Invisible Higgs boson decays in spontaneously broken R parity

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The Higgs boson may decay mainly to an invisible mode characterized by missing energy, instead of the standard model channels. This is a generic feature of many models where neutrino masses arise from the spontaneous breaking of ungauged lepton number at relatively low scales, such as spontaneously broken R-parity models. Taking these models as framework, we reanalyze this striking suggestion in view of the recent data on neutrino oscillations that indicate nonzero neutrino masses. We show that, despite the smallness of neutrino masses, the Higgs boson can decay mainly to the invisible Goldstone boson associated to the spontaneous breaking of lepton number. This requires a gauge singlet superfield coupling to the electroweak doublet Higgses, as in the next to minimal supersymmetric standard model scenario for solving the μ problem. The search for invisibly decaying Higgs bosons should be taken into account in the planning of future accelerators, such as the Large Hadron Collider and the Next Linear Collider.

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I. INTRODUCTION

Understanding the origin of mass is the main open puzzle in particle physics today. In the standard model, all masses arise as a result of the spontaneous breaking of the $SU(2) \otimes U(1)$ gauge symmetry. This implies the existence of an elementary Higgs boson, not yet found. Stabilizing the mass of the Higgs most likely requires new physics, and supersymmetry has thus far been the leading contender. Another aspect of this problem is the smallness of neutrino masses. Despite the tremendous effort that has led to the discovery of neutrino mass [1–3], the mechanism of neutrino mass generation will remain open for years to come (a detailed analysis of the three-neutrino oscillation parameters can be found in [4]). The most popular mechanism to generate neutrino masses is the seesaw mechanism [5–9]. Although the seesaw fits naturally in SO(10) unification models, we currently have no clear hints that uniquely point towards any unification scheme. Therefore it may well be that neutrino masses arise from garden-variety physics having nothing to do with unification, such as certain seesaw variants [10], and models with radiative generation [11,12]. In such models, the physics of neutrino mass would then be characterized by much lower scales [13], potentially affecting the decay properties of the Higgs boson. This is especially so if neutrino masses arise due to the spontaneous violation of ungauged lepton number. In this broad class of models, the Higgs boson will have an important decay channel into the singlet Goldstone boson (called majoron) associated to lepton-number violation [14],

$$h \to JJ. \tag{1}$$

Here we focus on the specific case of low-energy supersymmetry with spontaneous violation of R parity, as the origin of neutrino mass. R parity is defined as $R_p = (-1)^{3B+L+2S}$ with S, B, and L denoting spin, baryon, and lepton numbers, respectively [15]. In this model R-parity violation takes place "a la Higgs," i.e., spontaneously, due to nonzero sneutrino vacuum expectation values (vevs) [16–18]. In this case one of the neutral CP-odd scalars is identified with the majoron. In contrast with the seesaw majoron, ours is characterized by a small scale (TeV-like) and carries only one unit of lepton number. This scheme leads to the bilinear R-parity violation model, the simplest effective description of R-parity violation [19] (for calculations including also trilinear terms, see, for example, [20,21]). The model not only accounts for the observed pattern of neutrino masses and mixing [22–25], but also makes predictions for the decay branching ratios of the lightest supersymmetric particle [26–29] from the current measurements of neutrino mixing angles [4].

In previous studies [30], it was noted that the spontaneously broken R-parity (SBRP) model leads to the possibility of invisibly decaying Higgs bosons, provided there is an $SU(2) \otimes U(1)$ singlet superfield Φ coupling to the electroweak doublet Higgses, the same that appears in the next to minimal supersymmetric standard model (NMSSM).

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In this paper we reanalyze this issue, taking into account the small masses indicated by current neutrino oscillation data [4].[1] We focus on the lowest-lying neutral CP-even scalar boson of the model. We show explicitly that the presence of the SU(2) ⊗ U(1) singlet superfield Φ plays a triple role: (i) It gives a model where neutrino masses are obtained from first principles without any type of fine-tuning, even when radiative corrections are negligible, (ii) it solves the μ problem "a la NMSSM,"[2] and (iii) it makes the invisible Higgs boson decay in Eq. (1) potentially the most important mode of Higgs boson decay. The latter is remarkable, given the smallness of neutrino masses required to fit current neutrino oscillation data. We also verify that the production of such a Higgs boson in e^+e^- annihilation can be as large as that characterizing the standard case, and that therefore this situation should be taken as part of the agenda of future accelerators probing the mechanism of mass generation.

II. MODEL WITH SPONTANEOUSLY BROKEN R PARITY

The most general superpotential terms involving the minimal supersymmetric standard model (MSSM) superfields in the presence of the SU(2) ⊗ U(1) singlet superfields $(\hat{\nu}^c_i, \hat{S}_i, \hat{\Phi})$ carrying a conserved lepton number assigned as $(-1, 1, 0)$, respectively, are given as [32]

$$\mathcal{W} = \varepsilon_{ab}[h_U^{ij}\hat{Q}_i^a\hat{U}_j\hat{H}_u^b + h_D^{ij}\hat{Q}_i^b\hat{D}_j\hat{H}_d^a + h_E^{ij}\hat{L}_i^b\hat{E}_j\hat{H}_d^a + h_\nu^{ij}\hat{L}_i^a\hat{\nu}_j^c\hat{H}_u^b - \hat{\mu}\hat{H}_d^a\hat{H}_u^b - (h_0\hat{H}_d^a\hat{H}_u^b + \delta^2)\hat{\Phi}] + h^{ij}\hat{\Phi}\hat{\nu}_i^c\hat{S}_j + M_R^{ij}\hat{\nu}_i^c\hat{S}_j + \frac{1}{2}M_\Phi\hat{\Phi}^2 + \frac{\lambda}{3!}\hat{\Phi}^3. \tag{2}$$

The first three terms together with the $\hat{\mu}$ term define the R-parity conserving MSSM, the terms in the last row only involve the SU(2) ⊗ U(1) singlet superfields $(\hat{\nu}^c_i, \hat{S}_i, \hat{\Phi})$,[3] while the remaining terms couple the singlets to the MSSM fields. We stress the importance of the Dirac-Yukawa term which connects the right-handed neutrino superfields to the lepton doublet superfields, thus fixing lepton number.

A. Spontaneous symmetry breaking

The presence of singlets in the model is essential in order to drive the spontaneous violation of R parity and electroweak symmetries in a phenomenologically consistent way. Like all other Yukawa couplings h_U, h_D, h_E, we assume that h_ν is an arbitrary nonsymmetric complex matrix in generation space. For technical simplicity, we take the simplest case with just one pair of lepton-number-carrying SU(2) ⊗ U(1) singlet superfields, $\hat{\nu}^c$ and $\hat{S}$, in order to avoid inessential complication. This in turn implies $h_{ij} \to h$ and $h_\nu^{ij} \to h_\nu^i$.

