B meson wave function in k_T factorization

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We study the asymptotic behavior of the B meson wave function in the framework of k_T factorization theorem. We first construct a definition of the k_T-dependent B meson wave function, which is free of light-cone divergences. Next-to-leading-order corrections are then calculated based on this definition. The treatment of different types of logarithms in the above corrections, including the Sudakov logarithms, and those depending on a renormalization scale and on an infrared regulator, is summarized. The criticism raised in the literature on our resummation formalism and Sudakov effect is responded. We show that the B meson wave function remains normalizable after taking into account renormalization-group evolution effects, contrary to the observation derived in the collinear factorization theorem.

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I. INTRODUCTION

The B meson distribution amplitude $\phi_+(k^+)$ plays an essential role in a perturbative analysis of exclusive B meson decays based on collinear factorization theorem [1–5], where k^+ is the momentum carried by the light spectator quark. Its behavior certainly matters, and has been investigated in various approaches recently. Models of ϕ_+ with an exponential tail in the large k^+ region has been proposed [6]. Neglecting three-parton distribution amplitudes in a study by means of equations of motion [7,8], ϕ_+ was found to be proportional to a step function with a sharp drop at large k^+ [9]. The asymptotic behavior of ϕ_+ was also extracted from an renormalization-group (RG) evolution equation derived in the framework of collinear factorization theorem, which exhibits a decrease slower than $1/k^+$ [10]. That is, the B meson distribution amplitude is not normalizable. This striking feature has been confirmed in a QCD sum rule analysis [11], which includes next-to-leading-order (NLO) perturbative corrections. For a summary of the above progress, refer to [12]. A similar divergent normalization of the B meson distribution function involved in inclusive decays has been observed recently [13].

A non-normalizable B meson distribution amplitude does not cause a problem in practice [14]. In a leading-order collinear factorization formula, only the first inverse moment $\lambda_B^{-1}(\mu) \equiv \int dk^+ \phi_+(k^+)/k^+$ is relevant [15,16], which is a convergent quantity. The factor $1/k^+$ comes from a hard kernel of a decay mode. Note that a hard kernel would not be as simple as $1/k^+$ at higher orders, and information of more moments is required. However, the non-normalizability does introduce an ambiguity in defining the B meson decay constant f_B. The ambiguity can be understood through the matrix element,

$$\begin{aligned}\langle 0|\bar{q}(y)W_y(n_-)^\dagger W_0(n_-)\Gamma \not{n}_- h(0)|\bar{B}(v)\rangle \\ = -\frac{iF(\mu)}{2}\tilde{\phi}_+(v\cdot y,\mu)\mathrm{tr}\left(\Gamma \not{n}_- \frac{1+\not{v}}{2}\gamma_5\right),\end{aligned} \tag{1}$$

where the coordinate of the antiquark field $\bar{q}$, $y = (0, y^-, \mathbf{0}_T)$, is parallel to the null vector $n_- = (0,1,\mathbf{0}_T)$, h the rescaled b quark field characterized by the B meson velocity v, μ the renormalization scale, and Γ represents a Dirac matrix. The factor $W_y(n_-)$ denotes the Wilson line operator,

$$W_y(n_-) = P\exp\left[-ig\int_0^\infty d\lambda n_-\cdot A(y+\lambda n_-)\right]. \tag{2}$$

The quantity $F(\mu)$ is the heavy quark effective theory matrix element corresponding to the asymptotic value of the product $f_B\sqrt{m_B}$ in the heavy-quark limit. If the normalization $\tilde{\phi}_+(v\cdot y = 0,\mu)$ is divergent, the definition of f_B demands a further arbitrary renormalization [17].

We shall show that the above undesirable feature of $\tilde{\phi}_+$ is a consequence of adopting the collinear factorization theorem. It has been known that the collinear factorization formulas of many exclusive B meson decays suffer end-point singularities [18]. We regard these singularities as an indication [19] that the k_T factorization theorem [20–25] is more appropriate for studying these decays than the collinear factorization theorem. The perturbative QCD (PQCD) approach [26–29] based on the k_T factorization theorem has been developed. Retaining the parton transverse momenta k_T [30], the end-point singularities disappear [31], and the resultant predictions are in agreement with most of experimental data [32]. Viewing these merits, it is very tempting to reanalyze the RG evolution effect on the B meson wave function (or the unintegrated B meson distribution amplitude) in the k_T factorization theorem. Our conclusion is that the evolution effect does not drive the asymptotic behavior of the B meson wave function into $1/k^+$. Therefore, the B meson wave function is normalizable, and the B meson decay constant is well defined.

In Sec. II we find out a legitimate definition of the k_T-dependent B meson wave function. The NLO corrections to the B meson wave function are computed and compared to those in [10] in Sec. III. We respond to the criticism raised in [33,34] on the PQCD formalism and on the Sudakov effect in Sec. IV. Sec. V is the conclusion.

II. DEFINITIONS OF A WAVE FUNCTION

We first construct the definition of the B meson wave function in the k_T factorization theorem, which is nontrivial. Hence, our formalism for the NLO calculation differs from those in the literature [35,36], which also involve the parton transverse degrees of freedom. A gauge-invariant definition of the B meson wave functions $\tilde{\Phi}_+(v \cdot y, b, \mu)$ is given via the nonlocal matrix element [25],

$$\langle 0|\bar{q}(y)W_y(n_-)^\dagger I_{n_-;y,0}W_0(n_-)\not{n}_-\Gamma h(0)|\bar{B}(v)\rangle, \qquad (3)$$

as a naive extension of Eq. (1) with $y = (0, y^-, \mathbf{b})$. The two Wilson lines $W_y(n_-)$ and $W_0(n_-)$ must be connected by a link $I_{n_-;y,0}$, at infinity in this case [25,37].

As pointed out in [38], Eq. (3) contains additional collinear divergences from the region with a loop momentum parallel to n_-. These light-cone divergences, cancelling each other as $b \to 0$, that is, as $\tilde{\Phi}_+(v \cdot y, b, \mu) \to \tilde{\phi}_+(v \cdot y, \mu)$, do not cause a problem in the collinear factorization theorem. In the k_T factorization theorem, however, they must be subtracted in a gauge-invariant way. Two modified definitions have been proposed in [38]:

$$\langle 0|\bar{q}(y)W_y(u)^\dagger I_{u;y,0}W_0(u) \quad \not{n}_-\Gamma h(0)|\bar{B}(v)\rangle, \qquad (4)$$

$$\frac{\langle 0|\bar{q}(y)W_y(n_-)^\dagger I_{n_-;y,0}W_0(n_-)\not{n}_-\Gamma h(0)|\bar{B}(v)\rangle}{\langle 0|W_y(n_-)^\dagger W_y(u')I_{n_-;y,0}I^\dagger_{u';y,0}W_0(n_-)W_0(u')^\dagger|0\rangle}. \qquad (5)$$

In Eq. (4) a non-lightlike vector u has been substituted for the null vector n_-, so that no collinear divergence is associated with the Wilson lines. In Eq. (5) n_- is maintained, but the light-cone divergences are regularized by the denominator, which contains the same light-cone divergences as in the numerator. As a gluon travels along n_-, it does not resolve the detail of the valence quarks, which can then be replaced by the Wilson lines in an arbitrary direction u' [39,40]. Both the above modifications with the appropriate Wilson links are gauge-invariant. Nevertheless, the universality of the B meson wave function is broken due to the appearance of the auxiliary scale, for example, $\zeta = (k \cdot u)/\sqrt{u^2}$ from Eq. (4). Fortunately, the evolution in ζ, the so-called Sudakov evolution [38], can be derived using the k_T-resummation technique [39,41], such that the initial condition of the evolution remains universal. Note that Eqs. (4) and (5) do not approach the B meson distribution amplitude directly in the limit $b \to 0$ for general u and u', but convolutions of hard kernels with the B meson distribution amplitude [38,42].

