

Neutrino masses and mixing, quark-lepton symmetry, and strong right-handed neutrino hierarchy

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Assuming the same form of all mass matrices as motivated by quark-lepton symmetry, we discuss conditions under which bilarge mixing in the lepton sector can be obtained with a minimal amount of fine-tuning requirements for possible models. We assume hierarchical mass matrices, dominated by the 3-3 element, with off-diagonal elements much smaller than the larger neighboring diagonal element. Characteristic features of this scenario are strong hierarchy in masses of right-handed neutrinos, and comparable contributions of both lighter right-handed neutrinos to the resulting left-handed neutrino Majorana mass matrix. Because of obvious quark-lepton symmetry, this approach can be embedded into grand unified theories. The mass of the lightest neutrino does not depend on details of a model in the leading order. The right-handed neutrino scale can be identified with the GUT scale in which case the mass of the lightest neutrino is given as $(m_{\text{top}}^2/M_{\text{GUT}})|U_{\tau 1}|^2$.

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I. INTRODUCTION

A global analysis of neutrino oscillation data [1,2] gives the best fit to the neutrino mass-squared differences: $\Delta m_{\text{sol}}^2 \equiv m_{\nu_2}^2 - m_{\nu_1}^2 \simeq 6.9 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\text{atm}}^2 \equiv m_{\nu_3}^2 - m_{\nu_1}^2 \simeq 2.6 \times 10^{-3} \text{ eV}^2$, which, in the case $m_{\nu_1} \ll m_{\nu_2}, m_{\nu_3}$, can be interpreted as

$$m_{\nu_2} \simeq \sqrt{\Delta m_{\text{sol}}^2} \simeq 8.3 \times 10^{-3} \text{ eV}, \quad (1)$$

$$m_{\nu_3} \simeq \sqrt{\Delta m_{\text{atm}}^2} \simeq 5.1 \times 10^{-2} \text{ eV}. \quad (2)$$

The 3σ ranges for 1-2 and 2-3 mixing angles are

$$0.23 \leq \sin^2 \theta_{\text{sol}} \leq 0.39, \quad (3)$$

$$0.31 \leq \sin^2 \theta_{\text{atm}} \leq 0.72, \quad (4)$$

and the 3σ upper bound on the 1-3 mixing angle is

$$\sin^2 \theta_{13} \leq 0.054. \quad (5)$$

In grand unified theories [GUTs], both quarks and leptons originate from the same multiplets of unified gauge symmetry. For example, one generation of standard model quarks and leptons together with the right-handed neutrino naturally fits into the 16-dimensional representation of SO(10). Therefore, one would like to relate the results above to what we observe in the quark sector and understand them as a consequence of assumptions, the same for both quarks and leptons, we make about the structure of mass matrices. However, we know that mixing angles in the quark sector are small. The Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{CKM} is close to the identity matrix with off-diagonal elements

dominated by small Cabibbo angle (1-2 mixing). On the other hand, as we see from the results above, mixing in the lepton sector is large. The 2-3 mixing is close to maximal, 1-2 mixing is large, somewhat smaller than maximal, and 1-3 mixing is small, close to zero. There is no obvious quark-lepton symmetry in this pattern which makes the unified understanding of mass matrices quite challenging.

It has been realized that the lepton mixing can be enhanced by the seesaw mechanism even if the Dirac Yukawa couplings of leptons have a similar structure to that in the quark sector [3,4].

In a democratic approach (where all elements of mass matrices are equal in the leading order), it has been shown that a bilarge mixing can be obtained while assuming the same form of all Yukawa matrices and the right-handed neutrino Majorana mass matrix [5,6]. Even the form of perturbations can be the same for all mass matrices. This makes the approach manifestly quark-lepton symmetric and so it can be embedded into GUT models. The lepton mixing matrix is predominantly given by the matrix diagonalizing the charged lepton Yukawa matrix. Furthermore, there exists a well-defined framework (without exactly specifying perturbations) in which the left-handed neutrino mass matrix contributes the minimal amount of mixing to the lepton mixing matrix. In this case the lepton mixing matrix is given in terms of two parameters (neglecting phases) and the value of one mixing angle, $\sin \theta_{13}$, can be predicted [6]. If embedded into grand unified theories, the third generation Yukawa coupling unification is a generic feature (without necessity of distinguishing the third generation from the other two by family symmetries or in any other way), while masses of the first two generations of charged fermions depend on small perturbations. In the neutrino sector, the heavier two neutrinos are model dependent, while the

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mass of the lightest neutrino in this approach does not depend on perturbations in the leading order. Finally, the right-handed neutrino mass scale can be identified with the GUT scale in which case the mass of the lightest neutrino is given as $(m_{\text{top}}^2/M_{\text{GUT}})|U_{\tau 1}|^2$ [6]. This framework has everything one could wish for: obvious quark-lepton symmetry, third generation Yukawa coupling unification, bilarge lepton mixing with a prediction for $\sin\theta_{13}$ in the minimal case, and, more importantly, no need for an intermediate right-handed neutrino scale and with that associated prediction for the mass of the lightest neutrino. There is one problem, however: It is not straightforward to build a concrete model with the usual use of family symmetries (for discussion see [6]).

