Predictions for $e^+e^- \to J/\psi\eta_c$ with light-cone wave functions

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Predictions for $e^+e^- \to J/\psi\eta_c$ from previous studies are made by taking charmonia as a nonrelativistic bound state and by using nonrelativistic QCD (NRQCD) approach. The predicted cross section is smaller by an order of magnitude than the experimentally observed cross section. We study the process by taking charm quark as a light quark and use light-cone wave functions to parameterize nonperturbative effects related to charmonia. The total cross section of $e^+e^- \to J/\psi\eta_c$ can be predicted, if these wave functions are known. Motivated by studies of light-cone wave functions of light hadrons, we make a reasonable assumption of the forms of light-cone wave functions. With these light-cone wave functions we can obtain the cross section which is closer to the experimentally observed than that from the NRQCD approach. We also discuss in detail the difference between two approaches.

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At Belle the production of double charmonia at e^+e^- collider with $\sqrt{s} = 10.6$ GeV has been studied. The experimental result is given as [1]:

$$\sigma(e^+e^- \to J/\psi\eta_c)\mathrm{Br}(\eta_c \to 4 \text{ charged particles}) = 33^{+7}_{-6} \pm 9 \text{ fb}. \quad (1)$$

Since a branching ratio is smaller than 1, the above experimental result gives a lower bound for the cross section. This experimental result is in conflict with theoretical predictions. Theoretical predictions are made by taking a charmonium as a bound state of a $c\bar{c}$ quark. Employing nonrelativistic wave functions for such a bound state one can predict production rates like the one measured at e^+e^- colliders. Starting from this, the process were studied in [2–4]. From these studies the cross section is about 2–5 fb. Comparing with Eq. (1) the experimentally measured cross section is about an order of magnitude larger than theoretical predictions.

If one takes charm quarks as heavy quarks, a charmonium system can be thought of as a bound state consisting mainly of a $c\bar{c}$ quark, in which the c- and $\bar{c}$-quark move with a small velocity. This fact enables us to describe such a system by an expansion in the small velocity. A systematic expansion can be achieved by using nonrelativistic QCD (NRQCD) [5]. Within this framework inclusive decays and inclusive productions of single quarkonium can be studied consistently and rigorously, where a factorization of nonperturbative effects can be completed. But it is expected that theoretical predictions for charmonia can have large uncertainties in general. There are two important sources of corrections. One is of relativistic correction. Because the velocity of a c quark in a charmonium is not very small, the relativistic correction is large. This has been shown in different processes studied in [6–8]. Another source of correction is from high order of α_s. If one works with NRQCD factorization for the process $e^+e^- \to \eta_c J/\psi$, the charm quark mass m_c can not be neglected and large logarithms like $\ln(m_c^2/s)$ will appear. Those large logarithms can spoil the perturbative expansion in α_s and a resummation is needed. However it is not expected that these large uncertainties can result in such a large discrepancy. To explain the discrepancy it was suggested that the experimental signals for the final state of $J/\psi\eta_c$ may contain those of double J/ψ [9] and the initial state interaction can enhance the cross section of $J/\psi\eta_c$ [10]. Indeed, the cross section becomes large if these suggested effects are taken into account. But the discrepancy still remains large.

It should be noted that for a process involving two quarkonia there is no rigorous theory based on NRQCD in the sense of factorization of nonperturbative effects. Although a proof of the factorization has not been given yet, it could be the case that the factorization does not hold. The reason is the following: NRQCD is only applicable for a quarkonium in its rest frame. For a process involving only one quarkonium one can always find such a rest frame through a Lorentz boost. While for a process involving two or more quarkonia one can not find in general a frame in which all quarkonia are in rest. Therefore, a complete factorization may not be achieved.

If the center-mass energy $\sqrt{s}$ is very large, i.e., $\sqrt{s} \gg m_c$, one can take c-quark as a light quark. Then one can use light-cone wave functions to describe nonperturbative effects of charmonia and a factorized form of the production amplitude in terms of these wave functions and a perturbative part can be obtained. Such an approach for exclusive processes was proposed a long time ago [11].