The full scalar potential along neutral directions is given by

$$\begin{aligned} V_{\rm total} = {}& |h\Phi\tilde{S} + h_\nu^i\tilde{\nu}_iH_u + M_R\tilde{S}|^2 + |h_0\Phi H_u + \hat{\mu}H_u|^2 + |h\Phi\tilde{\nu}^c + M_R\tilde{\nu}^c|^2 + |-h_0\Phi H_d - \hat{\mu}H_d + h_\nu^i\tilde{\nu}_i\tilde{\nu}^c|^2 \\ & + \left|-h_0H_uH_d + h\tilde{\nu}^c\tilde{S} - \delta^2 + M_\Phi\Phi + \frac{\lambda}{2}\Phi^2\right|^2 + \sum_{i=1}^3|h_\nu^i\tilde{\nu}^cH_u|^2 + \Big[A_hh\Phi\tilde{\nu}^c\tilde{S} - A_{h_0}h_0\Phi H_uH_d \\ & + A_{h_\nu}h_\nu^i\tilde{\nu}_iH_u\tilde{\nu}^c - B\hat{\mu}H_uH_d - C_\delta\delta^2\Phi + B_{M_R}M_R\tilde{\nu}^c\tilde{S} + \frac{1}{2}B_{M_\Phi}M_\Phi\Phi^2 + \frac{1}{3!}A_\lambda\lambda\Phi^3 + \text{H.c.}\Big] \\ & + \sum_\alpha\tilde{m}_\alpha^2|z_\alpha|^2 + \frac{1}{8}(g^2 + g'^2)\Big(|H_u|^2 - |H_d|^2 - \sum_{i=1}^3|\tilde{\nu}_i|^2\Big)^2, \end{aligned} \tag{3}$$

where z_α denotes any neutral scalar field in the theory.

The pattern of spontaneous symmetry breaking of both electroweak and R-parity symmetries works in a very simple way. The spontaneous breaking of R parity is driven by nonzero vevs for the scalar neutrinos. The scale characterizing R-parity breaking is set by the isosinglet vevs

$$\langle\tilde{\nu}^c\rangle = \frac{v_R}{\sqrt{2}}, \qquad \langle\tilde{S}\rangle = \frac{v_S}{\sqrt{2}}, \tag{4}$$

and

$$\langle\Phi\rangle = \frac{v_\Phi}{\sqrt{2}}. \tag{5}$$

We also have very small left-handed sneutrino vacuum expectation values

[1]Reference [30] assumed MeV scale for the heaviest neutrino mass, inconsistent with the atmospheric data which points towards $m_\nu \sim 0.05$ eV.

[2]Provided domain walls are either eliminated by imposing a Z_2 R symmetry on the nonrenormalizable operators [31], or that they are simply inflated away.

[3]The term linear in Φ has been included in the first row as it is relevant in electroweak breaking.

$$\langle \tilde{\nu}_{Li} \rangle = \frac{v_{Li}}{\sqrt{2}}. \tag{6}$$

The spontaneous breaking of R parity also entails the spontaneous violation of total lepton number. This implies that one of the neutral CP-odd scalars, which we call majoron, and which is given by the imaginary part of

$$\frac{\sum_i v_{Li}^2}{Vv^2}(v_u H_u - v_d H_d) + \sum_i \frac{v_{Li}}{V}\tilde{\nu}_i + \frac{v_S}{V}S - \frac{v_R}{V}\tilde{\nu}^c, \tag{7}$$

remains massless, as it is the Nambu-Goldstone boson associated to the breaking of lepton number. Note that this majoron is quite different from the one that emerges in the seesaw majoron model, as it is characterized by a different lepton number (one unit instead of two) and by a different scale, determined by the combination $V = \sqrt{v_R^2 + v_S^2} \sim \text{TeV}$. Note that Eq. (4) is the origin of lepton-number violation in this model and plays a crucial role in determining the neutrino masses.

On the other hand, electroweak breaking is driven by the isodoublet vevs $\langle H_u \rangle = v_u/\sqrt{2}$ and $\langle H_d \rangle = v_d/\sqrt{2}$, with the combination $v^2 = v_u^2 + v_d^2 + \sum_i v_{Li}^2$ fixed by the W mass

$$m_W^2 = \frac{g^2 v^2}{4}, \tag{8}$$

while the ratio of isodoublet vevs yields

$$\tan\beta = \frac{v_u}{v_d}. \tag{9}$$

This basically recovers the standard tree-level spontaneous breaking of the electroweak symmetry in the MSSM [33].[4]

B. Neutrino masses

Since neutrino masses are so much smaller than all other fermion mass terms in the model, one can find the effective neutrino mass matrix in a seesaw-type approximation. From the full neutral fermion mass matrix, see Eq. (A2), one calculates the effective 3×3 neutrino mass matrix $(\mathbf{m}_{\nu\nu}^{\text{eff}})$ as

$$\mathbf{m}_{\nu\nu}^{\text{eff}} = -\mathbf{M}_\mathbf{D}^\mathbf{T}\mathbf{M}_\mathbf{H}^{-1}\mathbf{M}_\mathbf{D}, \tag{10}$$

where $\mathbf{M_H}$ is the 7×7 matrix of all other neutral fermion states, see Eq. (A2), and the 3×7 matrix $\mathbf{m}_{\chi^0\nu}^\mathbf{T}$ is given as

$$\mathbf{M}_\mathbf{D}^\mathbf{T} = (\mathbf{m}_{\chi^0\nu}^\mathbf{T} \quad \mathbf{m_D} \quad \mathbf{0} \quad \mathbf{0}), \tag{11}$$

where the matrices $\mathbf{m}_{\chi^0\nu}^\mathbf{T}$ and $\mathbf{m_D}$ are given in Eqs. (A4) and (A7). The inverse of $\mathbf{M_H}$ is too long to be given explicitly here.

[4]We have verified explicitly, however, that radiative electroweak breaking may also occur.

