Manifestation of the $a_0^0(980) - f_0(980)$ mixing in the reaction $\pi^- p \to \eta\pi^0 n$ on a polarized target

N. N. Achasov* and G. N. Shestakov†

Laboratory of Theoretical Physics, Sobolev Institute for Mathematics, Novosibirsk, 630090, Russia

(Received 12 May 2004; published 14 October 2004)

It is argued that the single-spin asymmetry in the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ is extremely sensitive to the mixing of the $a_0^0(980)$ and $f_0(980)$ resonances. It is shown that at low momentum transfers (namely, in any one of the intervals $0 \le -t \le 0.025, ..., 0.1$ GeV^2) the normalized asymmetry, which can take the values from -1 to 1, must undergo a jump in magnitude close to 1 in the $\eta\pi^0$ invariant mass region between 0.965 GeV and 1.01 GeV. The strong asymmetry jump is the straightforward manifestation of the $a_0^0(980) - f_0(980)$ mixing. For observing the jump, any very high quality $\eta\pi^0$ mass resolution is not required. The energy dependence of the polarization effect is expected to be rather weak. Therefore, it can be investigated at any high energy, for example, in the range from eight to 100 GeV.

DOI: 10.1103/PhysRevD.70.074015 PACS numbers: 14.40.Cs, 13.75.Gx, 13.88.+e

I. INTRODUCTION

Study of the nature of light scalar resonances has become a central problem of nonperturbative QCD. The point is that the elucidation of their nature is important for understanding both the confinement physics and the chiral symmetry realization way in the low-energy region, i.e., the main consequences of QCD in the hadron world. The nontrivial nature of the well established lightest scalar resonances is no longer denied practically anybody. In particular, there exist numerous evidences in favor of the four-quark ($q^2\bar{q}^2$) structure of these states, see, for example, Ref. [1] and references therein. In a recent paper [2], we suggested a new method of the investigation of the $a_0(980)$ and $f_0(980)$ resonances with use of the polarization phenomena closely related to the $a_0^0(980) - f_0(980)$ mixing effect that carries important information on the nature of these puzzling states, in particular, about their coupling to the $K\bar{K}$ channels.

The mixing between the $a_0^0(980)$ and $f_0(980)$ resonances was discovered theoretically as a threshold phenomenon (induced by the K^+ and K^0 meson mass difference) in the late 1970s [3]. The cross section of the process $\pi^+\pi^- \to \eta\pi^0$, which is forbidden by G-parity but appears because of $a_0^0(980) - f_0(980)$ mixing, was calculated for the first time [3]. Moreover, the reactions $\pi^\pm N \to \eta\pi^0(N, \Delta)$, $KN \to [(\eta\pi^0), (\pi^+\pi^-)] \times [\Lambda, \Sigma, \Sigma(1385)]$ on unpolarized targets, the $f_1(1285) \to a_0^0(980)\pi^0 \to 3\pi$ decay, and the $\bar{p}n \to (\pi^-, \rho^-)f_0(980) \to (\pi^-, \rho^-)\eta\pi^0$ annihilations at rest, in which the $a_0^0(980) - f_0(980)$ mixing could be detected, were considered in detail, and effects of violation of isospin invariance caused by this mixing were estimated in the $\eta\pi^0$ and $\pi\pi$ mass spectra and differential cross sections [3].

Recently, interest in the $a_0^0(980) - f_0(980)$ mixing was renewed, and its possible manifestations in various reactions are intensively discussed in the literature [4–19]. However, this mixing has not unambiguously been identified yet in corresponding experiments. For example, in Ref. [8] it was suggested that the data on the centrally produced $a_0^0(980)$ resonance in the reaction $pp \to p_s(\eta\pi^0)p_f$, in principle, can be interpreted in favor of the existence of $a_0^0(980) - f_0(980)$ mixing. In Ref. [11] it was noted that within the experimental errors and the model uncertainty in the $f_0(980)$ production cross section the result obtained in Ref. [8] does not contradict to the predictions made in Ref. [3]. However, the experimental confirmation of such a scenario requires measuring the reaction $pp \to p_s(\eta\pi^0)p_f$ at a much higher energy to exclude a possible effect of the secondary Regge trajectories, for which the $\eta\pi^0$ production is not forbidden by G-parity.

A qualitatively new proposal concerning a search for the $a_0^0(980) - f_0(980)$ mixing effect was given in Ref. [2]. We suggested performing the polarized target experiments on the reaction $\pi^- p \to \eta\pi^0 n$ at high energy in which the fact of the existence of the $a_0^0(980) - f_0(980)$ mixing can be unambiguously and very easily established through the presence of a strong jump in the azimuthal (single-spin) asymmetry of the S-wave $\eta\pi^0$ production cross section near the $K\bar{K}$ thresholds [2].

In this paper, the expected polarization effect in the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ is considered in more detail and all the quantitative estimates are discussed as thoroughly as possible (here and in what follows $(\eta\pi^0)_S$ denotes the $\eta\pi^0$ system with the relative orbital angular momentum $L = 0$). It should be noted

*Electronic address: achasov@math.nsc.ru
†Electronic address: shestako@math.nsc.ru

that the main features of the predicted interference pattern are to a great extent model independent. We show that at low momentum transfers the normalized asymmetry, which can take the values from -1 to 1, must undergo a jump in magnitude close to 1 in the $\eta\pi^0$ invariant mass region between 0.965 GeV and 1.01 GeV. The strong asymmetry jump is the exclusive consequence of isospin breaking due to the $a_0^0(980) - f_0(980)$ mixing. We also emphasize that observing the asymmetry jump does not require at all any very high quality $\eta\pi^0$ mass resolution that would be absolutely necessary to recognize the $a_0^0(980) - f_0(980)$ mixing manifestation in the $\eta\pi^0$ mass spectrum in unpolarized experiments. Furthermore, the energy dependence of the polarization effect is expected to be rather weak. Therefore, it can be investigated at any high energy, for example, in the range from eight to 100 GeV.

The paper is organized as follows. In Sec. II, the definitions of the cross section and asymmetry for the reaction $\pi^- p \to (\eta\pi^0)_S n$ on a polarized target are given. Section III is purely technical. It contains the detail formulae for the $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ reaction amplitudes at high energies. These amplitudes correspond to the ρ_2, b_1, and π Regge pole exchange mechanisms. The one-pion exchange mechanism violates the G-parity conservation and arises owing to the $a_0^0(980) - f_0(980)$ mixing. This section demonstrates also the specific features of the $a_0^0(980) - f_0(980)$ transition amplitude, which are of crucial importance for polarization phenomena. In Sec. IV, the data from the recent unpolarized experiments on the reaction $\pi^- p \to \eta\pi^0 n$ are discussed and the quantitative estimates of the ρ_2 and b_1 exchange contributions to the $(\eta\pi^0)_S$ production cross section are presented. In Secs. V and VI, the results of the calculation of the polarization effect due to the $a_0^0(980) - f_0(980)$ mixing are given in the ρ_2 and π exchange model and in the ρ_2, b_1, and π exchange one, respectively. The conclusions based on these results are briefly formulated in Sec. VII.

II. CROSS SECTION AND ASYMMETRY

Owing to parity conservation, the differential cross section of the reaction $\pi^- p \to (\eta\pi^0)_S n$ on a polarized proton target at fixed incident pion laboratory momentum, $P_{lab}^{\pi^-}$, has the form

$$d^3\sigma/dtdmd\psi = [d^2\sigma/dtdm + I(t,m)P\cos\psi]/2\pi, \quad (1)$$

where t is the square of the four-momentum transferred from the incident π^- to the outgoing $\eta\pi^0$ system, m is the $\eta\pi^0$ invariant mass, ψ is the angle between the normal to the reaction plain, formed by the momenta of the π^- and $\eta\pi^0$ system, and the transverse (to the π^- beam axis) polarization of the protons, P is a degree of this polarization, $d^2\sigma/dtdm = |M_{++}|^2 + |M_{+-}|^2$ is the unpolarized differential cross section, M_{+-} and M_{++} are the s-channel helicity amplitudes with and without nucleon helicity flip, $I(t,m) = 2\mathrm{Im}(M_{++}M_{+-}^*)$ describes the interference contribution responsible for the azimuthal (or single-spin) asymmetry of the cross section. In terms of the directly measurable quantities $I(t,m)$ and $d^2\sigma/dtdm$, one can also define the dimensionless, normalized asymmetry $A(t,m) = I(t,m)/[d^2\sigma/dtdm]$, $-1 \le A(t,m) \le 1$. The asymmetry pertaining to some interval of $-t$, $A(-t_1 \le -t \le -t_2, m)$, or to some interval of m, $A(t, m_1 \le m \le m_2)$, is defined by the ratio of the corresponding integrals of $I(t,m)$ and $d^2\sigma/dtdm$ over t or over m. Here we shall be interested in the region of $m \approx 1$ GeV. The available data from unpolarized target experiments on the reaction $\pi^- p \to \eta\pi^0 n$ [4,20–23] shows that the $(\eta\pi^0)_S$ mass spectrum in this region of m is dominated by the production of the $a_0^0(980)$ resonance, $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$.

