Heavy-quark symmetry and the electromagnetic decays of excited charmed strange mesons

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Heavy-hadron chiral perturbation theory (HHχPT) is applied to the decays of the even-parity charmed strange mesons, $D_{s0}(2317)$ and $D_{s1}(2460)$. Heavy-quark spin-symmetry predicts the branching fractions for the three electromagnetic decays of these states to the ground states D_s and D_s^* in terms of a single parameter. The resulting predictions for two of the branching fractions are significantly higher than current upper limits from the CLEO experiment. Leading corrections to the branching ratios from chiral loop diagrams and spin-symmetry violating operators in the HHχPT Lagrangian can naturally account for this discrepancy. Finally the proposal that the $D_{s0}(2317)$ ($D_{s1}(2460)$) is a hadronic bound state of a $D(D^*)$ meson and a kaon is considered. Leading order predictions for electromagnetic branching ratios in this molecular scenario are in very poor agreement with existing data.

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I. INTRODUCTION

The discovery of the $D_{s0}(2317)$ [1] and $D_{s1}(2460)$ [2] has revived interest in excited charmed mesons. The dominant decay modes of these states are $D_{s0}(2317) \to D_s\pi^0$ and $D_{s1}(2460) \to D_s^*\pi^0$, with widths less than 7 MeV [2]. There is experimental evidence indicating that $D_{s0}(2317)$ and $D_{s1}(2460)$ are $J^P = 0^+$ and 1^+ states, respectively [3,4]. Had the masses of the 0^+ and 1^+ states been above the threshold for the S-wave decay into D mesons and kaons, as anticipated in quark model [5,6] as well as lattice calculations [7–9], they would have had widths of a few hundred MeV. In reality, the unexpectedly low masses make those decays kinematically impossible. The only available strong decay modes violate isospin, accounting for the narrow widths.

The $D_{s0}(2317)$ and $D_{s1}(2460)$ can also decay electromagnetically. The possible decays are $D_{s1}(2460) \to D_s^*\gamma$, $D_{s1}(2460) \to D_s\gamma$, and $D_{s0}(2317) \to D_s^*\gamma$. The decay $D_{s0}(2317) \to D_s\gamma$ is forbidden by angular momentum conservation. In the heavy-quark limit, both the three electromagnetic decays and the two strong decays are related by heavy-quark spin symmetry [10]. Belle has observed the decay $D_{s1}(2460) \to D_s\gamma$ from $D_{s1}(2460)$ produced in the decays of B mesons [3] and from continuum e^+e^- production [4]. The ratio of the electromagnetic branching fraction to the isospin violating one-pion decay reported by the experiment is

$$\frac{\mathrm{Br}[D_{s1}(2460) \to D_s\gamma]}{\mathrm{Br}[D_{s1}(2460) \to D_s^*\pi^0]} = \begin{cases} 0.38 \pm 0.11 \pm 0.04 & [3] \\ 0.55 \pm 0.13 \pm 0.08 & [4] \end{cases}. \quad (1)$$

In each case the first error is statistical and the second systematic. The other electromagnetic decays have not been observed. CLEO quotes the following bounds on the branching fraction ratios [2]:

$$\frac{\mathrm{Br}[D_{s1}(2460) \to D_s^*\gamma]}{\mathrm{Br}[D_{s1}(2460) \to D_s^*\pi^0]} < 0.16$$
$$\frac{\mathrm{Br}[D_{s0}(2317) \to D_s^*\gamma]}{\mathrm{Br}[D_{s0}(2317) \to D_s\pi^0]} < 0.059. \quad (2)$$

(The BELLE collaboration quotes weaker lower bounds of 0.31 and 0.18, respectively, for these ratios [4].)

In this paper the decays of the $D_{s0}(2317)$ and $D_{s1}(2460)$ are analyzed using heavy-hadron chiral perturbation theory (HHχPT) [11]. HHχPT is an effective theory applicable to the low energy strong and electromagnetic interactions of particles containing a heavy quark. It incorporates the approximate heavy-quark and chiral symmetries of QCD. Corrections to leading order predictions can be computed in an expansion in $\Lambda_{\mathrm{QCD}}/m_Q$, M/Λ_χ, and p/Λ_χ, where m_Q is the heavy-quark mass, M is a Goldstone boson mass, p is the typical momentum in the decay, and Λ_χ is the chiral symmetry breaking scale.

In Sec. II, the leading order HHχPT predictions for the branching ratios are derived:

$$\frac{\mathrm{Br}[D_{s1}(2460) \to D_s^*\gamma]}{\mathrm{Br}[D_{s1}(2460) \to D_s^*\pi^0]} = 0.37 \pm 0.07$$
$$\frac{\mathrm{Br}[D_{s0}(2317) \to D_s^*\gamma]}{\mathrm{Br}[D_{s0}(2317) \to D_s\pi^0]} = 0.13 \pm 0.03. \quad (3)$$

(Leading order calculations of strong and electromagnetic decays were first done in Refs. [12–14].) These predictions deviate significantly from the CLEO limits. At next-to-leading order (NLO) there are $O(1/m_Q)$ suppressed heavy-quark spin-symmetry violating operators as well as one-loop chiral corrections to the electromagnetic decays. Once these effects are included, predictions for the ratios in Eq. (2) can be made consistent with the present experimental bounds with coupling constants in the Lagrangian of natural size.

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The splitting between the even- and odd-parity doublets should be approximately the same for both bottom strange and charmed strange mesons. Therefore it is likely that the B_s even-parity states will be below threshold for decay into kaons and narrow like their charm counterparts. The calculations of this paper can also be applied to the electromagnetic and strong decays of even-parity B_s mesons when these states are discovered.

The leading order HHχPT Lagrangian used in this paper is invariant under nonlinearly realized chiral $SU(3)_L \times SU(3)_R$ and no further assumptions are made about the mechanism of chiral symmetry breaking. Models that treat the $D_{s0}(2317)$ and $D_{s1}(2460)$ as the 0^+ and 1^+ chiral partners of the ground state charm strange mesons are proposed in Refs. [13,15–17]. In these models, referred to as parity-doubling models, the even-parity and odd-parity mesons are placed in a linear representation of chiral $SU(3)_L \times SU(3)_R$. These fields couple in a chirally invariant manner to a field Σ that transforms in the $(3, \bar{3})$ of $SU(3)_L \times SU(3)_R$. The Σ field develops a vacuum expectation value that breaks the chiral symmetry. The resulting nonlinear sigma model of Goldstone bosons coupled to heavy mesons has the same operators as the HHχPT Lagrangian used in this paper. The assumed mechanism of chiral symmetry breaking in parity-doubling models predicts relationships between coupling constants in the HHχPT Lagrangian. For example, the parity-doubling models predict that the hyperfine splittings of the even- and odd-parity doublets are equal. This is in agreement with experimental observations. Other relationships between coupling constants in HHχPT are predicted [18,19] by the theory of algebraic realizations of chiral symmetry [20], in which hadrons are placed in reducible representations of $SU(3)_L \times SU(3)_R$. QCD sum rules have also been used to calculate some of the HHχPT couplings [21]. When more data on the electromagnetic decays of even-parity D_s and B_s mesons becomes available, the formulae derived in this paper can be used to extract the relevant couplings and test these theories.

The low mass of the $D_{s0}(2317)$ and $D_{s1}(2460)$ has prompted reexamination of quark models [22–26] as well as speculation that these states are exotic. Possibilities include DK molecules [27–29], $D_s\pi$ molecules [30], and tetraquarks [28,31–35]. Masses have been calculated in lattice QCD [36–38], heavy-quark effective theory (HQET) sum rules [39,40], and potential as well as other models [22,24–26,41,42]. The results of some of these papers are contradictory. For example, the lattice calculation of Ref. [36] yields a $0^+ - 0^-$ mass splitting about 120 MeV greater than experimentally observed, quotes errors of about 50 MeV, and argues this is evidence for an exotic interpretation of the state. On the other hand, the lattice calculation of Ref. [37] obtains similar numerical results but concludes that uncertainties in the calculation are large enough to be consistent with a conventional $c\bar{s}$ P-wave state. Some quark model analyses [25,26,41] conclude that interpreting the states as conventional $c\bar{s}$ P-wave mesons naturally fits the observed data; others reach the opposite conclusion [22,42].

There have been some attempts to determine the nature of the $D_{s0}(2317)$ and $D_{s1}(2460)$ from the observed pattern of decays [23] as well as their production in b-hadron decays [43–46]. Ref. [23] argues that the total width and electromagnetic branching ratios can distinguish between $c\bar{s}$ P-wave states and DK molecules, and gives predictions for these branching ratios calculated in the quark model. Refs. [43,44] argue that the observed branching fractions for $B \to D_{s0}(2317)D^{(*)}$ and $B \to D_{s1}(2460)D^{(*)}$ are smaller then expected for $c\bar{s}$ P-wave states, suggesting that these states are exotic. These analyses assume an unproven (but plausible) factorization conjecture for the decays as well as quark model estimates for the $D_{s0}(2317)$ and $D_{s1}(2460)$ decay constants, and have recently been extended to Λ_b [45] and semileptonic B_s decays [46].

