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New integrable structures in large-N QCD

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We study the anomalous dimensions of single trace operators composed of field strengths $F_{\mu\nu}$ in large-N QCD. The matrix of anomalous dimensions is the Hamiltonian of a compact spin chain with two spin one representations at each vertex corresponding to the self-dual and anti-self-dual components of $F_{\mu\nu}$. Because of the special form of the interaction it is possible to study separately renormalization of purely self-dual components. In this sector the Hamiltonian is integrable and can be exactly solved by Bethe ansatz. Its continuum limit is described by the level two SU(2) Wess-Zumino-Witten model.

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The anomalous dimensions of local operators are important physical quantities which are indispensable in describing logarithmic scaling violation in QCD [1] and which have many uses in QCD phenomenology. There are however other theoretical reasons for the study of anomalous dimensions. We hope that the study of anomalous dimensions and operator mixing will help in understanding the large-N limit of QCD [2] and in elucidating its relationship to string theory [3].

It is not clear at present what kind of string theory, if any, describes the large-N limit of QCD, but for its superconformal cousin, $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory, the answer is widely believed to be known. The SYM is exactly dual to type IIB superstring theory in $AdS_5 \times S^5$, according to the conjectured anti- de Sitter (AdS)/conformal field theory (CFT) duality [4]. The scaling dimensions of local operators play an important role in the AdS/CFT correspondence, since they are dual to the energies of the string states [5]. The spectrum of scaling dimensions in the field theory thus coincides with the string spectrum. The latter is poorly known because of the difficulties in quantizing strings in $AdS_5 \times S^5$, but the semiclassical string states with large energies [6,7] can be studied in much detail. Such states are dual to operators with large dimensions that often contain many constituent fields [6]. Computation of anomalous dimensions of such large operators is a challenging problem, even at one loop, because of the operator mixing. This problem drastically simplifies in the large-N limit and reduces to diagonalization of a Hamiltonian of a certain spin chain.

The spin chain that computes the complete one-loop matrix of anomalous dimensions in $\mathcal{N} = 4$ SYM possesses the amazing property of complete integrability [8]. (Some results also extend to the first few higher loops for a restricted class of operators.) The spectrum of the spin chain can be computed exactly by means of the Bethe ansatz [9,10]. Comparison of the semiclassical Bethe states with the classical strings in $AdS_5 \times S^5$ has led to many quantitative tests of the AdS/CFT correspondence [11]. More recently, the equivalence between the spin chain and strings was made more explicit at the level of effective actions [12]. It turns out that the effective action of the one-loop spin chain can be interpreted as the string action in a certain gauge, and thus the mixing matrix of large operators in principle carries some information about the world sheet dynamics of the dual string theory. We do not know the string dual of QCD, but it should be possible to compute the mixing matrix of local operators at one loop, which can shed new light on the string description of the large-N limit.

Integrable structures arise in many instances in QCD. They were first found in the analysis of the parton evolution in the Regge regime [13]. In a subsequent development, the mixing matrices of Wilson operators with various quantum numbers were identified with Hamiltonians of noncompact spin chains [14,15], which at large N turn out to be integrable for maximal-helicity operators.

We will study a different set of operators built from the gluon field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$:

$$\mathcal{O} = \mathrm{tr} F_{\mu_1\nu_1} \ldots F_{\mu_L\nu_L}. \qquad (1)$$

Multiplicatively renormalizable operators are linear combinations of those. In particular, we can contract indices of different $F_{\mu\nu}$'s or contract indices with $\varepsilon^{\mu\nu\lambda\rho}$ thus allowing for pseudotensors characterized by the presence of one $\tilde{F}_{\mu\nu}$. More general operators may contain multiple traces, quarks, and covariant derivatives and in principle mix with pure field strength operators. However, the mixing with quark and multitrace operators is suppressed by powers of $1/N$. The mixing with operators that con-

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tain derivatives is not suppressed, but we shall explain below that the mixing with derivative operators starts at two loops. Hence, pure field strength operators form a closed sector at one loop.