We have investigated the $O(\alpha_s)$ diagrams in Fig. 1 according to both modifications, and found that Eq. (4) would change the ultraviolet structure of the quark-Wilson-line vertex correction in Eq. (3). This problem can be explained using the pole term obtained from Fig. 1(c) [see Eq. (A16) in the Appendix],

$$N_c^{(1)} \approx \frac{\alpha_s C_F}{4\pi}\Gamma(\epsilon)\left[2 - \left(\frac{4\zeta^2}{m_g^2}\right)^{-\epsilon}\right], \qquad (6)$$

m_g being an infrared regulator. If taking the $u \to n_-$, i.e., $\zeta \to \infty$ limit in the above expression before making the expansion in ϵ, only the first term contributes to the ultraviolet pole, which is $2/\epsilon$ in unit of $\alpha_s C_F/(4\pi)$, the same as in Eq. (3). If expanding the factor $(4\zeta^2/m_g^2)^{-\epsilon}$ first (note that this expansion makes sense for $u^2 \neq 0$, i.e., $\zeta \not\to \infty$), the second term also contributes, and changes the ultraviolet pole into $1/\epsilon$. Consequently, Eq. (4) is governed by a RG evolution different from that of Eq. (3). In this work we shall adopt Eq. (5), and demonstrate that the freedom in choosing the vector u' allows a correct RG evolution of the B meson wave function.

III. $O(\alpha_S)$ CORRECTIONS

The lowest-order evolution kernel for $\Phi_+(k^+, b, \mu)$ is written as

$$K^{(0)}(k^+, k'^+, b, \mu) = \delta(k^+ - k'^+), \qquad (7)$$

which implies that the light spectator quark, carrying only a longitudinal momentum, is initially on shell. It acquires the transverse degrees of freedom through collinear gluon exchanges, before participating a hard scattering [25]. As indicated in Eq. (7), we perform k_T

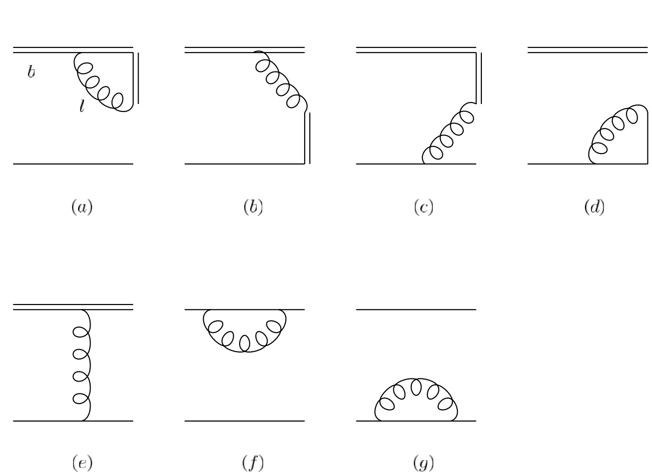

FIG. 1. $O(\alpha_s)$ diagrams for the B meson wave function.

factorization in the conjugate b space. We then calculate the $O(\alpha_s)$ corrections to Eq. (7) in dimensional regularization. A gluon mass m_g is introduced to regularize the infrared divergences, so that we can clearly distinguish the ultraviolet poles $1/\epsilon$ from the infrared divergences represented by $\ln m_g$. As suggested in [38], a small plus component is added to the null vector n_- in Eq. (5) at the intermediate step of the calculation. That is, we start with the Wilson line in a non-lightlike direction u for the numerator, and take the $u \to n_-$ limit eventually. Figs. 1(a)–1(g) contribute to the numerator of Eq. (5), and Figs. 1(a)–1(d) with the quark lines being replaced by the Wilson lines along u' contribute to the denominator.

To highlight the difference between the collinear and k_T factorizations, we present the loop integrals associated with Figs. 1(a) and 1(b) and with Figs. 1(c) and 1(d),

$$\begin{aligned} N_a^{(1)} + N_b^{(1)} = & -ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \\ & \times \frac{u \cdot v}{(v \cdot l + i\epsilon)(l^2 - m_g^2 + i\epsilon)(u \cdot l + i\epsilon)} \\ & \times [\delta(k^+ - k'^+) - \delta(k^+ - k'^+ + l^+) \\ & \times \exp(-i\mathbf{l}_T \cdot \mathbf{b})], \end{aligned} \tag{8}$$

$$\begin{aligned} N_c^{(1)} + N_d^{(1)} = & \frac{i}{4} g^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} tr \\ & \times \left[\gamma_\nu \frac{\not{k}' - \not{l}}{(k' - l)^2 + i\epsilon} \gamma_5 \not{n}_- \not{n}_+ \gamma_5 \right] \\ & \times \frac{1}{l^2 - m_g^2 + i\epsilon} \frac{u^\nu}{u \cdot l + i\epsilon} \times [\delta(k^+ - k'^+) \\ & - \delta(k^+ - k'^+ + l^+) \exp(-i\mathbf{l}_T \cdot \mathbf{b})], \end{aligned} \tag{9}$$

respectively, where the arbitrary Dirac matrix Γ has been set to γ_5. In the above integrals we have made explicit the $i\epsilon$ prescription in the propagators $1/v \cdot l$ and $1/u \cdot l$, which follows the eikonal approximation of the quark or gluon propagators the loop momentum l flows through [23]. Note the additional Fourier factor $\exp(-i\mathbf{l}_T \cdot \mathbf{b})$ associated with Figs. 1(b) and 1(d) [23,41]. In the collinear factorization theorem this Fourier factor disappears, corresponding to the $b \to 0$ limit. Moreover, for $u = n_-$, we obtain, from Eqs. (8) and (9), the counterterm identical to that found in the collinear factorization theorem [10],

$$-\frac{\alpha_s C_F}{4\pi} \frac{2}{\epsilon} \left[\frac{k^+}{k'^+} \frac{\theta(k'^+ - k^+)}{(k'^+ - k^+)_+} + \frac{\theta(k^+ - k'^+)}{(k^+ - k'^+)_+} \right]. \tag{10}$$

The resultant anomalous dimension contributes to the splitting kernel in the RG evolution equation, that determines the asymptotic behavior of the B meson distribution amplitude [10]. The ultraviolet pole $1/\epsilon$ arises from the integration over the transverse loop momentum l_T up to infinity. It is then expected that Eq. (10) will be absent in the k_T factorization theorem due to the suppression in the large l_T region from $\exp(-i\mathbf{l}_T \cdot \mathbf{b})$.

The $O(\alpha_s)$ corrections $N_i^{(1)}(k^+, k'^+, b, \mu)$ from Fig. 1 (i), $i = a \cdots e$, to the numerator of Eq. (5) are summarized below:

$$N_a^{(1)} = -\frac{\alpha_s C_F}{4\pi} \ln(2\nu^2) \left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{m_g^2 e^{\gamma_E}} \right) \delta(k^+ - k'^+), \tag{11}$$

$$\begin{aligned} N_b^{(1)} = & -\frac{\alpha_s C_F}{4\pi} \left\{ \ln(2\nu^2) \ln\frac{m_g^2 b^2 e^{2\gamma_E}}{4} \delta(k^+ - k'^+) \right. \\ & + 4 \frac{\theta(k'^+ - k^+)}{(k'^+ - k^+)_+} K_0(2\nu(k'^+ - k^+)b) \\ & \left. - 4 \frac{\theta(k^+ - k'^+)}{(k^+ - k'^+)_+} K_0(\sqrt{2}(k^+ - k'^+)b) \right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} N_c^{(1)} = & \frac{\alpha_s C_F}{4\pi} \left[\frac{1}{\epsilon} - 2\ln^2\frac{2\nu k^+}{m_g} + 2\ln\frac{2\nu k^+}{m_g} + \ln\frac{4\pi\mu^2}{m_g^2 e^{\gamma_E}} \right. \\ & \left. - \frac{5}{6}\pi^2 \right] \delta(k^+ - k'^+), \end{aligned} \tag{13}$$