It follows from the G-parity conservation that at high energies and small $-t$ the amplitudes M_{+-} and M_{++} are defined by the t-channel exchanges with the quantum numbers of the b_1 and ρ_2 Regge poles, respectively [5] (hereinafter they are denoted by $M_{+-}^{b_1}$ and $M_{++}^{\rho_2}$).[1] In addition, there arises the possibility of the π Regge pole exchange in the reaction $\pi^- p \to (\eta\pi^0)_S n$ by virtue of the process $\pi^- p \to f_0(908)n \to a_0^0(980)n \to (\eta\pi^0)_S n$, which is stipulated by the $a_0^0(980) - f_0(980)$ mixing violating G-parity [3,5].[2] As is well known, the amplitude of the π exchange in the reaction $\pi^- p \to (\pi\pi)_S n$ is large in the low $-t$ region. Moreover, both the modulus and the phase of the $a_0^0(980) - f_0(980)$ transition amplitude dramatically change as functions of m near the $K\bar{K}$ thresholds. As we shall see, all of these features lead in the reaction $\pi^- p \to (\eta\pi^0)_S n$ to rather impressive consequences, which can be easily revealed in polarized target experiments, because they make possible direct measurements of the interference between the ρ_2 and π exchange amplitudes.

III. AMPLITUDES OF THE REACTION $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$

Let us use the Regge pole model and write the ρ_2, b_1, and π exchange amplitudes for the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ in the following form:

[1] History of the ρ_2 Regge exchange has been reviewed in Ref. [5]. Recall that the lower-lying representative of the ρ_2 Regge trajectory has the quantum numbers $I^G(J^{PC}) = 1^+(2^{--})$ which belong to the $^3D_2$ $q\bar{q}$ family [24].

[2] The process $\pi^- p \to f_0(908)n \to a_0^0(980)n \to (\eta\pi^0)_S n$ can also occur via the a_1 exchange. The estimates fulfilled on the basis of Ref. [25] show, however, that, in this case, the amplitude $M_{++}^{a_1}$ can be neglected in comparison with the other ones for small $-t$.

$$M_{++}^{\rho_2} = e^{-i\pi\alpha_{\rho_2}(t)/2} e^{\Lambda_{\rho_2} t/2} (s/s_0)^{\alpha_{\rho_2}(0)-1} a_{\rho_2} G_{a_0}(m) \times [2m^2 \Gamma_{a_0\eta\pi^0}(m)/\pi]^{1/2}, \qquad (2)$$

$$M_{+-}^{b_1} = i e^{-i\pi\alpha_{b_1}(t)/2} \sqrt{-t} e^{\Lambda_{b_1} t/2} (s/s_0)^{\alpha_{b_1}(0)-1} a_{b_1} G_{a_0}(m) \times [2m^2 \Gamma_{a_0\eta\pi^0}(m)/\pi]^{1/2}, \qquad (3)$$

$$M_{+-}^{\pi} = e^{-i\pi\alpha_\pi(t)/2} \frac{\sqrt{-t}}{t - m_\pi^2} e^{\Lambda_\pi(t-m_\pi^2)/2} a_\pi e^{i\delta_B(m)} G_{a_0 f_0}(m) \times [2m^2 \Gamma_{a_0\eta\pi^0}(m)/\pi]^{1/2}. \qquad (4)$$

It should be immediately emphasized that the π exchange amplitude M_{+-}^{π}, which is forbidden by G-parity considerations, is essentially well known theoretically [3,5,25,26]. In Eqs. (2)–(4), $\alpha_j(t) = \alpha_j(0) + \alpha_j' t$, a_j, and $\Lambda_j/2 = \Lambda_j^0/2 + \alpha_j' \ln(s/s_0)$ are the trajectory, residue, and slope of the j-th Regge pole [as a guideline one can accept $\alpha_\pi(t) \approx 0.8(t - m_\pi^2)/\mathrm{GeV}^2$, $\alpha_{b_1}(t) \approx -0.21 + 0.8t/\mathrm{GeV}^2$, and $\alpha_{\rho_2}(t) \approx -0.31 + 0.8t/\mathrm{GeV}^2$], $s \approx 2m_p P_{lab}^{\pi^-}$, $s_0 = 1\ \mathrm{GeV}^2$, $G_{a_0}(m) = D_{f_0}(m)/[D_{a_0}(m)D_{f_0}(m) - \Pi_{a_0f_0}^2(m)]$ is the propagator of the mixed $a_0^0(980)$ resonance [3], $G_{a_0f_0}(m) = \Pi_{a_0f_0}(m)/[D_{a_0}(m)D_{f_0}(m) - \Pi_{a_0f_0}^2(m)]$, $\Pi_{a_0f_0}(m)$ is the nondiagonal element of the polarization operator describing the $a_0^0(980) - f_0(980)$ transition amplitude [3], $1/D_r(m)$ is the propagator of an unmixed resonance r with a mass m_r, $D_r(m) = m_r^2 - m^2 + \sum_{ab}[\mathrm{Re}\Pi_r^{ab}(m_{f_0}) - \Pi_r^{ab}(m)]$, $r = [a_0(980), f_0(980)]$, $ab = (\eta\pi^0, K^+K^-, K^0\bar{K}^0)$ for $r = a_0(980)$, and $ab = (\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0)$ for $r = f_0(980)$, $\Pi_r^{ab}(m)$ is the diagonal element of the polarization operator for the resonance r corresponding to the contribution of the ab intermediate state [25,26], for $m \geq m_a + m_b$

$$\Pi_r^{ab}(m) = \frac{g_{rab}^2}{16\pi}\left[\frac{m_+ m_-}{\pi m^2} \ln\frac{m_b}{m_a} + \rho_{ab}(m)\left(i - \frac{1}{\pi} \times \ln\frac{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}}\right)\right],$$

where g_{rab} is the coupling constant of r to the ab channel (here for identical π^0 mesons $g_{f_0\pi^0\pi^0}^2 = g_{f_0\pi^+\pi^-}^2/2$), $\rho_{ab}(m) = [(m^2 - m_+^2)(m^2 - m_-^2)]^{1/2}/m^2$, $m_\pm = m_a \pm m_b$, $m_a \geq m_b$, and $\Gamma_{rab}(m) = \mathrm{Im}[\Pi_r^{ab}(m)]/m = g_{rab}^2 \rho_{ab}(m)/16\pi m$ is the width of the $r \to ab$ decay; if $m_- \leq m \leq m_+$, then $(m^2 - m_+^2)^{1/2}$ should be replaced by $i(m_+^2 - m^2)^{1/2}$. In Eq. (4), $a_\pi = g_{\pi NN} g_{f_0\pi^+\pi^-}/\sqrt{8\pi s}$, $g_{\pi NN}^2/4\pi \approx 14.3$, and $\delta_B(m)$ is a smooth and large phase (of about 90° for $m \approx 1$ GeV) of the elastic background accompanying the $f_0(980)$ resonance in the S-wave reaction $\pi\pi \to \pi\pi$ in the channel with isospin $I = 0$ [3,25,26].

The amplitude of the $a_0^0(980) - f_0(980)$ transition, $\Pi_{a_0f_0}(m)$, must be determined to a considerable extent by the K^+K^- and $K^0\bar{K}^0$ intermediate states [3] because of the proximity of the $a_0^0(980)$ and $f_0(980)$ resonances to the $K\bar{K}$ thresholds and their strong coupling to the $K\bar{K}$ channels. The sum of the one-loop diagrams $f_0(980) \to K^+K^- \to a_0^0(980)$ and $f_0(980) \to K^0\bar{K}^0 \to a_0^0(980)$, with isotopic symmetry for coupling constants, gives [3]

$$\Pi_{a_0f_0}(m) = \frac{g_{a_0K^+K^-} g_{f_0K^+K^-}}{16\pi}\left[i(\rho_{K^+K^-}(m) - \rho_{K^0\bar{K}^0}(m)) - \frac{\rho_{K^+K^-}(m)}{\pi} \ln\frac{1 + \rho_{K^+K^-}(m)}{1 - \rho_{K^+K^-}(m)} + \frac{\rho_{K^0\bar{K}^0}(m)}{\pi} \ln\frac{1 + \rho_{K^0\bar{K}^0}(m)}{1 - \rho_{K^0\bar{K}^0}(m)}\right], \qquad (5)$$