The mixing matrix is defined as a logarithmic derivative of the renormalization factor in the UV cutoff: $\Gamma = Z^{-1} \cdot dZ/d\ln\Lambda$. The renormalization factor is defined by the requirement that multiplication by Z makes correlation functions of composite operators $\mathcal{O}$ finite: $\mathcal{O}^{\boldsymbol{\mu}}_{\text{ren}} = Z^{\boldsymbol{\mu}}{}_{\boldsymbol{\nu}}\mathcal{O}^{\boldsymbol{\nu}}$. Here $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ are multi-indices that parametrize all possible operators of the form (1). The eigenvectors of the mixing matrix are multiplicatively renormalizable operators whose anomalous dimensions are given by the eigenvalues. The size of the mixing matrix rapidly grows with L and the resolution of the operator mixing becomes a more and more complicated problem.

Fortunately, a useful reformulation of this problem drastically simplifies it in the large-N limit. The idea is to identify the operators (1) and their linear combinations with the states in the Hilbert space of a periodic spin chain with L sites, one site for each $F_{\mu_l\nu_l}$. The sites are naturally ordered by the trace over the color indices. The mixing matrix is a Hermitian operator in this Hilbert space and thus can be regarded as a Hamiltonian of the spin chain. The mixing matrix is determined by the divergent pieces of the diagrams one of which is shown in Fig. 1. In more general operators containing covariant derivatives, $D_{\lambda_1} \cdots D_{\lambda_n} F_{\mu_l\nu_l}$ should be identified with a single site of the lattice (as in [14,15]), which is clear from the structure of the diagrams ($D_{\lambda_1} \cdots D_{\lambda_n} F_{\mu_l\nu_l}$ emits a single gluon to the lowest order in perturbation theory).

It is also clear that the number of sites can diminish but cannot grow at one loop, because a divergent diagram with $L + 1$ external legs contains at least three three-gluon vertices or one four-gluon and one three-gluon vertex and consequently appears at $O(g^3)$. A careful analysis [16] shows that operators with derivatives, containing the equations of motion, may appear at $O(g^2)$ in counterterms for pure field strength operators, all other

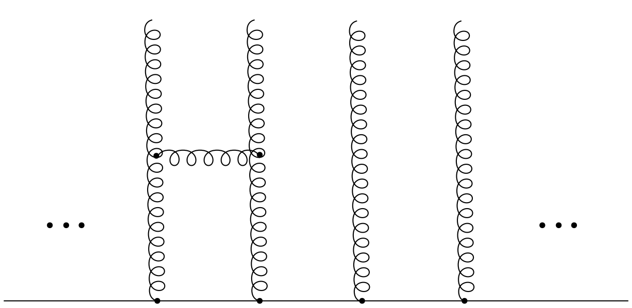

FIG. 1. One of the diagrams that contribute to the mixing matrix at one loop. The operator is depicted by a horizontal bar with gluon vertices ordered by an overall trace over the color indices.

mixings being $O(g^3)$. Thus, the one-loop mixing matrix is block upper triangular (block diagonal if one neglects operators vanishing by the equations of motion) and the anomalous dimensions of pure field strength operators are entirely determined by mixing among themselves. The other crucial property of the one-loop mixing matrix is that is has only nearest-neighbor interactions at large N:

$$\Gamma = \frac{g^2 N}{48\pi^2} \sum_{l=1}^{L} H_{l,l+1}, \tag{2}$$

where $H_{l,l+1}$ acts on the two adjacent sites. Nonlocal interactions correspond to nonplanar diagrams and are suppressed at large N.

The two-body Hamiltonian $H_{l,l+1}$ acts on two $F_{\mu\nu}$'s and thus is a matrix with eight Lorentz indices, which in principle can be computed from the diagrams. Fortunately, known anomalous dimensions of several particular operators make this calculation unnecessary. The Lorentz invariance severely restricts the structure of the mixing matrix and significantly reduces the number of independent matrix elements. The field strength transforms in the reducible representation $(1, 0) \oplus (0, 1)$ of the Lorentz group. The irreducible components are self-dual and anti-self-dual parts of $F_{\mu\nu}$: $F_{\mu\nu} = \sigma_{\mu\nu}{}^{\alpha\beta} f_{\alpha\beta} + \bar{\sigma}_{\mu\nu}{}^{\dot{\alpha}\dot{\beta}} \bar{f}_{\dot{\alpha}\dot{\beta}}$, where $\sigma_{\mu\nu} = i\sigma_2(\sigma_\mu\bar{\sigma}_\nu - \sigma_\nu\bar{\sigma}_\mu)/4$, $\bar{\sigma}_{\mu\nu} = -i(\bar{\sigma}_\mu\sigma_\nu - \bar{\sigma}_\nu\sigma_\mu)\sigma_2/4$, $\sigma_\mu = (1, \boldsymbol{\sigma})$, $\bar{\sigma}_\mu = (1, -\boldsymbol{\sigma})$.

