R}_-^0 \equiv \frac{\bar{\lambda}^2 \bar{\Gamma}(B^0 \to \rho^- K^+)}{\bar{\Gamma}(B^0 \to \rho^-\pi^+)},$$
$$\mathcal{R}_-^+ \equiv \frac{\bar{\lambda}^2 \bar{\Gamma}(B^+ \to \rho^+ K^0)}{\bar{\Gamma}(B^0 \to \rho^-\pi^+)}, \qquad (30)$$

where superscripts and subscripts denote the charges of the B and ρ mesons. Using Eqs. (1)–(3), (27), and (28), the following expressions are obtained in terms of the hadronic parameters $r_{\pm}$ and $\delta_{\pm}$ and the weak phase $\beta + \alpha$:

$$\mathcal{R}_{\pm}^0 = \frac{r_{\pm}^2 + 2r_{\pm}\bar{\lambda}_{\pm}^2 z_{\pm} + \bar{\lambda}_{\pm}^4}{1 - 2r_{\pm}z_{\pm} + r_{\pm}^2},$$
$$\mathcal{R}_{\pm}^{\pm} = \frac{r_{\pm}^2}{1 - 2r_{\pm}z_{\pm} + r_{\pm}^2}, \qquad (31)$$

where

$$z_{\pm} \equiv \cos\delta_{\pm}\cos(\beta+\alpha), \qquad \bar{\lambda}_+ \equiv \sqrt{\frac{f_{K^*}}{f_\rho}}\bar{\lambda} = 0.235,$$
$$\bar{\lambda}_- \equiv \sqrt{\frac{f_K}{f_\pi}}\bar{\lambda} = 0.254. \qquad (32)$$

Each of these four measurables provides an additional constraint on appropriate parameters. [CP asymmetries in $B^0 \to K^{*+}\pi^-$ and $B^0 \to \rho^- K^+$ do not provide additional information but can be used to test SU(3); see Eqs. (60)

and (61) below.] This leads to an overconstrained system from which α can be determined. That is, the six observables, given in Eqs. (12), (15), and (16) and in one pair of Eqs. (31), can be used to solve for the six unknowns, $r_\pm, \delta_\pm, \delta_t$ and α, as discussed in more detail in Sec. VII. In the present section we study bounds on $r_\pm$ and on $\alpha_{\rm eff} - \alpha$ which follow from the four observables $\mathcal{R}_\pm^{0,+}$.

Each of the four expressions (31) may be inverted to write $r_\pm$ in terms of $z_\pm$ and a corresponding ratio of rates,

$$r_\pm = \frac{\sqrt{(\mathcal{R}_\pm^0 + \bar\lambda_\pm^2)^2 z_\pm^2 + (1-\mathcal{R}_\pm^0)(\mathcal{R}_\pm^0 - \bar\lambda_\pm^4)} - (\mathcal{R}_\pm^0 + \bar\lambda_\pm^2) z_\pm}{1-\mathcal{R}_\pm^0} = \frac{\sqrt{\mathcal{R}_\pm^{+2} z_\pm^2 + (1-\mathcal{R}_\pm^+)\mathcal{R}_\pm^+} - \mathcal{R}_\pm^+ z_\pm}{1-\mathcal{R}_\pm^+}. \tag{33}$$

The four expressions are monotonically decreasing functions of $z_\pm$ having their minima and maxima at $z_\pm = 1$ and $z_\pm = -1$, respectively,

$$\frac{\sqrt{\mathcal{R}_\pm^0} - \bar\lambda_\pm^2}{1+\sqrt{\mathcal{R}_\pm^0}} \le r_\pm \le \frac{\sqrt{\mathcal{R}_\pm^0} + \bar\lambda_\pm^2}{1-\sqrt{\mathcal{R}_\pm^0}}, \tag{34}$$

$$\frac{\sqrt{\mathcal{R}_\pm^+}}{1+\sqrt{\mathcal{R}_\pm^+}} \le r_\pm \le \frac{\sqrt{\mathcal{R}_\pm^+}}{1-\sqrt{\mathcal{R}_\pm^+}}. \tag{35}$$

Using current constraints on γ [15], $38^\circ \le \gamma \le 80^\circ$ (at 95% confidence level), the lowest and highest allowed value of $z_\pm$ are -0.79 and 0.79, respectively. This determines slightly smaller ranges of $r_\pm$ than given by (34) and (35) in terms of measured values of $\mathcal{R}_\pm^{0,+}$.

We note that one may use ratios of separate rates for B or $\overline{B}$ mesons instead of the ratios of charge averaged rates defined in (29) and (30). The above considerations and the bounds on $r_\pm$ apply almost equally to these ratios. Instead of factors $z_\pm \equiv -\cos\delta_\pm \cos\gamma$ one now has factors $\cos(\delta_\pm - \gamma)$ or $\cos(\delta_\pm + \gamma)$, which are constrained to lie in a range between -1 to 1. These then imply bounds of the form (34) and (35). For given measurements of rates and asymmetries, as specified in the next section, one may then compare the three types of ranges obtained for $r_\pm$ and choose the most restrictive ones.