A similar approach, also assuming the same form of all mass matrices in the leading order, was recently discussed in Ref. [7]. Instead of democratic mass matrices, the starting point is a singular matrix of the form $(\lambda^2, \lambda, 1)^T \cdot (\lambda^2, \lambda, 1)$.

Motivated by these findings, we want to identify the corresponding picture in the hierarchical approach. After all, a democratic mass matrix [and a matrix of the type $(\lambda^2, \lambda, 1)^T \cdot (\lambda^2, \lambda, 1)$ as well] is equivalent to a matrix with the 3-3 element only. This is the usual starting point of hierarchical models.

In this letter we assume hierarchical mass matrices, dominated by the 3-3 element, with off-diagonal elements much smaller than the larger neighboring diagonal element:

$$Y_f \sim \begin{pmatrix} 0 & \delta_f & \delta_f \\ \delta_f & \epsilon_f & \epsilon_f \\ \delta_f & \epsilon_f & 1 \end{pmatrix} \lambda_f, \quad f = u, d, e, \nu, \quad (6)$$

where $0 \ll \delta_f \ll \epsilon_f \ll 1$ represent only orders of magnitude of different elements of Yukawa matrices. Clebsch-Gordan coefficients (or order one couplings) necessary to explain quark-lepton mass relations of the first two generations are understood. Yukawa matrices of this type naturally explain hierarchy in masses of charged fermions and mixing in the quark sector. Rather than starting with a specific model or texture, we discuss conditions under which bilarge mixing in the lepton sector can be obtained assuming the same generic structure of the neutrino Yukawa matrix. We find that the characteristic feature of this scenario is a strong hierarchy in masses of right-handed neutrinos and comparable contributions of both lighter right-handed neutrinos to the resulting left-handed neutrino mass matrix. Furthermore, the heaviest right-handed neutrinos have to contribute negligibly which leads to a prediction for the mass of the lightest neutrino. The right-handed neutrino scale can be identified with the GUT scale in which case (assuming third generation Yukawa coupling unification) the mass of the lightest neutrino is given as $(m_{\text{top}}^2/M_{\text{GUT}})|U_{\tau 1}|^2$. It does not depend on details of a model (ϵ 's and δ 's). Finally, we

discuss how a suitable right-handed neutrino sector can be naturally obtained in SO(10) models.

II. CONDITIONS FOR BILARGE LEPTON MIXING

We assume that the right-handed neutrino Majorana mass matrix has the same hierarchical structure as Yukawa matrices (this is not necessarily required by quark-lepton symmetry, but it is well motivated, see the discussion later). For simplicity, we work in the basis where the right-handed neutrino Majorana mass matrix is diagonal, defined as

$$M_{\nu_R} = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} M_0, \quad (7)$$

where $r_1 \ll r_2 \ll 1$, and M_0 is the scale at which right-handed neutrino masses are generated. Later we take $M_0 = M_{\text{GUT}}$. In this basis the neutrino Yukawa matrix (defined with doublets on the left) is in general given as

$$Y_\nu = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & 1 \end{pmatrix} \lambda_\nu, \quad (8)$$

where we assume the same order of perturbations ϵ_{ij} as in Eq. (6), namely, $\epsilon_{11} \ll \epsilon_{21} \sim \epsilon_{31} \ll \epsilon_{22} \sim \epsilon_{32} \ll 1$ with $\epsilon_{ij} \sim \epsilon_{ji}$. The inverse of the right-handed neutrino Majorana mass matrix is given as

$$M_{\nu_R}^{-1} = \frac{1}{r_1 r_2 M_0} \begin{pmatrix} r_2 & 0 & 0 \\ 0 & r_1 & 0 \\ 0 & 0 & r_1 r_2 \end{pmatrix}. \quad (9)$$