where $m \geq 2m_{K^0}$; in the region $0 \leq m \leq 2m_K$, $\rho_{K\bar{K}}(m)$ should be replaced by $i|\rho_{K\bar{K}}(m)|$. The "resonancelike" behavior of the modulus and phase of the amplitude $\Pi_{a_0f_0}(m)$, induced by the K^+ and K^0 meson mass difference, is clearly illustrated in Figs. 1(a) and 1(b). Note that in the region between the K^+K^- and $K^0\bar{K}^0$ thresholds, which is eight MeV wide, $|\Pi_{a_0f_0}(m)| \approx |g_{a_0K^+K^-} g_{f_0K^+K^-}/16\pi|[(m_{K^0}^2 - m_{K^+}^2)/m_{K^0}^2]^{1/2} \approx 0.1265|g_{a_0K^+K^-} g_{f_0K^+K^-}/16\pi|$, i.e., is of the order of $m_K\sqrt{m_{K^0}^2 - m_{K^+}^2} \approx \sqrt{\alpha} m_K^2$ [3].[3] From Eqs. (4) and (5) it follows also that the contribution of M_{+-}^{π} to $d^2\sigma/dtdm$, in this mass region, is controlled mainly by the production of the ratios of coupling constants, i.e., $|M_{+-}^{\pi}|^2 \propto \sigma(\pi^+\pi^- \to \eta\pi^0) \propto (g_{f_0K^+K^-}^2/g_{f_0\pi^+\pi^-}^2)(g_{a_0K^+K^-}^2/g_{a_0\eta\pi^0}^2)$.

When constructing the curve for $|\Pi_{a_0f_0}(m)|$ in Fig. 1(a) and obtaining the quantitative estimates presented below for the polarization effect, we used the following tentative values of the $f_0(980)$ and $a_0(980)$ resonance parameters: $m_{f_0} \approx 0.980$ GeV, $g_{f_0\pi^+\pi^-}^2/16\pi \approx \frac{2}{3} 0.1\ \mathrm{GeV}^2$, $g_{f_0K^+K^-}^2/16\pi \approx \frac{1}{2} 0.4\ \mathrm{GeV}^2$, $\delta_B(m) \approx 35.5° + 47° m/\mathrm{GeV}$, $m_{a_0} \approx 0.9847$ GeV, $g_{a_0K^+K^-}^2/16\pi \approx g_{f_0K^+K^-}^2/16\pi \approx \frac{1}{2} 0.4\ \mathrm{GeV}^2$, and $g_{a_0\eta\pi^0}^2/16\pi \approx 0.25\ \mathrm{GeV}^2$; in addition, see also Refs. [3,24–30]. Figures 1(c) and 1(d) show the $\pi^+\pi^-$ and $\eta\pi^0$ mass spectra $dN[f_0(980) \to \pi^+\pi^-]/dm = 2m^2\Gamma_{f_0\pi^+\pi^-}(m)/\pi|D_{f_0}(m)|^2$ and $dN[a_0(980) \to \eta\pi^0]/dm = 2m^2\Gamma_{a_0\eta\pi^0}(m)/\pi|D_{a_0}(m)|^2$ for these values of the

[3]It is the unique effect of the $\sqrt{m_d - m_u} \sim \sqrt{\alpha}$ order which dominates in our consideration. As for effects of the $m_d - m_u \sim \alpha$ order, they are small. Such effects were considered partly in Ref. [9], $a_0^0(908) \to \eta\pi^0 \to \pi^0\pi^0 \to f_0(980)$. A clear idea of the magnitude of effects of the $m_d - m_u \sim \alpha$ order gives $|\Pi_{a_0f_0}(m)|$ at $m < 0.95$ GeV and $m > 1.05$ GeV, see Fig. 1(a).

parameters.[4] Note that, while $\Gamma_{f_0\pi\pi}(m_{f_0}) = \frac{3}{2}\Gamma_{f_0\pi^+\pi^-}(m_{f_0}) \approx 98$ MeV and $\Gamma_{a_0\eta\pi^0}(m_{f_0}) \approx 166$ MeV, the visible (effective) widths of the corresponding peaks at their half maxima are approximately equal to 42 MeV and 68 MeV, respectively. This is so owing to the couplings of the resonances to the $K\bar{K}$ channels. In its turn, Figs. 1(e) and 1(f) give an idea of the absolute values and the typical shapes of the differential cross sections due to the π exchange, $d\sigma^\pi/dt = \int |M^\pi_{+-}|^2 dm$ and $d\sigma^\pi/dm = \int |M^\pi_{+-}|^2 dt$, corresponding to the integration regions over m from 0.8 to 1.2 GeV and over t from -0.025 GeV^2 to 0, respectively, and $P^{\pi^-}_{lab} = 18.3$ GeV (i.e., the Brookhaven National Laboratory energy (BNL) [4]), at which $\Lambda_\pi/2 \approx 4.5$ GeV^2 [31,32]. Integrating $d\sigma^\pi/dt$ presented in Fig. 1(e) over t, we find that the total cross section of the reaction $\pi^- p \to a^0_0(980)n \to (\eta\pi^0)_S n$ caused by the π exchange, σ^π, is approximately equal to 10.9 nb. Based on the previous investigations [3,5], one can conclude that the indicated value of σ^π should be considered as its rather reliable lower bound. Thus, using the above values of the $a^0_0(980)$ and $f_0(980)$ resonance parameters, we present the most conservative estimates of the expected polarization effect. Finally, it should be particularly emphasized that the sharp and strong variation (by about 90°) of the phase of the amplitude $\Pi_{a_0f_0}(m)$ between the $K\bar{K}$ thresholds, being crucial for polarization phenomena, is generally independent of the $f_0(980)$ and $a^0_0(980)$ resonance parameters [see Fig. 1(b) and Eq. (5)].

To estimate quantitatively the ρ_2 and b_1 exchange contributions to the $(\eta\pi^0)_S$ production cross section, and to clarify a question about the relative role of the π exchange, it is necessary to turn to the available experimental data.

IV. DATA FROM UNPOLARIZED EXPERIMENTS

The experiments on the reaction $\pi^- p \to \eta\pi^0 n$ on unpolarized targets were performed at $P^{\pi^-}_{lab} = 18.3$ GeV at BNL [4,20,21], 38 GeV at Institute High Energy Physics (IHEP Protvino)[22,23], 32 GeV at IHEP [23], and 100 GeV at CERN [23]. The present situation is rather interesting. The point is that, in general, the available data from BNL [4], IHEP [22,23], and CERN [23] do not require at all the introduction of the b_1 exchange amplitude $M^{b_1}_{+-}$ to describe the t distributions (dN/dt) of the $\pi^- p \to a^0_0(980)n \to (\eta\pi^0)_S n$ reaction events in the $a^0_0(890)$ mass region. All the data for $0 \le -t \le (0.6 - 0.8)$ GeV^2 are excellently approximated by the simplest exponential form $C\exp(\Lambda t)$ [5,22,23] corresponding to the amplitude $M^{\rho_2}_{++}$ nonvanishing for $t \to 0$ [5]. For example, such a fit to the normalized BNL data [4,5] for the differential cross section $d\sigma/dt$ of the reaction $\pi^- p \to a^0_0(980)n \to (\eta\pi^0)_S n$, shown in Fig. 2 by the solid curve, gives $\chi^2/n.d.f. = 15.75/22$ and $d\sigma/dt = [(945.8 \pm 46.3)\text{nb/GeV}^2]\exp[t(4.729 \pm 0.217)/\text{GeV}^2]$. Here it is necessary to clarify that the experimental points shown in Fig. 2 correspond to the BNL data for dN/dt at $P^{\pi^-}_{lab} = 18.3$ GeV [4] normalized to the $a^0_2(1320)$ formation cross section in the reaction $\pi^- p \to a^0_2(1320)n$ in such a way as it was done in Ref. [5]. According to this estimate, the total cross section σ for the reaction $\pi^- p \to a^0_0(980)n \to (\eta\pi^0)_S n$ at 18.3 GeV is approximately equal to 200 nb [5]. This number we refer to the m region from 0.8 to 1.2 GeV and to the whole region of $t \le 0$. Note that the above value of σ is in close agreement with the estimate presented in Ref. [22]. Comparing the indicated values of σ and $d\sigma/dt$ with the values of σ^π and $d\sigma^\pi/dt$ estimated in the previous section,[5] we obtain that $\sigma^\pi \approx 10.9$ nb makes up about 5.5% of the total reaction cross section $\sigma \approx 200$ nb, and that $d\sigma^\pi/dt$ at the maximum, located near $t \approx -0.0149$ GeV^2, ≈ 139 nb/GeV^2 accounts for approximately 14.7% of $(d\sigma/dt)|_{t\approx 0}$, see Fig. 2. However, the main point is that the whole value of $d\sigma^\pi/dt$ at given t, in fact, comes from the narrow region of m near the $K\bar{K}$ thresholds, see Figs. 1(a) and 1(f), whereas the values of the total differential cross section $d\sigma/dt$ are assembled over the m region which is at least by an order of magnitude wider, see, for example, Fig. 1(d). Thus, at low $-t$ and m near the $K\bar{K}$ thresholds, the π exchange contribution can be quite comparable with that of the G-parity conserving ρ_2 exchange.