Section III addresses the question of whether a model independent analysis of the decays can provide insight into the nature of the $D_{s0}(2317)$ and $D_{s1}(2460)$. In HHχPT the fields describing the 0^+ and 1^+ mesons are added to the Lagrangian by hand. The only assumption made about these states is that the light degrees of freedom in the hadron are in the $\bar{3}$ of $SU(3)$ and have $j^p = \frac{1}{2}^+$. (In this paper, J^P refers to the angular momentum and parity of a heavy meson, and j^p to the angular momentum and parity of the light degrees of freedom.) Light degrees of freedom with these quantum numbers could be an $\bar{s}$ quark in an orbital P-wave or $\bar{s}q\bar{q}$ quarks all in an S-wave. Therefore, a conventional quark model $c\bar{s}$ P-wave state and an unconventional $c\bar{s}\,\bar{q}\,q$ tetraquark state will be represented by fields having the same transformation properties in the HHχPT Lagrangian. HHχPT predictions for the ratios in Eqs. (1)-(2) are valid for either interpretation, and so cannot distinguish between these two scenarios.

However, if $D_{s0}(2317)$ ($D_{s1}(2460)$) is modeled as a bound state of a D (D^*) meson and a kaon the predictions for the electromagnetic branching ratios will be different. In this scenario, instead of adding the even-parity heavy-quark doublet to the Lagrangian by hand, the dynamics of the theory containing only the ground state heavy-quark doublet and Goldstone bosons generate the observed $D_{s0}(2317)$ and $D_{s1}(2460)$. This interpretation has been pursued in Refs. [47–51]. In this scenario the binding energy is only about 40 MeV, so the mesons in the hadronic bound state are nonrelativistic. The decay rates can be calculated by convolving the unknown nonrelativistic wavefunction with leading order HHχPT amplitudes for $D^{(*)}K \to D_s^{(*)}\gamma$ and $D^{(*)}K \to D_s^{(*)}\pi^0$. Dependence on the bound state wavefunction drops out of the ratios in Eqs. (1) and (2). The resulting predictions for these branching ratios are much larger than experiment. Furthermore, the branching ratio for $D_{s1}(2460) \to D_s\gamma$ is predicted to be the smallest of the three, in direct conflict with experimental observations. A DK molecular interpretation of the $D_{s0}(2317)$ and $D_{s1}(2460)$ is disfavored by the existing data on electromagnetic branching fractions.

II. ELECTROMAGNETIC AND STRONG DECAYS IN HHχPT

In the heavy-quark limit, hadrons containing a single heavy quark fall into doublets of the $SU(2)$ heavy-quark spin-symmetry group. Heavy hadrons can be classified by the total angular momentum and parity quantum numbers of their light degrees of freedom, j^p. The ground state doublet has $j^p = \frac{1}{2}^-$ and therefore the mesons in the doublet are 0^- and 1^- states. In HHχPT, these states are combined into a single field [11]

$$H_a = \frac{1+\not{v}}{2}(H_a^\mu \gamma_\mu - H_a\gamma_5), \tag{4}$$

where a is an $SU(3)$ index. In the charm sector, H_a consists of the $(D^0, D^+, D_s^+) \sim (c\bar{u}, c\bar{d}, c\bar{s})$ pseudoscalar mesons and H_a^μ are the $(D^{0*}, D^{+*}, D_s^{+*})$ vector mesons. The doublet with light degrees of freedom $j^p = \frac{1}{2}^+$ consists of mesons whose quantum numbers are 0^+ and 1^+. These are combined into the field [52]

$$S_a = \frac{1+\not{v}}{2}(S_a^\mu \gamma_\mu\gamma_5 - S_a), \tag{5}$$

where the scalar states in the charm sector are $S_a = D_{0a}$ and the axial vectors are $S_a^\mu = D_{1a}$.

The relevant strong interaction terms in the HHχPT chiral Lagrangian are [11,53]

$$\begin{aligned}\mathcal{L} = {}& \frac{f^2}{8}\mathrm{Tr}[\partial_\mu\Sigma\partial^\mu\Sigma^\dagger] + \frac{f^2 B_0}{4}\mathrm{Tr}[m_q\Sigma + \Sigma^\dagger m_q] - \mathrm{Tr}[\overline{H}_a iv\cdot D_{ba}H_b] + \mathrm{Tr}[\overline{S}_a(iv\cdot D_{ba} - \delta_{\mathrm{SH}}\delta_{ab})S_b] \\ & + g\mathrm{Tr}[\overline{H}_a H_b \not{A}_{ba}\gamma_5] + g'\mathrm{Tr}[\overline{S}_a S_b \not{A}_{ba}\gamma_5] + h(\mathrm{Tr}[\overline{H}_a S_b \not{A}_{ba}\gamma_5] + \mathrm{h.c.}) - \frac{\Delta_H}{8}\mathrm{Tr}[\overline{H}_a\sigma^{\mu\nu}H_a\sigma_{\mu\nu}] \\ & + \frac{\Delta_S}{8}\mathrm{Tr}[\overline{S}_a\sigma^{\mu\nu}S_a\sigma_{\mu\nu}].\end{aligned} \tag{6}$$

The first two terms in Eq. (6) are the leading order chiral Lagrangian for the octet of Goldstone bosons. f is the octet meson decay constant. The conventions for defining Σ in terms of meson fields, the chiral covariant derivative, D_{ab}, and the axial vector field, A_{ab}^μ, are identical to those of Ref. [54]. Here m_q is the light quark mass matrix. The third and fourth terms in Eq. (6) contain the kinetic terms for the fields H_a and S_a and the couplings to two and more pions determined by chiral symmetry. The parameter δ_{SH} is the residual mass of the S_a field. The H_a residual mass can be set to zero by an appropriate definition of the H_a field, and this convention is adopted here. Then δ_{SH} is the difference between the spin-averaged masses of the even- and odd-parity doublets in the heavy quark limit. The second line contains the couplings of H_a and S_a to the axial vector field $A_{ab}^\mu = -\partial^\mu\pi_{ab}/f + \ldots$. These terms are responsible for transitions involving a single pion. The couplings g, g', and h are parameters that are not determined by the HHχPT symmetries. The last two terms in Eq. (6) are operators that give rise to $1^- - 0^-$ and $1^+ - 0^+$ hyperfine splittings, which are Δ_H and Δ_S, respectively. Since the splittings should vanish in the heavy-quark limit, $\Delta_S \sim \Delta_H \sim \Lambda_{\mathrm{QCD}}^2/m_Q$. The parameters Δ_S and Δ_H are independent in HHχPT so there is no relation between the hyperfine splitting in the even- and odd-parity doublets. In parity-doubling models, $\Delta_H = \Delta_S$ at tree level, in agreement with the observation that hyperfine splittings are equal to within 2 MeV.

Electromagnetic effects are incorporated by gauging the $U(1)_{\mathrm{em}}$ subgroup of $SU(3)_L \times SU(3)_R$ and adding terms to the Lagrangian involving the gauge invariant field strength, $F_{\mu\nu}$. Gauging derivatives in Eq. (6) does not yield terms which can mediate the $(0^+, 1^+) \to (0^-, 1^-)$ electromagnetic decays at tree level. The leading contribution to these decays comes from the operator

$$\mathcal{L} = \frac{e\tilde{\beta}}{4}\mathrm{Tr}[\overline{H}_a S_b\sigma^{\mu\nu}]F_{\mu\nu}Q_{ba}^\xi, \tag{7}$$

where $Q_{ba}^\xi = \frac{1}{2}(\xi Q\xi^\dagger + \xi^\dagger Q\xi)_{ba}$, $\xi^2 = \Sigma$, and $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ is the light quark electric charge matrix. A tree level calculation of the decay rates using Eq. (7) shows that

$$\begin{aligned}\Gamma[1_a^+ \to 1_a^-\gamma] &= \frac{2}{3}\alpha e_q^2\tilde{\beta}^2\frac{m_{1_a^-}}{m_{1_a^+}}|\mathbf{k}_\gamma|^3 \\ \Gamma[1_a^+ \to 0_a^-\gamma] &= \frac{1}{3}\alpha e_q^2\tilde{\beta}^2\frac{m_{0_a^-}}{m_{1_a^+}}|\mathbf{k}_\gamma|^3 \\ \Gamma[0_a^+ \to 1_a^-\gamma] &= \alpha e_q^2\tilde{\beta}^2\frac{m_{1_a^-}}{m_{0_a^+}}|\mathbf{k}_\gamma|^3,\end{aligned} \tag{8}$$

where e_q is the electric charge of the light valence quark, α is the fine-structure constant, and $\tilde{\beta}$ is the unknown parameter in Eq. (7). The three-momentum of the photon in the decay is $\mathbf{k}_\gamma$ and $m_{J_a^P}$ is the mass of the heavy-meson with quantum numbers J_a^P. In the heavy-quark limit, the members of each doublet are degenerate and the phase space is the same for all three decays. If differences in phase space are neglected the decay rate ratios are $\Gamma[1_a^+ \to 1_a^- \gamma]:\Gamma[1_a^+ \to 0_a^- \gamma]:\Gamma[0_a^+ \to 1_a^- \gamma] = 2:1:3$. Differences in the phase space factors are formally $O(1/m_Q)$ but in practice it is critical to include these effects to make sensible predictions. For the charmed strange mesons using the physical masses gives

$$\begin{aligned}\Gamma[D_{s1}(2460) \to D_s^* \gamma] &= (\tilde{\beta}\ \text{GeV})^2 15.6\ \text{keV} \\ \Gamma[D_{s1}(2460) \to D_s \gamma] &= (\tilde{\beta}\ \text{GeV})^2 18.7\ \text{keV} \\ \Gamma[D_{s0}(2317) \to D_s^* \gamma] &= (\tilde{\beta}\ \text{GeV})^2 5.6\ \text{keV}.\end{aligned} \tag{9}$$