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Recently, light-cone wave functions have been employed for charmonia to study their production in B-decay [12,13] and in photoproduction [14]. In comparison with the approach based on NRQCD for the process $e^+e^- \to J/\psi\eta_c$, where the expansion parameter is the velocity, the approach with light-cone wave function is with the expansion parameters as $\Lambda/\sqrt{s}$, where Λ is a soft scale and can be $\Lambda_{\rm QCD}$, m_c and masses of charmonia. In this paper we will use this approach to study the process $e^+e^- \to J/\psi\eta_c$. We will work at the leading order of the expansion. The correction to our result is at order of $\Lambda/\sqrt{s}$ or $(\Lambda/\sqrt{s})^2$. Taking Λ to be the mass of J/ψ, one can expect that our result at $\sqrt{s} = 10$ GeV has an uncertainty at a level of 30% or smaller. Since an expansion in masses is used, there is only one large scale $\sqrt{s}$ in the perturbative part and our prediction will not contain large logarithms like $\ln(m_c^2/s)$ if one takes higher orders in α_s into account. However, such large logarithms like $\ln(m_c^2/s)$ and $\ln(\Lambda_{\rm QCD}^2/s)$ will appear in light-cone wave functions. These large logarithms can be resummed with evolution equations of wave functions, like the Efremov-Radyushkin-Brodsky-Lepage evolution equation [15].

In principle the $c\bar{c}g$ components of charmonia will also contribute, if one take light-cone wave functions at twist 3 into account. Although NRQCD factorization may not directly be applicable to the process studied here, it can be used to study light-cone wave functions of charmonia as those introduced below, in which nonperturbative effects can be factorized into NRQCD matrix elements and light-cone wave functions will be proportional to NRQCD matrix elements [16]. The light-cone wave functions of $c\bar{c}g$ components will be proportional to NRQCD matrix elements containing gluon fields explicitly. It is known that NRQCD matrix elements containing gluon fields are small by NRQCD power counting [5], although they can be important in some processes because of some mechanism of enhancement. In our approach such an enhancement does not exist, hence we neglect contributions from these $c\bar{c}g$ components. We will only consider contributions from $c\bar{c}$ components.

We consider the exclusive process:

$$e^+(p_1) + e^-(p_2) \to \gamma^*(q) \to J/\psi(p) + \eta_c(k), \qquad (2)$$

where momenta are indicated in the brackets. The amplitude can be written as

$$\mathcal{T} = ie\bar{u}(p_1)\gamma_\mu v(p_2)\frac{1}{q^2}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(p)p_\alpha k_\beta \mathcal{F}(q^2), \qquad (3)$$

where $\varepsilon_\nu^*(p)$ is the polarization vector of J/ψ and $\mathcal{F}(q^2)$ is the form factor defined as

$$\langle J/\psi(p)\eta_c(k)|J^\mu|0\rangle = iQ_c e\varepsilon^{\mu\nu\alpha\beta}\varepsilon_\nu^*(p)p_\alpha k_\beta\mathcal{F}(q^2), \qquad (4)$$

where Q_c is the charge fraction of c-quark in unit of e. From the above the produced J/ψ is transversally polarized. With the form factor the cross section can be calculated as

$$\sigma(e^+e^- \to J/\psi\eta_c) = 4\pi\alpha^2 Q_c^2 \frac{|\mathcal{F}(s)|^2}{64}\left(1 - \frac{4m_h^2}{s}\right)^{3/2} \times \int_{-1}^{+1} dx(1 + x^2), \qquad (5)$$

where $x = \cos\theta$ and θ is the angle between J/ψ and e^+. We neglect the small mass difference between J/ψ and η_c:

$$m_h \approx m_{J/\psi} \approx m_{\eta_c}. \qquad (6)$$

From Eq. (4) it is easy to see that the helicity in the process is not conserved. For helicity-conserving processes one can use the power-counting rule in [17] to determine the asymptotic behavior of relevant form factors when $q^2 \to \infty$. For a process in which the helicity conservation is violated, one can use the generalized power-counting rule in [18] to determine the asymptotic behavior of relevant form factors. In our case one can obtain that $\mathcal{F}(s) \sim s^{-2}$ when $s \to \infty$.