Certainly, the b_1 exchange contribution cannot be fully rejected only on the basis of a good quality of the fit to

---

[4]There are many reactions in which a simplest shape of the solitary $f_0(980)$ resonance line, shown in Fig. 1(c), is drastically distorted at the expense of the interference of the $f_0(980)$ with accompanying background contributions. Thus, the $I = 0$ S wave amplitude of the reaction $\pi\pi \to \pi\pi$ in the $f_0(980)$ region has the form $T^0_0 = (e^{2i\delta_B(m)} - 1)/2i + e^{2i\delta_B(m)} m\Gamma_{f_0\pi\pi}(m)/D_{f_0}(m)$, where the phase of the smooth, elastic background, $\delta_B(m)$, is close to 90°. It is this circumstance that leads to the fact that the $f_0(980)$ resonance is observed in the corresponding $\pi\pi \to \pi\pi$ cross section as an interference dip. In our case, the amplitude M^π_{+-} includes the amplitude of the reaction $\pi^+\pi^- \to f_0(980) \to K\bar{K} \to a^0_0(980) \to \eta\pi^0$, which, along with the resonance phase, must possess the additional phase of the elastic nonresonant background in the $\pi\pi$ channel. That is the reason why the factor $e^{i\delta_B(m)}$ was introduced in Eq. (4). In similar situations, this is the simplest (and the conventional) way to account for the nonresonance contributions in accordance with the unitarity condition. Since the phase $\delta_B(m)$ is large, it is very important to take it into account. We know nothing about an analogous background phase in the $\eta\pi^0$ channel. However, in this case, the phase is common for the amplitudes in Eqs. (2)–(4), and hence it is absolutely inessential. The additional details to the aforesaid discussion see, for example, in Refs. [3,25–27].

[5]The curve for $d\sigma^\pi/dt$ shown in Fig. 1(e) has been reproduced also in Fig. 2 for convenience of the comparison of $d\sigma/dt$ and $d\sigma^\pi/dt$.

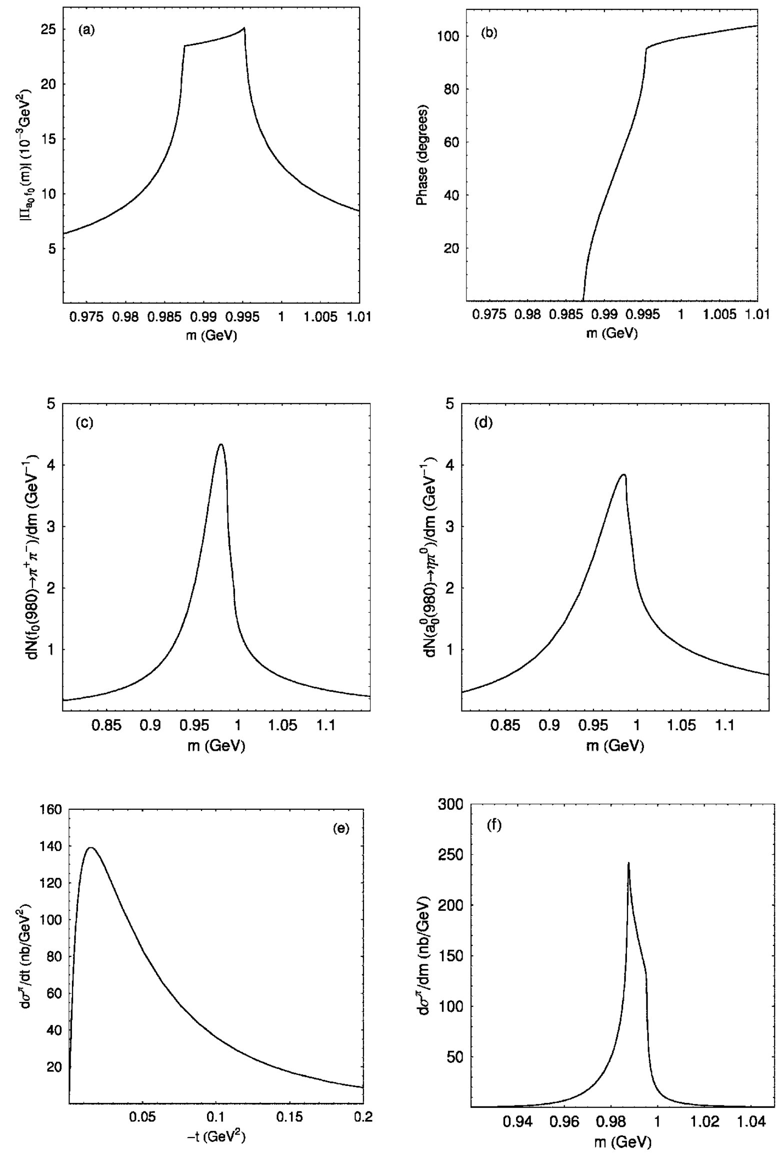

FIG. 1. (a) An example of the modulus of the $a_0^0(980) - f_0(980)$ transition amplitude $\Pi_{a_0 f_0}(m)$, see Eq. (5). (b) The phase of the $a_0^0(980) - f_0(980)$ transition amplitude $\Pi_{a_0 f_0}(m)$, see Eq. (5). (c) An example of the $\pi^+\pi^-$ mass spectrum corresponding to the solitary $f_0(980)$ resonance. (d) An example of the $\eta\pi^0$ mass spectrum corresponding to the solitary $a_0^0(980)$ resonance. (e) The differential cross section $d\sigma^\pi/dt$ for the reaction $\pi^- p \to f_0(980)n \to a_0^0(980)n \to (\eta\pi^0)_S n$, due to the π exchange mechanism, at $P_{lab}^{\pi^-} = 18.3$ GeV and for the region $0.8 \le m \le 1.2$ GeV. (f) $d\sigma^\pi/dm$ for the same reaction and $P_{lab}^{\pi^-}$ corresponding to the interval $0 \le -t \le 0.025$ GeV^2.

the measured t distributions [4,22,23] with the simplest function $C\exp(\Lambda t)$. In principle, the unpolarized data [4,22,23] are fitted equally well by using the expression $dN/dt = C_1 \exp(\Lambda_1 t) - tC_2 \exp(\Lambda_2 t)$ [5,22], where the first term can be identified with the contribution of the amplitude $M_{++}^{\rho_2}$ and the second one with that of the amplitude $M_{+-}^{b_1}$, see Eqs. (2) and (3). The particular example of such an approximation of the BNL data is shown in Fig. 2. The dotted curve in this figure presents $d\sigma/dt = d\sigma^{\rho_2}/dt + d\sigma^{b_1}/dt$, where $d\sigma^{\rho_2}/dt = 958.1(\text{nb/GeV}^2)\exp(7.6t/\text{GeV}^2)$ and $d\sigma^{b_1}/dt = -t2486.6(\text{nb/GeV}^4)\exp(5.8t/\text{GeV}^2)$ are shown by the long and short-dashed curves, respectively. For this fit $\chi^2 = 15.7$ and, as is seen, the dotted curve practically coincides with the solid one corresponding to the fit using the ρ_2 exchange model, for which $\chi^2 = 15.75$. In this example, the b_1 exchange yields approximately 37% of the integrated cross section. Thus, the χ^2 test does not allow to select between the two models.[6] The additional information on the ρ_2 and b_1 exchange model can be

[6]Let us emphasize that only polarized target experiments would allow us to elucidate unambiguously a question about the contribution of the b_1 exchange amplitude because they would make possible direct measurements the interference term $\text{Im}(M_{++}M_{+-}^*)$, together with the sum $|M_{++}|^2 + |M_{+-}|^2$.