After some algebraic manipulation, the effective neutrino mass matrix can be cast into a very simple form:

$$(\mathbf{m}_{\nu\nu}^{\text{eff}})_{ij} = a\Lambda_i\Lambda_j + b(\epsilon_i\Lambda_j + \epsilon_j\Lambda_i) + c\epsilon_i\epsilon_j, \tag{12}$$

where one can define the effective bilinear R-parity violating parameters ϵ_i and Λ_i as

$$\epsilon_i = h_\nu^i \frac{v_R}{\sqrt{2}} \tag{13}$$

and

$$\Lambda_i = \epsilon_i v_d + \mu v_{L_i}. \tag{14}$$

Here the parameter μ is

$$\mu = \hat{\mu} + h_0 \frac{v_\Phi}{\sqrt{2}}, \tag{15}$$

while the coefficients appearing in Eq. (12) are given by

$$a = \frac{1}{4\mu\,\text{Det}(\mathbf{M_H})}[m_\gamma \hat{M}_R(-h^2 v_R v_S \mu + \hat{M}_\Phi \hat{M}_R \mu + h_0^2 \hat{M}_R v_d v_u)], \tag{16}$$

$$b = \frac{1}{8\mu\,\text{Det}(\mathbf{M_H})}[h_0 m_\gamma \hat{M}_R(h_0\hat{M}_R + h\mu)v_u(v_u^2 - v_d^2)], \tag{17}$$

$$c = \frac{1}{4\mu\,\text{Det}(\mathbf{M_H})}[(h_0\hat{M}_R + h\mu)^2 v_u^2 (2M_1M_2\mu - m_\gamma v_d v_u)], \tag{18}$$

and $\text{Det}(\mathbf{M_H})$ is given as

$$\text{Det}(\mathbf{M_H}) = \tfrac{1}{8}\hat{M}_R\{8M_1M_2\mu(\hat{M}_\Phi\hat{M}_R\mu - h^2\mu v_R v_S + h_0^2\hat{M}_R v_d v_u) - m_\gamma[4\mu v_d(\hat{M}_\Phi\hat{M}_R - h^2 v_R v_S)v_u + h_0^2\hat{M}_R(v_d^2 + v_u^2)^2]\}. \tag{19}$$

Note that $\hat{M}_R$ and $\hat{M}_\Phi$ above are defined as

$$\hat{M}_R = M_R + h\frac{v_\Phi}{\sqrt{2}}, \qquad \hat{M}_\Phi = M_\Phi + \lambda\frac{v_\Phi}{\sqrt{2}}. \tag{20}$$

The "photino" mass parameter is defined as $m_\gamma = g^2 M_1 + g'^2 M_2$.

Equation (12) resembles very closely the corresponding expression for the explicit bilinear R-parity breaking model [19–24], once the dominant one-loop corrections are taken into account. Note that the tree-level result of the explicit bilinear model can be recovered in the limit $\hat{M}_R, \hat{M}_\Phi \to \infty$. In this limit the coefficients b and c go to zero, while

$$a = \frac{m_\gamma}{4\,\text{Det}(\mathbf{M}_{\chi^0})}. \tag{21}$$

In this limit only one nonzero neutrino mass remains. Whether the one-loop corrections or the contribution from the singlet fields are more important in determining the neutrino masses depends essentially on the relative size of the coefficient c in Eq. (12) compared to the corresponding one-loop coefficient. Both extremes can be realized in our model. We note, however, that as discussed below large branching ratios of the Higgs into invisible final states require sizable values of h and h_0 (as well as singlets not being too heavy). For such choices of parameters we have found that the "singlino" contribution to Eq. (12) is usually much more important than the one-loop corrections to the neutrino masses.

Note also that the model does not predict whether the atmospheric (solar) mass scale is mainly due to the first (third) term in Eq. (12) or vice versa. We have checked numerically that both possibilities can be realized and "good" points (in the sense of being appropriate for neutrino physics) can be found easily in either case.

C. Scalar mass matrices

With the above choices and definitions, we can obtain the neutral scalar boson mass matrices as in Ref. [17] by evaluating the second derivatives of the scalar potential in Eq. (3) at the minimum. This results in 8×8 mass matrices for the real and imaginary parts of the neutral scalars.[5] We have checked, in particular, that in the CP-odd sector we find both the Goldstone "eaten" by the Z^0 as well as the Goldstone boson corresponding to the spontaneous breaking of R parity, namely, the majoron, Eq. (7). In the basis $A_0' = (H_d^{0I}, H_u^{0I}, \tilde{\nu}^{1I}, \tilde{\nu}^{2I}, \tilde{\nu}^{3I}, \Phi^I, \tilde{S}^I, \tilde{\nu}^{cI})$, these fields are given as

$$\begin{aligned} G_0 &= (N_0 v_d, -N_0 v_u, N_0 v_{L1}, N_0 v_{L2}, N_0 v_{L3}, 0, 0, 0), \\ J &= N_4(-N_1 v_d, N_1 v_u, N_2 v_{L1}, N_2 v_{L2}, N_2 v_{L3}, 0, N_3 v_S, \\ &\quad -N_3 v_R), \end{aligned} \tag{22}$$

where the normalization constants N_i are given as

$$\begin{aligned} N_0 &= \frac{1}{\sqrt{v_d^2 + v_u^2 + v_{L1}^2 + v_{L2}^2 + v_{L3}^2}}, \\ N_1 &= v_{L1}^2 + v_{L2}^2 + v_{L3}^2, \\ N_2 &= v_d^2 + v_u^2, \\ N_3 &= N_1 + N_2, \\ N_4 &= \frac{1}{\sqrt{N_1^2 N_2 + N_2^2 N_1 + N_3^2(v_R^2 + v_S^2)}}, \end{aligned} \tag{23}$$

and can easily be checked to be orthogonal; i.e., they satisfy $G_0 \cdot J = 0$.

In order to study the phenomenology of the scalar sector, we need some information about the parameters of the SBRP model. Broadly speaking, there are four types of parameters that are to a large extent undetermined. First, there are Yukawa couplings, such as h, h_0, and λ. In contrast to h_U, h_D, and h_E, these are not fixed by fermion masses. Then there are MSSM parameters such as $\tan\beta$, the effective Higgsino mixing parameter μ, the supersymmetry breaking scalar mass parameters m_0 and A_0. These are partially restricted by negative collider searches for supersymmetric particles [34]. Then there are singlet-sector mass parameters, such as M_R, M_Φ, and δ^2. Finally, there is the important Yukawa coupling h_ν, which determines the strength of effective R-parity breaking parameters, through Eq. (13). This is constrained by neutrino oscillation data. In Sec. IV, we will discuss our strategy to choose the parameters in such a way that the results can be easily interpreted. We will also show there that a fully cubic superpotential, without any mass scale parameter such as the $\hat{\mu} H_u H_d$ term, also leads to a realistic model [35] consistent with neutrino oscillation data. Before that, however, we consider the corresponding Higgs boson phenomenology, focusing on Higgs boson production and decays, and stressing the potentially large invisible decay branching ratio.