The chiral pieces of the field strength $f_{\alpha\beta}$ and $\bar{f}_{\dot{\alpha}\dot{\beta}}$ transform in the spin one representations of $SU_L(2)$ and $SU_R(2)$. It is convenient to introduce the vectors $f_A = (\sigma_2\sigma_A)^{\alpha\beta} f_{\alpha\beta}$ and $\bar{f}_{\dot{A}} = (\sigma_2\sigma_{\dot{A}})^{\dot{\alpha}\dot{\beta}} \bar{f}_{\dot{\alpha}\dot{\beta}}$, where $A, \dot{A} = 1, 2, 3$ and σ_A, $\sigma_{\dot{A}}$ are ordinary Pauli matrices. Since the Hamiltonian $H_{l,l+1}$ commutes with Lorentz transformations, it takes constant values on irreducible representations that appear in the tensor product $[(1, 0) \oplus (0, 1)] \otimes [(1, 0) \oplus (0, 1)] = (2, 0) \oplus (0, 2) \oplus (1, 0) \oplus (0, 1) \oplus (0, 0)^+ \oplus (0, 0)^- \oplus (1, 1)^+ \oplus (1, 1)^-$, where the superscripts denote parity. The anomalous dimensions of representations related by parity should be the same, so the Hamiltonian contains six independent structures:

$$\begin{aligned} H = {} & a(\mathcal{P}_{(2,0)} + \mathcal{P}_{(0,2)}) + b(\mathcal{P}_{(1,0)} + \mathcal{P}_{(0,1)}) + c\mathcal{P}_{(0,0)^+} \\ & + d\mathcal{P}_{(0,0)^-} + e\mathcal{P}_{(1,1)^+} + f\mathcal{P}_{(1,1)^-}, \end{aligned} \tag{3}$$

where $\mathcal{P}_R$ are projectors on irreducible representations. The coefficients a–f can be fixed by comparing the energies of the eigenstates for short chains ($L = 2, 3, 4$) with known anomalous dimensions.

We start with dimension four operators that correspond to the spin chain with two sites connected by two links. The Hamiltonian (2) acts on each of the two links in the same way. Because of the cyclicity of trace, only symmetric representations survive. The simplest operator is the energy-momentum tensor $T_{\mu\nu}$ which belongs to

$(1,1)^+$. Since the energy-momentum tensor has zero anomalous dimension, $e = 0$.

Next, the action and the topological density are renormalization-group invariant to one loop when multiplied by g^2 [16,17]. The anomalous dimensions of the canonically normalized operators are thus equal to the beta function:

$$\mathrm{tr}F_{\mu\nu}F_{\mu\nu}, \qquad \mathrm{tr}F_{\mu\nu}\tilde{F}_{\mu\nu}: \qquad \gamma = -\frac{11g^2N}{24\pi^2}. \quad (4)$$

Acting by (2) and (3) on the scalar and on the pseudoscalar we obtain the anomalous dimensions $2cg^2N/48\pi^2$ and $2dg^2N/48\pi^2$, respectively, which implies that $c = d = -11$. The anomalous dimension of the $(2,0)\oplus(0,2)$ operator was computed in [18]:

$$\mathrm{tr}F_{\mu\nu}F_{\lambda\rho} - \text{index contractions}: \qquad \gamma = \frac{7g^2N}{24\pi^2}. \quad (5)$$

Comparing to the eigenvalue of the spin chain Hamiltonian, we find: $a = 7$.

The anomalous dimensions of the unique scalar and pseudoscalar operators of dimension six are also known [17]:

$$\mathrm{tr}F_{\mu\nu}F_{\nu\lambda}F_{\lambda\mu}, \qquad \mathrm{tr}\tilde{F}_{\mu\nu}F_{\nu\lambda}F_{\lambda\mu}: \qquad \gamma = \frac{g^2N}{16\pi^2}. \quad (6)$$

The calculation of the eigenvalues in the spin chain is no more tricky than for dimension four states. The chain now contains three sites. The Hilbert space consists of three copies of $(1,0)\oplus(0,1)$. In order to get a scalar state, we should keep only $(1,0)$ and $(0,1)$ in the tensor product of basic representations, since only $(1,0)$ and $(0,1)$ can produce a scalar when tensored with the remaining $(1,0)\oplus(0,1)$ on the third site. Hence, only the projectors $\mathcal{P}_{(1,0)}$ and $\mathcal{P}_{(0,1)}$ in the Hamiltonian contribute to the energy. Thus, we get $3bg^2N/48\pi^2$ for the anomalous dimension. Consequently, $b = 1$.