$$\begin{aligned} N_d^{(1)} = & -\frac{\alpha_s C_F}{4\pi} \left(\left[\ln\frac{2\nu^2 k^{+2} b e^{\gamma_E}}{m_g} \ln\frac{m_g^2 b^2 e^{2\gamma_E}}{4} + \frac{\pi^2}{3} \right] \delta(k^+ \right. \\ & - k'^+) - 4 \frac{k^+ \theta(k'^+ - k^+)}{k'^+ (k'^+ - k^+)_+} \left\{ K_0\left(\sqrt{k^+/k'^+} m_g b \right) \right. \\ & \left. \left. - K_0[2\nu(k'^+ - k^+)b] \right\} \right), \end{aligned} \tag{14}$$

$$\begin{aligned} N_e^{(1)} = & \frac{\alpha_s C_F}{4\pi} \left(\left[\ln\frac{k^{+2} b e^{\gamma_E}}{m_g} \ln\frac{m_g^2 b^2 e^{2\gamma_E}}{4} + \frac{\pi^2}{3} \right] \delta(k^+ \right. \\ & - k'^+) - 4 \frac{k^+ \theta(k'^+ - k^+)}{k'^+ (k'^+ - k^+)_+} \left\{ K_0\left(\sqrt{k^+/k'^+} m_g b \right) \right. \\ & \left. \left. - K_0[\sqrt{2}(k'^+ - k^+)b] \right\} \right). \end{aligned} \tag{15}$$

For their detailed derivation, refer to the Appendix. The auxiliary parameter $\nu = (n_+ \cdot u)/\sqrt{u^2}$, defined via $\zeta \equiv \nu k^+$, denotes the u dependence, γ_E is the Euler constant, and the subscript "+" in the factor $1/(k'^+ - k^+)_+$ represents the "plus" distribution. The self-energy corrections to the heavy-quark field h and to the light spectator quark $\bar{q}$ are

$$N_f^{(1)} = \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{m_g^2 e^{\gamma_E}}\right)\delta(k^+ - k'^+), \tag{16}$$

$$N_g^{(1)} = -\frac{\alpha_s C_F}{4\pi}\left(\frac{1}{2\epsilon} + \frac{1}{2}\ln\frac{4\pi\mu^2}{m_g^2 e^{\gamma_E}} - \frac{1}{4}\right)\delta(k^+ - k'^+). \tag{17}$$

Some remarks are in order. The logarithms $\ln(2\nu^2)$ denote the light-cone collinear divergences mentioned before. Equation (11) does not contain a double pole $1/\epsilon^2$ observed in [10] due to the replacement of the null vector n_- by the non-lightlike vector u. For a similar reason, the single-pole term $\ln(\mu/k^+)/\epsilon$, which leads to the type of Sudakov logarithms in the collinear factorization theorem [10], does not exist. As expected, Eq. (12), with the suppression from the Fourier factor, does not generate the ultraviolet pole in Eq. (10). Because of the Bessel function K_0, the splitting effect from the plus distribution is negligible in the asymptotic region with large k^+. Equation (13) produces the double logarithm $\ln^2(k^+/m_g)$, which is not yet in the form of Sudakov logarithms in the k_T factorization theorem. After combining Eqs. (13) and (14), we derive the standard k_T-dependent infrared-finite Sudakov logarithms,

$$-\frac{\alpha_s C_F}{2\pi}[\ln^2(k^+b) - (1 - 2\gamma_E)\ln(k^+b)], \tag{18}$$

with the first (second) term being leading (next-to-leading). The ν dependence is not made explicit, since it will be cancelled by that from the denominator. These logarithms should be resummed to all orders using the technique in [26]. The double infrared logarithm $\ln^2 m_g$ in Eq. (15) is new, which does not exist in radiative corrections to a light meson process [35]. The important splitting effects, proportional to $K_0(\sqrt{k^+/k'^+}m_g b) \approx \ln(m_g b)$, cancel between Eqs. (14) and (15).

The $O(\alpha_s)$ corrections to the denominator are computed in a similar way, but with $\delta(k^+ - k'^+)$ being substituted for $\delta(k^+ - k'^+ + l^+)$. This substitution is made in that the denominator is to remove the light-cone divergences arising from $l^+ \to 0$. Hence, the splitting terms, i.e., the plus distributions in Eqs. (11)–(15), disappear. We choose $u' = v$ for the incoming Wilson line (along the b quark), and a different u' for the outgoing Wilson line, such that the ultraviolet structure of the quark-Wilson-line vertex correction the same as in Eq. (3) is recovered. We emphasize that other choices of u' are equivalent, in view that the resultant B meson wave functions all collect the same soft structure of an exclusive decay. The expressions are summarized below:

$$D_a^{(1)} = N_a^{(1)}, \tag{19}$$

$$D_b^{(1)} = -\frac{\alpha_s C_F}{4\pi}\ln(2\nu^2)\ln\frac{m_g^2 b^2 e^{2\gamma_E}}{4}\delta(k^+ - k'^+), \tag{20}$$

$$D_c^{(1)} = -\frac{\alpha_s C_F}{4\pi}\ln(4\nu^2\nu'^2)\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu^2}{m_g^2 e^{\gamma_E}}\right)\delta(k^+ - k'^+), \tag{21}$$

$$D_d^{(1)} = -\frac{\alpha_s C_F}{4\pi}\ln(4\nu^2\nu'^2)\ln\frac{m_g^2 b^2 e^{2\gamma_E}}{4}\delta(k^+ - k'^+), \tag{22}$$

with the auxiliary parameter $\nu' = (u' \cdot n_-)/\sqrt{u'^2}$. It is easy to check that the sum of the above corrections is free of the infrared cutoff m_g. That is, the denominator in Eq. (5) does not alter the soft structure of the numerator, i.e., of Eq. (3), as requested above. According to our prescription, we set $\ln(4\nu'^2)$ to unity.

The total one-loop correction $K^{(1)}$ to Eq. (7) is then written as,

$$\begin{aligned} K^{(1)} &= \sum_{j=a}^{g} N_j^{(1)} - \sum_{j=a}^{d} D_j^{(1)}, \\ &= \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{5}{2} + \ln\nu^2\right)\left[\frac{1}{\epsilon} + \ln(\pi e^{\gamma_E}\mu^2 b^2)\right]\right. \\ &\quad -2\ln^2(\nu k^+ b) + 2(1 - 2\gamma_E)\ln(\nu k^+ b) \\ &\quad -(5 - 2\gamma_E)\ln\left(\frac{m_g b}{2}\right) + 2\ln\frac{k^{+2}b}{m_g}\ln\frac{m_g b e^{\gamma_E}}{2} \\ &\quad \left. + \frac{1}{4} - \frac{5}{6}\pi^2 - 3\gamma_E\right\}. \end{aligned} \tag{23}$$

The ultraviolet pole $5/(2\epsilon)$ in unit of $\alpha_s C_F/(4\pi)$ is the same as the corresponding one derived in Eq. (8) of [10] under our prescription for fixing u'. Note that it differs from $3/\epsilon$ in [26], since it is the b quark field, instead of the rescaled one, that was adopted to define the B meson wave function in [26]. The pole $5/(2\epsilon)$ should be partitioned in the way that $3/(2\epsilon)$ contributes to the factor $F(\mu)$ in Eq. (1) and $1/\epsilon$ to $\Phi_+(k^+, b, \mu)$. Here we do not perform such a partition. The splitting terms, which either vanish in the asymptotic region with large k^+ or cancel between Eqs. (14) and (15), have been dropped.