III. HIGGS BOSON PRODUCTION AND DECAYS

Supersymmetric Higgs bosons can be produced at the e^+e^- collider through their couplings to Z, via the so-called Bjorken process. In our SBRP model there are eight neutral CP-even states H_i and six neutral CP-odd Higgs bosons A_i, in addition to the majoron J. One must diagonalize the scalar boson mass matrix in order to find the coupling of the massive scalars to the Z. The Lagrangian is

$$\mathcal{L}_{HZZ} = \sum_{i=1}^{8} (\sqrt{2} G_F)^{1/2} M_Z^2 Z_\mu Z^\mu \eta_i H_i, \tag{24}$$

with each η_i given as a weighted combination of the five SU(2) ⊗ U(1) doublet scalars,

$$\eta_i = \frac{v_d}{v} R_{i1}^S + \frac{v_u}{v} R_{i2}^S + \sum_{j=1}^{3} \frac{v_{Lj}}{v} R_{ij+2}^S, \tag{25}$$

where R_{ij}^S is the 8×8 rotation matrix for the CP-even scalars. Note that we leave the discussion of the CP-odd scalars for elsewhere. Moreover, here we focus mainly on the production of the lightest CP-even supersymmetric Higgs boson $h \equiv H_1$. The main difference between the production of this state and the lightest CP-even Higgs boson of the MSSM is the fact that ours contains an admixture of the SU(2) ⊗ U(1) singlet scalar fields $\tilde{\nu}^c$ and $\tilde{S}$, and its coupling to the Z is correspondingly reduced by a factor

$$\eta \equiv \eta_1 \leq 1, \tag{26}$$

in comparison with the standard model case.[6] When the lightest CP-even Higgs boson is mainly singlet its production cross section in e^+e^- annihilation will be suppressed.

[5]As already mentioned, we assume for technical simplicity that we have just one pair of lepton-number-carrying SU(2) ⊗ U(1) singlet superfields.

[6]For the MSSM we have a reduction given by $\eta = \frac{v_d}{v} R_{11}^S + \frac{v_u}{v} R_{12}^S = \sin(\beta - \alpha)$ in the usual notation.

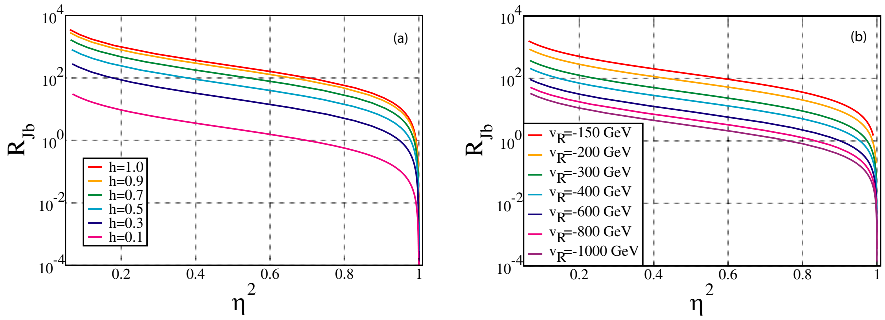

FIG. 1 (color online). Ratio R_{Jb}, defined in Eq. (27), as a function of η^2. (a) To the left, for different values of the parameter h, from top to bottom: $h = 1, 0.9, 0.7, 0.5, 0.3, 0.1$. (b) To the right, for different values of the parameter $v_R = v_S$: $-v_R = 150, 200, 300, 400, 600, 800, 1000$ GeV. The plots show explicitly that $R_{Jb} > 1$ is possible even for $\eta \simeq 1$. This is the main result of the current paper.

We now turn to the lightest Higgs boson decays. Given that other MSSM decay modes are less important, we are particularly interested here in the ratio

$$R_{Jb} = \frac{\Gamma(h \to JJ)}{\Gamma(h \to b\bar{b})} \tag{27}$$

of the invisible decay to the standard model decay into b jets. For this we have to look separately at the decay widths,

$$\Gamma(h \to JJ) = \frac{g_{hJJ}^2}{32\pi m_h} \tag{28}$$

and

$$\Gamma(h \to b\bar{b}) = \frac{3\sqrt{2}G_F}{8\pi\cos^2\beta}(R_{11}^S)^2 m_h m_b^2\left[1 - 4\left(\frac{m_b}{m_h}\right)^2\right]^{3/2}. \tag{29}$$

From these expressions, we see that $\Gamma(h \to b\bar{b})$ will be small if the component of the lightest Higgs boson along H_d^0 is small. On the other hand, the magnitude of $\Gamma(h \to JJ)$ will depend on the g_{hJJ} coupling. This is in general given by a complicated expression, but for the situation that we are considering here with

$$v_{Li} \ll v_d, v_u \ll v_R, v_S, \tag{30}$$

we have to a very good approximation

$$J \simeq \left(0, 0, 0, 0, 0, 0, \frac{v_S}{V}, -\frac{v_R}{V}\right), \tag{31}$$

where $V^2 = v_S^2 + v_R^2$. Under this approximation, we can write the coupling g_i' for the vertex $h_i' JJ$ of the Majoron with the *unrotated* Higgs boson h_i', in the following form:

$$\begin{aligned} g_1' &= h h_0 v_u \frac{v_S v_R}{V^2}, \qquad g_2' = h h_0 v_d \frac{v_S v_R}{V^2} - \frac{2v_u}{V^2}\sum_{j=1}^{3}\epsilon_j^2, \\ g_i' &= -\frac{2\epsilon_{i-2}}{V^2}\sum_{j=1}^{3}\epsilon_j v_{Lj} \quad (i = 3, 4, 5), \\ g_6' &= -\sqrt{2}h(A_h + \hat{M}_\Phi)\frac{v_S v_R}{V^2} - \sqrt{2}h\hat{M}_R, \\ g_7' &= -h^2\frac{v_S v_R^2}{V^2}, \qquad g_8' = -h^2\frac{v_S^2 v_R}{V^2}, \end{aligned} \tag{32}$$

where $\hat{M}_R$ and $\hat{M}_\Phi$ have been defined in Eq. (20).

From these expressions we conclude that g_{hJJ} can be large in two situations. The first is, of course, if the lightest Higgs boson is mainly a combination of the $\tilde{\nu}^c$ and $\tilde{S}$ fields. In this case not only g_{hJJ} will be large, but also $\Gamma(h \to b\bar{b})$ will be small suppressing $h \to b\bar{b}$. Unfortunately, the production would be suppressed, as

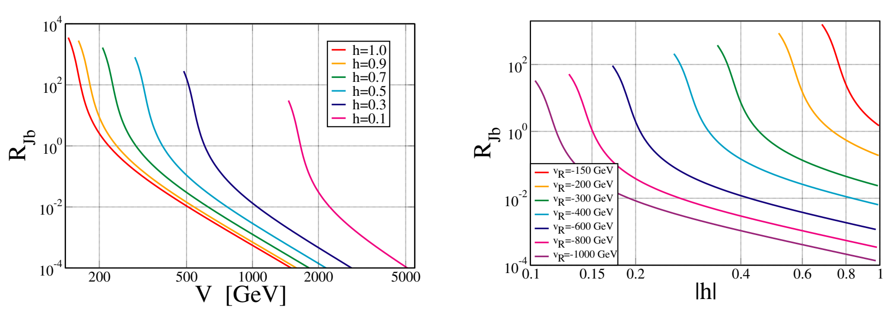

FIG. 2 (color online). Ratio R_{Jb}, defined in Eq. (27), as a function of V (left) and as a function of h (to the right). Small (large) values of V (h) lead to large values of R_{Jb}.