The four strangeness changing processes (27) and (28), which are expected to be dominated by penguin amplitudes, can also be used to set an upper bound on $|\alpha_{\rm eff} - \alpha|$. For the two charged B decays one has

$$\cos 2(\alpha_{\rm eff}^\pm - \alpha) = \frac{1 - 2\mathcal{R}_\pm^+ \sin^2(\beta+\alpha)}{\sqrt{1-\mathcal{A}_{CP}^{\pm 2}}} \ge \frac{1-2\mathcal{R}_\pm^+}{\sqrt{1-\mathcal{A}_{CP}^{\pm 2}}}. \tag{36}$$

For the two processes involving neutral B decays one finds

$$\frac{\bar\lambda^2 \overline{\Gamma}(B^0 \to K^{*+}\pi^-)}{|p_+|^2} = 1 + \bar\lambda_+^4 r_+^{-2} + 2\bar\lambda_+^2 r_+^{-1} z_+ \ge \sin^2\gamma, \tag{37}$$

$$\frac{\bar\lambda^2 \overline{\Gamma}(B^0 \to \rho^- K^+)}{|p_-|^2} = 1 + \bar\lambda_-^4 r_-^{-2} + 2\bar\lambda_-^2 r_-^{-1} z_- \ge \sin^2\gamma, \tag{38}$$

where the two inequalities follow simply from the identity and the inequality $1 + x^2 + 2x\cos\delta\cos\gamma = 1 - \cos^2\delta\cos^2\gamma + (x + \cos\delta\cos\gamma)^2 \ge \sin^2\gamma$. Combining these inequalities with (22), we find [14,15]

$$\cos 2(\alpha_{\rm eff}^\pm - \alpha) \ge \frac{1-2\mathcal{R}_\pm^0}{\sqrt{1-\mathcal{A}_{CP}^{\pm 2}}}. \tag{39}$$

Thus, measured branching ratios and asymmetries, appearing on the right-handside of (36) and (39) and listed in the next section, provide upper bounds on $|\alpha_{\rm eff}^\pm - \alpha|$ and, using (21), they imply upper bounds on $|\alpha_{\rm eff} - \alpha|$.

VI. CURRENT RATES, ASYMMETRIES AND BOUNDS ON $r_\pm$ AND $\alpha_{\rm eff} - \alpha$

The current measured branching ratios and asymmetries in $B^0 \to \rho^\pm\pi^\mp$ and in SU(3) related processes are summarized in Table I. For ratios of B^+ and B^0 decay rates we will use the lifetime ratio [39] $\tau(B^+)/\tau(B^0) = 1.077 \pm 0.013$. The *BABAR* [8,15,30] and BELLE [9] collaborations measured in $B^0(t) \to \rho^\pm\pi^\mp$ also the four quantities,

$$C = \begin{cases} 0.34 \pm 0.12 \\ 0.25 \pm 0.17 \\ 0.31 \pm 0.10 \end{cases} \qquad \Delta C = \begin{cases} 0.15 \pm 0.11 \\ 0.38 \pm 0.18 \\ 0.21 \pm 0.10 \end{cases}$$
$$S = \begin{cases} -0.10 \pm 0.15 \\ -0.28 \pm 0.25 \\ -0.15 \pm 0.13 \end{cases} \qquad \Delta S = \begin{cases} 0.22 \pm 0.15 \\ -0.30 \pm 0.26 \\ 0.08 \pm 0.23 (S = 1.7), \end{cases} \tag{40}$$

where the first values were obtained by *BABAR*, the second by BELLE, and the third are their averages. Statistical and systematic errors were added in quadrature, and a scaling factor $S = 1.7$ is used for the error on the averaged value of ΔS. Using these values and the definitions (13) and (14), one finds

$$\overline{S} = \begin{cases} -0.11 \pm 0.13 \\ -0.17 \pm 0.18 \end{cases} \qquad \Delta\overline{S} = \begin{cases} 0.21 \pm 0.14 \\ -0.19 \pm 0.19, \end{cases} \tag{41}$$

for *BABAR* and BELLE, respectively. In view of the

TABLE I. Experimental charge averaged branching ratios and CP asymmetries of selected $\Delta S = 0$ and $\Delta S = 1$ B meson decays. For each process, the first line gives the branching ratio in units of 10^{-6}, while the second line quotes the CP asymmetry. Note, that the averages for $\rho^+\pi^-$ and $\rho^-\pi^+$ final states were obtained using also the value for the summed branching ratio from CLEO and are thus not a simple average of *BABAR* and Belle columns.