The treatment of each term in Eq. (23) is explained as follows. The ultraviolet pole together with the constants are subtracted in a renormalization procedure. The logarithm $\ln(\mu b)$ is then summed to all orders using a standard RG evolution equation [10], giving an exponential $R(b, \mu, \nu)$. The Sudakov logarithms $\alpha_s \ln^2(\nu k^+ b)$ and $\alpha_s \ln(\nu k^+ b)$ are resummed, leading to the Sudakov factor $S(k^+, b, \nu)$ [26]. The evolution of the B meson wave function from Eq. (5) is then given by

$$\begin{aligned} \Phi_+(k^+, b, \mu) &= S(k^+, b, \nu)\phi_+(k^+, b, \mu), \\ \phi_+(k^+, b, \mu) &= R(b, \mu, \nu)\phi_+(k^+, b, \mu = 1/b). \end{aligned} \tag{24}$$

The logarithms $\ln(m_g b)$ and $\ln(k^{+2}b/m_g)\ln(m_g b)$, representing the soft structure of the B meson wave function, are absorbed into the initial condition $\phi_+(k^+, b, \mu = 1/b)$ of the above evolution. They are then used to subtract the infrared divergences in the evaluation of hard kernels, i.e., in the so-called "matching" procedure.

The exponentials in Eq. (24) are quoted from [26,39] as

$$S(k^+, b, \nu) = \exp\left\{-\int_{1/b}^{k^+} \frac{d\bar{\mu}}{\bar{\mu}}\left[\ln\frac{k^+}{\bar{\mu}} A(\alpha_s(\bar{\mu})) + B(\nu, \alpha_s(\bar{\mu}))\right]\right\}, \tag{25}$$

$$R(b, \mu, \nu) = \exp\left[-\int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu}))\right], \tag{26}$$

with the one-loop anomalous dimensions,

$$A = \frac{\alpha_s}{\pi} C_F, \tag{27}$$

$$B = \frac{\alpha_s}{2\pi} C_F \ln(\nu^2 e^{2\gamma_E - 1}), \tag{28}$$

$$\gamma = -\frac{\alpha_s}{4\pi} C_F(5 + 2\ln\nu^2), \tag{29}$$

where the running of the coupling constant α_s has been taken into account. It can be confirmed trivially that the exponent of Eq. (25) is identical to the Sudakov logarithms in Eq. (23), if the running of α_s is frozen. In a practical analysis, the scale μ is set to a hard scale, which is usually of order k^+. The ν-dependences then cancel between $S(k^+, b, \nu)$ and $R(b, \mu = k^+, \nu)$, such that the B meson wave function $\Phi_+(k^+, b, \mu = k^+)$ does not depend ν. This cancellation is equivalent to that of the light-cone divergences, in agreement with the speculation in [38]. After the above cancellation, the Sudakov exponent in Eq. (25) reduces to the logarithms in Eq. (18), if neglecting the running of α_s, and the anomalous dimension γ is equal to -5 in unit of $\alpha_s/(4\pi)$. We then bring the Wilson line direction u back to the null vector n_- as stated before.

At last, we discuss the normalization of the B meson wave function $\Phi_+(k^+, b, \mu)$ in the k_T factorization theorem, which is defined as

$$\int_0^\infty dk^+ \lim_{b\to 1/k^+} \Phi_+(k^+, b, \mu) = \int_0^\infty dk^+ \phi_+(k^+, b = 1/k^+, \mu). \tag{30}$$

The Sudakov factor in Eq. (24) becomes identity in the limit $b \to 1/k^+$, which approaches zero in the heavy-quark limit for a fixed momentum fraction $x \equiv k^+/(m_B v^+)$. In the above limit the splitting terms proportional to the Bessel function K_0 remain finite, and will not contribute to the evolution kernel. It is then obvious from Eq. (24) that the normalizability of the B meson distribution amplitude is not spoiled by the RG evolution effect, when evaluated according to Eq. (30).

IV. RESPONSE TO THE CRITICISM

After completing the NLO calculation for the B meson wave function in the k_T factorization theorem, we are ready to respond to the criticism raised by Descotes-Genon and Sachrajda [33] and by Lange and Neubert [34], which concerned the PQCD formalism and the Sudakov effect. We fully recognize that it is not easy to appreciate the delicacies of different approaches, and that misunderstandings are unavoidable. We hope that this section helps clarify these misunderstandings.

It was concluded that a heavy-to-light form factor is not calculable in practice due to the ignorance of the heavy meson wave functions [33]. The word "calculable" is perhaps confusing. It is more appropriate to use "factorizable," which means that a physical quantity can be written to all orders of α_s as a factorization formula containing a hard kernel (Wilson coefficient) and wave functions, plus jet functions, and Sudakov factors $\cdots$. Then a form factor is factorizable in the PQCD approach based on the k_T factorization theorem because of the absence of the end-point singularities. A form factor is not factorizable in QCD-improved factorization (QCDF) (considering only the leading contribution) [15], and partially factorizable in soft-collinear effective theory (SCET) [43], since both factorizable and nonfactorizable pieces exist at leading level. Therefore, a heavy meson wave function plays the role of an input in the PQCD approach the same as the form factor does in QCDF, and is determined by the value of the form factor from experimental data, lattice QCD, or sum rules. The inappropriate criticism in [33] is thus a conceptual misunderstanding of the PQCD approach.

A difference between the pion form factor and the $B \to \pi$ transition form factor was pointed out [33]: the former does not contain an end-point singularity in the collinear factorization theorem, but the latter does. Hence, it was questioned whether the PQCD approach, working for the former, can be extended to the latter. Our opinion is that both collinear and k_T factorization theorems are applicable to the pion form factor or to the decay $B \to \gamma l \nu$ [44], and the numerical outcomes are not very different, because there is no end-point singularity. However, the end-point singularity in the collinear factorization formula of the $B \to \pi$ form factor demands the use of the k_T factorization theorem, which is more conservative than the collinear one: the parton transverse momenta should not be treated as a pure higher-power effect, when the end-point region of a parton momentum fraction is important. This is the motivation to develop the PQCD approach, and it makes sense to compare its predictions for two-body nonleptonic B meson decays with experimental data.

It was stated that the b quark line a collinear gluon attaches could not be approximated by a Wilson line in the direction v, that is, it could not be replaced by the rescaled b quark line [33]. In fact, the approximation holds, no matter a soft or collinear gluon attaches the b quark, in that it does not change the soft or collinear divergence of a loop integral. This is exactly the idea to remove the light-cone divergences in the numerator of Eq. (5) by introducing the denominator, where a Wilson line is substituted for the b quark.

It has been argued that Fig. 1(e) does not contain the double logarithm $\ln^2(k \cdot v/\sqrt{v^2})$, and the invariant $(k \cdot v)^2/v^2$ is irrelevant in the Sudakov resummation [26]. This argument was doubted in [33]. The explicit $O(\alpha_s)$ result in Eq. (15) has clarified the issue: ζ is the only relevant invariant. It is well known that a double logarithm arises from a vertex correction [45], such as Fig. 1(c), instead of from the type of corrections like Fig. 1(e).

An expression equivalent to Eq. (25) was given by [33]

$$S(k^+, b, \nu) = \exp\left\{-\int_{C_1/b}^{C_2\nu k^+} \frac{d\bar{\mu}}{\bar{\mu}}\left[\ln\left(\frac{C_2\nu k^+}{\bar{\mu}}\right)A(C_1, \alpha_s(\bar{\mu})) + B(C_1, C_2, \alpha_s(\bar{\mu}))\right]\right\}, \tag{31}$$

where $A(C_1, \alpha_s)$ is equal to $A(\alpha_s)$ in Eq. (27) at one-loop level, and

$$B(C_1, C_2, \alpha_s) = \frac{\alpha_s}{2\pi} C_F \ln\left(\frac{e^{2\gamma_E - 1}C_1^2}{C_2^2}\right). \tag{32}$$

Our choice $C_1 = C_2 = 1$ [26,28], questioned in [33], is now justified, since it indeed reproduces the logarithms in Eq. (23). Other choices of C_1 and C_2 are equally fine: they lead to a change only in the next-to-leading logarithms, which can be compensated by the corresponding change in hard kernels. Hence, the choice of C_1 and C_2 is not a problem from the viewpoint of factorization theorem.