The rates are then in the following ratios:

$$\begin{aligned}\Gamma[D_{s1}(2460) \to D_s^* \gamma]&:\Gamma[D_{s1}(2460) \to D_s \gamma] \\ &:\Gamma[D_{s0}(2317) \to D_s^* \gamma] \\ &= 0.83:1.0:0.30.\end{aligned} \tag{10}$$

Note that the rate for $D_{s1}(2460) \to D_s\gamma$, smallest in the exact heavy-quark limit, is actually the largest when phase space effects are included since $|\mathbf{k}_\gamma|$ is largest for this decay.

To compare with the ratio measured by Belle, the isospin violating strong decays must be calculated. These decays proceed through $\eta - \pi^0$ mixing. The result is [14]

$$\begin{aligned}\Gamma[D_{s1}(2460) \to D_s^* \pi^0] &= \frac{h^2\theta^2}{3\pi f^2}\frac{m_{D_s^*}}{m_{D_{s1}(2460)}} E_{\pi^0}^2 |\mathbf{p}_{\pi^0}| \\ &= h^2 \begin{cases} 17.0\ \text{keV} & \text{if} \quad f = f_\pi = 130\ \text{MeV} \\ 9.8\ \text{keV} & \text{if} \quad f = f_\eta = 171\ \text{MeV}\end{cases} \\ \Gamma[D_{s0}(2317) \to D_s \pi^0] &= \frac{h^2\theta^2}{3\pi f^2}\frac{m_{D_s}}{m_{D_{s0}(2317)}} E_{\pi^0}^2 |\mathbf{p}_{\pi^0}| \\ &= h^2 \begin{cases} 16.9\ \text{keV} & \text{if} \quad f = f_\pi = 130\ \text{MeV} \\ 9.8\ \text{keV} & \text{if} \quad f = f_\eta = 171\ \text{MeV}\end{cases},\end{aligned} \tag{11}$$

where $\theta = \sqrt{3}/2(m_d - m_u)/(2m_s - m_d - m_u) = 0.01$ is the $\eta - \pi^0$ mixing angle. E_{π^0} and $\mathbf{p}_{\pi^0}$ are the energy and three momentum of the π^0, respectively. At tree level $f = f_\pi = f_\eta$. The difference between the two predictions provides an estimate of the uncertainty due to higher order $SU(3)$ violating effects.

The branching fraction ratios in Eqs. (1) and (2) depend only on the ratio $\tilde{\beta}^2/h^2$ at leading order in HHχPT. To obtain h^2 separately, a measurement of an excited strong decay width is needed. Currently h^2 cannot be extracted from the strange sector because only loose experimental bounds on $\Gamma[D_{s0}(2317)]$ and $\Gamma[D_{s1}(2460)]$ exist. Until measurements of these widths are dramatically improved, h^2 can be estimated using data on nonstrange even-parity D meson widths. CLEO [55] has observed preliminary evidence for the D_1^0 ($J^P = 1^+$) meson. (Here the superscript refers to the particle charge.) More recently, Belle [56] has reported observing even-parity D_0^0 ($J^P = 0^+$) and D_1^0 states. Finally, FOCUS [57] has observed broad structures in excess of background in the $D^+\pi^-$ and $D^0\pi^+$ invariant mass spectra. FOCUS does not claim to observe an excited charm resonance but does fit the excess with a Breit-Wigner to determine the resonance properties required to explain their data. The masses and widths reported by all three experiments are collected in Table I. The experiments all quote several errors which have been combined in quadrature for simplicity. Note that the CLEO and Belle measurements of the D_1^0 are consistent with each other while the central value of the D_0^0 mass obtained by FOCUS is 99 MeV higher than the central value of the Belle measurement. Furthermore, the FOCUS D_0^0 mass is actually greater than the mass of the $D_{s0}(2317)$. If the effect observed by FOCUS is a scalar D resonance, it seems implausible that this resonance is related to the $D_{s0}(2317)$ by $SU(3)$ symmetry. Therefore, the FOCUS data will not be used to estimate h^2. Even the masses obtained by CLEO and Belle are large compared to expectations based on $SU(3)$ symmetry. Combining the strange sector $0^+ - 0^-$ and $1^+ - 1^-$ mass splittings with $SU(3)$ symmetry leads to the prediction that the D_0^0 mass is 2212 MeV and the D_1^0 mass is 2355 MeV [13].

Applying the leading order expression for the decay widths of the nonstrange 0^+ and 1^+ mesons

$$\begin{aligned}\Gamma[D_1^0] &= \frac{h^2}{2\pi f^2}\left(\frac{m_{D^{+*}}}{m_{D_1^0}} E_{\pi^-}^2 |\mathbf{p}_{\pi^-}| + \frac{1}{2}\frac{m_{D^{0*}}}{m_{D_1^0}} E_{\pi^0}^2 |\mathbf{p}_{\pi^0}|\right) \\ \Gamma[D_0^0] &= \frac{h^2}{2\pi f^2}\left(\frac{m_{D^+}}{m_{D_0^0}} E_{\pi^-}^2 |\mathbf{p}_{\pi^-}| + \frac{1}{2}\frac{m_{D^0}}{m_{D_0^0}} E_{\pi^0}^2 |\mathbf{p}_{\pi^0}|\right),\end{aligned} \tag{12}$$

to the CLEO and Belle data yields $h^2 = 0.39 \pm 0.13$ from the 0^+ decays and $h^2 = 0.49 \pm 0.14$ from the 1^+ decay. The error in each case is obtained by adding in quadrature the uncertainty in the decay rate from varying the mass

TABLE I. Masses and widths of even-parity nonstrange charmed mesons, D_J^Q, where Q is the electric charge.

Experiment	Particle(J^P)	Mass (MeV)	Width (MeV)
CLEO [55]	$D_1^0(1^+)$	2461^{+53}_{-48}	290^{+110}_{-91}
Belle [56]	$D_0^0(0^+)$	2308 ± 36	276 ± 66
	$D_1^0(1^+)$	2427 ± 36	384^{+130}_{-105}
FOCUS [57]	$D_0^0(0^+)$	2407 ± 41	240 ± 81
	$D_0^+(0^+)$	2403 ± 38	283 ± 42

within the allowed range and the experimental error in the decay rate. If the two results are averaged $h^2 = 0.44 \pm 0.11$. This estimate of h^2 is consistent with the bound $h^2 \leq 0.86$ extracted from an Adler-Weisberger type sum rule for πB scattering [58]. (To obtain this bound $g = 0.27$ [54] is used in the result of Ref. [58].) It is also consistent with a calculation of $h = -0.52 \pm 0.17$ obtained using QCD sum rules in Ref. [21].

The lowest order HHχPT prediction for the ratio measured by the Belle collaboration is

$$\frac{\mathrm{Br}[D_{s1}(2460) \to D_s \gamma]}{\mathrm{Br}[D_{s1}(2460) \to D_s^* \pi^0]} = \left(\frac{\tilde{\beta}\ \mathrm{GeV}}{h}\right)^2 \times \begin{cases} 1.1 & \text{if} \quad f = f_\pi = 130\ \mathrm{MeV} \\ 1.9 & \text{if} \quad f = f_\eta = 171\ \mathrm{MeV} \end{cases}. \tag{13}$$

Averaging the results of the two Belle measurements, $(\tilde{\beta}\ \mathrm{GeV}/h)^2 = 0.40 \pm 0.08(0.23 \pm 0.05)$ or $|\tilde{\beta}| = 0.42 \pm 0.07(0.32 \pm 0.05)\ \mathrm{GeV}^{-1}$, where $f = f_\pi (f = f_\eta)$. The error is estimated by first combining the statistical and systematic errors in quadrature for each measurement, then combining the two measurements assuming they are independent. The extracted values for $\tilde{\beta}$ and h are consistent with expectations based on naturalness. Plugging the value of $(\tilde{\beta}\ \mathrm{GeV}/h)^2$ into expressions for the unobserved electromagnetic decays yields the following predictions for the branching fraction ratios

$$\begin{aligned} \frac{\mathrm{Br}[D_{s1}(2460) \to D_s^* \gamma]}{\mathrm{Br}[D_{s1}(2460) \to D_s^* \pi^0]} &= 0.37 \pm 0.07 \\ \frac{\mathrm{Br}[D_{s0}(2317) \to D_s^* \gamma]}{\mathrm{Br}[D_{s0}(2317) \to D_s \pi^0]} &= 0.13 \pm 0.03. \end{aligned} \tag{14}$$

Both predictions are in excess of bounds quoted by the CLEO experiment. Heavy-quark spin symmetry predicts branching ratios for the electromagnetic decays $D_{s1}(2460) \to D_s^* \gamma$ and $D_{s0}(2317) \to D_s^* \gamma$ that are more than a factor of 2 in excess of the experimental upper limits.