	Mode	CLEO	*BABAR*	BELLE	Avg.
$B^0 \to$	$\rho^{\pm}\pi^{\mp}$	$27.6^{+8.4}_{-7.4} \pm 4.2$ [28]	$22.6 \pm 1.8 \pm 2.2$ [8]	$29.1^{+5.0}_{-4.9} \pm 4.0$ [29]	24.0 ± 2.5
		$\cdots$	$-0.088 \pm 0.049 \pm 0.013$ [30]	$-0.16 \pm 0.10 \pm 0.02$ [9]	-0.10 ± 0.05
	$\rho^+\pi^-$	$\cdots$	12.7 ± 2.0	19.5 ± 5.0	14.2 ± 1.9
		$\cdots$	-0.21 ± 0.12	-0.02 ± 0.16	-0.16 ± 0.09
	$\rho^-\pi^+$	$\cdots$	9.9 ± 1.8	9.6 ± 3.4	9.8 ± 1.5
		$\cdots$	-0.47 ± 0.15	-0.53 ± 0.30	-0.48 ± 0.14
$B^0 \to$	$K^{*+}\pi^-$	$16^{+6}_{-5} \pm 2$ [31]	11.9 ± 2.0 [32,33]	$14.8^{+4.6+1.5+2.4}_{-4.4-1.0-0.9}$ [34]	12.7 ± 1.8
		$0.26^{+0.33+0.10}_{-0.34-0.08}$ [35]	-0.03 ± 0.24 [a] [32,33]	$\cdots$	0.06 ± 0.20
	ρ^-K^+	$16.0^{+7.6}_{-6.4} \pm 2.8$ [28]	$8.6 \pm 1.4 \pm 1.0$ [33]	$15.1^{+3.4+1.4+2.0}_{-3.3-1.5-2.1}$ [34]	9.9 ± 1.9 [b]
		$\cdots$	$0.13^{+0.14}_{-0.17} \pm 0.04 \pm 0.13$ [33]	$0.22^{+0.22+0.06}_{-0.23-0.02}$ [34]	0.17 ± 0.15
$B^+ \to$	$K^{*0}\pi^+$	$7.6^{+3.5}_{-3.0} \pm 1.6$ [28]	$10.5 \pm 2.0 \pm 1.4$ [36]	$9.83 \pm 0.90^{+1.06}_{-1.24}$ [37]	9.8 ± 1.2
	ρ^+K^0	<48 [38]	$\cdots$	$\cdots$	<48

[a]The two measurements of the CP asymmetry entering this average have opposite signs, $\mathcal{A}_{\rm CP} = 0.23 \pm 0.18^{+0.09}_{-0.06}$ [32] and $\mathcal{A}_{\rm CP} = -0.25 \pm 0.17 \pm 0.02 \pm 0.02$ [33]. The combined error includes a scaling factor $S = 1.9$.
[b]The error includes a scaling factor $S = 1.2$.

difference between the values of ΔS measured by *BABAR* and BELLE, we will not only take their average in the discussion below but will also treat them separately.

Let us now consider SU(3) bounds on the penguin pollution parameters $r_{\pm}$ and $\alpha^{\pm}_{\rm eff} - \alpha$. Using the definitions (29) and (30) and taking branching ratios from Table I, one obtains the following values:

$$\mathcal{R}^0_+ = 0.048 \pm 0.010, \qquad \mathcal{R}^+_+ = 0.034 \pm 0.006, \qquad \mathcal{R}^0_- = 0.053 \pm 0.014, \tag{42}$$

but only an upper bound on $\mathcal{R}^+_-$. Applying (33) and assuming Gaussian distributions, these values lead to allowed ranges for $r_{\pm}$. The most stringent bounds on r_+ follow from $\mathcal{R}^+_+$, which gives at 90% confidence level:

$$0.14(0.16) \leq r_+ \leq 0.25\ (0.22). \tag{43}$$

Values in parentheses are obtained by using the central value of $\mathcal{R}^+_+$. The most stringent bounds on r_- are obtained by using the ratios $\Gamma(B^0 \to \rho^- K^+)/\Gamma(B^0 \to \rho^-\pi^+)$ and their charge conjugates, instead of relying on the ratio of the charge averaged rates. Using the branching ratios *and asymmetries* of Table I, one finds the following range at 90% confidence level,

$$0.14(0.21) \leq r_- \leq 0.34\ (0.29). \tag{44}$$

The bounds (43) and (44) are expected to be modified by additional SU(3) breaking effects which were not included in the analysis. In (25) we assumed exact SU(3) for penguin amplitudes. Somewhat smaller values of $r_{\pm}$ are obtained if SU(3) breaking enhances $p'_{\pm}$ relative to $p_{\pm}$, as it would, for instance, by assuming factorization for these amplitudes. The above bounds are somewhat wider than and, as expected, consistent with values obtained in a global SU(3) fit to *all $B \to VP$ decays* [17], $r_+ = 0.17 \pm 0.02$ and $r_- = 0.29 \pm 0.04$, obtained when $|p_+|$ and $|p_-|$ were not assumed to be equal. (Somewhat smaller values, $r_- = 0.25 \pm 0.03$, were obtained in the global fit when assuming $p_+ = -p_-$, as proposed in [40].) Values on the low side, $r_+ = 0.10^{+0.06}_{-0.04}$ and $r_- = 0.10^{+0.09}_{-0.05}$, were calculated in QCD factorization [19].