It was claimed that the wave function in Eq. (4) defined in terms of the non-lightlike vector u could not be related to the standard definition in terms of the null vector n_- [33]. As explained in Sec. II, the ν dependence can be grouped into the Sudakov factor, and the initial condition of the Sudakov evolution is identified as the standard definition. The validity of Eq. (24) in the large b region was also challenged, because it suffers a large $O[\alpha_s(1/b)]$ correction. The treatment of this correction, not multiplied by a logarithm, is a matter of factorization scheme: it corresponds to the constant terms in Eq. (23), and is allowed to shift freely between a wave function and a hard kernel. This shift is similar to that of the next-to-leading Sudakov logarithms resulted in by varying the parameters $C_{1,2}$ in Eq. (31). Therefore, it is always possible to choose a factorization scheme for a NLO evaluation of a hard kernel, such that Eq. (24) holds.

The explicit ν dependence of the Sudakov factor derived from Eq. (4) [26] was pointed out in [33]. It has been known that this ν dependence is cancelled by that of a soft function, which collects irreducible soft gluons to all orders. The cancellation has been demonstrated in the processes including Landshoff scattering [23], deep inelastic scattering [39], Drell-Yan production [39], inclusive semileptonic B meson decays [39], dijet production [46], and the $B \to D\pi$ decays [47]. We believe that such a cancellation is general, though having not yet explored all other processes, since a physical quantity should not depend on this artificial dependence.

There are two leading-twist B meson wave functions Φ_+ and Φ_- [6], the latter being defined by, for example, the matrix element in Eq. (5) with $\not{n}_-$ replaced by $\not{n}_+$, where $n_+ = (1, 0, \mathbf{0}_T)$ is another null vector. It was claimed [33] that the equality $\Phi_+ = \Phi_-$ was assumed in the PQCD approach [28]. We make clear that this equality was never postulated. Precisely speaking, the B meson wave function Φ_B adopted in [28] is identified as Φ_+ discussed here, and another wave function $\bar{\Phi}_B$, appearing as the combination $\Phi_+ - \Phi_-$, is numerically negligible as confirmed in [48]. The above combination was found to be important, when its contribution to a single term in the full expression of the form factor was investigated [33]. An observation based on such an incomplete analysis is certainly not solid. Because the wave function Φ_- does not appear in the leading PQCD formalism, it is not urgent to discuss the corresponding Sudakov resummation.

It was concluded [34] that the Sudakov effect is not important for the soft contribution to the $B \to \pi$ form factor. First, we emphasize that the Sudakov logarithms studied in SCET [34] differ from what we discussed here and adopted in the PQCD approach [28]: the former appear in the Wilson coefficient associated with the soft form factor in the collinear factorization theorem, while the latter come from the wave functions in the k_T factorization theorem. The difference manifests itself in the explicit expressions of the Sudakov factors: the latter is k_T-dependent, but the former is not. Second, there is no conflict between the conclusions in [28] and in [34]. The weak Sudakov suppression in [34] refers to that on the whole form factor. The strong suppression in PQCD applies only to the end-point region of a momentum fraction (a form factor is factorizable in PQCD), and the suppression away from the end point is weak. Note that the strong Sudakov effect has been confirmed in [33] (see page 271) for the model of the B meson wave function proposed in [28,49]. Speaking of the whole form factor, whose contribution mainly arises from the non-end-point region, the suppression studied in PQCD is not significant either. For a more detailed explanation on this issue, refer to [50].

V. CONCLUSION

In this paper we have surveyed the definitions of a wave function in the k_T factorization theorem given in Eqs. (3)–(5). The naive one in Eq. (3) contains additional light-cone collinear divergences, which cancel in a distribution amplitude in the collinear factorization theorem. These light-cone divergences are removed in the modified definitions of Eqs. (4) and (5) in a gauge-invariant way. However, Eq. (4), in which the Wilson line has been rotated away from the light-cone to an arbitrary direction u, alters the ultraviolet structure of Eq. (3). Certainly, this change is not a problem, similar to the fact that the ultraviolet structure of a heavy-light current is changed under the heavy quark effective theory approximation. All the definitions of the B meson wave function are equivalent, as long as they collect the same soft structure of an exclusive decay. We have found that it is possible to maintain the ultraviolet structure by adopting Eq. (5). The dependence on a general u or u' can be factored into the Sudakov factor, such that the wave function, as the initial condition of the Sudakov evolution, is gauge-invariant and universal [38]. Eventually, the u or u' dependence of the Sudakov factor will be cancelled by that of a soft function as making predictions for a physical quantity.

We have calculated the $O(\alpha_s)$ corrections from Figs. 1(a)–1(g) to the B meson wave function following the definition in Eq. (5), which contain three types of logarithms $\ln(k^+b)$, $\ln(\mu b)$ and $\ln(m_g b)$. The leading and next-to-leading infrared-finite Sudakov logarithms $\ln(k^+b)$ have been verified, which are consistent with the Sudakov exponent adopted in our previous works. It has been observed that Figs. 1(b) and 1(d) do not generate the ultraviolet poles from the integration over the transverse loop momenta l_T due to the suppression from the Fourier factor $\exp(-i\boldsymbol{l}_T\cdot\mathbf{b})$. Hence, the RG evolution from the summation of $\ln(\mu b)$ is trivial. We have explained that the small b limit should be taken as $b\to 1/k^+$ in the k_T factorization theorem, under which the Sudakov evolution factor becomes identity, and Figs. 1(b) and 1(d) and 1(e) remain ultraviolet finite. As a consequence, the RG evolution effect does not spoil the normalizability of the B meson wave function, when evaluated according to Eq. (30). This is another indication that the k_T factorization theorem is a more appropriate framework for studying exclusive B meson decays than the collinear factorization theorem. At last, the infrared logarithms $\ln(m_g b)$ are used to subtract the corresponding infrared divergences in the computation of hard kernels.

Our NLO calculation is similar to that performed in [35], where the conjugate b space was also introduced. However, it was the $\gamma^*\gamma\to\pi^0$ amplitude, instead of the pion wave function, that was studied in [35]. Therefore, the issues of the undesirable light-cone collinear divergences and of a legitimate definition of a k_T-dependent wave function were not addressed. The $O(\alpha_s)$ corrections to the $B\to\gamma l\nu$ decay amplitude were computed in a different way in [36]. First, the issues mentioned above were not addressed either. Second, a parton was assumed to carry a transverse momentum initially, and the conjugate b space was not introduced. Third, a different type of double logarithms $\ln^2(m_B/k^+)$ was resummed, leading to the so-called threshold resummation [51]. Our opinion for proceeding a NLO analysis in the k_T factorization theorem is that one must define a valid k_T-dependent wave function first (under a factorization scheme as stated in Sec. IV). Next, one computes the $O(\alpha_s)$ corrections to the full parton-level amplitude, from which the wave function also evaluated at $O(\alpha_s)$ is subtracted. This subtraction results in an infrared-finite hard kernel, which is then substituted into a k_T factorization formula to estimate the NLO effect. Therefore, our work provides a basis of the above systematic procedure.