At the leading order of α_s the contribution to the form factor comes from four Feynman diagrams, one of them is given in Fig. 1. The contribution can be written as

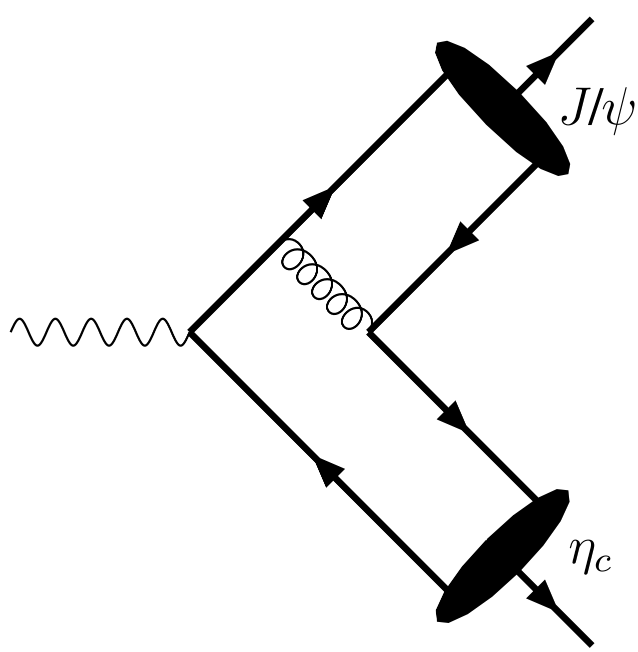

FIG. 1. One of the four Feynman diagrams for the amplitude.

$$\langle J/\psi(p)\eta_c(k)|J^\mu|0\rangle = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}H^\mu_{ij,kl}(k_1,k_2,m_c) \times \int d^4x e^{-ik_1\cdot x}\langle J/\psi(p)|\bar{c}_i(x)c_j(0)|0\rangle \cdot \int d^4y e^{-ik_2\cdot y}\langle\eta_c(k)|\bar{c}_k(y)c_l(0)|0\rangle, \tag{7}$$

where the hard part $H^\mu_{ij,kl}(k_1, k_2, m_c)$ is the amplitude for

$$\gamma^*(q) \to c_i(k_1) + c_k(k_2) + \bar{c}_j(p-k_1) + \bar{c}_l(k-k_2), \tag{8}$$

and all quarks can be off shell. The hard part reads:

$$H^\mu_{ij,kl} = \frac{ig_s^2}{(p-k_1+k_2)^2}(\gamma_\nu T^a)_{kj}\left(\gamma^\nu T^a\frac{\gamma\cdot(p+k_2)+m_c}{(p+k_2)^2-m_c^2}\gamma^\mu + \gamma^\mu\frac{\gamma\cdot(k_1-q)+m_c}{(k_1-q)^2-m_c^2}\gamma^\nu T^a\right)_{il} + \frac{ig_s^2}{(k-k_2+k_1)^2}(\gamma_\nu T^a)_{il} \times\left(\gamma^\mu\frac{\gamma\cdot(k_2-q)+m_c}{(k_2-q)^2-m_c^2}\gamma^\nu T^a + \gamma^\nu T^a\frac{\gamma\cdot(k+k_1)+m_c}{(k+k_1)^2-m_c^2}\gamma^\mu\right)_{kj}, \tag{9}$$

where $ijkl$ are indices for color and spin.

We will use light-cone wave functions to describe the two charmonia, in which a collinear expansion is used. We take an coordinate system in which J/ψ moves in the z-direction and η_c moves in $-z$-direction. The momenta of these charmonia in the light-cone coordinate system read:

$$p^\mu = \left(p^+, \frac{m_h^2}{2p^+}, 0, 0\right) = p^+l^\mu + \frac{m_h^2}{2p^+}n^\mu, \quad k^\mu = \left(\frac{m_h^2}{2k^-}, k^-, 0, 0\right) = \frac{m_h^2}{2k^-}l^\mu + k^-n^\mu, \tag{10}$$