In the rest of this section the leading corrections to both electromagnetic and strong decays are analyzed. Because $\Lambda_{\mathrm{QCD}}/m_c \sim 1/3$, corrections to heavy-quark spin-symmetry predictions can be rather large for charm hadrons. These corrections can be systematically analyzed using HHχPT. For example, the pattern of deviations from heavy-quark spin-symmetry predictions for the one-pion decays of excited D-wave charm mesons [59] can be understood by analyzing the structure of spin-symmetry violating operators appearing at $O(1/m_c)$ in the HHχPT Lagrangian [60]. A ratio of decay widths for which the $O(1/m_c)$ correction vanishes agrees well with data. There is another ratio for which the leading order heavy-quark spin-symmetry prediction fails rather badly. In this case the leading $O(1/m_c)$ correction is multiplied by a large numerical coefficient. Thus HHχPT is a useful tool for determining the robustness of predictions based on heavy-quark spin symmetry.

Spin-symmetry violating operators that contribute to $S \to H$ transitions must have the Dirac structure $\mathrm{Tr}[\overline{H}\sigma^{\mu\nu} S \gamma_5]$ or $\mathrm{Tr}[\overline{H}\sigma^{\mu\nu} S \gamma^\alpha]$. Operators with $\overline{H}S$ conserve spin symmetry, while operators with $\overline{H}\sigma^{\mu\nu} S$ violate spin symmetry. Operators with $\overline{H}\gamma^\mu S$ and $\overline{H}\gamma^\mu \gamma_5 S$ are redundant since $\overline{H}\gamma^\mu S = \overline{H}\frac{1}{2}\{\gamma^\mu, \not{v}\} S = v^\mu \overline{H} S$ and $\overline{H}\sigma^{\mu\nu} S = \overline{H}\frac{1}{2}\{\sigma^{\mu\nu}, \not{v}\} S = -\epsilon_{\mu\nu\alpha\beta} v^\alpha \overline{H}\gamma^\beta \gamma_5 S$, while $\overline{H}\gamma_5 S = 0$. Spin-symmetry violating operators will be of the form $\mathrm{Tr}[\overline{H}\sigma^{\mu\nu} S \Gamma]$. Γ must be γ^α or γ_5 since the trace vanishes for $\Gamma = 1$, while $\Gamma = \gamma^\alpha \gamma_5$ and $\sigma^{\alpha\beta}$ are redundant because

$$\begin{aligned} \mathrm{Tr}[\overline{H}\sigma^{\mu\nu} S \gamma^\alpha \gamma_5] &= v^\alpha \mathrm{Tr}[\overline{H}\sigma^{\mu\nu} S \gamma_5] \\ \mathrm{Tr}[\overline{H}\sigma^{\mu\nu} S \sigma^{\alpha\beta}] &= i \mathrm{Tr}[\overline{H}\sigma^{\mu\nu} S (v^\alpha \gamma^\beta - v^\beta \gamma^\alpha)]. \end{aligned}$$

Reparametrization invariance [61] forbids operators with derivatives acting on the H or S fields [53]. For $S \to H\gamma$ decays, the lowest dimension, parity conserving, spin-symmetry violating operators are

$$\mathcal{L} = \frac{iee_Q \tilde{\beta}'}{8m_Q} \mathrm{Tr}[\overline{H}_a \sigma^{\mu\nu} S_a \gamma_5] F^{\alpha\beta} \epsilon_{\mu\nu\alpha\beta} + \frac{ee_Q \tilde{\beta}''}{8m_Q} \mathrm{Tr}[\overline{H}_a \sigma^{\mu\nu} S_a \gamma^\alpha] i\partial_\alpha F_{\mu\nu} + \text{h.c.}. \tag{15}$$

The $1/m_Q$ dependence (expected for any operator which violates heavy-quark spin symmetry) is explicit. The factors of i are required by time reversal invariance. The first operator in Eq. (15) and the leading operator in Eq. (7) have mass dimension five. The second operator in Eq. (15) has mass dimension six, so $\tilde{\beta}''$ has mass dimension -1 and is expected to scale like $1/\Lambda_\chi \sim \mathrm{GeV}^{-1}$. Since $2\sigma^{\mu\nu}\partial_\mu F_{\alpha\nu} = \sigma^{\mu\nu}\partial_\alpha F_{\mu\nu}$ for an abelian field strength there is a unique way of contracting indices in this operator. Note that there is a unique dimension six, spin-symmetry conserving operator proportional to $\mathrm{Tr}[\overline{H}_a S_b \sigma^{\mu\nu} Q^\xi_{ba}] iv \cdot \partial F_{\mu\nu}$. This operator gives slight deviations from the ratios in Eq. (10) since its contribution is suppressed by $|\mathbf{k}_\gamma|/\Lambda_\chi$ and $|\mathbf{k}_\gamma|$ differs for the three decays due to hyperfine splittings. These corrections should be smaller than corrections coming from operators in Eq. (15) so they are neglected in what follows.

Power counting is used to determine the importance of higher dimension operators in the Lagrangian. HHχPT is a double expansion in $\Lambda_{\mathrm{QCD}}/m_Q$ and Q/Λ_χ, where $Q \sim p \sim m_\pi \sim m_K$. Two additional mass scales appearing in the Lagrangian of Eq. (6) are the mass splitting between the H and S doublet fields, δ_{SH}, and the hyperfine splittings within each doublet. In the heavy-quark limit, the S field propagator is proportional to

$$\frac{i}{2(v \cdot k - \delta_{SH})}. \tag{16}$$

If δ_{SH} were to scale as Q^0, then the S propagator could be expanded in powers of $v \cdot k/\delta_{SH}$ since $v \cdot k \sim Q$. In the strange sector loops receive important contributions from momenta $\sim m_K = 495$ MeV. Numerically, $\delta_{SH} \approx 350$ MeV in the strange quark sector and ≈ 430 MeV in the nonstrange sector, so expanding in $v \cdot k/\delta_{SH}$ is a poor approximation. Therefore, $\delta_{SH} \sim Q$ is required. The hyperfine splittings are also treated as $\sim Q$ since numerically these splittings are ≈ 140 MeV which is $\sim m_\pi$.

There are $SU(3)$ violating corrections to the decay rates from operators such as $\mathrm{Tr}[\overline{H}_a S_b \sigma^{\mu\nu}] F_{\mu\nu} Q^\xi_{bc} m^\xi_{ca}$. These operators will give the same correction to all three electromagnetic decays in Eqs. (1) and (2), so their effect can be absorbed into the definition of $\tilde{\beta}$. However, if one were interested in relating the electromagnetic decays of strange and nonstrange heavy mesons these operators must be included explicitly.

The leading operator in Eq. (7) is order Q because of the derivative in $F_{\mu\nu}$. The first operator in Eq. (15) is treated as $\sim Q^2$ because it is suppressed by $\Lambda_{\mathrm{QCD}}/m_Q$ relative to the leading operator. The second operator in Eq. (15) has two derivatives and is also $1/m_Q$ suppressed. It is treated as $\sim Q^2$. The correctness of this power counting is confirmed by the calculation of one-loop corrections to the decay, since in order to properly renormalize these diagrams both counterterms in Eq. (15) are needed. In loop diagrams, integrals scale as Q^4, the propagators of the H and S fields as $\sim Q^{-1}$ and the propagators of Goldstone bosons as $\sim Q^{-2}$. The leading couplings of the H and S fields to kaons and pions are $\sim Q$ and the couplings of the photon to the kaons and pions are $\sim Q$. Calculations of the loop corrections are performed in $v \cdot A = 0$ gauge where the leading coupling of the photon to the heavy meson fields vanishes. Finally, there is an $\sim Q^0$ coupling of two heavy-meson fields to a Goldstone boson and photon which comes from gauging the derivative couplings of the heavy-meson fields to Goldstone bosons. The $HS\pi\gamma$ vertex obtained by gauging the derivative on the pion field in the leading $HS\pi$ coupling vanishes in $v \cdot A = 0$ gauge. With these power counting rules, the loop diagrams shown in Fig. 1 give an $O(Q^2)$ contribution to the $S \to H\gamma$ decays. Double lines are S fields, solid lines are H fields and the dotted lines are Goldstone bosons. For D_s decays the Goldstone bosons in these loops are K^+ and the virtual heavy mesons are neutral D's.