To compute the last factor f we must consider operators of dimension eight. There are four scalars at this level: $\mathrm{tr}F_{\mu\nu}F_{\mu\nu}F_{\rho\lambda}F_{\rho\lambda}$, $\mathrm{tr}F_{\mu\nu}F_{\rho\lambda}F_{\mu\nu}F_{\rho\lambda}$, $\mathrm{tr}F_{\mu\nu}F_{\nu\rho}F_{\rho\lambda}F_{\lambda\mu}$, $\mathrm{tr}F_{\mu\nu}F_{\nu\lambda}F_{\mu\rho}F_{\lambda\rho}$, and their mixing matrix can be found in [19]:

$$\Gamma = \frac{g^2N}{48\pi^2}\begin{pmatrix} -17 & -2 & -18 & -14 \\ -2 & 8 & -20 & -12 \\ -11 & 0 & 2 & 2 \\ -1 & -6 & 8 & -16 \end{pmatrix}. \quad (7)$$

It is then a straightforward but slightly tedious exercise to show that this result implies $f = 3$.

We have independently checked all the above results. Also, the mixing matrix (7), in addition to fixing $f = 3$, yields several algebraic relations among the remaining coefficients in the Hamiltonian all of which are satisfied by their numerical values computed earlier.

To summarize, the spin chain Hamiltonian is

$$H = 7(\mathcal{P}_{(2,0)} + \mathcal{P}_{(0,2)}) + \mathcal{P}_{(1,0)} + \mathcal{P}_{(0,1)} - 11(\mathcal{P}_{(0,0)^+} + \mathcal{P}_{(0,0)^-}) + 3\mathcal{P}_{(1,1)^-}. \quad (8)$$

An important point to notice here is the degeneracy of the scalar representations of opposite parity. Because of this degeneracy, definite-parity projectors can be replaced by projectors on the left and right scalars ($f_A f_A$ and $\bar{f}_{\dot{A}}\bar{f}_{\dot{A}}$). As a result, only the last term in the Hamiltonian mixes f_A with $\bar{f}_{\dot{A}}$ and even this term conserves the number of f_A's (and of $\bar{f}_{\dot{A}}$'s). Hence, the number of self-dual components of the field strength in an operator is preserved by renormalization. This follows from a "chiral" type of symmetry rotating f_A and $\bar{f}_{\dot{A}}$ by opposite phases. The Lagrangian is not invariant under this transformation but its variation is proportional to the topological density that integrates to zero in the absence of instantons. This symmetry is responsible for the closure of the chiral sector $\mathrm{tr}(f_{A_1}\ldots f_{A_L})$. Such structure of the mixing matrix produces parity degeneracies for operators of higher dimension as well. For instance, parity-odd and parity-even operators in $(L,0)+(0,L)$ and $(L,0)-(0,L)$ are degenerate. The first example of such parity degeneracy is (6).

Let us consider the chiral sector of operators built from the self-dual field strength f_A. The mixing matrix in this sector is a Hamiltonian of a spin one $\mathrm{SU}_L(2)$ spin chain. It can be expressed in terms of permutation and trace operators acting on three-dimensional vectors as $Pv\otimes u = u\otimes v$, $Kv\otimes u = (v,u)\mathbf{1}$. These operators are SU(2) invariant and can be expressed in terms of projectors on irreducible representations: $P = \mathcal{P}_0 - \mathcal{P}_1 + \mathcal{P}_2$, $K = 3\mathcal{P}_0$. In terms of the permutation and trace operators, the mixing matrix of chiral operators is

$$\Gamma = \frac{g^2N}{48\pi^2}\sum_{l=1}^{L}(4 + 3P_{l,l+1} - 6K_{l,l+1}).$$

The same matrix can also be expressed in terms of ordinary SU(2) generators (spins) that act on each site of the lattice:

$$\Gamma = \frac{g^2N}{48\pi^2}\sum_{l=1}^{L}[7 + 3\mathbf{S}_l\cdot\mathbf{S}_{l+1} - 3(\mathbf{S}_l\cdot\mathbf{S}_{l+1})^2].$$