where $l^\mu = (1,0,0,0)$ and $n^\mu = (0,1,0,0)$ are two lightlike vectors. The transverse directions to the lightlike directions are denoted with the subscriber $\perp$. In Eq. (7) the quark pair of $c(k_1)\bar{c}(p-k_1)$ is transited into the J/ψ. In general one expects that the dominant contributions for the integration over k_1 are with $k_{1\perp} \sim \Lambda$ and $k_1^- \sim \Lambda^2/2k_1^+$, where Λ is the soft scale. Similarly the dominant contributions for the integration over k_2 are with $k_{2\perp} \sim \Lambda$ and $k_2^+ \sim \Lambda^2/2k_2^-$. Hence for these integrations one can expand the hard part around $k_1^\mu = (k_1^+, 0, 0, 0)$ and $k_2^\mu = (0, k_2^-, 0, 0)$ and in any soft scale:

$$H^\mu_{ij,kl}(k_1,k_2,m_c) \approx H^\mu_{ij,kl}(z_1p^+l, z_2k^-n, 0) + k^\rho_{1\perp}\left(\frac{\partial H^\mu_{ij,kl}}{\partial k^\rho_{1\perp}}\right)(z_1p^+l, z_2k^-n, 0) + k^\rho_{2\perp}\left(\frac{\partial H^\mu_{ij,kl}}{\partial k^\rho_{2\perp}}\right) \times (z_1p^+l, z_2k^-n, 0) + m_c\left(\frac{\partial H^\mu_{ij,kl}}{\partial m_c}\right)(z_1p^+l, z_2k^-n, 0) + \cdots. \tag{11}$$

This is equivalent to expanding the two matrix elements in Eq. (7) along light cones, i.e., the matrix element $\langle\eta_c(k)|\bar{c}_k(y)c_l(0)|0\rangle$ is expanded around $x^\mu = (0, x^-, 0, 0)$ and $\langle\eta_c(k)|\bar{c}_k(y)c_l(0)|0\rangle$ is expanded around $y^\mu = (y^+, 0, 0, 0)$. However, the expansion is not systematic in the sense that the leading term in Eq. (11) will also lead to some contributions which are at the same order of Λ as those from higher orders. The reason for this is clear: A Dirac field like $c(x)$ can be decomposed along a light cone into a "good" and "bad" component. The bad component can be solved with the good component with equation of motion and will lead to a contribution which is suppressed by Λ in comparison with that from the good component. This problem can be solved by expanding the matrix elements according to twists of operators. Matrix elements of operators with a given twist are Fourier-transformed light-cone wave functions. For those matrix elements with light hadrons, the expansion in terms of light-cone wave functions have been studied in detail [19,20]. One can use the results in [19,20] to write down the expansion for quarkonia. Up to twist 3 the expansion is [12,19,20]

$$
\begin{aligned}
\langle \eta_c(k)|\bar{c}_k(y)c_l(0)|0\rangle &= \frac{i}{12} f_{\eta_c}\{[\gamma\cdot n\gamma_5]_{lk}k^- \int dz_2 e^{iz_2k^-y^+}\phi^{[2]}(z_2) - \frac{m_{\eta_c}^2}{2m_c}[\gamma_5]_{lk}\int dz_2 e^{iz_2k^-y^+}\phi_p^{[3]}(z_2) \\
&\quad + [\sigma^{\mu\nu}\gamma_5]_{lk}n_\mu y_{\perp\nu}\int dz_2 e^{iz_2k^-y^+}\phi_\sigma^{[3]}(z_2)\} + \cdots, \\
\langle J/\psi(p)|\bar{c}_i(x)c_j(0)|0\rangle &= \frac{1}{12}\{if_{J/\psi}^T[\sigma^{\mu\nu}]_{ji}l_\mu \varepsilon^*_{\perp\nu}(p)p^+\int dz_1 e^{iz_1p^+x^-}\psi_\perp^{[2]}(z_1) + f_{J/\psi}m_{J/\psi}[\gamma^\mu]_{ji}\varepsilon^*_{\perp\mu}(p) \\
&\quad \times \int dz_1 e^{iz_1p^+x^-}\psi_\perp^{[3]}(z_1)\} + \cdots,
\end{aligned}
\tag{12}
$$