When right-handed neutrinos are integrated out, we obtain the left-handed neutrino Majorana mass matrix given by the seesaw formula [8]:

$$M_{\nu_L} = -v_u^2 Y_\nu M_{\nu_R}^{-1} Y_\nu^T. \quad (10)$$

In our basis it can be written in three terms, each corresponding to the contribution of one right-handed neutrino:

$$M_{\nu_L} = -\frac{\lambda^2 v_u^2}{r_1 r_2 M_0} (\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3), \quad (11)$$

where

$$\mathcal{M}_1 = r_2 \begin{pmatrix} \epsilon_{11}^2 & \epsilon_{11}\epsilon_{21} & \epsilon_{11}\epsilon_{31} \\ \epsilon_{21}\epsilon_{11} & \epsilon_{21}^2 & \epsilon_{21}\epsilon_{31} \\ \epsilon_{31}\epsilon_{11} & \epsilon_{31}\epsilon_{21} & \epsilon_{31}^2 \end{pmatrix} = r_2 \vec{e}_1 \cdot \vec{e}_1^T, \quad (12)$$

$$\mathcal{M}_2 = r_1 \begin{pmatrix} \epsilon_{12}^2 & \epsilon_{12}\epsilon_{22} & \epsilon_{12}\epsilon_{32} \\ \epsilon_{22}\epsilon_{12} & \epsilon_{22}^2 & \epsilon_{22}\epsilon_{32} \\ \epsilon_{32}\epsilon_{12} & \epsilon_{32}\epsilon_{22} & \epsilon_{32}^2 \end{pmatrix} = r_1 \vec{e}_2 \cdot \vec{e}_2^T, \quad (13)$$

$$\mathcal{M}_3 = r_1 r_2 \begin{pmatrix} \epsilon_{13}^2 & \epsilon_{13} \epsilon_{23} & \epsilon_{13} \\ \epsilon_{23} \epsilon_{13} & \epsilon_{23}^2 & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & 1 \end{pmatrix} = r_1 r_2 \vec{e}_3 \cdot \vec{e}_3^T, \quad (14)$$

and \vec{e}_i is the i th column of the neutrino Yukawa matrix. Each of these three matrices have two zero eigenvalues and a combination of any two of them still has one zero eigenvalue. The left-handed neutrino mass matrix can be diagonalized by a single unitary matrix,

$$M_{\nu_L}^{\text{diag}} = U_{\nu_L} M_{\nu_L} U_{\nu_L}^T. \quad (15)$$

Finally, the lepton mixing matrix which appears in the charged current Lagrangian is given as

$$U = U_e U_{\nu_L}^\dagger, \quad (16)$$

where U_e is the matrix diagonalizing the charged lepton Yukawa matrix, $Y_e^{\text{diag}} = U_e Y_e V_e^\dagger$. Because of the hierarchical nature of the charged lepton Yukawa matrix, $U_e \simeq 1$, and we will neglect it in our discussion.

A. Step 1: The dominant mass matrix

It is clear that if \mathcal{M}_3 dominates it is not possible to achieve large 2-3 mixing under the assumption of 3-3 dominance. The maximal 2-3 mixing corresponds to $\epsilon_{23} = 1$ which goes against our motivation. Several models of this type, usually called ‘‘lopsided,’’ were constructed [9]. The right-handed neutrino scale M_0 cannot be identified with the GUT scale, since the resulting mass of the heaviest neutrino would be too small, $m_{\nu_3} \sim m_{\text{top}}^2/M_{\text{GUT}} \sim 10^{-3}$ eV.

Let us assume that the \mathcal{M}_2 matrix is the dominant contribution to M_{ν_L} and let us neglect for a moment \mathcal{M}_1 and \mathcal{M}_3 . In this case maximal 2-3 mixing corresponds to $\epsilon_{32} = \epsilon_{22}$. This is actually quite good, since ϵ_{32} and ϵ_{22} are of the same order of magnitude in many models. At this point only the mass of the heaviest neutrino is generated. The eigenvalues of \mathcal{M}_2 are $\{0, 0, r_1 |\vec{e}_2|^2\}$. The eigenvector corresponding to m_{ν_3} is \vec{e}_2 , and since $U \simeq U_{\nu_L}^\dagger$ it will appear (properly normalized) in the third column of the lepton mixing matrix. As a consequence of degenerate zero eigenvalues, the first two columns of the lepton mixing matrix are not uniquely specified. In general, they can be any orthogonal linear combinations of two unit vectors orthogonal to \vec{e}_2 . Therefore, at this stage the 1-2 mixing angle is not specified, and comparing \vec{e}_2 with the third column of U , parametrized in general as $(s_{13}, s_{23}c_{13}, c_{23}c_{13})^T$, where $s_{13} \equiv \sin\theta_{13}$ and so on, the 2-3 and 1-3 mixing angles are given as

$$\tan\theta_{23} = \frac{e_{22}}{e_{32}} \quad \text{or} \quad \sin^2\theta_{23} = \frac{e_{22}^2}{e_{22}^2 + e_{32}^2}, \quad (17)$$