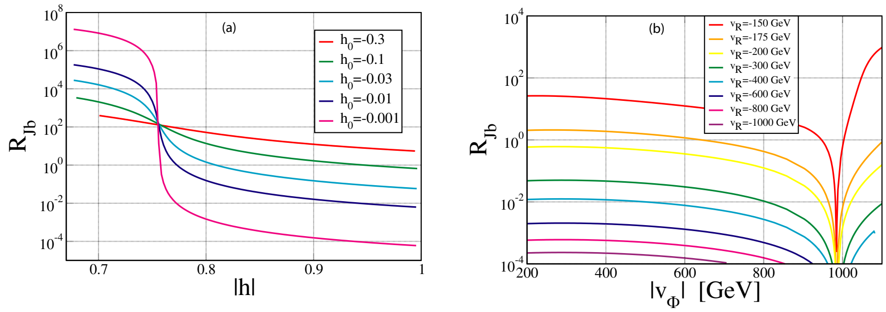

FIG. 3 (color online). Ratio R_{Jb}, defined in Eq. (27), as (a) left figure: function of $|h|$ for $-h_0 = 0.3, 0.1, 0.03, 0.01, 0.001$ (on the right part of the plot from top to bottom). The right panel (b) gives R_{Jb} as function of $|v_\Phi|$ for different values of the parameter $v_R = v_S$ for $-v_R = 150, 175, 200, 300, 400, 600, 800, 1000$ GeV.

singlets do not couple to the Z. The phenomenologically novel and interesting situation is when h and h_0 are large. In this case, the Higgs boson behaves as the lightest MSSM Higgs boson (with moderately reduced production cross section) but with a large branching to the invisible channel $h \to JJ$.

The sensitivities of LEP experiments to the invisible channel $h \to JJ$ have been discussed since long ago [36,37] and the current status has been presented in Ref. [38]. In order to evaluate the experimental sensitivities to the parameters of the model, we must take into account both the production as well as Higgs decays.

IV. NUMERICAL RESULTS

In this section we discuss the numerical results on the invisible decay of the Higgs boson in our model. We start with a brief discussion of the SBRP parameters.

Unknown parameters of the spontaneous R-parity breaking model fall into three different groups. First, there are the MSSM parameters, mainly the unknown soft SUSY breaking terms. The second group of parameters are the ϵ_i and left-handed sneutrino vevs v_{L_i}. We trade the latter for the parameters Λ_i using Eq. (14). These six parameters occur also in the explicit bilinear model. Finally, there are the parameters of the singlet sector, namely, singlet vevs v_R, v_S, and v_Φ, Yukawa couplings h, h_0, and λ, and the singlet mass terms M_R, M_Φ, δ^2, as well as the corresponding soft terms.

We have checked by a rather generous scan that the results presented below qualitatively do not depend on the choice of MSSM parameters, as expected. Thus, for definiteness we will fix the MSSM parameters in the following to the SPS1a benchmark point [39], defined by

$$m_0 = 100 \text{ GeV}, \qquad m_{1/2} = 250 \text{ GeV}, \qquad \tan\beta = 10,$$
$$A_0 = -100 \text{ GeV}, \qquad \mu < 0. \tag{33}$$

We have run down this set of parameters to the electroweak scale using the program package SPheno [40]. We stress again that different choices of MSSM parameters will not lead to qualitatively different results.

A. General case

We first consider the general model defined by the superpotential in Eq. (2) reduced to one generation of ν^c and S fields. For the singlet parameters, we choose as a starting point $v_R = v_S = v_\Phi = -150$ GeV and $M_R = -M_\Phi = \delta = 10^3$ GeV, as well as $h = 0.8$, $h_0 = -0.15$, and $\lambda = 0.1$. We have tried other values of parameters

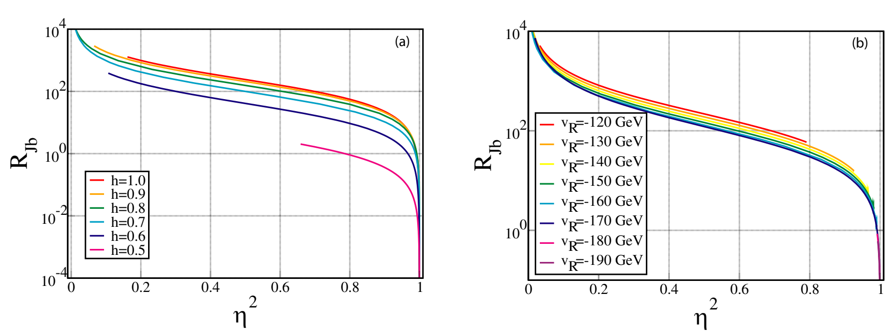

FIG. 4 (color online). Ratio R_{Jb}, defined in Eq. (27), as a function of η^2, (a) to the left, for different values of the parameter h and (b) to the right, for different values of the parameter $v_R = v_S$. As in the general case (Fig. 1), large values of R_{Jb} can be found even for $\eta \simeq 1$ also in the cubic-only case.

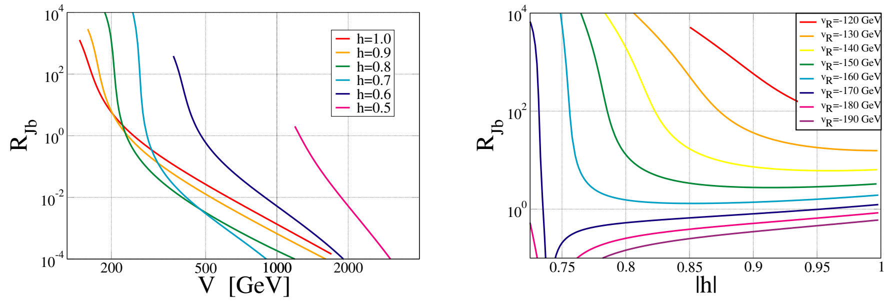

FIG. 5 (color online). Ratio R_{Jb}, defined in Eq. (27), as a function of the parameter V (left) and as a function of h (right). The qualitative behavior is similar to the general case; compare to Fig. 2.

and obtained qualitatively similar results to the ones discussed below.