Assuming $\mathcal{A}^{\pm}_{\rm CP} \approx 0$, Eqs. (36) and (39) imply

$$\begin{aligned} |\sin(\alpha^+_{\rm eff} - \alpha)| &\leq \sqrt{\mathcal{R}^0_+}, \\ |\sin(\alpha^+_{\rm eff} - \alpha)| &\leq \sqrt{\mathcal{R}^+_+}|\sin(\beta + \alpha)|, \\ |\sin(\alpha^-_{\rm eff} - \alpha)| &\leq \sqrt{\mathcal{R}^0_-}. \end{aligned} \tag{45}$$

Note that in (39) $\mathcal{A}^-_{\rm CP}$ is 3.3σ away from zero; nonzero asymmetries would improve the above bounds. Currently we find the following upper limits at 90% confidence level,

$$\begin{aligned} \mathcal{R}^0_+ &\Rightarrow |\alpha^+_{\rm eff} - \alpha| \leq 14.2^\circ, \\ \mathcal{R}^+_+ &\Rightarrow |\alpha^+_{\rm eff} - \alpha| \leq 7.3^\circ - 11.7^\circ, \\ \mathcal{R}^0_- &\Rightarrow |\alpha^-_{\rm eff} - \alpha| \leq 15.4^\circ, \end{aligned} \tag{46}$$

where the central upper limit (obtained by using $38^\circ \leq \gamma \leq 80^\circ$) is shown to improve slightly as α increases in the range $78^\circ \leq \alpha \leq 122^\circ$ [15]. Using (21), the second and third upper limits imply

$$|\alpha_{\rm eff} - \alpha| \leq 11.3^\circ - 13.5^\circ, \tag{47}$$

where the bound is improved slightly as α becomes larger within the above range. A recent study of α in time-dependent asymmetries in $B^0 \to \pi^+\pi^-$ [13] favors the upper part of this range.

The upper bound (47), which may be improved by a few degrees through more precise measurements of branching ratios, including a first observation of $B^+ \to \rho^+ K^0$, is expected to be modified by SU(3) breaking. For instance, if SU(3) breaking enhances $p'_\pm$ relative to $p_\pm$ (as it would by assuming factorization for these amplitudes), then the upper bound becomes stronger. In this respect these upper limits may be considered conservative. In any event, even if SU(3) breaking suppresses $p'_\pm$ relative to $p_\pm$ by 20 or 30%, one expects the upper bounds to change by this amount. This result provides an important conclusion, implying that *in time-dependent decays* $B^0 \to \rho^\pm\pi^\mp$ α *may be measured through* $\alpha_{\rm eff}$ *with a precision of about* $\pm 15°$:

$$|\alpha_{\rm eff} - \alpha| \leq 15° [\text{including SU(3) breaking}]. \qquad (48)$$

This accuracy is comparable to that of measuring α through $\alpha_{\rm eff}$ in time-dependent $B^0 \to \rho^+\rho^-$ decays [41]. Here the shift caused by the penguin amplitude is constrained in the isospin analysis [1,42,43] by measured branching ratios of $B^0 \to \rho^+\rho^-$, $B^+ \to \rho^+\rho^0$, and by an upper bound on $B^0 \to \rho^0\rho^0$ [44] to a range, $|\alpha_{\rm eff} - \alpha| \leq 17°$, at 90% confidence level.

Using Eq. (21), the current data (40) may now be translated into solutions for $\alpha_{\rm eff}$. In order to reduce discrete ambiguities caused by the few branches of the two arcsin functions in (21), we note that

$$(2\alpha_{\rm eff}^+ + \hat{\delta}) - (2\alpha_{\rm eff}^- - \hat{\delta}) = 2\delta_t + \mathcal{O}(r_\pm). \qquad (49)$$

We will make a conservative assumption that the two angles on the left-hand side differ by much less than 180°, which is equivalent to assuming that δ_t is much smaller than 90°. (Note that QCD factorization predicts a very small value, $\delta_t = (1 \pm 3)°$ [19], while a global SU(3) fit finds $\delta_t \simeq -20°$ [17].) This mild assumption can be checked experimentally by measuring the phase difference $\arg(A_-/A_+)$ using the overlap of the ρ^+ and ρ^- resonance bands in the $B^0 \to \pi^+\pi^-\pi^0$ Dalitz plot [30]. (This measurement is expected to be feasible much before a complete isospin and Dalitz plot analysis [6] can be performed.) The measurable phase difference $\arg(A_-/A_+)$ is dominated by $\delta_t = \arg(t_-/t_+)$. The difference, $|\arg(A_-/A_+) - \delta_t|$, is governed by the subdominant amplitudes $p_\pm$. We have checked that this difference is less then 25° at 90% confidence level, when r_+ is in the range (43), consistent with (33) and (42) and when r_- is in the range (44), consistent with (33) where R^0_- is replaced by $\Gamma(B^0 \to \rho^- K^+)/\Gamma(B^0 \to \rho^-\pi^+)$. Therefore, a small measured value of $\arg(A_-/A_+)$ would imply that δ_t is much smaller than 90°, confirming our assumption.