and

$$\sin\theta_{13} = \frac{e_{12}}{|\vec{e}_2|}. \quad (18)$$

Finally, if \mathcal{M}_1 dominates, the discussion is similar to the case of \mathcal{M}_2 dominance with \vec{e}_2 being replaced by \vec{e}_1 . Namely, the eigenvalues of \mathcal{M}_1 are $\{0, 0, r_2 |\vec{e}_1|^2\}$ and the 2-3 and 1-3 mixing angles are given as

$$\tan\theta_{23} = \frac{e_{21}}{e_{31}} \quad \text{or} \quad \sin^2\theta_{23} = \frac{e_{21}^2}{e_{21}^2 + e_{31}^2}, \quad (19)$$

and

$$\sin\theta_{13} = \frac{e_{11}}{|\vec{e}_1|}. \quad (20)$$

If the 1-3 mixing angle turns out to be much smaller than the present experimental upper bound, Eq. (5), this possibility will become strongly favored, since e_{11} can be arbitrarily small, and typically, $e_{11} = 0$ in many models.

B. Step 2: Masses of the heavier two neutrinos and the mixing matrix

The second eigenvalue is lifted and the lepton mixing matrix is specified when the contribution from the sub-leading right-handed neutrino is taken into account. To determine which matrix should be next to dominant, let us look at the eigenvector corresponding to the massless eigenvalue. The eigenvector corresponding to the largest eigenvalue (\vec{e}_1 if \mathcal{M}_1 dominates and \vec{e}_2 if \mathcal{M}_2 dominates) which represents the third column of the lepton mixing matrix is $\vec{e} \sim (s_{13}, c_{13}s_{23}, c_{13}c_{23})^T$. Let \vec{g} be \vec{e}_i in the case \mathcal{M}_i is next to dominant. The normalized eigenvector corresponding to the zero eigenvalue is perpendicular to both \vec{e} and \vec{g} and so it can be written as

$$\vec{v}_0 = \frac{\vec{g} \times \vec{e}}{|\vec{g} \times \vec{e}|}. \quad (21)$$

This vector will appear in the first column of the lepton mixing matrix. The first component of this vector (the 1-1 component of the lepton mixing matrix) is given as

$$(\vec{v}_0)_1 = \frac{c_{13}(g_2 c_{23} - g_3 s_{23})}{|\vec{g}| \sin\alpha}, \quad (22)$$

where α is the angle between \vec{e} and \vec{g} ,

$$\cos\alpha = \frac{1}{|\vec{g}|} (g_1 s_{13} + g_2 c_{13} s_{23} + g_3 c_{13} c_{23}). \quad (23)$$

Comparing $(\vec{v}_0)_1$ with the 1-1 element of U , parametrized in general as $c_{12}c_{13}$, we can read out $\cos\theta_{12}$, and with the use of Eq. (23) we find

$$\sin\theta_{12} = \frac{g_1 c_{13} - (g_2 s_{23} + g_3 c_{23}) s_{13}}{|\vec{g}| \sin\alpha}, \quad (24)$$

or equivalently,

$$\tan\theta_{12} = \frac{g_1 c_{13} - (g_2 s_{23} + g_3 c_{23}) s_{13}}{g_2 c_{23} - g_3 s_{23}}. \quad (25)$$

This relation was derived in a different way in Ref. [10].

Since $s_{23} \sim c_{23}$, $c_{13} \sim 1$, and $s_{13} \ll 1$, in order to generate large 1-2 mixing, we need either $g_1 \simeq g_2 - g_3$ or $g_2 + g_3 \gg g_2 - g_3$ assuming nonzero s_{13} . This cannot be satisfied for $\vec{g} = \vec{e}_3$. In this case we need $\epsilon_{23} \simeq 1$ and we would be back to lopsided textures. So \mathcal{M}_3 cannot be next to dominant for the same reason it cannot be dominant. On the other hand, both \mathcal{M}_1 and \mathcal{M}_2 can easily satisfy the above relations. For example, in the case \mathcal{M}_2 is next to dominant, $\vec{g} = \vec{e}_2$, and we need $\epsilon_{12} \simeq \epsilon_{22} - \epsilon_{32}$, which can be satisfied for $\epsilon_{12} \ll \epsilon_{22}$, and $\epsilon_{22} \sim \epsilon_{32}$ which we already assume anyway.