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APPENDIX: DETAIL OF CALCULATION

We present the details of the $O(\alpha_s)$ calculation in this Appendix. We assume the small plus component added to n_- to be negative, $u^+<0$, for convenience. One can always work out a loop integral, whose result is a function of u^2, in the $u^2<0$ ($u^2>0$) region, and then analytically continue the result into the $u^2>0$ ($u^2<0$) region. Applying contour integration in the light-cone coordinates, we obtain the integral for the numerator of Eq. (5) associated with Fig. 1(a),

$$N_a^{(1)} = ig^2C_F\mu^{2\epsilon}\frac{2\pi i u\cdot v}{u^+v^- - u^-v^+}\int\frac{d^{2-2\epsilon}l_T}{(2\pi)^{4-2\epsilon}}\times\left\{\int_0^\infty dl^+\frac{u^+}{l^+[2u^-l^{+2}+u^+(l_T^2+m_g^2)]}+\int_{-\infty}^0 dl^+\frac{v^+}{l^+[2v^-l^{+2}+v^+(l_T^2+m_g^2)]}\right\}\times\delta(k^+-k'^+), \tag{A1}$$

where the first term corresponds to the pole $l^-=-u^-l^+/u^++i\epsilon$ for $l^+>0$, and the second term to the pole $l^-=-v^-l^+/v^+-i\epsilon$ for $l^+<0$. The integration over l_T leads to

$$N_a^{(1)} = -\frac{\alpha_s C_F}{2\pi}\frac{u\cdot v}{u^- v^+ - u^+ v^-}\left(\frac{4\pi\mu^2}{m_g^2}\right)^\epsilon \Gamma(\epsilon) \times \int_0^\infty \frac{dt}{t^{1-2\delta}}\left[\left(\frac{2v^-}{v^+}t^2+1\right)^{-\epsilon} - \left(\frac{2u^-}{u^+}t^2+1\right)^{-\epsilon}\right]\delta(k^+ - k'^+), \tag{A2}$$

where the variable change $l^+ = m_g t$ has been applied. It is easy to observe that the soft divergences in the above two terms cancel. Hence, the small parameter δ, introduced for convenience, will approach zero eventually. Working out the integration over t, we have

$$N_a^{(1)} = -\frac{\alpha_s C_F}{4\pi}\frac{u\cdot v}{\sqrt{(u\cdot v)^2 - u^2 v^2}}\left(\frac{4\pi\mu^2}{m_g^2}\right)^\epsilon \Gamma(\epsilon) \times B(\delta, \epsilon-\delta)\left[\left(\frac{v^+}{2v^-}\right)^\delta - \left(\frac{u^+}{2u^-}\right)^\delta\right]\delta(k^+ - k'^+), \tag{A3}$$

whose $\delta \to 0$ limit is given by

$$N_a^{(1)} = -\frac{\alpha_s C_F}{4\pi}\frac{u\cdot v}{\sqrt{(u\cdot v)^2 - u^2 v^2}}\left(\frac{4\pi\mu^2}{m_g^2}\right)^\epsilon \Gamma(\epsilon) \times \ln\frac{u^- v^+}{u^+ v^-}\delta(k^+ - k'^+). \tag{A4}$$

The above expression can be further simplified into Eq. (11) as $u^2 \to 0$.

Following the reasoning for Eq. (A1), the loop integral associated with Fig. 1(b) is written as

$$N_b^{(1)} = ig^2 C_F \frac{2\pi i u\cdot v}{u^- v^+ - u^+ v^-}\int \frac{d^2 l_T}{(2\pi)^4} \times\left\{\frac{u^+\theta(k'^+ - k^+)\exp(-i\mathbf{l}_T\cdot \mathbf{b})}{(k'^+ - k^+)[2u^-(k'^+ - k^+)^2 + u^+(l_T^2 + m_g^2)]} - \frac{v^+\theta(k^+ - k'^+)\exp(-i\mathbf{l}_T\cdot \mathbf{b})}{(k^+ - k'^+)[2v^-(k^+ - k'^+)^2 + v^+(l_T^2 + m_g^2)]}\right\}, \tag{A5}$$

where the θ-functions $\theta(k'^+ - k^+)$ and $\theta(k^+ - k'^+)$ correspond to the integration ranges $l^+ > 0$ and $l^+ < 0$, respectively. The integration over l_T gives

$$N_b^{(1)} = -\frac{\alpha_s C_F}{\pi}\frac{u\cdot v}{\sqrt{(u\cdot v)^2 - u^2 v^2}} \times\left\{\frac{\theta(k'^+ - k^+)}{k'^+ - k^+}K_0\left(\sqrt{\frac{2u^-}{u^+}(k'^+ - k^+)^2 + m_g^2}\,b\right) - \frac{\theta(k^+ - k'^+)}{k^+ - k'^+}K_0\left(\sqrt{\frac{2v^-}{v^+}(k^+ - k'^+)^2 + m_g^2}\,b\right)\right\}. \tag{A6}$$

We split the above expression into

$$N_b^{(1)}(k^+, k'^+, b, \mu) = \delta(k^+ - k'^+)\int_0^\infty dy N_b^{(1)}(k^+, y, b, \mu) + N_{b+}^{(1)}(k^+, k'^+, b, \mu), \tag{A7}$$

where the first term can be combined with $N_a^{(1)}$, and the second term defines the plus distribution. The first term is rewritten, by applying the variable changes $y = (t+1)k^+$ for the first Bessel function and $y = (1-t)k^+$ for the second Bessel function, as

$$\int_0^\infty dy N_b^{(1)}(k^+, y, b, \mu) = -\frac{\alpha_s C_F}{\pi}\frac{u\cdot v}{\sqrt{(u\cdot v)^2 - u^2 v^2}} \times\left\{\int_0^\infty \frac{dt}{t^{1-2\delta}}K_0\left(\sqrt{4\zeta^2 t^2 + m_g^2}\,b\right) - \int_0^1 \frac{dt}{t^{1-2\delta}}K_0\left(\sqrt{2k^{+2}t^2 + m_g^2}\,b\right)\right\}, \tag{A8}$$

where the denominators t have been replaced by $t^{1-2\delta}$ as in Eq. (A2). In the asymptotic region with large k^+, it is legitimate to extend the upper bound of t in the second integral from one to ∞. Using the relation

$$\int_0^\infty x^{2\mu+1}K_0\left(\alpha\sqrt{x^2+z^2}\right)dx = 2^\mu \Gamma(\mu+1) \times\left(\frac{z}{\alpha}\right)^{\mu+1}K_{\mu+1}(\alpha z), \tag{A9}$$

Eq. (A8) becomes

$$\int_0^\infty dy N_b^{(1)}(k^+, y, b, \mu) = -\frac{\alpha_s C_F}{\pi}\frac{u\cdot v}{\sqrt{(u\cdot v)^2 - u^2 v^2}}2^{-1+\delta}\Gamma(\delta)K_\delta(m_g b) \times\left[\left(\frac{m_g}{4\zeta^2 b}\right)^\delta - \left(\frac{m_g}{2k^{+2}b}\right)^\delta\right]. \tag{A10}$$

Taking the limit $\delta \to 0$, the $1/\delta$ poles cancel between the above two integrals, and Eq. (A10) reduces to

$$\int_0^\infty dy N_b^{(1)}(k^+, y, b, \mu) = -\frac{\alpha_s C_F}{2\pi}\frac{u\cdot v}{\sqrt{(u\cdot v)^2 - u^2 v^2}} \times \ln\frac{u^- v^+}{u^+ v^-}\ln\frac{m_g b e^{\gamma_E}}{2}. \tag{A11}$$

The second term in Eq. (A7) is written as

$$N_{b+}^{(1)}(k^+, k'^+, b, \mu) = -\frac{\alpha_s C_F}{\pi} \frac{u \cdot v}{\sqrt{(u \cdot v)^2 - u^2 v^2}} \times \left\{ \frac{\theta(k'^+ - k^+)}{(k'^+ - k^+)_+} K_0[2\nu(k'^+ - k^+)b] - \frac{\theta(k^+ - k'^+)}{(k^+ - k'^+)_+} K_0[\sqrt{2}(k^+ - k'^+)b] \right\}, \tag{A12}$$

where the infrared regulators m_g^2 have been dropped, since a plus distribution is infrared-finite. The combination of Eqs. (A11) and (A12) then gives Eq. (12).