Including both the loop diagrams and tree level insertions of the operators in Eq. (15) yields:

$$\begin{aligned}
\hat{\Gamma}[1^+_a \to 1^-_a \gamma] &= 1 + \frac{2}{e_q \tilde{\beta}}\left(-\frac{e_Q \tilde{\beta}'}{m_Q} + F[m_{1^+_a} - m_{1^-_b}, m_{1^+_a} - m_{1^+_b}, |\mathbf{k}_\gamma|, M, \mu]\right) \\
\hat{\Gamma}[1^+_a \to 0^-_a \gamma] &= 1 + \frac{2}{e_q \tilde{\beta}}\left(\frac{e_Q \tilde{\beta}'}{m_Q} + \frac{e_Q \tilde{\beta}'' |\mathbf{k}_\gamma \mathbf{V}}{2m_Q} + F[m_{1^+_a} - m_{1^-_b}, m_{1^+_a} - m_{0^+_b}, |\mathbf{k}_\gamma|, M, \mu]\right) \qquad (17) \\
\hat{\Gamma}[0^+_a \to 1^-_a \gamma] &= 1 + \frac{2}{e_q \tilde{\beta}}\left(\frac{e_Q \tilde{\beta}'}{m_Q} - \frac{e_Q \tilde{\beta}'' |\mathbf{k}_\gamma|}{2m_Q} + F[m_{0^+_a} - m_{0^-_b}, m_{0^+_a} - m_{1^+_b}, |\mathbf{k}_\gamma|, M, \mu]\right).
\end{aligned}$$

Here $\hat{\Gamma} = \Gamma^{NLO}/\Gamma^{LO}$, where the Γ^{LO} are given in Eq. (8). The $SU(3)$ index a refers to the external heavy mesons while the index b refers to the mesons inside the loop. M is the mass of the virtual Goldstone boson. For heavy-strange decays the external particles are heavy-strange mesons; $a = 3$, the Goldstone boson is a K^+, and the heavy mesons inside the loops are neutral heavy mesons with $b = 1$. $\hat{\Gamma}$ is expanded to $O(Q)$. The function $F[\Delta_1, \Delta_2, |\mathbf{k}_\gamma|, M, \mu]$ is given in the Appendix. The loop graphs are regulated in dimensional regularization, counterterms are defined in the $\overline{MS}$ scheme and the dimensional regularization parameter is μ. All μ dependence is canceled by the implicit μ dependence of the renormalized couplings $\tilde{\beta}$, $\tilde{\beta}'$, and $\tilde{\beta}''$.

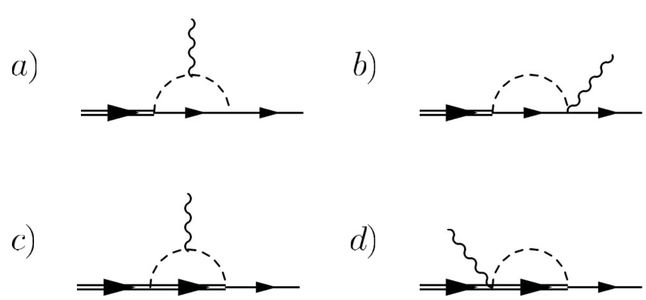

FIG. 1. One-loop chiral corrections to the electromagnetic decays $S \to H\gamma$ in $v \cdot A = 0$ gauge. Double lines are S mesons, solid lines are H mesons, dashed lines are Goldstone bosons and the wavy line is the photon.

An NLO calculation of the electromagnetic branching ratios also requires $O(1/m_c)$ corrections to the decays $\Gamma[D_{s0}(2317) \to D_s \pi^0]$ and $\Gamma[D_{s1}(2460) \to D_s^* \pi^0]$. The leading spin-symmetry violating operator contributing to these decays is

$$\mathcal{L} = \frac{h'}{2m_Q} \mathrm{Tr}[\overline{H}_a \sigma^{\mu\nu} S_b \gamma^\alpha] A^\beta_{ba} \epsilon_{\mu\nu\alpha\beta}. \qquad (18)$$

Because of the $1/m_Q$ suppression this operator is considered $O(Q^2)$. The one-loop diagrams contributing to $S \to$

$H\pi$ transitions are subleading at $O(Q^3)$. The decay rates to NLO are

$$\Gamma[1_3^+ \to 1_3^- \pi^0] = \left(h - \frac{h'}{m_Q}\right)^2 \frac{\theta^2}{3\pi f^2} \frac{m_{1_3^-}}{m_{1_3^+}} E_{\pi^0}^2 |\mathbf{p}_{\pi^0}|$$
$$\Gamma[0_3^+ \to 0_3^- \pi^0] = \left(h + 3\frac{h'}{m_Q}\right)^2 \frac{\theta^2}{3\pi f^2} \frac{m_{0_3^-}}{m_{0_3^+}} E_{\pi^0}^2 |\mathbf{p}_{\pi^0}|. \quad (19)$$

Earlier in this section the data from D_0^0 and D_1^0 decays was averaged to extract h^2. Including the leading correction it is possible to fit h and h' separately, extracting $h = 0.69 \pm 0.09$ and $h'/m_c = -0.019 \pm 0.034$.

The NLO expression for the branching fraction ratios of heavy-strange mesons is obtained by combining Eq. (19) with Eq. (17) and (8). The result is

$$\frac{\mathrm{Br}[1_3^+ \to 1_3^- \gamma]}{\mathrm{Br}[1_3^+ \to 1_3^- \pi^0]} = \frac{2\pi\alpha f^2 \tilde{\beta}^2}{9\theta^2 h^2} \frac{|\mathbf{k}_\gamma|^3}{E_{\pi^0}^2 p_{\pi^0}} \times \left(1 + \frac{2h'}{hm_Q} + \frac{6e_Q\tilde{\beta}'}{\tilde{\beta} m_Q} - \frac{6}{\tilde{\beta}} F[m_{1_a^+} - m_{1_b^-}, m_{1_a^+} - m_{1_b^+}, |\mathbf{k}_\gamma|, m_{K^+}, \mu]\right)$$
$$\frac{\mathrm{Br}[1_3^+ \to 0_3^- \gamma]}{\mathrm{Br}[1_3^+ \to 1_3^- \pi^0]} = \frac{\pi\alpha f^2 \tilde{\beta}^2}{9\theta^2 h^2} \frac{m_{0_3^-}}{m_{1_3^-}} \frac{|\mathbf{k}_\gamma|^3}{E_{\pi^0}^2 p_{\pi^0}} \times \left(1 + \frac{2h'}{hm_Q} - \frac{6e_Q\tilde{\beta}'}{\tilde{\beta} m_Q} - \frac{3e_Q\tilde{\beta}''|\mathbf{k}_\gamma|}{\tilde{\beta} m_Q} - \frac{6}{\tilde{\beta}} F[m_{1_a^+} - m_{1_b^-}, m_{1_a^+} - m_{0_b^+}, |\mathbf{k}_\gamma|, m_{K^+}, \mu]\right) \quad (20)$$
$$\frac{\mathrm{Br}[0_3^+ \to 1_3^- \gamma]}{\mathrm{Br}[0_3^+ \to 0_3^- \pi^0]} = \frac{\pi\alpha f^2 \tilde{\beta}^2}{3\theta^2 h^2} \frac{m_{1_3^-}}{m_{0_3^-}} \frac{|\mathbf{k}_\gamma|^3}{E_{\pi^0}^2 p_{\pi^0}} \times \left(1 - \frac{6h'}{hm_Q} - \frac{6e_Q\tilde{\beta}'}{\tilde{\beta} m_Q} + \frac{3e_Q\tilde{\beta}''|\mathbf{k}_\gamma|}{\tilde{\beta} m_Q} - \frac{6}{\tilde{\beta}} F[m_{0_a^+} - m_{0_b^-}, m_{0_a^+} - m_{1_b^+}, |\mathbf{k}_\gamma|, m_{K^+}, \mu]\right),$$

where e_q has been set to $e_s = -1/3$. Applying the formulae in Eq. (20) to the experimentally observed ratios gives

$$\frac{\mathrm{Br}[D_{s1}(2460) \to D_s^* \gamma]}{\mathrm{Br}[D_{s1}(2460) \to D_s^* \pi^0]} = 1.58 \frac{\tilde{\beta}^2}{h^2} \times \left(1 + \frac{1.43h'}{h} + \frac{2.86\tilde{\beta}'}{\tilde{\beta}} + \frac{0.18gh}{\tilde{\beta}} - \frac{2.94^{+0.70}_{-0.58} g'h}{\tilde{\beta}}\right) < 0.16$$
$$\frac{\mathrm{Br}[D_{s1}(2460) \to D_s \gamma]}{\mathrm{Br}[D_{s1}(2460) \to D_s^* \pi^0]} = 1.90 \frac{\tilde{\beta}^2}{h^2} \times \left(1 + \frac{1.43h'}{h} - \frac{2.86\tilde{\beta}'}{\tilde{\beta}} - \frac{0.63\tilde{\beta}''}{\tilde{\beta}} - \frac{0.03gh}{\tilde{\beta}} - \frac{2.40^{+0.73}_{-0.59} g'h}{\tilde{\beta}}\right) = 0.44 \pm 0.09 \quad (21)$$
$$\frac{\mathrm{Br}[D_{s0}(2317) \to D_s \gamma]}{\mathrm{Br}[D_{s0}(2317) \to D_s \pi^0]} = 0.57 \frac{\tilde{\beta}^2}{h^2} \times \left(1 - \frac{4.29h'}{h} - \frac{2.86\tilde{\beta}'}{\tilde{\beta}} - \frac{0.28\tilde{\beta}''}{\tilde{\beta}} + \frac{0.37gh}{\tilde{\beta}} - \frac{3.81^{+0.90}_{-0.75} g'h}{\tilde{\beta}}\right) < 0.059.$$