Let us summarize. If \mathcal{M}_1 dominates, large 2-3 mixing is generated for $\epsilon_{21} \sim \epsilon_{31} \sim \delta$, and at this point, the 1-3 mixing is zero or very small as a consequence of $\epsilon_{11} \sim 0$. Large 1-2 mixing is generated when subdominant \mathcal{M}_2 is taken into account and it occurs for $\epsilon_{12} \simeq \epsilon_{22} - \epsilon_{32}$. Alternatively, if \mathcal{M}_2 dominates, large 2-3 mixing is generated for $\epsilon_{22} \sim \epsilon_{32} \sim \epsilon$, and at this point the 1-3 mixing is small, $\sin\theta_{13} \simeq \epsilon_{12}/\epsilon_{22} \simeq \delta/\epsilon$. Large 1-2 mixing is generated when subdominant \mathcal{M}_1 is taken into account, and it occurs for $\epsilon_{21} \simeq \epsilon_{31}$. In this case $\tan\theta_{12} \simeq s_{13}(\epsilon_{21} + \epsilon_{31})/(\epsilon_{21} - \epsilon_{31})$. Both situations are viable. However, we will see shortly that it is not possible to generate the observed hierarchy in neutrino masses under the assumption of single right-handed neutrino dominance while keeping the structure of mass matrices strictly hierarchical as given in Eq. (6).

The nonzero eigenvalues of $\mathcal{M}_1 + \mathcal{M}_2$ can be written as

$$m_{\pm} = \frac{1}{2} t \left(1 \pm \sqrt{1 - \frac{4d}{t^2}} \right), \quad (26)$$

where

$$t = r_2 |\vec{e}_1|^2 + r_1 |\vec{e}_2|^2, \quad (27)$$

$$d = r_1 r_2 |\vec{e}_1 \times \vec{e}_2|^2 = r_1 r_2 |\vec{e}_1|^2 |\vec{e}_2|^2 \sin^2 \alpha, \quad (28)$$

and α is the angle between \vec{e}_1 and \vec{e}_2 . If \mathcal{M}_1 dominates, $r_1 |\vec{e}_2|^2 \ll r_2 |\vec{e}_1|^2$ and

$$\frac{m_-}{m_+} \simeq \frac{d}{t^2} \simeq \frac{r_1 |\vec{e}_2|^2 \sin^2 \alpha}{r_2 |\vec{e}_1|^2}. \quad (29)$$

Numerically, from Eqs. (1) and (2), we find $m_-/m_+ \simeq 0.16$. However, examining Eq. (24) we see that, in order to get large 1-2 mixing,

$$\sin\alpha \sim \frac{\epsilon_{12}}{\epsilon_{22}}, \quad (30)$$

which is naturally of order δ/ϵ and not larger than about 0.2. Therefore, the naturally generated hierarchy in

masses of the heavier two neutrinos is at least of order 100 (assuming single right-handed neutrino dominance).

The tension between generating large 1-2 mixing and mild hierarchy in masses of the heavier two neutrinos can be relieved when assuming $\epsilon_{12} \simeq \epsilon_{22}$, while keeping $\epsilon_{12} \simeq \epsilon_{22} - \epsilon_{32}$. For a model of this type, see Ref. [11]. This solution, however, goes against our motivation and is technically the same as the lopsided texture.

In the next section we propose another solution, which keeps the desired form of mass matrices given in Eq. (6). Rather we abandon the idea of single right-handed neutrino dominance.

C. Comparable contribution of \mathcal{M}_1 and \mathcal{M}_2

In the previous two sections, we showed that in order to obtain large 2-3 and 1-2 mixing angles it is necessary that the contribution of \mathcal{M}_3 is neither dominant nor next to dominant. However, the relative contribution \mathcal{M}_1 and \mathcal{M}_2 to the resulting left-handed neutrino mass matrix was not important at all. No matter which one dominates, the observed pattern of mixing angles can be recovered. In the case \mathcal{M}_1 dominates, interpreting the condition for large 1-2 mixing angle as $\epsilon_{12} \simeq \epsilon_{22} - \epsilon_{32}$, with $\epsilon_{12} \ll \epsilon_{22}$ rather than $\epsilon_{12} \sim \epsilon_{22} \sim \epsilon_{32}$ coincides with the condition for large 2-3 mixing angle if the \mathcal{M}_2 matrix dominates. Therefore, as far as 2-3 mixing is concerned the contribution from \mathcal{M}_1 and \mathcal{M}_2 to the left-handed neutrino mass matrix can be equal. The masses of the heavier two neutrinos can be then anything, from highly hierarchical to degenerate [which happens when t in Eq. (26) is zero].