where the subscriber $\perp$ denote the transverse direction to the lightlike directions. It should be noted that the space-time coordinate x and y in the matrix elements is not on light cones, in the right-hand side of the above equations the expansion along light cones is done. The decay constants are defined as:

$$
\begin{aligned}
\langle \eta_c(k)|\bar{c}(0)\gamma^\mu\gamma_5 c(0)|0\rangle &= -if_{\eta_c}k^\mu, \\
\langle J/\psi(p)|\bar{c}(0)\gamma^\mu c(0)|0\rangle &= f_{J/\psi}m_{J/\psi}\varepsilon^{*\mu}(p), \\
2f_{J/\psi}m_c &= f_{J/\psi}^T m_{J/\psi}.
\end{aligned}
\tag{13}
$$

In Eq. (12) the numbers in the bracket $[\cdots]$ as subscribers indicate twists. The $\cdots$ denote twist 4 terms and those twist 3 terms which are proportional to the factor

$$
f_{J/\psi} - \frac{2m_c}{m_{J/\psi}}f_{J/\psi}^T = f_{J/\psi}\left(1 - \frac{4m_c^2}{m_{J/\psi}^2}\right) \approx 0, \tag{14}
$$

which vanishes when the quark mass m_c goes to infinite. We will neglect contributions proportional to this factor. The above wave functions are normalized, i.e.,

$$
\int_0^1 dz\{\psi_\perp^{[2]}, \psi_\perp^{[3]}, \phi^{[2]}, \phi_p^{[3]}, \phi_\sigma^{[3]}\}(z) = 1. \tag{15}
$$

With the expansion in Eq. (12) one can calculate the form factor in terms of these light-cone wave functions. If one only takes twist 2 wave functions and neglects the quark mass m_c, the form factor is zero, reflecting the fact that the helicity is not conserved. This also implies that the contribution with twist 2 wave functions only is proportional to m_c and it is at the same order of those contributions in which one of twist 3 wave functions is involved. We keep the contribution of twist 2 by taking the finite quark mass into account. It is straightforward to evaluate the form factor in terms of these wave functions. We obtain:

$$
\begin{aligned}
\mathcal{F}(s) &= \frac{8\pi\alpha_s(s)}{9}f_{\eta_c}f_{J/\psi}\frac{1}{s^2}\int_0^1 dz_1dz_2\Big\{\frac{-2m_c^2}{m_{J/\psi}}\psi_\perp^{[2]}(z_1)\phi^{[2]}(z_2)\Big[\frac{1}{(1-z_1)^2z_2} + \frac{1}{(1-z_2)z_1^2}\Big] \\
&\quad + m_{J/\psi}\psi_\perp^{[3]}(z_1)\phi^{[2]}(z_2)\Big[\frac{1}{z_2^2(1-z_1)} - \frac{1}{z_2(1-z_1)} + \frac{1}{z_1(1-z_2)^2} - \frac{1}{z_1(1-z_2)}\Big] \\
&\quad + \frac{2m_{\eta_c}^2}{m_{J/\psi}}\psi_\perp^{[2]}(z_1)\phi_p^{[3]}(z_2)\Big[\frac{1}{z_2(1-z_1)^2} + \frac{1}{z_1^2(1-z_2)}\Big]\Big\}\cdot\Big[1 + \mathcal{O}\Big(\frac{\Lambda}{\sqrt{s}}\Big)\Big].
\end{aligned}
\tag{16}
$$

It is interesting to note that the wave function $\phi_\sigma^{[3]}$ does not contribute at the considered order, as shown by our calculation. The correction to our result in Eq. (16) is suppressed by the power of $\Lambda/\sqrt{s}$ or Λ^2/s, where Λ can be the QCD parameter $\Lambda_{\rm QCD}$, the quark mass m_c and masses of quarkonia.