The loop integral associated with Fig. 1(c) from Eq. (9) is rewritten, in the light-cone coordinates, as

$$N_c^{(1)} = ig^2 C_F \mu^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \frac{2(k'^+ - l^+)u^-}{[2(l^+ - k'^+)l^- - l_T^2](2l^+ l^- - l_T^2 - m_g^2)(u^- l^+ + u^+ l^-)} \delta(k^+ - k'^+). \tag{A13}$$

For $0 < l^+ < k'^+$, we enclose the contour in the l^- plane over the pole $l^- = (l_T^2 + m_g^2)/(2l^+) - i\epsilon$. For $k'^+ < l^+$, we enclose the contour over the pole $l^- = -u^- l^+/u^+ + i\epsilon$. For $l^+ < 0$, there is no pinch singularity (noticing $u^+ < 0$ in our choice), and the contour integration vanishes. Hence, Eq. (A13) becomes

$$N_c^{(1)} = ig^2 C_F \mu^{2\epsilon} 4\pi i \int \frac{d^{2-2\epsilon} l_T}{(2\pi)^{4-2\epsilon}} \left\{ \int_0^{k'^+} dl^+ \frac{(k'^+ - l^+) l^+ u^-}{[k'^+ l_T^2 + (k'^+ - l^+) m_g^2][u^+(l_T^2 + m_g^2) + 2u^- l^{+2}]} - \int_{k'^+}^{\infty} dl^+ \frac{(l^+ - k'^+) u^+ u^-}{[u^+ l_T^2 + 2u^- l^+(l^+ - k'^+)][u^+(l_T^2 + m_g^2) + 2u^- l^{+2}]} \right\} \delta(k^+ - k'^+). \tag{A14}$$

The integration over l_T leads to

$$N_c^{(1)} = -\frac{\alpha_s C_F}{2\pi} \left(\frac{4\pi\mu^2}{m_g^2}\right)^{\epsilon} \Gamma(\epsilon) \left\{ \int_0^1 dt t^{-1+2\delta}(1-t)^{-\epsilon} - \int_0^{\infty} dt t^{-1+2\delta} \left(\frac{4\zeta^2}{m_g^2} t^2 + 1\right)^{-\epsilon} + \left(\frac{4\zeta^2}{m_g^2}\right)^{-\epsilon} \times \left[\int_0^{\infty} dt t^{-2\epsilon} - \int_1^{\infty} dt t^{-\epsilon}(t-1)^{-\epsilon} \right] + \left(\frac{4\zeta^2}{m_g^2}\right)^{-\epsilon} \times \int_1^{\infty} dt t^{-1-\epsilon}(t-1)^{-\epsilon} - \int_0^1 dt (1-t)^{-\epsilon} \right\} \delta(k^+ - k'^+), \tag{A15}$$

where the variable change $l^+ = k'^+ t$ has been made. The above expression has been arranged in a way that the infrared divergences from $t \to 0$ cancel in the first line, and the linear ultraviolet divergences cancel in the second line. We have

$$N_c^{(1)} = -\frac{\alpha_s C_F}{2\pi} \left(\frac{4\pi\mu^2}{m_g^2}\right)^{\epsilon} \Gamma(\epsilon) \left\{ B(2\delta, 1-\epsilon) - \frac{1}{2} \times \left(\frac{4\zeta^2}{m_g^2}\right)^{-\delta} B(\delta, \epsilon - \delta) + \left(\frac{4\zeta^2}{m_g^2}\right)^{-\epsilon} \frac{\epsilon}{1-2\epsilon} B(1-\epsilon, 2\epsilon) + \left(\frac{4\zeta^2}{m_g^2}\right)^{-\epsilon} B(1-\epsilon, 2\epsilon) - \frac{1}{1-\epsilon} \right\} \delta(k^+ - k'^+), \tag{A16}$$

which leads to Eq. (13) by employing the expansion,

$$\Gamma(\epsilon) \approx \frac{1}{\epsilon}\left[1 - \gamma_E \epsilon + \left(\frac{\gamma_E^2}{2} + \frac{\pi^2}{12}\right)\epsilon^2\right]. \tag{A17}$$

We calculate the correction from Fig. 1(d) in the light-cone coordinates,

$$N_d^{(1)} = -ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{2(k'^+ - l^+)u^-}{[2(l^+ - k'^+)l^- - l_T^2](2l^+ l^- - l_T^2 - m_g^2)(u^- l^+ + u^+ l^-)} \times \delta(k^+ - k'^+ + l^+) \exp(-i\boldsymbol{l}_T \cdot \mathbf{b}). \tag{A18}$$

Applying the reasoning for Eq. (A13), we have

$$N_d^{(1)} = -ig^2 C_F 4\pi i\theta(k'^+ - k^+) \int \frac{d^2 l_T}{(2\pi)^4} \left\{ \frac{k^+(k'^+ - k^+)u^- \exp(-i\boldsymbol{l}_T \cdot \mathbf{b})}{[k'^+ l_T^2 + k^+ m_g^2][u^+(l_T^2 + m_g^2) + 2u^-(k'^+ - k^+)^2]} + \frac{k^+ u^+ u^- \exp(-i\boldsymbol{l}_T \cdot \mathbf{b})}{[u^+ l_T^2 - 2u^- k^+(k'^+ - k^+)][u^+(l_T^2 + m_g^2) + 2u^-(k'^+ - k^+)^2]} \right\}. \tag{A19}$$

The integration over l_T gives

$$N_d^{(1)} = \frac{\alpha_s C_F}{\pi} \frac{k^+ \theta(k'^+ - k^+)}{k'^+(k'^+ - k^+)} \left[K_0\left(\sqrt{k^+/k'^+} m_g b\right) - K_0\left(\sqrt{\frac{2u^-}{u^+}(k'^+ - k^+)^2 + m_g^2} b\right)\right]. \tag{A20}$$

We then adopt the splitting similar to Eq. (A7), whose first term is rewritten, by applying the variable change $y = k^+/(1-t)$ for the first Bessel function and $y = (t+1)k^+$ for the second Bessel function, as

$$\begin{aligned} \int_0^\infty dy N_d^{(1)}(k^+, y, b, \mu) &= \frac{\alpha_s C_F}{\pi} \left[\int_0^1 dt t^{-1+2\delta} K_0(\sqrt{1-t} m_g b) - \int_0^\infty dt t^{-1+2\delta} K_0\left(\sqrt{4\zeta^2 t^2 + m_g^2} b\right) \right. \\ &\quad \left. + \int_0^\infty \frac{dt}{t+1} K_0(2\zeta b t)\right]. \end{aligned} \tag{A21}$$

It is easy to show, using the relation,

$$\int_0^\infty \frac{K_\nu(\alpha x)}{x+1} dx = \frac{\pi^2}{2} \csc^2(\nu\pi)[I_\nu(\alpha) + I_{-\nu}(\alpha) - e^{-i\nu\pi/2}\mathbf{J}_\nu(i\alpha) - e^{i\nu\pi/2}\mathbf{J}_{-\nu}(i\alpha)], \tag{A22}$$

where $\mathbf{J}_0$ denotes the Anger function, that the third term is given by $\pi/(4\zeta b)$ in the asymptotic region with large k^+. Employing Eq. (A9), Eq. (A21) becomes

$$\int_0^\infty dy N_d^{(1)}(k^+, y, b, \mu) = -\frac{\alpha_s C_F}{2\pi} \left[\ln\frac{2\zeta^2 b e^{\gamma_E}}{m_g} \ln\frac{m_g b e^{\gamma_E}}{2} + \frac{\pi^2}{6}\right]. \tag{A23}$$

The second term in the splitting with the plus distribution can be obtained in a way similar to Eq. (A12).