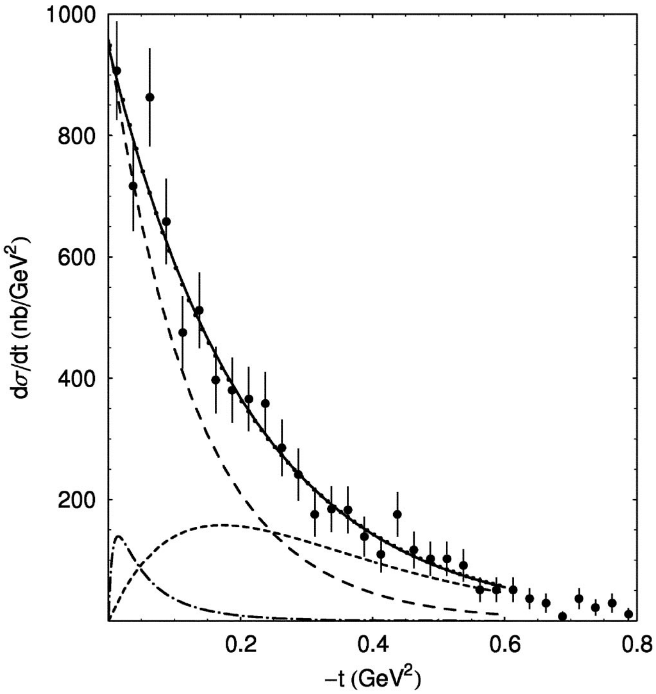

FIG. 2. The experimental points are the normalized BNL data [4] for $d\sigma/dt$ of the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ at $P_{lab}^{\pi^-} = 18.3$ [5]. The solid curve corresponds to the best fit of the data obtained in the ρ_2 exchange model. The dotted curve, which practically coincides with the solid one, gives an example of the approximation of the data with use of the ρ_2 and b_1 exchange model; in so doing, the long-dashed and short-dashed curves show the ρ_2 and b_1 exchange contributions to $d\sigma/dt$, respectively. The dot-dashed curve is the differential cross section $d\sigma^\pi/dt$ corresponding to Fig. 1(e).

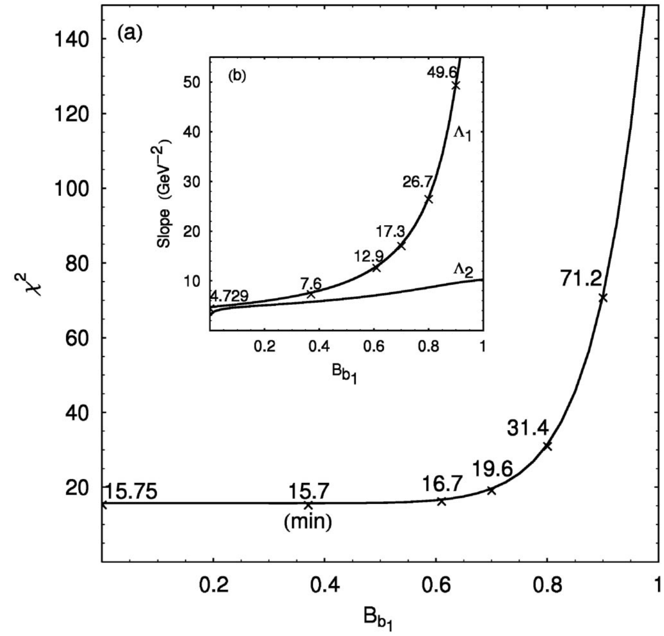

FIG. 3. The χ^2, Λ_1, and Λ_2 as functions of B_{b_1} obtained by fitting to the BNL data (shown in Fig. 2) with use of Eq. (6). The marked values of χ^2 in (a) and Λ_1 in (b) correspond (from left to right) to the values of $B_{b_1} = 0$, 0.37, 0.61, 0.7, 0.8, and 0.9.

obtained by fitting to the BNL data with use of the following parametrization:

$$d\sigma/dt = \sigma[(1 - B_{b_1})\Lambda_1 e^{\Lambda_1 t} - tB_{b_1}\Lambda_2^2 e^{\Lambda_2 t}], \quad (6)$$

where $\sigma = 200$ nb, Λ_1 and Λ_2 are free parameters, and B_{b_1} is a portion of the integrated cross section caused by the b_1 exchange. The resulting values of χ^2, Λ_1, and Λ_2 as functions of B_{b_1} are plotted in Fig. 3; the marked points on the χ^2 and Λ_1 curves correspond (from left to right) to the values of $B_{b_1} = 0$, 0.37, 0.61, 0.7, 0.8, and 0.9. As is seen from Fig. 3(a), χ^2 has a wide plateau. It remains practically unchanged with increasing B_{b_1} from 0 to 0.37 [7] and rises by one only when B_{b_1} reaches 0.61. At $B_{b_1} = 0.7$ $\chi^2 \approx 19.6$ and then increases very rapidly with B_{b_1}, see Fig. 3(a). The approximation of the BNL data obtained in the fit using Eq. (6) at $B_{b_1} = 0.7$ is shown in Fig. 4. Formally, this approximation is very similar to the approximations shown in Fig. 2 for $B_{b_1} = 0$ (the solid curve) and $B_{b_1} = 0.37$ (the dotted curve). The crucial difference between these examples is, however, in the values of the slope Λ_1 corresponding to the ρ_2 exchange contribution. Figure 3(b) shows the very strong increase of Λ_1 with B_{b_1}. If the values of $\Lambda_1 \approx 4.729$ GeV^{-2} (at $B_{b_1} = 0$) and $\Lambda_1 \approx 7.6$ GeV^{-2} (at $B_{b_1} = 0.37$) are quite reasonable from the Regge phenomenology standpoint for the secondary Regge exchanges, then the slope $\Lambda_1 \approx 17.3$ GeV^{-2} (at $B_{b_1} = 0.7$) is already very unlikely, as well as its larger values. Nevertheless, for completeness sake we consider the polarization effect in this case also, see Sec. VI.

[7]The χ^2 passes through its "invisible" minimum at $B_{b_1} \approx 0.37$.

V. POLARIZATION EFFECT IN THE ρ_2 AND π EXCHANGE MODEL

Taking into account the above uncertainty of information on the b_1 exchange contribution, it is reasonable to present in the first place the results of the calculation of the polarization effect due to the $a_0^0(980) - f_0(980)$ mixing in the model including only the ρ_2 and π exchange mechanisms.

In Figs. 5(a)–5(c) are shown $d\sigma/dm = \int[|M_{++}^{\rho_2}|^2 + |M_{+-}^{\pi}|^2]dt$, $d\sigma^{\rho_2}/dm = \int|M_{++}^{\rho_2}|^2 dt$, $I(m) = \int I(t,m)dt = \int 2\text{Im}[M_{++}^{\rho_2}(M_{+-}^{\pi})^*]dt$, pertaining to the $-t$ interval from 0 to 0.025 GeV^2 at $P_{lab}^{\pi^-} = 18.3$ GeV, and the corresponding asymmetry $A(0 \le -t \le 0.025\text{GeV}^2, m)$. Figs. 5(d)–5(f) show the same values but pertaining to the $-t$ interval from 0 to 0.2 GeV^2. In so doing, the parameters of the ρ_2 exchange, which we

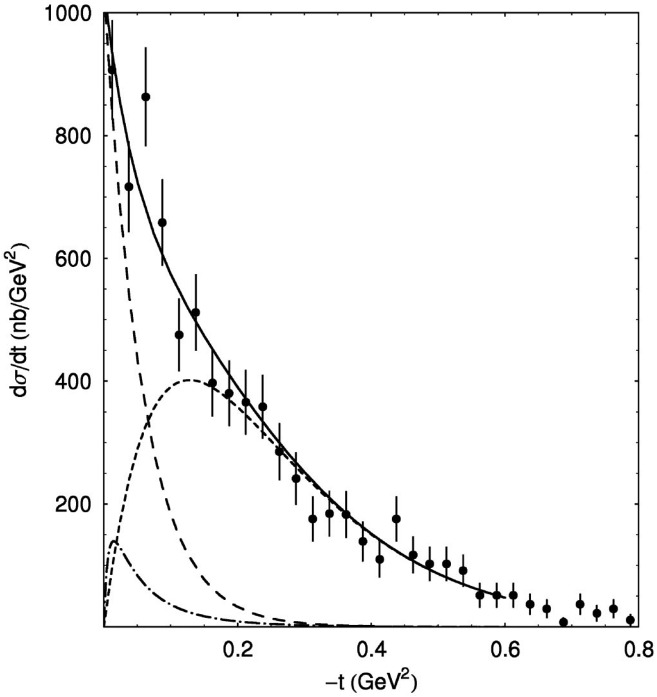

FIG. 4. The experimental data and the dot-dashed curve are the same as in Fig. 2. The solid curve corresponds to the fit to the data using Eq. (6) at $B_{b_1} = 0.7$. The long-dashed and short-dashed curves show the ρ_2 and b_1 exchange contributions to $d\sigma/dt$, respectively.

substitute in Eq. (2), correspond to the above-mentioned fit to the BNL data, shown by the solid curve in Fig. 2. Notice that the $I(m)$ and asymmetry are determined only up to the sign because the relative sign of the ρ_2 and π exchanges is unknown. Let us also note that, in the ρ_2 and π exchange model, the values $I(m)$ and asymmetry for the reaction $\pi^- p \to (\eta\pi^0)_S n$ and for the charge-symmetric reaction $\pi^+ n \to (\eta\pi^0)_S p$ are equal in magnitude but opposite in sign. Figure 5 shows that the polarization effect caused by the interference between the amplitudes $M^{\rho_2}_{++}$ and M^{π}_{+-} is quite considerable in any one of the intervals $0 \le -t \le 0.025, \ldots, 0.2$ GeV^2. A natural measure of the effect is the magnitude of a distinctive jump of the asymmetry, which takes place in the m region from 0.965 to 1.01 GeV. As is seen from Figs. 5(c) and 5(f), the corresponding difference between the maximal and minimal values of the asymmetry smoothed at the expense of the finite $\eta\pi^0$ mass resolution[8] turns out to be approximately equal to 0.95 in this mass region for the interval $0 \le -t \le 0.025$ GeV^2, see the dotted curve in Fig. 5(c), and ≈ 0.75 for the interval $0 \le -t \le 0.2$ GeV^2, see the dotted curve in Fig. 5(f). It is quite clear that the jump of the asymmetry is the exclusive consequence of the sharp variation, by 90°, of the phase of the $a_0^0(980) - f_0(980)$ transition amplitude between the K^+K^- and $K^0\bar{K}^0$ thresholds. The large magnitude of the jump is due to both the considerable value of the modulus of the amplitude $\Pi_{a_0 f_0}(m)$ and the enhancement of its manifestation owing to the π exchange mechanism.