The explicit bilinear parameters are then fixed approximately such that neutrino masses and mixing angles [4] are in agreement with experimental data [1–3]. Slightly different values of parameters are found, depending on whether the first or the third term in Eq. (12) is responsible for the atmospheric neutrino mass scale. Both possibilities lead to very similar results for the invisible decay of the Higgs. This can be understood quite easily. The ratio of the atmospheric and solar neutrino mass scale is only of the order of (4–7)[7] and the changes in parameters $\vec{\Lambda}$ and $\vec{\epsilon}$ are only of the order of the square root of this number. Such a small change can always be compensated by a slight adjustment of other parameters, leading to the same (or very similar) final result.

After having defined our "preferred" choice of parameters in the following, we will vary one unknown parameter at a time. We now turn to a discussion of the results. In Fig. 1, we show the ratio R_{Jb} as a function of η^2 for different choices of h (left) and for different choices of v_R (right) and all other parameters fixed. Larger values of R_{Jb} are found for smaller values of η, as expected. However, one sees explicitly that even for values of $\eta \simeq 1$, R_{Jb} can be larger than 1. This means that the lightest Higgs can decay mainly invisibly, even when the cross section for its production is essentially equal to the usual (MSSM) doublet Higgs boson cross section. This is the main result of this work.

In Fig. 2 we show R_{Jb} as function of $V = \sqrt{v_R^2 + v_S^2}$ (left) and as function of h (to the right). The figure shows that large values of R_{Jb} are obtained for small values of V and for large values of h. The decreasing of R_{Jb} with increasing values of V can be easily understood, since in the limit $V \to \infty$ the majoron should obviously decouple.

Other singlet-sector parameters also can have an important impact on R_{Jb}, as demonstrated in Fig. 3. As shown in the left panel of this figure, larger values of h_0 lead to larger values of R_{Jb}. For values of h smaller than about $h \simeq 0.75$ (for our specific choice of the other parameters), the order of the lines is exchanged. This is due to a level crossing in the eigenvalues. Below this value, the lightest Higgs is mainly a singlet and thus even though it decays dominantly invisibly its production cross section is very much reduced.

On the other hand, the right panel of Fig. 3 shows that the value of v_Φ is normally somewhat less important than the value of V in determining R_{Jb}. Again, this can be qualitatively understood since V is the parameter whose magnitude determines the breaking of lepton number [indeed, with the help of the approximate couplings g_i' in Eq. (31) one can see that the parameters h, h_0, v_R, and v_S should be the most important ones].

As a summary of this section, we conclude that large branching ratios of the doubletlike Higgs boson into invisible final states are possible in the SBRP model, despite the smallness of the neutrino masses indicated by oscillation data. Large values of R_{Jb} occur for large values of the Yukawa couplings and for small values of v_R. The presence of the field Φ plays a crucial role in getting the invisible Higgs boson decays that are not suppressed by the small neutrino masses.

B. Cubic-only superpotential

Before concluding, we illustrate the results we have obtained for the case of a restricted SBRP model described by the superpotential in Eq. (2) containing only cubic terms [35]. The restricted model provides a potential "solution" to the μ problem in the context of spontaneous R-parity violation. We give results for the same parameter choices as above, except that no mass parameters are now present in the basic superpotential.

Even though acceptable physical solutions consistent with experiment (supersymmetric particle searches as well as neutrino oscillation data) are somewhat harder to find, they exist. Figures 4 and 5 show R_{Jb} as a function of η^2 and as a function of h and V for the cubic-only case, compared to Figs. 1 and 2 for the general case. As can be

[7] In a hierarchical model, such as the one discussed here, the square roots of the Δm^2_{ij} are approximately equal to the larger mass.

seen, the qualitative behavior is very similar in all cases, although the parameters for which acceptable solutions are found are usually restricted to narrower ranges in the cubic-only case. These figures demonstrate that also in the cubic-only case large production cross section and large invisible branching ratios for the lightest Higgs decay can occur at the same time.

V. DISCUSSION

We have discussed the possibility of an invisibly decaying Higgs boson in the context of the spontaneously broken R-parity model. One of the neutral CP-odd scalars in this model corresponds to the $SU(2) \otimes U(1)$ singlet Nambu-Goldstone boson associated with the breaking of lepton number. In contrast to the MSSM, where the Higgs boson can decay invisibly only to supersymmetric states (in regions of parameters where the Higgs is heavier than twice the lightest neutralino mass), in our case the Higgs can decay mainly due to Eq. (1), instead of the standard model channels, over large regions of parameters, given that there is no kinematical barrier for this decay. We have reanalyzed this striking suggestion in view of the recent data on neutrino oscillations that indicate nonzero but small neutrino masses. We have explicitly shown that (i) despite the smallness of neutrino masses, invisible Higgs boson decay may indeed provide the most important mode of Higgs boson decays and (ii) its production cross section need not be suppressed with respect to that characterizing the standard MSSM case. As a result, our analysis indicates that invisibly decaying Higgs bosons should be an important topic in the agenda of future accelerators, such as the Large Hadron Collider and the Next Linear Collider. In fact, the interest on this possibility goes beyond the model we have taken as framework; it is much more general. However, the SBRP model provides an attractive explanation for the origin of the neutrino masses that can also be probed at future collider experiments through the predicted pattern of the lightest supersymmetric particle decays which directly traces the experimentally observed neutrino mixing angles.

ACKNOWLEDGMENTS

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APPENDIX A: NEUTRINO-NEUTRALINO-SINGLINO MASS MATRIX

In the basis

$$(-i\lambda', -i\lambda^3, \tilde{H}_d, \tilde{H}_u, \nu_e, \nu_\mu, \nu_\tau, \nu^c, S, \tilde{\Phi}), \tag{A1}$$

the mass matrix of the neutral fermions following from Eq. (2) can be written as

$$\mathbf{M_N} = \begin{pmatrix} \mathbf{M}_{\chi^0} & \mathbf{m}_{\chi^0\nu} & \mathbf{m}_{\chi^0\nu^c} & \mathbf{0} & \mathbf{m}_{\chi^0\Phi} \\ \mathbf{m}^{\mathbf{T}}_{\chi^0\nu} & \mathbf{0} & \mathbf{m_D} & \mathbf{0} & \mathbf{0} \\ \mathbf{m}^{\mathbf{T}}_{\chi^0\nu^c} & \mathbf{m}^{\mathbf{T}}_{\mathbf{D}} & \mathbf{0} & \mathbf{M}_{\nu^c\mathbf{S}} & \mathbf{M}_{\nu^c\Phi} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}^{\mathbf{T}}_{\nu^c\mathbf{S}} & \mathbf{0} & \mathbf{M}_{\mathbf{S}\Phi} \\ \mathbf{m}^{\mathbf{T}}_{\chi^0\Phi} & \mathbf{0} & \mathbf{M}^{\mathbf{T}}_{\nu^c\Phi} & \mathbf{M}^{\mathbf{T}}_{\mathbf{S}\Phi} & \mathbf{M}_\Phi \end{pmatrix}. \tag{A2}$$