Here $\tilde{\beta}$ and $\tilde{\beta}''$ are measured in units of GeV^{-1} and h' in units of GeV. All other quantities are dimensionless. The charm quark mass is $m_c = 1.4$ GeV, the renormalization scale is $\mu = 1$ GeV, and $e_Q = e_c = 2/3$. For the loop corrections with kaons the meson decay constant is $f = f_K$, while for the strong decays $f = f_\eta$. (If f_π is used in the strong decays, then the branching fraction ratios in Eq. (20) should be multiplied by $f_\pi^2/f_\eta^2 = 0.58$.) The masses used for the virtual nonstrange even-parity heavy mesons in the loops are $m_{0_1^+} = 2308 \pm 36$ MeV and $m_{1_1^+} = 2438 \pm 29$ MeV, where the first number is the nonstrange 0^+ mass measured by Belle and the second is the average of the nonstrange 1^+ mass measured by CLEO and Belle. The uncertainty in the coefficient of $g'h/\tilde{\beta}$ in Eq. (21) is due to the uncertainty in the masses of the D_0^0 and D_1^0.

The result depends on seven parameters: g, g', h, h', $\tilde{\beta}$, $\tilde{\beta}'$, and $\tilde{\beta}''$. The coupling g is constrained to be $0.27^{+.06}_{-.03}$ from a next-to-leading order HHχPT analysis of D^* decays [54]. h and h' are extracted from the nonstrange decays, leaving four unknown parameters. Since there are only three constraints coming from experiment, further analysis requires additional assumptions to constrain the parameter space.

To illustrate how the current data is consistent with natural size parameters the following situation is considered. The contribution from $\tilde{\beta}''$ is neglected since in Eq. (21) $\tilde{\beta}''$ is multiplied by a coefficient that is much smaller than the coefficients multiplying h' and $\tilde{\beta}'$. (The smallness of this coefficient is due to the factor $|\mathbf{k}_\gamma|/m_c$.) g, h, h', and the branching fraction ratio measured by Belle are set to their central values: 0.27, 0.69, $-0.019m_c$, and 0.44, respectively. Ranges for the remaining parameters ($\tilde{\beta}$, $\tilde{\beta}'$, and g') are extracted by varying the branching ratios in Eq. (2) between zero and their upper limits. There are two solutions since the formulae for the electromagnetic decay rate is quadratic in $\tilde{\beta}$. The results are

$$0.70 \le \tilde{\beta}\ \text{GeV} \le 0.86 \qquad -0.01 \le \tilde{\beta}' \le 0.01$$
$$0.32 \le g' \le 0.40;$$
$$-0.62 \le \tilde{\beta}\ \text{GeV} \le -0.46 \qquad -0.01 \le \tilde{\beta}' \le 0.02$$
$$-0.25 \le g' \le -0.16. \tag{22}$$

Note that the ranges quoted in Eq. (22) do not include errors due to the uncertainties in the parameters g, h, and h' or the masses of the D_0^0 and D_1^0. h' is highly uncertain because of the uncertainty in the masses and widths of the D_0^0 and D_1^0 used to extract it. The loop contribution proportional to $g'h/\tilde{\beta}$ is also sensitive to the masses of the D_0^0 and D_1^0 that appear as intermediate states. The ranges given in Eq. (22) do not reflect these uncertainties and do not exhaust the possible parameter space. Instead, they are simply illustrative of natural size parameters consistent with existing data.

When more data on excited heavy meson systems becomes available, the formulae in Eq. (20) could be used to test models that make predictions for the parameters in HHχPT. In parity-doubling models, $g = -g'$ and $h = 1$ at tree level [13]. The authors of Ref. [13] note that h can be renormalized away from its tree level value and allow this parameter to vary in their analysis of strong decays. The tree level result $h = 1$ exceeds the value extracted from excited nonstrange decays in HHχPT. Another theoretical framework which makes similar predictions for the coupling constants g, g', and h is the algebraic realization of chiral symmetry [20]. Applying this theory to heavy mesons [18,19] leads to the predictions $g' = -g$ and $g^2 + h^2 = 1$. Using $g = 0.27$ in this relation gives $h^2 = 0.93$ which is also larger than extracted from Eq. (19). While the predictions for h are not in agreement with available data, the condition $g = -g'$ is consistent with available data but not required.

Eventually the even-parity B_s states will be observed and all electromagnetic branching fractions for heavy-strange mesons will be measured. Then the parameter space will be overconstrained and HHχPT for excited heavy mesons can be tested decisively. Furthermore, the extracted values for g, g' and h can be compared with predictions from parity-doubling models and algebraic realizations of chiral symmetry. At the present time, observed violations of leading heavy-quark spin-symmetry predictions are consistent with what is expected from loop effects and higher order operators appearing in the HHχPT Lagrangian.

III. ELECTROMAGNETIC DECAYS AND D K MOLECULES

The unexpectedly low masses of the $D_{s0}(2317)$ and $D_{s1}(2460)$ have prompted speculation that these states are unconventional. Two common proposals are that these mesons are $c\bar{s}q\bar{q}$ tetraquarks or hadronic bound states of D and K mesons. This section addresses the question of what the decays reveal about the internal structure of the $D_{s0}(2317)$ and $D_{s1}(2460)$. In the analysis of the previous section the only information about the states needed to construct the HHχPT Lagrangian is the assumed $SU(3)$ and j^p quantum numbers of the light degrees of freedom in the hadrons. A constituent quark in a P-wave or an exotic with two light quarks and an antiquark both have $j^p = \frac{1}{2}^-$. Both states are represented by a field like that in Eq. (5). Analysis of electromagnetic and strong decays within HHχPT is identical for both states, though the coupling constants $\tilde{\beta}$, $\tilde{\beta}'$, h, etc., would be different for the two states. Since these coupling constants are unknown in either case, the HHχPT predictions for electromagnetic and strong decays cannot distinguish between exotic $c\bar{s}q\bar{q}$ and conventional $c\bar{s}$ P-wave states. Of course, if the $D_{s0}(2317)$ and $D_{s1}(2460)$ are $c\bar{s}q\bar{q}$ states then in the quark model there should be distinct $c\bar{s}$ P-wave mesons with the same quantum numbers. These states could be very hard to detect, however, if they are above the DK threshold. Mixing between the conventional and exotic mesons is also likely [28,62].

However, if the $D_{s0}(2317)$ [$D_{s1}(2460)$] is a bound state of $D^{(*)}$ and K mesons then the HHχPT predictions for electromagnetic and strong decays will be different. For a hadronic bound state of a D or D^* and a kaon, one could in principle calculate the bound state masses and other properties from the HHχPT Lagrangian with the field H_a alone. There have been attempts to generate the $D_{s0}(2317)$ and $D_{s1}(2460)$ as resonances in a unitarized meson model [47,48] as well as by solving Bethe-Salpeter equations in relativistic, unitarized chiral perturbation theory [49–51]. Producing a bound state requires resumming an infinite number of Feynman graphs in HHχPT and the renormalization of these graphs requires introducing higher order operators whose renormalized coefficients are unknown. Such a calculation will not be attempted in this paper. Instead the DK molecular picture will be tested by simply assuming that strong forces between $D^{(*)}$ and K mesons give rise to the $D_{s0}(2317)$ and $D_{s1}(2460)$ and determining what this implies for the decay rates. If the $D_{s0}(2317)$ ($D_{s1}(2460)$) are bound states of $D^{(*)}K$ then the characteristic momentum of the constituents is $p \sim \sqrt{2\mu B} \approx 190$ MeV, where μ is the reduced mass and B is the binding energy. The DK molecule can then be modeled as a nonrelativistic bound state since relativistic corrections are suppressed by $v^2 = p^2/M_K^2 = 0.15$. Strong and electromagnetic decays can be calculated in terms of the unknown bound state wavefunction. Even without any knowledge of these wavefunctions it is possible to make predictions for the decay ratios in Eqs. (1) and (2). It turns out that these predictions disagree with data so interpreting $D_{s0}(2317)$ and $D_{s1}(2460)$ as DK molecules is disfavored.