Note that any noticeable variation of the interference pattern does not arise if one refits the BNL data in Fig. 2 by adding the π exchange contribution, indicated in the same figure, to the ρ_2 exchange one.

VI. POLARIZATION EFFECT IN THE ρ_2, b_1, AND π EXCHANGE MODEL

Let us now see what is changed by including the b_1 exchange contribution.

In the first place we note that if the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ is determined only by the ρ_2 and b_1 exchange mechanism, then this would lead to a rather cheerless m dependence of $A(t, m)$. The asymmetry in this case would be independent of m in the $a_0(980)$ peak region for any t. Indeed, the phase of the production $M_{++}(M_{+-})^*$ in the ρ_2 and b_1 exchange model is defined only by the Regge signature factors of the amplitudes $M^{\rho_2}_{++}$ and $M^{b_1}_{+-}$, see Eqs. (2) and (3), and $A(t, m) = \pm\cos\{\pi[\alpha_{\rho_2}(t) - \alpha_{b_1}(t)]/2\} \times 2|M^{\rho_2}_{++}||M^{b_1}_{+-}|/[|M^{\rho_2}_{++}|^2 + |M^{b_1}_{+-}|^2]$, where the factors involving the resonance m dependence simply cancel. Here $\pm$ denotes that the relative sign of the ρ_2 and b_1 exchanges is unknown. However, it is clear that the absolutely different feature should be expected in the presence of the amplitude M^{π}_{+-}: for low $-t$, $A(t, m)$ as a function of m must sharply vary near the $K\bar{K}$ thresholds.

We performed the calculations in the ρ_2, b_1, and π exchange model using the example of the fit to the BNL data described in Sec. IV and shown in Fig. 2. Recall that, in this example, the b_1 exchange contribution makes up approximately 37% of the total cross section. The polarization effect corresponding to this case is demonstrated in Fig. 6. The solid (and dotted) curves in this figure show the asymmetry $A(0 \le -t \le -t_2, m)$, pertaining to three $-t$ intervals, $0 \le -t \le 0.025, 0.1, 0.2$ GeV^2, without (and with) a Gaussian smearing (with the dispersion of ten MeV). The left and right parts of Fig. 6 correspond to the different choices of the sign of the b_1 exchange amplitude. The overall sign of the asymmetry is unknown and was chosen arbitrarily as well as in the case of the ρ_2 and π exchange model. Note that if the interference pattern corresponding, for example, to the left (right) part of Fig. 6 is realized for the reaction $\pi^- p \to (\eta\pi^0)_S n$, then, for the charge-symmetric reaction $\pi^+ n \to$

[8]Smoothing the initial values $d\sigma/dt$ and $I(m)$ had been made by using a Gaussian distribution with the dispersion of ten MeV. The measured mass distributions are always smoothed by finite resolutions of spectrometers. Therefore, for instance, the dotted curves in Fig. 5 can be directly compared with corresponding experimental histograms having approximately 10-MeV-wide mass step and a good statistical accuracy. Obtaining similar high quality data in the unpolarized $\eta\pi^0$ production experiments has already become commonplace [20–23].

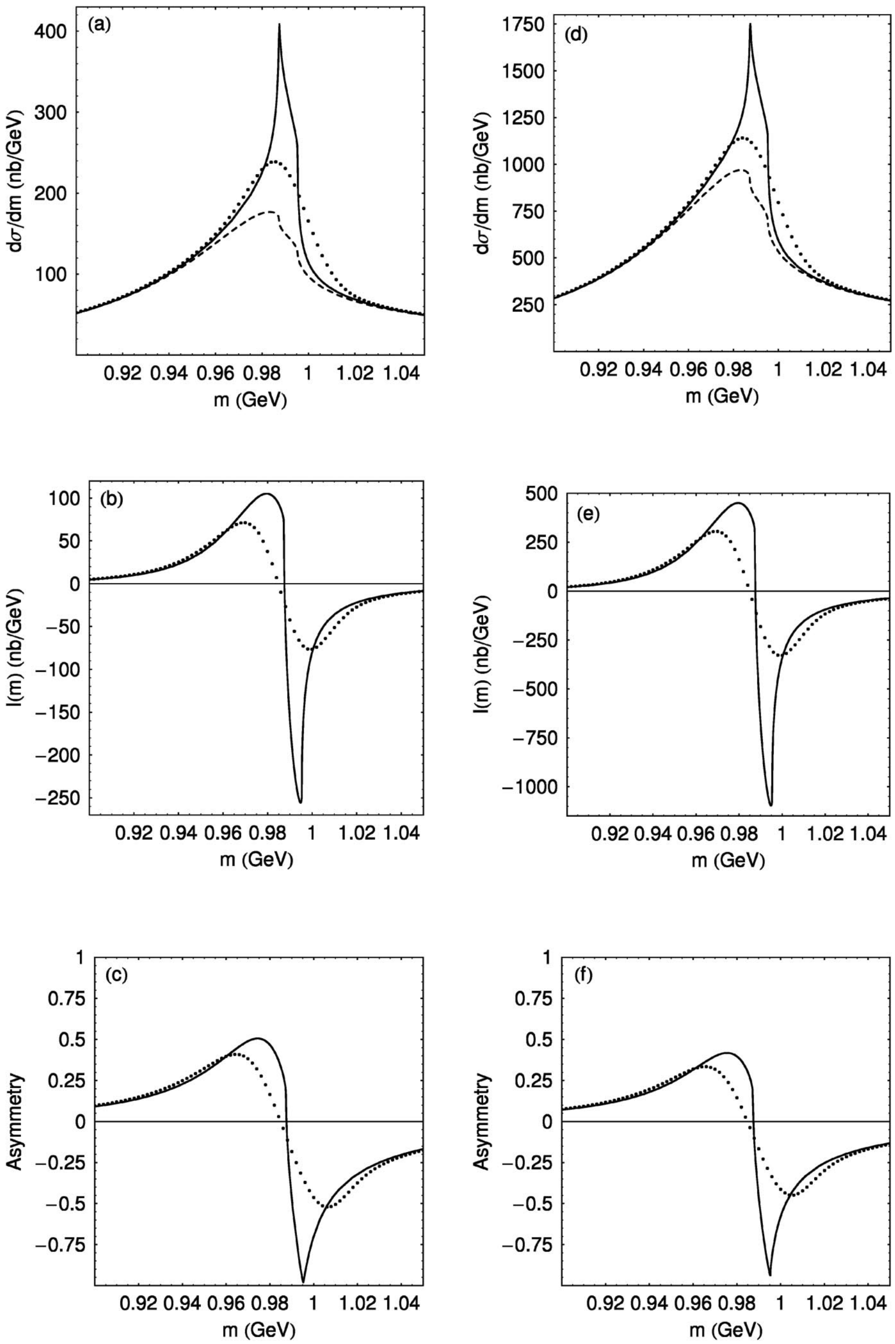

FIG. 5. Manifestation of the $a_0^0(980) - f_0(980)$ mixing effect in the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ on a polarized target at $P_{lab}^{\pi^-} = 18.3$ GeV in the ρ_2 and π exchange model. The solid curves in (a), (b), (c) show $d\sigma/dm$, $I(m)$, for the interval $0 \leq -t \leq 0.025$ GeV^2, and the corresponding asymmetry $A(0 \leq -t \leq 0.025\ \text{GeV}^2, m)$, respectively. The dashed curve in (a) shows the ρ_2 exchange contribution to $d\sigma/dm$. The dotted curves in (a), (b), (c) show $d\sigma/dm$, $I(m)$, smoothed with a Gaussian mass distribution with the dispersion of ten MeV, and the corresponding asymmetry, respectively. Plots (d), (e), (f) show the same as plots (a), (b), (c) but for the interval $0 \leq -t \leq 0.2$ GeV^2. The overall sign of the $I(m)$ and, consequently, the asymmetry is unknown and was chosen arbitrarily.