The $O(\alpha_s)$ correction from Fig. 1(e) is written as

$$\begin{aligned} N_e^{(1)} &= \frac{i}{4} g^2 C_F \int \frac{d^4 l}{(2\pi)^4} tr\left[\frac{\gamma^\nu(\not k' - \not l)}{(k'-l)^2(l^2 - m_g^2)} \gamma_5 \not n_- \not n_+ \gamma_5 \right] \frac{v_\nu}{v \cdot l} \times \delta(k^+ - k'^+ + l^+) \exp(-i\boldsymbol{l}_T \cdot \mathbf{b}), \\ &= ig^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{2(k'^+ - l^+)v^-}{[2(l^+ - k'^+)l^- - l_T^2](2l^+ l^- - l_T^2 - m_g^2)(v^- l^+ + v^+ l^-)} \times \delta(k^+ - k'^+ + l^+) \exp(-i\boldsymbol{l}_T \cdot \mathbf{b}). \end{aligned} \tag{A24}$$

The substitution of v for u introduces an additional pole $l^- = v^- l^+/v^+ - i\epsilon$ in the region $l^+ < k'^+$. It is straightforward to confirm that the contribution from this additional pole vanishes. Hence, the result of Fig. 1(e) is similar to that of Fig. 1(d), but with the variable ζ replaced by $k \cdot v/\sqrt{v^2}$, which is Eq. (15).

[1] G. P. Lepage and S. J. Brodsky, Phys. Lett. B **87**, 359 (1979); Phys. Rev. D **22**, 2157 (1980).
[2] A. V. Efremov and A. V. Radyushkin, Phys. Lett. B **94**, 245 (1980).
[3] V. L. Chernyak, A. R. Zhitnitsky, and V. G. Serbo, JETP Lett. **26**, 594 (1977).
[4] V. L. Chernyak and A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 544 (1980); Phys. Rep. **112**, 173 (1984).
[5] H.-N. Li, Phys. Rev. D **64**, 014019 (2001); M. Nagashima and H.-N. Li, hep-ph/0202127.
[6] A. G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997).
[7] V. M. Braun and I. E. Filyanov, Z. Phys. C **48**, 239 (1990); P. Ball, J. High Energy Phys. 01 (1999), 010.
[8] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. **B529**, 323 (1998).

[9] H. Kawamura, J. Kodaira, C. F. Qiao, and K. Tanaka, Phys. Lett. B **523**, 111 (2001); **536**, 344E (2002); Mod. Phys. Lett. A **18**, 799 (2003).
[10] B. O. Lange and M. Neubert, Phys. Rev. Lett. **91**, 102001 (2003).
[11] V. M. Braun, D. Yu. Ivanov, and G. P. Korchemsky, Phys. Rev. D **69**, 034014 (2004).
[12] H.-N. Li, Prog. Part. Nucl. Phys. **51**, 85 (2003); Czech. J. Phys. **53**, 657 (2003).
[13] C.W. Bauer and A.V. Manohar, Phys. Rev. D **70**, 034024 (2004).
[14] B. O. Lange, hep-ph/0311159.
[15] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000); Nucl. Phys. **B606**, 245 (2001).
[16] P. Ball and E. Kou, J. High Energy Phys. 04 (2003), 029.
[17] M. Neubert (private communication).
[18] A. Szczepaniak, E. M. Henley, and S. Brodsky, Phys. Lett. B **243**, 287 (1990).
[19] H.-N. Li, hep-ph/0304217.
[20] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B **242**, 97 (1990); Nucl. Phys. **B366**, 135 (1991).
[21] J. C. Collins and R. K. Ellis, Nucl. Phys. **B360**, 3 (1991).
[22] E. M. Levin, M. G. Ryskin, Yu. M. Shabelskii, and A. G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657 (1991).
[23] J. Botts and G. Sterman, Nucl. Phys. **B225**, 62 (1989).
[24] H.-N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
[25] M. Nagashima and H.-N. Li, Phys. Rev. D **67**, 034001 (2003).
[26] H.-N. Li and H. L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995); Phys. Rev. D **53**, 2480 (1996); H.-N. Li, Chin. J. Phys. (Taipei) **34**, 1047 (1996).
[27] C. H. Chang and H.-N. Li, Phys. Rev. D **55**, 5577 (1997); T.W. Yeh and H.-N. Li, Phys. Rev. D **56**, 1615 (1997).
[28] Y. Y. Keum, H.-N. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001); Y. Y. Keum and H.-N. Li, Phys. Rev. D **63**, 074006 (2001).
[29] C. D. Lü, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001).
[30] T. Huang and Q. X. Shen, Z. Phys. C **50**, 139 (1991); J. P. Ralston and B. Pire, Phys. Rev. Lett. **65**, 2343 (1990); R. Jakob and P. Kroll, Phys. Lett. B **315**, 463 (1993); **319**, 545E (1993).
[31] T. Kurimoto, H.-N. Li, and A. I. Sanda, Phys. Rev. D **65**, 014007 (2002); Phys. Rev. D **67**, 054028 (2003).
[32] Y. Y. Keum, H.-N. Li, and A. I. Sanda, *Ninth International Symposium on Heavy Flavor Physics*, AIP Conf. Proc. No. 618 (AIP, New York, 2002); Y. Y. Keum and A. I. Sanda, Phys. Rev. D **67**, 054009 (2003).
[33] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. **B625**, 239 (2002).
[34] B. O. Lange and M. Neubert, Nucl. Phys. **B690**, 249 (2004).
[35] I. V. Musatov and A. V. Radyushkin, Phys. Rev. D **56**, 2713 (1997) and references therein.
[36] G. P. Korchemsky, D. Pirjol, and T. M. Yan, Phys. Rev. D **61**, 114510 (2000) and references therein.
[37] X. Ji, and F. Yuan, Phys. Lett. B **543**, 66 (2002); A. V. Belitsky, X. Ji, and F. Yuan, Nucl. Phys. **B656**, 165 (2003).
[38] J. C. Collins, Acta Phys. Pol. B **34**, 3103 (2003).
[39] H.-N. Li, Phys. Rev. D **55**, 105 (1997).
[40] H.-N. Li, Phys. Lett. B **405**, 347 (1997); H.-N. Li, hep-ph/9703328; H.-N. Li and J. L. Lim, Eur. Phys. J. C **10**, 319 (1999).
[41] J. C. Collins and D. E. Soper, Nucl. Phys.**B193**, 381 (1981).
[42] H.-N. Li, hep-ph/9803202.
[43] C.W. Bauer, D. Pirjol, and I.W. Stewart, Phys. Rev. D **67**, 071502 (2003); R. J. Hill, T. Becher, S. J. Lee, and M. Neubert, hep-ph/0404217.
[44] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. **B650**, 356 (2003); E. Lunghi, D. Pirjol, and D. Wyler, Nucl. Phys. **B649**, 349 (2003).
[45] G. P. Korchemsky and A. V. Radyushkin, Nucl. Phys. **B283**, 342 (1987); I. A. Korchemskaya and G. P. Korchemsky, Phys. Lett. B **287**, 169 (1992).
[46] N. Kidonakis, G. Oderda, and G. Sterman, Nucl. Phys. **B531**, 365 (1998).
[47] H.-N. Li and B. Tseng, Phys. Rev. D **57**, 443 (1998): the ν dependence from the nonfactorizable soft gluon exchange in Fig. 3a cancels that in the Sudakov factor in an external W-exchange amplitude; B. Tseng and H.-N. Li, *Proceedings of the APCTP Workshop: Pacific Particle Physics Phenomenology, Seoul, Korea, 1997* (World Scientific, Singapore, 1999), p. 168.
[48] Z. T. Wei and M. Z. Yang, Nucl. Phys. **B642**, 263 (2002); C. D. Lu and M. Z. Yang, Eur. Phys. J. C **28**, 515 (2003).
[49] H.-N. Li and B. Melic, Eur. Phys. J. C **11**, 695 (1999).
[50] Y. Y. Keum, T. Kurimoto, H.-N. Li, C. D. Lu, and A. I. Sanda, Phys. Rev. D **69**, 094018 (2004).
[51] H.-N. Li, Phys. Rev. D **66**, 094010 (2002); H.-N. Li and K. Ukai, Phys. Lett. B **555**, 197 (2003).