Ref. [23] advocates using the radiative decays of the $D_{s0}(2317)$ and $D_{s1}(2460)$ to determine the nature of these states and calculates radiative and strong decays within a nonrelativistic quark model. Predictions for the branching fraction ratios in Eqs. (1) and (2) are in the same proportion as leading order heavy-quark symmetry predictions, though they are approximately 45% larger than the leading order predictions obtained in Sec. II. The quark model expectation for the total widths of the $D_{s0}(2317)$ and $D_{s1}(2460)$ is $O(10\text{ keV})$, consistent with Eq. (11). However, the conclusion of Ref. [23] states that a $D^{(*)}K$ molecule should have a width of $O(1\text{ MeV})$ and that the electromagnetic transitions should be absent. The analysis that follows is consistent with the first conclusion but not the second. Below it is demonstrated that the electromagnetic branching ratios of a $D^{(*)}K$ molecule are large and are in worse agreement with experiment than the nonrelativistic quark model.

The $D_{s0}(2317)$ ($D_{s1}(2460)$) is assumed to be an S-wave $I = 0$ bound state of $D^{(*)}$ and K mesons. The matrix elements for the electromagnetic decays of the $D_{s0}(2317)$ and $D_{s1}(2460)$ are given by

$$\mathcal{M}[D_{s0}(2317) \to D_s^*\gamma] = \sqrt{\frac{2}{m_{D_{s0}}}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \tilde{\psi}_{DK}(\mathbf{p}) \times \mathcal{M}[D(\mathbf{p})K(-\mathbf{p}) \to D_s\gamma]$$

$$\mathcal{M}[D_{s1}(2460) \to D_s^{(*)}\gamma] = \sqrt{\frac{2}{m_{D_{s1}}}} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \tilde{\psi}_{D^*K}(\mathbf{p}) \times \mathcal{M}[D^*(\mathbf{p})K(-\mathbf{p}) \to D_s^{(*)}\gamma]. \tag{23}$$

Here $\mathbf{p}$ is the three-momentum of the $D^{(*)}$ meson in the bound state and $\tilde{\psi}_{D^{(*)}K}(\mathbf{p})$ is the bound state momentum-space wavefunction. Although calculation of the bound state wavefunction is nonperturbative, the typical momentum is small enough that the amplitudes $\mathcal{M}[D^{(*)}K \to D_s^{(*)}\gamma]$ are perturbatively calculable in HHχPT. The leading order diagrams for the decay rates in Eq. (23) are shown in Fig. 2. The shaded oval on the left hand side of these Feynman diagrams represents the $D^{(*)}K$ molecule. In Figs. 2(a) and 2(b) the dashed line is a K^+, and the vertex involving the photon comes from gauging the $D^* - D_s - K^+$ coupling [Fig. 2(a)] or the K^+ kinetic term [Fig. 2(b)]. In Figs. 2(c) and 2(d) the photon coupling comes from gauging the heavy-meson kinetic term. There are also diagrams like Figs. 2(c) and 2(d) where the photon heavy-meson coupling comes from a term in the Lagrangian proportional to $\text{Tr}[\overline{H}_b H_a \sigma^{\mu\nu} Q^{\xi}_{ab}]F_{\mu\nu}$, but these only contribute in the P-wave channel.

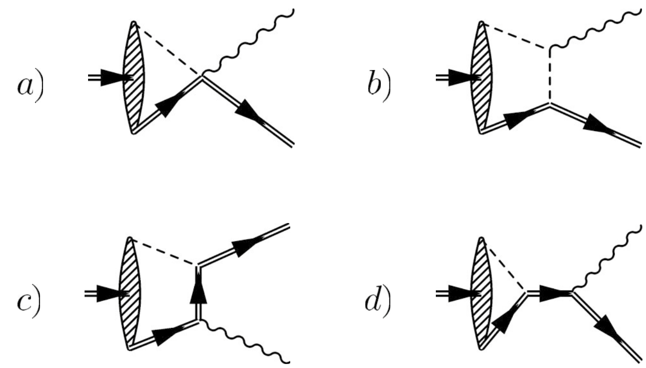

FIG. 2. Leading order diagrams for $D^{(*)}K$ bound states decaying into $D_s^{(*)}\gamma$. The shaded oval represents the $D^{(*)}K$ bound state wavefunction.

The graphs in Fig. 2(c) and 2(d) are nonvanishing in the K^0D^+ channel, but are equal and opposite in sign so the contribution in this channel vanishes. The graph in Fig. 2(c) vanishes in the K^+D^0 channel. The amplitudes are

$$\mathcal{M}[D^*(\mathbf{p})K(-\mathbf{p}) \to D_s^*\gamma] = -i\sqrt{m_{D^*}m_{D_s^*}}\frac{2eg}{f}\left[\frac{p_K^\mu(p_K - p_\gamma)^\delta}{p_K \cdot p_\gamma} + g^{\mu\delta} - \frac{p_K^\delta v^\mu}{v \cdot p_K}\right]\epsilon^*_\mu \epsilon_{\nu\beta\lambda\delta} v^\beta \epsilon_3^{*\nu}\epsilon_1^\lambda$$

$$\mathcal{M}[D^*(\mathbf{p})K(-\mathbf{p}) \to D_s\gamma] = \sqrt{m_{D^*}m_{D_s^*}}\frac{2eg}{f}\left[\frac{p_K^\mu(p_K - p_\gamma)^\nu}{p_K \cdot p_\gamma} + g^{\mu\nu} - \frac{v^\mu p_K^\nu}{v \cdot p_K}\right]\epsilon^*_\mu(\epsilon_1)_\nu$$

$$\mathcal{M}[D(\mathbf{p})K(-\mathbf{p}) \to D_s^*\gamma] = \sqrt{m_D m_{D_s^*}}\frac{2eg}{f}\left[\frac{p_K^\mu(p_K - p_\gamma)^\nu}{p_K \cdot p_\gamma} + g^{\mu\nu} - \frac{v^\mu p_K^\nu}{v \cdot p_K}\right]\epsilon^*_\mu(\epsilon_3^*)_\nu. \tag{24}$$

Here p_K and p_γ are the kaon and photon four momentum, respectively. The polarization vectors for the photon, D^*, and D_s^* are denoted ϵ, ϵ_1, and ϵ_3, respectively. It is easy to check that the amplitudes respect the QED Ward identity. These expressions are inserted into Eq. (23), p_K^μ is set to $E_K v^\mu + \mathbf{p}^\mu$, $\mathbf{p}^\mu = (0, -\mathbf{p})$, and the matrix element is expanded to lowest order in $\mathbf{p}$. Because of the rotational symmetry of the S-wave wavefunction, $\tilde{\psi}(\mathbf{p})$, terms linear in $\mathbf{p}$ vanish. There are corrections to the amplitudes from higher orders in chiral perturbation theory that are $O(m_K^2/\Lambda_\chi^2)$ and relativistic corrections of $O(v^2)$. The errors in the predictions for the decay rates could be as large as 50%.

The results for the decay rates are

$$\Gamma[D_{s1}(2460) \to D_s^* \gamma] = \frac{8g^2\alpha}{3f^2}\left(\frac{m_{D^{0*}} m_{D_s^*}}{m_{D_{s1}}^3}\right)|\psi_{D^*K}(0)|^2|\mathbf{k}_\gamma|$$

$$\Gamma[D_{s1}(2460) \to D_s \gamma] = \frac{4g^2\alpha}{3f^2}\left(\frac{m_{D^{0*}} m_{D_s}}{m_{D_{s1}}^3}\right)|\psi_{D^*K}(0)|^2|\mathbf{k}_\gamma|$$

$$\Gamma[D_{s0}(2317) \to D_s^* \gamma] = \frac{4g^2\alpha}{f^2}\left(\frac{m_{D^0} m_{D_s^*}}{m_{D_{s0}}^3}\right)|\psi_{DK}(0)|^2|\mathbf{k}_\gamma|. \tag{25}$$

Here $\psi_{DK}(0)[\psi_{D^*K}(0)]$ is the wavefunction at the origin for the $D_{s0}(2317)[D_{s1}(2460)]$. In the heavy-quark limit the partial width ratios are again $\Gamma[D_{s1}(2460) \to D_s^*\gamma]$:$\Gamma[D_{s1}(2460) \to D_s\gamma]$:$\Gamma[D_{s0}(2317) \to D_s^*\gamma] =$ 2:1:3. However, the decay rates are proportional to $|\mathbf{k}_\gamma|$ instead of $|\mathbf{k}_\gamma|^3$. This important difference in the kinematic factors leads to a very different prediction for the relative sizes of the partial widths than obtained in Eq. (10). In this case

$$\Gamma[D_{s1}(2460) \to D_s^*\gamma]:\Gamma[D_{s1}(2460) \to D_s\gamma] :\Gamma[D_{s0}(2317) \to D_s^*\gamma] = 1.57:1:R_\psi 1.58,$$

where $R_\psi = |\psi_{DK}(0)|^2/|\psi_{D^*K}(0)|^2$ is expected to be ≈ 1. In this scenario $\Gamma[D_{s1}(2460) \to D_s\gamma]$ is the smallest decay rate rather than the largest.