$(\eta\pi^0)_S p$, must be realized that corresponding to the right (left) part of Fig. 6, but with the opposite sign. Figure 6 clearly shows that the asymmetry pertaining to any interval of $0 \leq -t \leq 0.025, \ldots, 0.1$ GeV^2, as before, undergoes a jump of order one in the region $0.965 \leq m \leq 1.01$ GeV owing to the π exchange admixture. However, the jump takes place now not relative to the zeroth value of the asymmetry but relative to its value determined by the interference between the ρ_2 and b_1 exchange contributions. Thus, Figs. 5 and 6 together give already a quite exhaustive idea of the polarization effect caused by the $a_0^0(980) - f_0(980)$ mixing, which should be expected in

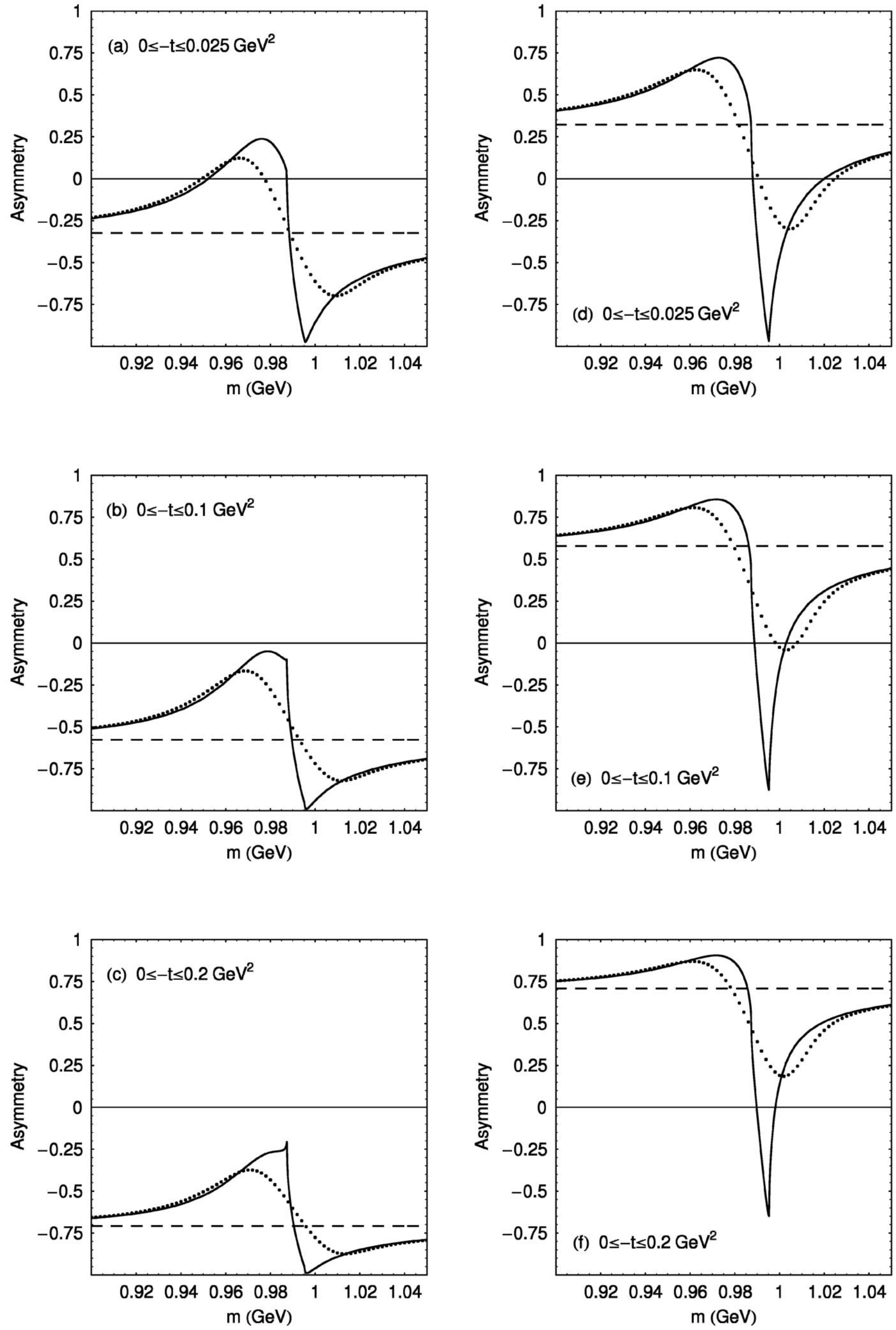

FIG. 6. The solid (and dotted) curves in (a), (b), (c) show the asymmetry $A(0 \leq -t \leq -t_2, m)$ for the different intervals of $-t$ in the ρ_2, π, and b_1 exchange model without (and with) a Gaussian mass smearing (with the dispersion of ten MeV). The dashed lines correspond to the asymmetry in the ρ_2 and b_1 exchange model. The overall sign of the asymmetry and the relative sign of the b_1 and π exchange amplitudes are unknown and were chosen arbitrarily. Plots (d), (e), (f) show the same as plots (a), (b), (c) but for the different choice of the sign of the b_1 exchange amplitude.

the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$. It should be mentioned in addition to this pattern that the polarization effect is found to be large in the low $-t$ region, as is clear from Fig. 7, even for such a practically improbable case when the b_1 exchange contribution makes up 70% of the total cross section (see discussion in Sec. IV).

Let us make yet some general remarks. First, it should be particularly emphasized that the reliable observation of the asymmetry jump does not require at all any very high quality $\eta\pi^0$ mass resolution that would be absolutely necessary to recognize the $a_0^0(980) - f_0(980)$ mixing manifestation in the $\eta\pi^0$ mass spectrum in unpolarized experiments. Really, the fine structure arising in $d\sigma/dm$ by the $a_0^0(980) - f_0(980)$ mixing is strongly shaded by a mass smearing, see Figs. 5(a) and 5(d), but in so doing the asymmetry jump remains, as is clear from Figs. 5(c),