Applying (21) separately to the *BABAR* and BELLE measurements, (which differ in their ΔS values by 2σ) and to their averages, we find that in all three cases the experimental errors in $C, \Delta C, S$, and ΔS translate into quite small errors in $\alpha_{\rm eff}$, $\pm 5°$, $\pm 13°$ and $\pm 4°$ respectively. In each case one finds for the central values of these measurements only two solutions in the range $0 \leq \alpha_{\rm eff} \leq \pi$:

$$\alpha_{\rm eff} = \begin{cases} \textit{BABAR}\text{: } 93°, 177° \\ \text{BELLE: } 102°, 168° \\ \text{Average: } 94°, 175°. \end{cases} \qquad (50)$$

Excluding by $\alpha + \beta < \pi$ the three values near 180°, corresponding to an ambiguity $\alpha \to 3\pi/2 - \alpha$, we find

$$\alpha = \begin{cases} (93 \pm 5 \pm 15)° \ \textit{BABAR} \\ (102 \pm 13 \pm 15)° \ \text{BELLE} \\ (94 \pm 4 \pm 15)° \ \text{Average,} \end{cases} \qquad (51)$$

where the first error is experimental and the second is theoretical, coming from the bound (48). Note the weak dependence of α on ΔS, the central values of which have opposite signs in the *BABAR* and BELLE measurements (40). Combining for simplicity the experimental and theoretical errors (51) in quadrature gives for the average

$$\alpha = (94 \pm 16)°. \qquad (52)$$

All the above results are in good agreement with the range $78° \leq \alpha \leq 122°$ obtained from other CKM constraints [15]. For comparison, we note that the world average CP asymmetries measured in $B^0 \to \pi^+\pi^-$ [45,46] have been recently studied in Ref. [13] and were shown to imply a comparable range, $\alpha = (103 \pm 17)°$, favoring large values of the weak phase in this range.

VII. EXTRACTING α

As mentioned, the observables in $B \to \rho^\pm\pi^\mp$ and the SU(3) related rates are sufficient for determining α. One has four independent observables $\bar{S}$, $\Delta\bar{S}$, $\mathcal{A}_{CP}^\pm$ in time-dependent decays $B \to \rho^\pm\pi^\mp$, and additional four independent observables $\mathcal{R}^0_\pm$, $\mathcal{R}^\pm_\pm$ in $\Delta S = 1$ decays (where currently only an upper bound on $\mathcal{R}^\pm_-$ exists). These eight observables provide an overdetermined set of conditions, as they depend on only six parameters, the hadronic parameters $r_\pm$, $\delta_\pm$, δ_t, and the weak angle α. The set of Eqs. (12), (15), (16), and (31) then allows for an extraction of α as well as all the hadronic parameters. No assumption is required about δ_t, thus relaxing the mild and experimentally testable assumption made in the previous section in order to obtain the unambiguous ranges (51).

A solution of Eqs. (12), (15), (16), and (31) under these general conditions is shown for illustration in Fig. 2, which plots confidence levels (CL) as functions of α for different levels of statistics. To obtain the plots we generated data for the observables $\bar{S}$, $\Delta\bar{S}$, $\mathcal{A}_{CP}^\pm$, $\mathcal{R}^0_\pm$ and $\mathcal{R}^\pm_\pm$,

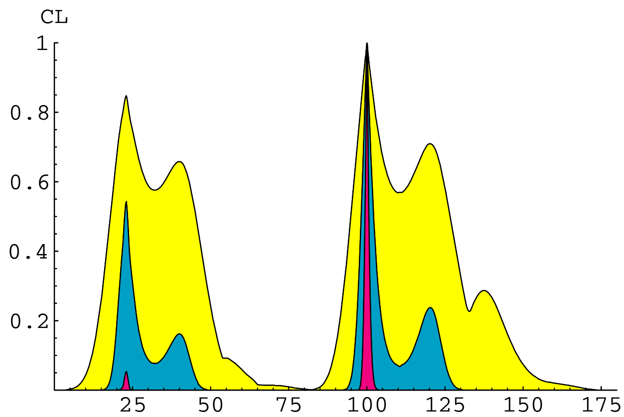

FIG. 2 (color online). Confidence level (CL) as a function of α for a generated set of data using a choice of parameters, $r_+ = 0.18$, $r_- = 0.23$, $\delta_+ = 30°$, $\delta_- = -55°$, $\delta_t = 170°$ and $\alpha_{\rm input} = 100°$. The interpretation of a confidence level is the same as in Ref. [15] when Gaussian errors are assumed. Errors used for χ^2 are the currently measured ones [yellow (light gray) region], those anticipated with 10 times statistics [cyan (gray)], and hundred times statistics [purple (dark gray)]. We assume an experimental error in $\mathcal{R}^{+}_{-}$ as in $\mathcal{R}^{0}_{-}$.