where the matrix $\mathbf{M}_{\chi^0}$ is the MSSM neutralino mass matrix:

$$\mathbf{M}_{\chi^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g'v_d & +\frac{1}{2}g'v_u \\ 0 & M_2 & +\frac{1}{2}gv_d & -\frac{1}{2}gv_u \\ -\frac{1}{2}g'v_d & +\frac{1}{2}gv_d & 0 & -\mu \\ +\frac{1}{2}g'v_u & -\frac{1}{2}gv_u & -\mu & 0 \end{pmatrix}. \tag{A3}$$

Here, $\mu = \hat{\mu} + h_0 v_\Phi/\sqrt{2}$. $\mathbf{m}_{\chi^0\nu}$ is the R-parity violating neutrino-neutralino mixing part, which also appears in explicit bilinear R-parity breaking models:

$$\mathbf{m}^{\mathbf{T}}_{\chi^0\nu} = \begin{pmatrix} -\frac{1}{2}g'v_{Le} & \frac{1}{2}gv_{Le} & 0 & \epsilon_e \\ -\frac{1}{2}g'v_{L\mu} & \frac{1}{2}gv_{L\mu} & 0 & \epsilon_\mu \\ -\frac{1}{2}g'v_{L\tau} & \frac{1}{2}gv_{L\tau} & 0 & \epsilon_\tau \end{pmatrix}, \tag{A4}$$

where v_{Li} are the vevs of the left sneutrinos, ϵ_i are defined by $\epsilon_i = (1/\sqrt{2})h^i_\nu v_R$, and v_R is the vev of the right sneutrino.

Here $\mathbf{m}_{\chi^0\nu^c}$ is given as

$$\mathbf{m}^{\mathbf{T}}_{\chi^0\nu^c} = \Big(0, 0, 0, \frac{1}{\sqrt{2}}\sum h^i_\nu v_{Li}\Big). \tag{A5}$$

and $\mathbf{m}^{\mathbf{T}}_{\chi^0\Phi}$ is

$$\mathbf{m}^{\mathbf{T}}_{\chi^0\Phi} = \Big(0, 0, -\frac{1}{\sqrt{2}}h_0 v_u, -\frac{1}{\sqrt{2}}h_0 v_d\Big). \tag{A6}$$

The "Dirac" mass matrix is defined in the usual way:

$$(\mathbf{m_D})_i = \frac{1}{\sqrt{2}}h^i_\nu v_u. \tag{A7}$$

The ν^c and S states are coupled by

$$(\mathbf{M}_{\nu^c\mathbf{S}}) = M_R + h\frac{v_\Phi}{\sqrt{2}}. \tag{A8}$$

$\mathbf{M}^{\mathbf{T}}_{\nu^c\Phi}$ and $\mathbf{M}^{\mathbf{T}}_{\mathbf{S}\Phi}$ are

$$\mathbf{M}^{\mathbf{T}}_{\nu^c\Phi} = (\langle v_S\rangle), \tag{A9}$$

$$\mathbf{M}^{\mathbf{T}}_{\mathbf{S}\Phi} = (\langle v_R \rangle). \tag{A10}$$

Here, $\langle v_R \rangle = h v_R$ and $\langle v_S \rangle = h v_S$. Finally $\mathbf{M}_\Phi$ is

$$\mathbf{M}_{\Phi} = M_\Phi + \lambda \frac{v_\Phi}{\sqrt{2}}. \tag{A11}$$

We briefly comment on the case of three generations of neutral fermions in the singlet sector. For three copies of ν^c and S fields, the mass matrix of the neutral fermions can be written in exactly the same form as given in Eq. (A2) with some rather straightforward generalizations of the above definitions. These changes are as follows: h and h^i_ν become 3×3 matrices h^{ij} and h^{ij}_ν. In Eq. (A5) the matrix becomes a 3×4 matrix, M_R is a symmetric 3×3 matrix, and Eqs. (A9) and (A10) have to be replaced by

$$\mathbf{M}^{\mathbf{T}}_{\nu^c\Phi} = (\langle v_{S_1} \rangle, \langle v_{S_2} \rangle, \langle v_{S_3} \rangle), \tag{A12}$$

$$\mathbf{M}^{\mathbf{T}}_{\mathbf{S}\Phi} = (\langle v_{R_1} \rangle, \langle v_{R_2} \rangle, \langle v_{R_3} \rangle), \tag{A13}$$

where $\langle v_{Ri} \rangle = \sum_j h^{ji} v^R_j$ and $\langle v_{S_i} \rangle = \sum_j h^{ij} v^S_j$.

Notice that even with three generations of ν^c and S fields, one neutrino mass is zero at the tree level.

APPENDIX B: THE NEUTRAL SCALAR MASS MATRIX

The 8×8 scalar mass matrix is a symmetric matrix that in the basis of the real part of $(H^0_d, H^0_u, \tilde{\nu}_i, \Phi, \tilde{S}, \tilde{\nu}^c)$ can be written in the form

$$M^{S^2} = \begin{bmatrix} M^{S^2}_{HH} & M^{S^2}_{H\tilde{L}} & M^{S^2}_{HS} \\ M^{S^2T}_{H\tilde{L}} & M^{S^2}_{\tilde{L}\tilde{L}} & M^{S^2}_{\tilde{L}S} \\ M^{S^2T}_{HS} & M^{S^2T}_{\tilde{L}S} & M^{S^2}_{SS} \end{bmatrix}, \tag{B1}$$

where $M^{S^2}_{HH}$ is a symmetric 2×2 matrix, $M^{S^2}_{\tilde{L}\tilde{L}}$ and $M^{S^2}_{SS}$ are symmetric 3×3 matrices, while $M^{S^2}_{H\tilde{L}}$ and $M^{S^2}_{HS}$ are 2×3 matrices and finally $M^{S^2}_{\tilde{L}S}$ is (a nonsymmetric) 3×3 matrix. In this notation $\tilde{L}$ denotes the sneutrinos and S the singlet fields.