Because of mild hierarchy in masses of the heavier two neutrinos, the simplest possibility is that \mathcal{M}_1 only slightly dominates, $r_2 |\vec{e}_1|^2 \gtrsim r_1 |\vec{e}_2|^2$. The relations for mixing angles, Eqs. (19) and (20), are now just a rough approximation. The 2-3 mixing will be close to maximal, as far as $\epsilon_{21} \sim \epsilon_{31}$ and $\epsilon_{22} \sim \epsilon_{32}$. The 1-3 mixing will be roughly given by the larger of Eqs. (18) and (20). Finally, the 1-2 mixing can be again obtained by looking at the first component of the eigenvector corresponding to the zero eigenvalue. Assuming $\epsilon_{11} = 0$ and $\epsilon_{21} = \epsilon_{31}$, we get

$$\vec{v}_0 \propto \vec{e}_1 \times \vec{e}_2 = (\epsilon_{32} - \epsilon_{22}, \epsilon_{12}, -\epsilon_{12})^T, \quad (31)$$

which does not depend on relative dominance of \mathcal{M}_1 and \mathcal{M}_2 . Comparing the first component of \vec{v}_0 with the 1-1 element of the lepton mixing matrix, $c_{12} c_{13}$, and taking $\cos\theta_{13} \simeq 1$, we find

$$\sin\theta_{12} \simeq \frac{\sqrt{2}\epsilon_{12}}{\sqrt{(\epsilon_{32} - \epsilon_{22})^2 + 2\epsilon_{12}^2}}, \quad (32)$$

or equivalently,

$$\tan\theta_{12} \simeq \frac{\sqrt{2}\epsilon_{12}}{\epsilon_{32} - \epsilon_{22}}. \quad (33)$$

Although these relations are similar to those in Eqs. (24) and (25) derived in the framework of single right-handed neutrino dominance, they are independent on any assumption about the dominance of \mathcal{M}_1 or \mathcal{M}_2 .

We see that we can successfully obtain close to maximal atmospheric mixing angle, large solar mixing angle, and small 1-3 mixing angle assuming that both \mathcal{M}_1 and \mathcal{M}_2 contribute comparably to the left-handed neutrino mass matrix. Furthermore, in this case mild hierarchy (or no hierarchy at all) in masses of the heavier two neutrinos can be achieved. We will demonstrate that this scenario works on a simple example. Before we do that, let us discuss what changes when the contribution from \mathcal{M}_3 is included.

D. Step 3: Mass of the lightest neutrino

The mass of the lightest neutrino is lifted when \mathcal{M}_3 is taken into account. Since we assume it is just a small correction to the first two terms, it can be treated as a perturbation. Adding this perturbation does not significantly affect the heavy two eigenvalues and the diagonalization matrix, but it is crucial for the lightest eigenvalue which is exactly zero in the limit when this term is ignored. In the case of nondegenerate eigenvalues, corrections to eigenvalues m_i of a matrix M generated by a matrix δM are given as

$$\delta m_i = u_i^\dagger \delta M u_i, \quad (34)$$

where u_i are normalized eigenvectors. In our case,

$$\delta M = -\frac{\lambda^2 v_u^2}{M_0} \begin{pmatrix} \epsilon_{13}^2 & \epsilon_{13}\epsilon_{23} & \epsilon_{13} \\ \epsilon_{23}\epsilon_{13} & \epsilon_{23}^2 & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & 1 \end{pmatrix}, \quad (35)$$

and the vector \vec{v}_0 corresponding to the zero eigenvalue is the first row of U_{ν_L} obtained from \mathcal{M}_1 and \mathcal{M}_2 . It corresponds to the first column of the lepton mixing matrix U , see Eq. (16), since $U_e \simeq 1$. Therefore, we have $\vec{v}_0 \simeq (U_{e1}, U_{\mu 1}, U_{\tau 1})^\dagger$. Since δM is strongly dominated by the 3-3 element, we get the mass of the lightest neutrino in the form

$$m_{\nu_1} = \frac{\lambda_\nu^2 v_u^2}{M_0} |U_{\tau 1}|^2, \quad (36)$$

which does not depend on perturbations (ϵ_{ij}) in the leading order.

Although we do not measure $U_{\tau 1}$, it is related to the observed mixing angles due to the unitarity of the lepton mixing matrix. In the case $\sin\theta_{13} \simeq 0$, it is simply given as $U_{\tau 1} \simeq \sin\theta_{23} \sin\theta_{12}$. A global analysis of neutrino oscillation data [1] gives the 3σ range: $0.20 \leq |U_{\tau 1}| \leq 0.58$.