To compare with the measured branching ratios the strong decays must also be calculated. The leading order diagrams are shown in Fig. 3. All three diagrams depict $D^{(*)}K \to D_s^{(*)}\eta$ followed by $\eta - \pi^0$ mixing, which is represented by a cross on the dashed line in the final state. The vertex for the graph in Fig. 3(a) comes from the chirally covariant derivative in the heavy-meson kinetic term. This graph contributes in the S-wave channel while Figs. 3(b) and 3(c) contribute to the P-wave channel only. The results for the decay rates are

$$\Gamma[D_{s1}(2460) \to D_s^*\pi^0] = \frac{3(m_K + E_{\pi^0})^2\theta^2}{4\pi f^4} \times\left(\frac{m_{D^*} m_{D_s^*}}{m_{D_{s1}}^3}\right)|\psi_{D^*K}(0)|^2|\mathbf{p}_{\pi^0}| \tag{26}$$

$$\Gamma[D_{s0}(2317) \to D_s\pi^0] = \frac{3(m_K + E_{\pi^0})^2\theta^2}{4\pi f^4}\left(\frac{m_D m_{D_s}}{m_{D_{s0}}^3}\right)|\psi_{DK}(0)|^2|\mathbf{p}_{\pi^0}|.$$

If $f = f_\pi = 130$ MeV then the bounds $\Gamma[D_{s1}(2460)] \le 7$ MeV and $\Gamma[D_{s0}(2317)] \le 7$ MeV imply $|\psi_{D^{(*)}K}(0)|^2 \le (52\text{ MeV})^3$. If instead $f = f_\eta = 171$ MeV then $|\psi_{D^{(*)}K}(0)|^2 \le (75\text{ MeV})^3$. In either case the bounds on the wavefunctions are somewhat smaller than expected: $|\psi_{D^{(*)}K}(0)|^2 \sim |\mathbf{p}|^3 \sim (190\text{ MeV})^3$. Since this is only an order of magnitude estimate, the bounds on $|\psi_{D^{(*)}K}(0)|^2$ are not a problem for the DK molecular interpretation. However, they do imply that if the $D_{s0}(2317)$ and $D_{s1}(2460)$ are $D^{(*)}K$ molecules the states should not be much narrower than the present upper limits [23,27].

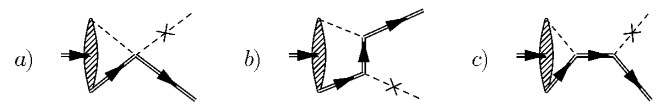

FIG. 3. Leading order diagram for $D^{(*)}K$ bound states decaying into $D_s^{(*)}\pi^0$. The dashed line from the bound state is a K, the dashed line in the final state is an η which mixes into a π^0.

Since the wavefunction squared cancels in the ratio of strong and electromagnetic decays the electromagnetic branching fractions can be predicted:

$$\frac{\text{Br}[D_{s1}(2460) \to D_s^*\gamma]}{\text{Br}[D_{s1}(2460) \to D_s^*\pi^0]} = 3.23$$
$$\frac{\text{Br}[D_{s1}(2460) \to D_s\gamma]}{\text{Br}[D_{s1}(2460) \to D_s^*\pi^0]} = 2.21 \tag{27}$$
$$\frac{\text{Br}[D_{s0}(2317) \to D_s\gamma]}{\text{Br}[D_{s0}(2317) \to D_s\pi^0]} = 2.96.$$

In this calculation $\alpha = 1/137$, $\theta = 0.01$, $g = 0.27$, $f_K = 159$ MeV in the electromagnetic decays and $f = f_\eta = 171$ MeV in the strong decays. If instead $f = f_\pi = 130$ MeV is used in the strong decays the predicted branching fraction ratios are smaller by a factor of 3. While the branching fraction ratios are quite sensitive to the choice of f, in any case they are much too large compared to experiment. Also, the relative sizes of the branching fraction ratios are in disagreement with experiment, since the second branching fraction ratio in Eq. (27) is predicted to be smallest, not largest. Note that the possibility of these states being mixtures of quark level bound states and DK molecules [28,62] is also disfavored since this would enhance the first and third ratios in Eq. (27) relative to the second, whereas in reality these ratios are suppressed relative to the leading order prediction in Eq. (10).

IV. CONCLUSIONS

In this paper, corrections to electromagnetic and strong decays of $D_{s0}(2317)$ and $D_{s1}(2460)$ are calculated in HHχPT. The corrections depend on a number of unknown or poorly determined coupling constants. These predictions can be consistent with Belle and CLEO data with coupling constants of natural size. Serious tests of the HHχPT description of the $D_{s0}(2317)$ and $D_{s1}(2460)$ will require more data on the electromagnetic branching ratios of the even-parity charmed strange mesons and the strong decays of their nonstrange partners as well as the decays of even-parity bottom strange mesons yet to be observed. The work in this paper provides further stimulus for better experimental measurements of charmed strange decays as well as discovery of their bottom strange counterparts. Once better data becomes available, it would be interesting to test models of chiral symmetry breaking which make specific predictions for the coupling constants appearing in the HHχPT Lagrangian.

This paper also tests the hypothesis that the $D_{s0}(2317)$ and $D_{s1}(2460)$ are molecular bound states of DK and D^*K molecules, respectively. In this scenario, these states are sufficiently nonrelativistic that HHχPT can be used to predict the decay rates at lowest order. Furthermore, bound state wavefunctions cancel out of predictions for the observed branching fraction ratios so absolute predictions can be made. These predictions are in much worse agreement with data than leading order HHχPT predictions. Specifically, predictions for all the branching fraction ratios are larger than observed and the branching fraction for the only observed electromagnetic decay is predicted to be the smallest of the three possible decays rather than the largest. Therefore, a molecular interpretation of these states is disfavored by available data on electromagnetic decays.

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APPENDIX

The function $F[\Delta_1, \Delta_2, |\mathbf{k}_\gamma|, M, \mu]$ is

$$F[\Delta_1, \Delta_2, |\mathbf{k}_\gamma|, M, \mu] = \frac{gh}{8\pi^2 f^2}\left\{\Delta_1\left[3 + \ln\left(\frac{\mu^2}{\Delta_1^2}\right)\right] + \frac{\Delta_1}{|\mathbf{k}|^2} G[\Delta_1, |\mathbf{k}_\gamma|, M]\right\} + \frac{g' h}{8\pi^2 f^2}\left[-(\Delta_2 - |\mathbf{k}_\gamma|)\ln\left(\frac{\mu^2}{\Delta_2^2}\right) + \frac{(\Delta_2 - |\mathbf{k}_\gamma|)}{|\mathbf{k}_\gamma|^2} H(\Delta_2, |\mathbf{k}_\gamma|, M)\right], \tag{28}$$

where

$$G(\Delta, |\mathbf{k}_\gamma|, M) = -2|\mathbf{k}_\gamma|\Delta - |\mathbf{k}_\gamma|^2\ln\left(\frac{M^2}{\Delta^2}\right) + (\Delta - |\mathbf{k}_\gamma|)^2 F_1\left(\frac{\Delta - |\mathbf{k}_\gamma|}{M}\right) - \Delta(\Delta - 2|\mathbf{k}_\gamma|)F_1\left(\frac{\Delta}{M}\right) + M^2\left[F_2\left(\frac{\Delta}{M}\right) - F_2\left(\frac{\Delta - |\mathbf{k}_\gamma|}{M}\right)\right],$$

and

$$H(\Delta, |\mathbf{k}_\gamma|, M) = -|\mathbf{k}_\gamma|^2 - 2|\mathbf{k}_\gamma|\Delta + |\mathbf{k}_\gamma|^2\ln\left(\frac{M^2}{\Delta^2}\right) + (\Delta^2 - |\mathbf{k}_\gamma|^2)F_1\left(\frac{\Delta - |\mathbf{k}_\gamma|}{M}\right) - \Delta^2 F_1\left(\frac{\Delta}{M}\right) + M^2\left[F_2\left(\frac{\Delta}{M}\right) - F_2\left(\frac{\Delta - |\mathbf{k}_\gamma|}{M}\right)\right].$$

The functions $F_{1,2}(x)$ are given by:

$$\begin{aligned} F_1(x) &= \frac{2\sqrt{1-x^2}}{x}\left[\frac{\pi}{2} - \arctan\left(\frac{x}{\sqrt{1-x^2}}\right)\right] \qquad |x| < 1 \\ &= -\frac{2\sqrt{x^2-1}}{x}\ln\left(x + \sqrt{x^2-1}\right) \qquad |x| > 1 \\ F_2(x) &= \left[\frac{\pi}{2} - \arctan\left(\frac{x}{\sqrt{1-x^2}}\right)\right]^2 \qquad |x| < 1 \\ &= -\ln^2\left(x + \sqrt{x^2-1}\right) \qquad |x| > 1. \end{aligned} \tag{29}$$

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