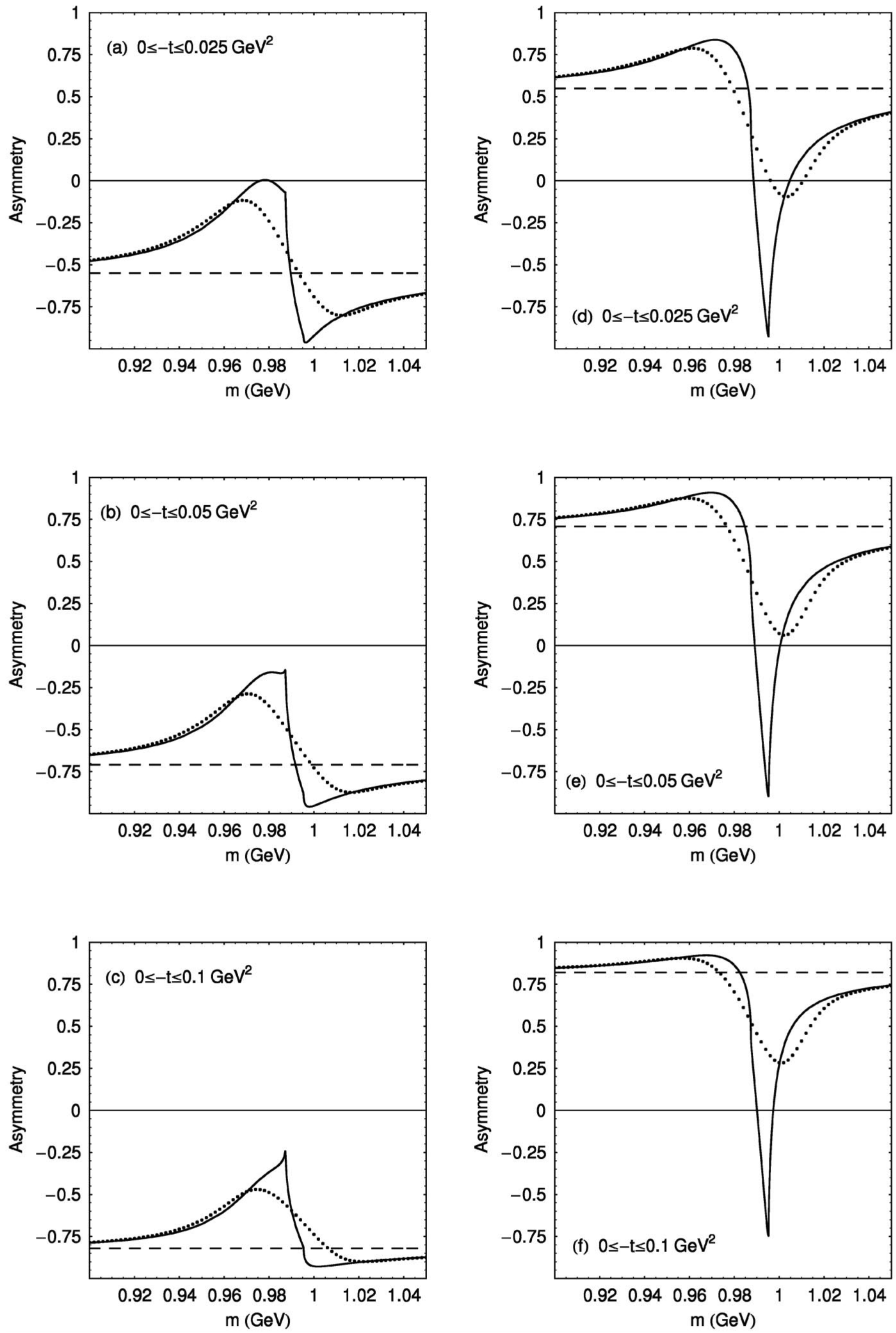

FIG. 7. The same as in Fig. 6 but for the ρ_2 and b_1 exchange contributions shown in Fig. 4.

5(f), 6, and 7, in spite of some smearing. Secondly, by virtue of the theoretically expected (and experimentally supported!) nearness of the π, ρ_2, and b_1 Regge trajectories, the energy dependence of the considered polarization effect (the asymmetry magnitude) should be expected to be rather weak. Therefore, the effect can be investigated, in fact, at any high energy, for example, in the range from eight to 100 GeV. Moreover, even if we slightly err, guiding by the simplest Regge pole model in constructing the G-parity conserving amplitudes (for example, in the choice of their phases), we are sure of that a jump of a single-spin asymmetry in the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ in the $\eta\pi^0$ invariant mass region between the K^+K^- and $K^0\bar{K}^0$ thresholds will necessarily take place owing to the specific m dependence of the $a_0^0(980) - f_0(980)$ transition amplitude [see Figs. 1(a) and 1(b), its enhancement due to the one-pion exchange mechanism of the $f_0(980)$ production, and the suppression of the b_1 exchange amplitude in the low $-t$ region.

VII. CONCLUSION

Thus, we conclude that the interference between the amplitudes M_{++} and M_{+-} in the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ at small $-t$, which can be measured in polarized target experiments, turns out to be extremely sensitive to the mixing of the $a_0^0(980)$ and $f_0(980)$ states. The asymmetry jump near the $K\bar{K}$ thresholds is the direct consequence of the $a_0^0(980) - f_0(980)$ mixing, and even very rough indications in the presence of such a jump will allow us to draw inferences about the existence of the mixing effect.

Currently, experimental investigations utilizing the polarized beams and targets are on the rise. Therefore, this analysis seems to be quite opportune. The relevant experiments on the reaction $\pi^- p \to \eta\pi^0 n$ on a polarized proton target, in principle, can be realized at High Energy Accelerator Research Organization (KEK, Tsukuba), BNL, IHEP, CERN (COMPASS), Fermi National Accelerator Laboratory (Batavia), Institute of Theoretical and Experimental Physics (Moscow), and Institut für Kernphysik in Jülich. Discovery of the $a_0^0(980) - f_0(980)$ mixing would open one more interesting page in investigation of the nature of the puzzling $a_0^0(980)$ and $f_0(980)$ states. Of course, the general idea of using polarization phenomena as an effective tool for the observation of the $a_0^0(980) - f_0(980)$ mixing connected with a great variation (by about 90°) of the phase of the $a_0^0(980) - f_0(980)$ mixing amplitude in the narrow energy region (8 MeV) between the K^+K^- and $K^0\bar{K}^0$ thresholds is also applicable to other reactions.

ACKNOWLEDGMENTS

This work was supported in part by the RFBR Grant No. 02-02-16061 and the Presidential Grant No. 2339.2003.2 for support of Leading Scientific Schools.

[1] N. N. Achasov, Nucl. Phys. **A728**, 425 (2003); Yad. Fiz. **65**, 573 (2002) [Phys. At. Nucl. **65**, 546 (2002)].

[2] N. N. Achasov and G. N. Shestakov, Phys. Rev. Lett. **92**, 182001 (2004). In this paper the factor $(s/s_o)^{-1}$ was omitted in Eqs. 4 and 5.

[3] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **88B**, 367 (1979); Yad. Fiz. **33**, 1337 (1981) [Sov. J. Nucl. Phys. **33**, 715 (1981)]; Usp. Fiz. Nauk **142**, 361 (1984) [**27**, 161 (1984)].

[4] A. R. Dzierba, in *Proceedings of the Second Workshop on Physics and Detectors for DAΦNE'95, Frascati, 1995*, edited by R. Baldini, F. Bossi, G. Capon, and G. Pancheri, (INFN, Frascati, 1996), p. 99.

[5] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **56**, 212 (1997); Yad. Fiz. **60**, 1669 (1997) [Phys. Atom. Nucl. **60**, 1522 (1997)].

[6] O. Krehl, R. Rapp, and J. Speth, Phys. Lett. B **390**, 23 (1997).

[7] B. Kerbikov and F. Tabakin, Phys. Rev. C **62**, 064601 (2000).

[8] F. E. Close and A. Kirk, Phys. Lett. B **489**, 24 (2000).

[9] A. E. Kudryavtsev and V. E. Tarasov, Pis'ma Zh. Eksp. Teor. Fiz. **72**, 589 (2000) [JETP Lett. **72**, 410 (2000)].

[10] V. Yu. Grishina *et al.*, Phys. Lett. B **521**, 217 (2001).

[11] N. N. Achasov and A. V. Kiselev, Phys. Lett. B **534**, 83 (2002).

[12] D. Black, M. Harada, and J. Schechter, Phys. Rev. Lett. **88**, 181603 (2002).

[13] A. E. Kudryavtsev *et al.*, Phys. Rev. C **66**, 015207 (2002); Yad. Fiz. **66**, 1994 (2003) [Phys. At. Nucl. **66**, 1946 (2003)].

[14] M. Buescher *et al.*, hep-ph/0301126.

[15] L. A. Kondratyuk *et al.*, Yad. Fiz. **66**, 155 (2003) [Phys. At. Nucl. **66**, 152 (2003)].

[16] C. Hanhart, in *Scalar Mesons: An Interesting Puzzle for QCD, Utica, NY, 2003*, edited by A. H. Fariborz, AIP Conf. Proc. No. 688 (AIP, New York, 2003), p. 61; C. Hanhart, Phys. Rep. **397**, 155 (2004).

[17] C. Amsler and N. A. Törnqvist, Phys. Rep. **389**, 61 (2004).

[18] M. Buescher, Acta Phys. Pol. B **35**, 1055 (2004); nucl-ex/0401010.

[19] Z.-G. Wang, W.-M. Yang, and S.-L. Wan, hep-ph/0401042.

[20] S. Teige *et al.*, Phys. Rev. D **59**, 012001 (1998).

[21] A. R. Dzierba *et al.*, Phys. Rev. D **67**, 094015 (2003).

[22] GAMS Collaboration, S. A. Sadovsky, in *Hadron Specroscopy, Seventh International Conference, Upton, NY, 1997*, edited by S. U. Chung and H. J. Willutzki, AIP Conf. Proc. No. 432 (AIP, New York, 1998).

[23] D. Alde *et al.*, Yad. Fiz. **62**, 462 (1999) [Phys. At. Nucl. **62**, 421 (1999)].

[24] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[25] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **58**, 054011 (1998); Yad. Fiz. **62**, 548 (1999) [Phys. At. Nucl. **62**, 505 (1999)].

[26] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Yad. Fiz. **32**, 1098 (1980) [Sov. J. Nucl. Phys. **32**, 566 (1980)].

[27] A. D. Martin, E. N. Ozmutlu, and E. J. Squires, Nucl. Phys. **B121**, 514 (1977).

[28] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **96B**, 168 (1980); N. N. Achasov and G. N. Shestakov, Z. Phys. C **41**, 309 (1988).

[29] N. N. Achasov and V. V. Gubin, Phys. Rev. D **63**, 094007 (2001); Yad. Fiz. **65**, 1566 (2002) [Phys. At. Nucl. **65**, 1528 (2002)]; N. N. Achasov and A. V. Kiselev, Phys. Rev. D **68**, 014006 (2003).

[30] P. Colangelo and F. De Fazio, Phys. Lett. B **559**, 49 (2003).

[31] J. Gunter *et al.*, Phys. Rev. D **64**, 072003 (2001).

[32] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **67**, 114018 (2003).