Because the spins repel, the vacuum of the system is antiferromagnetic, which means that the lowest anomalous dimension is negative. Remarkably, this spin chain is integrable [10,20] and the Hamiltonian can be diagonalized by Bethe ansatz [21]! The Bethe equations constitute a set of algebraic equations for rapidities of elementary excitations on the lattice:

$$\left(\frac{\lambda_k + i}{\lambda_k - i}\right)^L = \prod_{j\neq k}\frac{\lambda_k - \lambda_j + i}{\lambda_k - \lambda_j - i}.$$

The Bethe equations should be supplemented by the zeromomentum condition:

$$\prod_k \frac{\lambda_k + i}{\lambda_k - i} = 1.$$

This condition reflects the cyclicity of the trace in the operators. All possible solutions of Bethe equations yield the spectrum of the anomalous dimensions, computed for a given solution from the formula

$$\gamma = \frac{g^2 N}{48\pi^2}\left(7L - \sum_k \frac{12}{\lambda_k^2 + 1}\right).$$

Some sample anomalous dimensions can be readily calculated. The state with the largest possible spin corresponds to the maximally symmetric operator $\mathcal{O}_\Omega = \mathrm{tr} f_{(A_1} \ldots f_{A_L)}$ − index contractions. This operator corresponds to the pseudovacuum of the spin chain. Its anomalous dimension is large and positive: $\gamma_\Omega = 7g^2NL/48\pi^2$. At $L = 3$, this formula reproduces the known anomalous dimension of twist three dimension six gluon operator [15]. The real vacuum is a Lorentz scalar and has negative anomalous dimension (as an effect of the asymptotic freedom). In the thermodynamic limit of large L [21]: $\gamma_0 = -5g^2NL/48\pi^2 + O(L^0)$.

The known string representation of the superconformal Yang-Mills theory can be compared to the spin chain rather directly. The classical spin system obtained by taking the continuum limit of the relevant lattice model (which is of course different from the one considered here) agrees with that describing the semiclassical rotating string in the bulk spacetime [12]. This observation opens up the possibility of reconstructing the string description by analyzing the large-N renormalization of operators that consist of a large number of elementary fields. Needless to say, this map is far from complete since on the gauge theory side we have limited information on higher loop effects and the mapping becomes more involved as these effects are taken into account. Still, we believe it is of interest to consider the continuum limit of the spin chain we have obtained, hoping that it will provide some clue on the string description.

The continuum limit of the antiferromagnetic spin one chain is a relativistic theory in the integrable case, because the dispersion relation of low-lying excitations is linear [10,21,22]. The low-energy effective theory is the conformal SU(2) WZW model at level two [23]. This model has central charge $c = 3/2$ and can be described by a triplet of free fermions:

$$S = \int d^2\xi \sum_{a=1}^{3} (\chi_L^a \partial_+ \chi_L^a + \chi_R^a \partial_- \chi_R^a).$$

The fermion currents $J_L^a = i\varepsilon^{abc}\chi_L^b\chi_L^c/2$ (and similarly for J_R) obey a SU(2) Kac-Moody algebra at level two:

$$[J_L^a(x_-), J_L^b(y_-)] = i\epsilon^{abc} J_L^c(x_-)\delta(x_- - y_-) + 2\frac{i}{4\pi}\delta^{ab}\delta'(x_- - y_-).$$

The current algebra can be bosonized in the standard way [24] by introducing the chiral field $g(x) \in$ SU(2) with the WZW action

$$S = \frac{2}{16\pi}\int d^2\xi \mathrm{tr}\partial_\alpha g \partial^\alpha g^\dagger + \frac{2}{24\pi} \times \int d^3\xi \epsilon^{\alpha\beta\gamma} \mathrm{tr} g^\dagger \partial_\alpha g g^\dagger \partial_\beta g g^\dagger \partial_\gamma g. \tag{13}$$

Alternatively, one can trade two out of three fermions for a compact boson. In this representation, the model is manifestly supersymmetric. We are thus led to believe that some of the dynamics of the purely self-dual sector of QCD in the large N and L limit is captured by a WZW model, although higher loop corrections and the inclusion of derivatives need to be carefully analyzed (work in progress). We hope that these results will provide further clues on the construction of the elusive QCD string.

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