using a particular choice of values for the parameters $\delta_{\pm}$, $r_{\pm}$, δ_t (as specified in the figure caption) and an input value $\alpha = 100°$. The errors on the observables were taken to be the currently measured ones, apart from an error on $\mathcal{R}^{+}_{-}$, for which the error was taken to be the same as the current error on $\mathcal{R}^{0}_{-}$. Improvements in confidence level are shown for ten and hundred times more data than available today. One sees that with enough statistics only one solution at $\alpha = 100°$ survives in the range $0° \leq \alpha \leq 180°$. That is, for this particular choice of parameters 100 times the present statistics implies an uncertainty of $\pm 2°$ in the single value of α extracted at 95% CL. We checked that the situation presented in Fig. 2 is generic and applies to a large range of hadronic parameters.

The ambiguities in α, which are eventually resolved with high enough statistics, are seen in Fig. 2 to imply a large range of allowed values of α at current statistics. To get a quick insight into the origin of these ambiguities, let us first explore the case of $r_{\pm} = 0$. In this (oversimplified) case, involving merely mixing induced CP violation, the only observables carrying information on α are $\overline{S}$ and $\Delta\overline{S}$,

$$\overline{S} = \sin 2\alpha \cos\delta_t, \qquad \Delta\overline{S} = \cos 2\alpha \sin\delta_t. \tag{53}$$

Assuming that $\overline{S}$ and $\Delta\overline{S}$ are measured precisely, a solution for $\sin 2\alpha$ is given by

$$(\sin 2\alpha)^2 = \frac{1}{2}\Big\{1 + \overline{S}^2 - (\Delta\overline{S})^2 \pm \sqrt{[1 + \overline{S}^2 - (\Delta\overline{S})^2]^2 - 4\overline{S}^2}\Big\}. \tag{54}$$

The two signs in front of the square root of the discriminant correspond to the following map

$$P_t = \left\{\alpha \to \frac{\pi}{4} - \frac{\delta_t}{2}, \delta_t \to \frac{\pi}{2} - 2\alpha\right\}, \tag{55}$$

or equivalently to the following interchange in Eqs. (53):

$$\sin 2\alpha \leftrightarrow \cos\delta_t, \qquad \cos 2\alpha \leftrightarrow \sin\delta_t. \tag{56}$$

The other discrete ambiguities of (53) are

$$P_\pi = \{\alpha \to \alpha + \pi\}, \tag{57}$$

$$P_{\pi/2} = \left\{\alpha \to \frac{\pi}{2} - \alpha, \delta_t \to -\delta_t\right\}, \tag{58}$$

$$P_- = \{\alpha \to -\alpha, \delta_t \to \pi - \delta_t\}. \tag{59}$$

While none of these transformations relate to one another two values of α in the allowed range, $78° \leq \alpha \leq 122°$ [15], combinations of these transformations, such as $P_\pi P_{\pi/2} = \{\alpha \to 3\pi/2 - \alpha, \ldots\}$, are relevant to this range. However, this ambiguity, as well as the others, is resolved once higher order terms in $r_{\pm}$ are taken into account, as can be seen in Fig. 2. (See Appendix A for details.) Namely, the complete set of equations, (12), (15), (16), and (31) is not invariant under these transformations, which are violated by terms of order $r_{\pm}$. Note that although the ratios $\mathcal{R}^{0,+}_{\pm}$ are formally of order $r^2_{\pm}$ (because of the multiplicative factor $\bar{\lambda}^2$ in their definitions), they are in fact zeroth order. That is, they must be measured to an accuracy of order $r_{\pm}$ in order to resolve the ambiguities. It turns out that at least one pair of $(R^0_{\pm}, R^+_{\pm})$ ought to be measured to this precision. (See Appendix A.)

Since the extraction of α relies on a given scheme of broken flavor SU(3), one may wonder how SU(3) breaking effects other than those included may affect the value of α. Flavor SU(3) is used to fix the values of $r_{\pm}$, which we have shown to be small, $r_{\pm} \sim 0.2$. The extracted values of α are given to zeroth order in $r_{\pm}$ by (54). Terms of order $r_{\pm}$ are affected by SU(3) breaking corrections, which by themselves are approximately $r_{\pm}$. Therefore the overall SU(3) breaking effect in α is expected to be of order $r^2_{\pm}$.