In simple SO(10) models $\lambda_u = \lambda_\nu$, in which case the lightest and the heaviest fermion of the standard model are connected through the relation above where $\lambda_\nu^2 v_u^2$ is replaced by m_{top}^2 (actually to be precise, $\lambda_u = \lambda_\nu$ is a

relation at the GUT scale and the effects of the renormalization group running between the GUT scale and the electroweak scale should be taken into account). This is a very pleasant feature since we can further identify M_0 with the GUT scale, $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV, in which case we get

$$m_{\nu_1} = \frac{m_{\text{top}}^2}{M_{\text{GUT}}} |U_{\tau 1}|^2, \quad (37)$$

and predict the mass of the lightest neutrino to be between 5×10^{-5} and 5×10^{-4} eV depending on the value of $U_{\tau 1}$. This prediction does not depend on details of a model. It represents a realization of Yukawa coupling unification in the neutrino sector and adds to predictions of Yukawa coupling unification in quark and charged lepton sector [12].

E. A simple example

Let us demonstrate on a simple example that the scenario discussed in the above sections really works. Let us assign the following values to elements of the neutrino Yukawa matrix: $\epsilon_{11} = 0$, $\epsilon_{21} = \epsilon_{31} = \epsilon_{12} = \epsilon_{13} = \delta$, $\epsilon_{22} = \epsilon$, and $\epsilon_{32} = \epsilon_{23} = \epsilon + 2\delta$, where $\epsilon = 0.01$ and $\delta = 0.002$, and let us take $r_1 = 9 \times 10^{-8}$ and $r_2 = -5 \times 10^{-6}$. This choice of parameters certainly satisfies the desired texture in Eq. (6). For simplicity, we assume a symmetric Yukawa matrix; however, it is not crucial in any way. For example, exact values of ϵ_{23} , ϵ_{13} are not relevant at all, and would not significantly change numerical results below even if changed by a factor of 10. Also, ϵ_{21} does not have to be equal to ϵ_{12} . The only crucial relations are $\epsilon_{21} \simeq \epsilon_{31}$ and $\epsilon_{32} \simeq \epsilon + 2\delta$. Finally, let us make the minimal and the most interesting assumption that $M_0 = M_{\text{GUT}}$ and $\lambda_\nu = \lambda_u$. With these values of parameters we find

$$\sin^2\theta_{23} = 0.71, \quad (38)$$

$$\sin^2\theta_{12} = 0.35, \quad (39)$$

$$\sin^2\theta_{13} = 0.03, \quad (40)$$

and

$$m_{\nu_3} = 4.9 \times 10^{-2} \text{ eV}, \quad (41)$$

$$m_{\nu_2} = 7.8 \times 10^{-3} \text{ eV}, \quad (42)$$

$$m_{\nu_1} = 2.5 \times 10^{-4} \text{ eV}, \quad (43)$$

which is in good agreement with experimental values. The $\sin^2\theta_{23}$ is somewhat too large (but still within 3σ). We are not trying to provide the best fit to data, but rather demonstrate that bilarge mixing can be achieved with a simple choice of parameters with the neutrino Yukawa matrix of the form given in Eq. (6). Note, we parame-

trized the neutrino Yukawa matrix by two parameters and we specified values of all parameters to one digit only. Keeping this in mind, we actually find the results above quite remarkable. Relaxing the exact relations between elements of the neutrino Yukawa matrix, there is certainly enough freedom to fit all mixing angles and masses very accurately. Furthermore, in a realistic scenario one should take into account corrections from the right-handed neutrino mass matrix not being exactly diagonal, and also the contribution from diagonalization of the charged lepton Yukawa matrix.

Masses of the right-handed neutrinos in this example, $M_3 = M_{\text{GUT}}$, $M_2 \simeq 10^{11}$ GeV, and $M_1 \simeq 10^9$ GeV, are in an interesting range for leptogenesis [13]. The value of $r_2 |\vec{e}_1|^2 / r_1 |\vec{e}_2|^2 \simeq -1.5$ measures the relative contribution of \mathcal{M}_1 and \mathcal{M}_2 . It shows that the contributions of the two right-handed neutrinos to the left-handed neutrino mass matrix do not have to be extremely close.

Finally, we can check if the formula we derived for the mass of the lightest neutrino works. Plugging $U_{\tau 1} = 0.41$ and $m_{\text{top}}^2 / M_{\text{GUT}} = 0.0014$ eV (the same value used in the evaluation of the masses above) to Eq. (37), we find $m_{\nu_1} = 2.4 \times 10^{-4}$ eV which is in very good agreement with the numerical result in Eq. (43).