If we know these wave functions we can give an numerical result for the form factor and hence the cross section. Unfortunately, these wave functions are not well known at the energy scale we are interested in. If the energy scale is very large, these wave functions approach to their asymptotic form:

$$
\begin{aligned}
&\phi^{[2]}(z) \approx \phi_\sigma^{[3]}(z) \approx \psi_\perp^{[2]}(z) \approx 6z(1-z), \\
&\phi_p^{[3]}(z) \approx \psi_\perp^{[3]}(z) \approx 1.
\end{aligned}
\tag{17}
$$

If we take these asymptotic forms of wave functions to make predictions, we will have end-point singularities. These singularities may be regularized by introducing a momentum cut. We regularize the end-point singularities by change the integration range as:

$$
\int_0^1 dz_1\int_0^1 dz_2 \to \int_\varepsilon^{1-\varepsilon}dz_1\int_\varepsilon^{1-\varepsilon}dz_2, \tag{18}
$$

with $\varepsilon = m_c/\sqrt{s}$. For numerical predictions in this letter we take numerical values of parameters as:

$$
\begin{aligned}
&\sqrt{s} = 10.6\ \text{GeV}, \qquad \alpha_s(\sqrt{s}) = 0.1758, \\
&m_h \simeq 3.0\ \text{GeV}, \qquad m_c \simeq 1.6\ \text{GeV}, \\
&f_{\eta_c} \simeq 350\ \text{MeV}, \qquad f_{J/\psi}^T = \frac{2m_c}{m_h}f_{J/\psi}, \\
&f_{J/\psi} \simeq 405\ \text{MeV}.
\end{aligned}
\tag{19}
$$

Taking the asymptotic form of the light-cone wave functions and these parameters we obtain:

$$\sigma(e^+e^- \to J/\psi\eta_c) \simeq 1.31 \text{ fb}. \tag{20}$$

It is interesting to note that light-cone wave functions as defined in Eq. (12) can be calculated with NRQCD factorization, in which nonperturbative effects can be parameterized with NRQCD matrix elements [16]. It is easy to obtain the leading order results as:

$$\begin{aligned}\phi^{[2]}(z) &= \phi_\sigma^{[3]}(z) = \psi_\perp^{[2]}(z) = \phi_p^{[3]}(z) = \psi_\perp^{[3]}(z)\\ &= \delta(z - \tfrac{1}{2}).\end{aligned} \tag{21}$$

Using this type of wave functions we obtain for the cross section:

$$\sigma(e^+e^- \to J/\psi\eta_c) \simeq 0.706 \text{ fb}. \tag{22}$$

All predictions in the above two cases are too small in comparison with the experimental result in Eq. (1). However, the choice of forms of wave functions in the two cases is not reasonable, because the asymptotic form is only valid when the energy scale goes to infinity and the NRQCD predictions in Eq. (20) are only reliable at the energy scale to be m_c with possibly large corrections from higher orders in α_s and relativistic corrections. Here, we have an energy scale as $\sqrt{s} \approx 10$ GeV, which is not close to m_c and far from being infinity. In general, predictions are sensible to the form of wave functions. Light-cone wave functions are nonperturbative objects, which can be only determined with nonperturbative methods or extracted from experimental results. The most extensively studied one is the wave function of π and ρ(e.g., see [19–21]). Motivated by these studies, we can make some models of wave functions for charmonia.

A model for the twist 2 light-cone wave function ϕ_π of π was proposed long time ago in [21], it takes the form as

$$\phi_\pi(z) = 6z(1-z)\{1 + \tfrac{3}{2}c[5(1-2z)^2 - 1]\}, \tag{23}$$

with $c = 2/3$. A study with QCD sum rule gives $c = 0.44$ at $\mu = 1$ GeV [19]. It should be noted that the shape of ϕ_π with these nonzero values of c is dramatically different than the shape with $c = 0$, i.e., the shape of asymptotic form. Motivated by this observation we assume the twist 2 wave function for η_c to be

$$\phi^{[2]}(z) = 6z(1-z)\{1 + 0.44\tfrac{3}{2}[5(1-2z)^2 - 1]\}. \tag{24}$$

The twist 3 wave functions of π are also studied in [19]. Through a study of recursion relations of moments of $\phi_p^{[3,\pi]}(z)$ and with QCD sum rule the form of $\phi_p^\pi(z)$ is determined at $\mu = 1$ GeV as:

$$\begin{aligned}\phi_p^{[3,\pi]} = 1 &+ (0.39 - 2.5\rho_\pi^2)C_2^{1/2}(2z-1) + (0.117\\ &- 4.914\rho_\pi^2)C_4^{1/2}(2z-1),\end{aligned} \tag{25}$$

with $\rho_\pi = (m_u + m_d)^2/m_\pi^2$. $C_n^\lambda(x)$ denotes Gegenbauer polynomials. It should be noted that the terms with ρ_π represent the part of meson-mass correction. This part can be totally different in the case of η_c. Based on this fact we assume the wave function $\phi_p^{[3]}$ to be the form:

$$\begin{aligned}\phi_p^{[3]}(z) = 1 &+ (0.39 - 2.5\rho_{\eta_c}^2)C_2^{1/2}(2z-1)\\ &+ (0.117 - 4.914\rho_{\eta_c}^2)C_4^{1/2}(2z-1),\end{aligned} \tag{26}$$

where we simply replace ρ_π with ρ_{η_c} and ρ_{η_c} takes the form as $4m_c^2 a/M_{\eta_c}^2$ with a free parameter a. With similar technics the wave functions for ρ is determined at $\mu = 1$ GeV as [20]:

$$\begin{aligned}\psi_\perp^{[2,\rho]}(z) &= 6z(1-z)(1 + 0.3[5(2z-1)^2 - 1]),\\ \psi_\perp^{[3,\rho]}(z) &= 1 - 1.6248C_2^{1/2}(2z-1) - 0.413C_4^{1/2}(2z-1),\end{aligned} \tag{27}$$

We assume the wave functions of J/ψ to be:

$$\psi^{[2]}(z) = \psi_\perp^{[2,\rho]}(z), \qquad \psi_\perp^{[3]}(z) = \psi_\perp^{[3,\rho]}(z). \tag{28}$$

With these wave functions in Eq. (23), (25), and (27) we obtain the cross section for different values of a:

$$\begin{aligned}\sigma(e^+e^- \to J/\psi\eta_c) &\simeq 7.37 \text{ fb}, \qquad \text{for } a = 1,\\ \sigma(e^+e^- \to J/\psi\eta_c) &\simeq 20.1 \text{ fb}, \qquad \text{for } a = 1.5,\\ \sigma(e^+e^- \to J/\psi\eta_c) &\simeq 31.7\text{fb}, \qquad \text{for } a = 1.75.\end{aligned} \tag{29}$$

The cross section increases with increasing a. The predicted cross section with $a = 1.75$ is much larger than that predicted with the NRQCD approach and it is more comparable with the experimental result in Eq. (1). It should be emphasized that the forms of used wave functions are assumed without any solid arguments, although it is motivated with those of π and ρ. In this model we neglect the evolution effects of light-cone wave functions. The numbers in the light-cone wave functions we use are calculated with the QCD sum rules for π and ρ at $\mu = 1$ GeV. The physics reflected by these numbers have actually nothing to do the physics of charmonia. However, it shows the possibility to obtain a large cross section at the order of the experimentally observed with our approach. A detailed study of these light-cone wave functions of charmonia is needed to obtain a reliable prediction.

To summarize: We have studied the exclusive production of $e^+e^- \to J/\psi\eta_c$, in which we have taken charm quarks as light quarks and used light-cone wave functions to parameterize nonperturbative effects related to charmonia. In comparison with NRQCD factorization, the factorization of our approach may be achieved in a more clean way and the perturbative coefficients will not have corrections with large logarithms like $\ln(\sqrt{s}/m_c)$ from higher orders, while in the approach of NRQCD factorization, these large logarithms exist and call for resummation. The forms of these light-cone wave functions are known if the energy scale is close to m_c or is very large. Unfortunately, these wave functions at the considered energy scale, which is not close to m_c and far from being very large, are unknown. With a simple model of light-cone wave functions, we are able to predict the cross section which is at the same order of that measured by Belle. But this model may not represent the physics of charmonia. A systematic study of these light-cone wave functions is required to have a precise prediction.

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