This is demonstrated in the following way. We first generate values for observables by randomly varying δ_t, $\delta_{\pm}$ and α, and taking $r_{\pm}$ to vary in the allowed ranges (43) and (44). In addition to SU(3) breaking in tree amplitudes we now allow also for the SU(3) breaking in penguin amplitudes by introducing positive parameters, $0.7 < c_{\pm} < 1.3$, and writing $p'_{\pm} = -c_{\pm}\bar{\lambda}^{-1}p_{\pm}$ instead of (25). [We neglect SU(3) breaking in $\delta_{\pm}$ which is nonleading in the sense that the term $2r_{\pm}\bar{\lambda}^2_{\pm}z_{\pm}$ in the numerator of the first of Eq. (31) is smaller than the $r^2_{\pm}$ term.] Generating data under this assumption, we then extract α using Eqs. (12), (15), (16), and (31), where SU(3) was assumed to be present only in tree amplitudes. Running a Monte Carlo program for 10 000 different configurations shows that the local minimum in χ^2 shifts by only $\sqrt{\langle(\alpha^{\rm out} - \alpha^{\rm in})^2\rangle} \sim 2°$, which is indeed of order $r_\pm^2$.

There exist experimental tests of flavor SU(3) and SU(3) breaking corrections, in terms of equalities between CP rate differences in SU(3) related processes. Two of these relations follow from Eqs. (2) and (28) [17],

$$\Gamma(B^0 \to \rho^+\pi^-) - \Gamma(\overline{B}^0 \to \rho^-\pi^+) = \frac{f_\rho}{f_{K^*}}[\Gamma(\overline{B}^0 \to K^{*-}\pi^+) - \Gamma(B^0 \to K^{*+}\pi^-)], \tag{60}$$

$$\Gamma(B^0 \to \rho^-\pi^+) - \Gamma(\overline{B}^0 \to \rho^+\pi^-) = \frac{f_\pi}{f_K}[\Gamma(\overline{B}^0 \to \rho^+K^-) - \Gamma(B^0 \to \rho^-K^+)]. \tag{61}$$

These relations test the equality of products $|t_\pm^{(\prime)}||p_\pm^{(\prime)}|\sin\gamma\sin\delta_\pm$ in $\Delta S = 0$ and $\Delta S = 1$ decays. Using current values in Table I, one finds that, while the signs of the two asymmetries in the second equality confirm the SU(3) prediction, their absolute values differ by 2.0σ.

VIII. CONCLUSION

We have studied implications for α of time-dependent rate measurements in $B \to \rho^\pm\pi^\mp$ by proposing a parametrization which depends on a minimal number of hadronic parameters and observables. We have proposed one method based on factorization, which reduces by two the number of parameters to the number of observables. The limitation of this method was shown to be its sensitivity to terms quadratic in $r_\pm$, the two ratios of penguin and tree amplitudes.

Assuming broken flavor SU(3), which relates $B \to \rho^\pm\pi^\mp$ to four processes of the form $B^{0,+} \to K^*\pi$ and $B^{0,+} \to \rho K$, and using branching ratios measured for these processes, we calculated lower and upper bounds on $r_\pm$, slightly below and slightly above 0.2. Defining a measurable quantity $\alpha_{\rm eff}$, that becomes α in the limit of vanishing penguin amplitudes, we calculated upper bounds on $|\alpha_{\rm eff} - \alpha|$ in a range 11°–13°, which are expected to be at most about 15° when including unaccounted SU(3) breaking effects.

In order to resolve a discrete ambiguity in α, we assumed that the relative strong phase of two tree amplitudes, δ_t, is considerably smaller than 90°. This assumption, justified by QCD factorization and by a global SU(3) fit to $B \to VP$ decays, can be tested directly through a partial Dalitz plot analysis of $B \to \pi^+\pi^-\pi^0$. Using the *BABAR* and BELLE results for $B(t) \to \rho^\pm\pi^\mp$ this then implies single solutions, $\alpha = (93 \pm 16)°$ and $(102 \pm 20)°$, respectively, and an average $\alpha = (94 \pm 16)°$, taking into account an error scaling factor.

Finally, using a complete set of measurables, including CP asymmetries and avoiding any assumption about δ_t, we presented numerical studies demonstrating the feasibility of determining α and the reduction of discrete ambiguities with statistics. We have also shown that SU(3) breaking effects, which were not already included, are expected to be very small, of order $r_\pm^2$.