III. ORIGIN OF STRONG RIGHT-HANDED NEUTRINO HIERARCHY

In SO(10) models, the right-handed neutrino is part of the 16-dimensional representation. Therefore the Majorana mass term is not allowed by SO(10) symmetry and it is generated in the process of GUT symmetry breaking. A simple possibility is to assume that a 16, $\bar{16}$ pair of Higgs fields gets a vacuum expectation value in the right-handed neutrino direction. In models where the two Higgs doublets originate from 10-dimensional representation of SO(10), Yukawa couplings are generated from operators of the form

$$16_i (\dots)_{ij} 10 16_j, \quad i = 1, 2, 3, \quad (44)$$

where 10 contains the two Higgs doublets of minimal supersymmetric standard model, and $(\dots)_{ij}$ is a flavor dependent part responsible for generating the desired structure of Yukawa matrices. It typically contains flavon fields responsible for generating hierarchy between generations and other Higgs fields, for example, 45 of SO(10), responsible for the right quark-lepton mass relations of the first two generations. Besides these, there can also be operators where 10 of Higgs and one 16_i are replaced by $\langle \bar{16} \rangle_{\bar{p}}$ and SO(10) singlet fields N_i of the form

$$16_i (\dots)_{ij} \langle \bar{16} \rangle_{\bar{p}} N_j. \quad (45)$$

These operators generate a mass matrix, Y_N , between right-handed neutrinos and SO(10) singlets. It has naturally the same structure as other Yukawa matrices,

although it is not identical, because $(\dots)_{ij}$ distinguishes between different fields in 16-dimensional representation. A mass term for singlet fields effectively leads to a Majorana mass matrix for right-handed neutrinos:

$$M_{\nu_R} = Y_N M_N^{-1} Y_N^T. \quad (46)$$

A simple mass term for singlet fields of the form, $M_N \simeq \text{diag}(1, 1, 1) M_N$, where for simplicity we use M_N for both the matrix and the scale itself, automatically leads to a strong hierarchy in masses of right-handed neutrinos. If the hierarchy in mass eigenvalues of Y_N is $\sim (\delta_N, \epsilon_N, 1)$ (similar to eigenvalues of other Yukawa matrices), the hierarchy in right-handed neutrino masses is naturally doubled $\sim (\delta_N^2, \epsilon_N^2, 1)$.

The left-handed neutrino Majorana mass term can be written as

$$M_{\nu_L} = Y_\nu Y_N^{T-1} M_N Y_N^{-1} Y_\nu^T. \quad (47)$$

If Y_N is identical to Y_ν , then $M_{\nu_L} = M_N$ and all three neutrinos have almost the same mass. This is to demonstrate that it is actually very natural to expect that all three right-handed neutrinos contribute roughly equally to the resulting left-handed neutrino mass matrix. The desired situation when M_1 and M_2 contribute more is possible to achieve in two ways: either to manage ϵ_N and δ_N to be somewhat smaller than the corresponding perturbations in Y_ν , or simply assume that $M_{N_1} \simeq M_{N_2} > M_{N_3}$. From a model building point of view, the second choice does not represent a big challenge. As a by-product, the stronger hierarchy of M_{ν_R} makes it also closer to the identity matrix, and corrections to the discussion in previous sections from it not being exactly diagonal are smaller.

IV. CONCLUSIONS

We discussed conditions under which bilarge lepton mixing can be achieved in hierarchical models in which all mass matrices are dominated by the 3-3 element. Many features of this framework are similar to those in the democratic framework discussed in Ref. [6]. The obvious quark-lepton symmetry makes it easy to embed models of this type into GUTs. The right-handed neutrino mass scale can be identified with the GUT scale in which case the mass of the lightest neutrino is given as $(m_{\text{top}}^2 / M_{\text{GUT}}) |U_{\tau 1}|^2$, the same as in Ref. [6]. The third generation Yukawa coupling unification is obvious in this picture, since it is our starting point and we do not allow any large off-diagonal elements. This can be understood from two possible ways permutation symmetry can be used in model building. A matrix with a 3-3 element only can be also motivated by permutation symmetry under which the first two families transform as a doublet [14]. Both approaches require strong hierarchy in masses of right-handed neutrinos and negligible contri-

bution of the heaviest one to the left-handed neutrino mass matrix.

There are few major differences however. There is a tension between achieving the observed spectrum of heavier two neutrinos and bilarge mixing in the hierarchical approach, which is avoided when the two lighter right-handed neutrinos contribute comparably to the left-handed neutrino mass matrix. In the democratic approach there is no tension at all, since bilarge mixing originates predominantly from the matrix diagonalizing the charged lepton Yukawa matrix and so neutrino masses can be adjusted arbitrarily. Furthermore, in the democratic approach, there is a well-defined framework (without exactly specifying perturbations) in which the left-handed neutrino mass matrix contributes the mini-

mal amount of mixing to the lepton mixing matrix and the value of one mixing angle, $\sin\theta_{13}$, can be predicted [6]. We do not see the equivalent situation in the hierarchical approach. Finally, it seems to be much easier to build concrete GUT models with hierarchical Yukawa matrices of the type discussed here than democratic ones.

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