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APPENDIX: FURTHER DETAILS ON DISCRETE AMBIGUITIES

Discrete ambiguities were found in Sec. VII when order $r_\pm$ corrections to observables were neglected. This lead to a 16-fold ambiguity on α in the range $\alpha \in [0, 2\pi)$ spanned by the transformations P_t, P_π, $P_{\pi/2}$ and P_- given in (55) and (57)–(59). Let us now discuss how the higher order terms in $r_\pm$ affect the ambiguities. First of all, the (unphysical) transformation P_π is an exact symmetry of Eqs. (12), (15), (16), and (31), if extended to a transformation on strong phases

$$P_\pi = \{\alpha \to \alpha + \pi, \delta_\pm \to \delta_\pm + \pi, \delta_t \to \delta_t\}. \tag{A1}$$

The other symmetry transformations $P_i = \{P_t, P_{\pi/2}, P_-\}$ receive higher order corrections. To see under which conditions they remain ambiguities, let us expand Eqs. (12), (15), (16), and (31) to first order in $r_\pm$, where we count $\delta_t \sim r_\pm \sim \bar{\lambda} \ll 1$,

$$\begin{aligned}\overline{S} - \sin2\alpha = {} & \sin2\alpha[r_+\cos(\beta + \alpha + \delta_+) \\ & + r_-\cos(\beta + \alpha + \delta_-)] - \sin(\alpha - \beta) \\ & \times[r_+\cos\delta_+ + r_-\cos\delta_-],\end{aligned} \tag{A2}$$

$$\Delta\overline{S} = \cos2\alpha\sin\delta_t + (r_+\sin\delta_+ - r_-\sin\delta_-)\cos(\alpha - \beta), \tag{A3}$$

$$\mathcal{A}_{\rm CP}^\pm = -2r_\pm\sin\delta_\pm\sin(\beta + \alpha), \tag{A4}$$

$$\frac{1}{2}(\mathcal{R}_\pm^0 - \mathcal{R}_\pm^\pm) = r_\pm\bar{\lambda}_\pm^2\cos(\delta_\pm)\cos(\beta + \alpha). \tag{A5}$$

The parameters $r_\pm$ are obtained from

$$\sqrt{\mathcal{R}_\pm^0} = \sqrt{\mathcal{R}_\pm^\pm} = r_\pm. \tag{A6}$$

One is then left with six Eqs. (A2)–(A5) for four unknowns, $\delta_\pm$, δ_t, and α. In order to check for leftover ambiguities, let us assume that there exists at least one solution for (A2)–(A5), which we denote by $\delta_\pm^0$, δ_t^0, and α^0. The transformations P_i may give us another viable solution for α,

$$\alpha^{\rm new} = P_i\alpha^0 + \delta\alpha, \tag{A7}$$

together with new values for the other paremeters, $\delta_{\pm}^{\rm new}$, $\delta_t^{\rm new}$. From (A4) we get (to leading order in the small parameters, $r_{\pm}$, $\delta_t^{\rm new}$ and $\delta\alpha$),

$$\sin\delta_{\pm}^{\rm new} = -\frac{\mathcal{A}_{\rm CP}^{\pm}}{2r_{\pm}\sin(\beta + P_i\alpha^0)}, \tag{A8}$$

while (A5) gives

$$\cos\delta_{\pm}^{\rm new} = \frac{1}{2}\frac{(\mathcal{R}_{\pm}^0 - \mathcal{R}_{\pm}^{\pm})}{r_{\pm}\bar{\lambda}_{\pm}^2\cos(\beta + P_i\alpha^0)}. \tag{A9}$$

In general Eqs. (A8) and (A9) are not simultaneously satisfied which resolves the ambiguity.

In case that $\mathcal{R}_{\pm}^{0,+}$ is not measured to order $r_{\pm}$, i.e., to a precision of about 20°, the ambiguity is retained if the right-hand side of (A8) is not larger in magnitudes than one, leading to a solution for (A2) and (A3):

$$\delta_t^{\rm new} = \frac{1}{\cos(2P_i\alpha^0)}[\Delta\bar{S} - (r_+\sin\delta_+^{\rm new} - r_-\sin\delta_-^{\rm new}) \times\cos(P_i\alpha^0 - \beta)], \tag{A10}$$

$$\begin{aligned}\delta\alpha = \frac{1}{2\cos(2P_i\alpha^0)}\{&\bar{S} - \sin2\alpha^0 + \sin(P_i\alpha^0 - \beta)\\ &\times[r_+\cos\delta_+^{\rm new} + r_-\cos\delta_-^{\rm new}]\\ &- \sin2\alpha^0[r_+\cos(\beta + P_i\alpha^0 + \delta_+^{\rm new})\\ &+ r_-\cos(\beta + P_i\alpha^0 + \delta_-^{\rm new})]\}.\end{aligned} \tag{A11}$$

These expressions show the existence of further ambiguities in $\delta\alpha$ of order $r_{\pm}$ caused by twofold solutions for $\delta_{\pm}^{\rm new}$ in (A8) or (A9). [Note that $\delta_t^{\rm new}$ and $\delta\alpha$ are $O(r_{\pm})$ in accordance with our expansion.] This shows the importance of measuring $\mathcal{R}_{\pm}^{0,+}$ as precisely as possible.

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