We can write the mass matrix by giving the components of the various blocks. We get

(i) $M^{S^2}_{HH}$

$$M^{S^2}_{HH_{11}} = \frac{1}{4}(g^2 + g'^2)v_d^2 + \Omega \tan\beta + \frac{\sqrt{2}}{2}\mu \frac{v_R}{v_d} \times \sum_{i=1}^{3} h^i_\nu v_{Li}, \tag{B2}$$

$$M^{S^2}_{HH_{12}} = -\tfrac{1}{4}(g^2 + g'^2)v_d v_u - \Omega + h_0^2 v_u v_d, \tag{B3}$$

$$M^{S^2}_{HH_{22}} = \frac{1}{4}(g^2 + g'^2)v_u^2 + \Omega \cot\beta - \frac{\sqrt{2}}{2}\frac{v_R}{v_u} \times \sum_{i=1}^{3} A_{h_\nu} h^i_\nu v_{Li} - \frac{\sqrt{2}}{2}\hat{M}_R \frac{v_S}{v_u}\sum_{i=1}^{3} h^i_\nu v_{Li}, \tag{B4}$$

where

$$\Omega = B\mu - \delta^2 h_0 + \frac{\lambda}{4} h_0 v_\Phi^2 + \frac{1}{2} h h_0 v_R v_S + \frac{\sqrt{2}}{2} A_{h_0} h_0 v_\Phi + \frac{\sqrt{2}}{2} h_0 M_\Phi v_\Phi, \tag{B5}$$

and μ, $\hat{M}_R$, and $\hat{M}_\Phi$ are defined in Eqs. (15) and (20).

(ii) $M^{S^2}_{\tilde{L}\tilde{L}}$

$$M^{S^2}_{\tilde{L}\tilde{L}_{ij}} = \frac{1}{4}(g^2 + g'^2)v_{Li}v_{Lj} + \frac{1}{2}(v_R^2 + v_u^2)h^i_\nu h^j_\nu + \delta_{ij}\bigg(-\frac{\sqrt{2}}{2}\frac{v_u v_R}{v_{Li}} A_{h_\nu} h^i_\nu + \frac{\sqrt{2}}{2} \times \frac{v_d v_R}{v_{Li}} h^i_\nu \mu - \frac{1}{2}\frac{v_R^2 + v_u^2}{v_{Li}} h^i_\nu \sum_{k=1}^{3} h^k_\nu v_{Lk} - \frac{\sqrt{2}}{2}\hat{M}_R \frac{v_S v_u}{v_{Li}} h^i_\nu\bigg). \tag{B6}$$

(iii) $M^{S^2}_{\tilde{L}S}$

$$M^{S^2}_{\tilde{L}S_{i1}} = -\tfrac{1}{2} h_0 v_d v_R h^i_\nu + \tfrac{1}{2} h v_u v_S h^i_\nu, \tag{B7}$$

$$M^{S^2}_{\tilde{L}S_{i2}} = \frac{\sqrt{2}}{2}\hat{M}_R v_u h^i_\nu, \tag{B8}$$

$$M^{S^2}_{\tilde{L}S_{i3}} = \frac{\sqrt{2}}{2} v_u A_{h_\nu} h^i_\nu - \frac{\sqrt{2}}{2} h^i_\nu \mu v_d + h^i_\nu v_R \sum_{k=1}^{3} h^k_\nu v_{Lk}. \tag{B9}$$

(iv) $M^{S^2}_{H\tilde{L}}$

$$M^{S^2}_{H\tilde{L}_{1i}} = \frac{1}{4}(g^2 + g'^2)v_d v_{Li} - \frac{\sqrt{2}}{2}\mu v_R h^i_\nu, \tag{B10}$$

$$M^{S^2}_{H\tilde{L}_{2i}} = -\frac{1}{4}(g^2 + g'^2)v_u v_{Li} + \frac{\sqrt{2}}{2} v_R A_{h_\nu} h^i_\nu + \frac{\sqrt{2}}{2}\hat{M}_R v_S h^i_\nu + v_u h^i_\nu \sum_{k=1}^{3} h^k_\nu v_{Lk}. \quad \text{(B11)}$$

(v) $M^{S^2}_{HS}$

$$M^{S^2}_{HS_{11}} = \sqrt{2} h_0 \mu v_d - \frac{\sqrt{2}}{2} h_0 (A_{h_0} + \hat{M}_\Phi) v_u - \frac{1}{2} h_0 v_R \sum_{k=1}^{3} h^k_\nu v_{Lk}, \quad \text{(B12)}$$

$$M^{S^2}_{HS_{12}} = -\tfrac{1}{2} h h_0 v_R v_u, \quad \text{(B13)}$$

$$M^{S^2}_{HS_{13}} = -\frac{1}{2} h h_0 v_S v_u - \frac{\sqrt{2}}{2}\mu \sum_{k=1}^{3} h^k_\nu v_{Lk}, \quad \text{(B14)}$$

$$M^{S^2}_{HS_{21}} = \sqrt{2} h_0 \mu v_u - \frac{\sqrt{2}}{2} h_0 (A_{h_0} + \hat{M}_\Phi) v_d + \frac{1}{2} h v_S \sum_{k=1}^{3} h^k_\nu v_{Lk}, \quad \text{(B15)}$$

$$M^{S^2}_{HS_{22}} = -\frac{1}{2} h h_0 v_R v_d + \frac{\sqrt{2}}{2}\hat{M}_R \sum_{k=1}^{3} h^k_\nu v_{Lk}, \quad \text{(B16)}$$

$$M^{S^2}_{HS_{23}} = -\frac{1}{2} h h_0 v_S v_d + v_u v_R \times \sum_{k=1}^{3} h^k_\nu h^k_\nu + \frac{\sqrt{2}}{2} \sum_{k=1}^{3} A_{h_\nu} h^k_\nu v_{Lk}. \quad \text{(B17)}$$

(vi) $M^{S^2}_{SS}$

$$M^{S^2}_{SS_{11}} = \frac{1}{2}\lambda^2 v_\Phi^2 + \delta^2 (C_\delta + M_\Phi)\frac{\sqrt{2}}{v_\Phi} - \frac{\sqrt{2}}{2}(v_d^2 + v_u^2)\frac{h_0 \hat{\mu}}{v_\Phi} + \frac{\sqrt{2}}{4}\lambda(A_\lambda + 3M_\Phi) v_\Phi - \frac{\sqrt{2}}{2} h \times (A_h + M_\Phi)\frac{v_R v_S}{v_\Phi} + \frac{\sqrt{2}}{2} h_0 (A_{h_0} + M_\Phi) \times \frac{v_u v_d}{v_\Phi} + \frac{1}{2} h_0 \frac{v_d v_R}{v_\Phi}\sum_{k=1}^{3} h^k_\nu v_{Lk} - \frac{1}{2} h \frac{v_S v_u}{v_\Phi} \times \sum_{k=1}^{3} h^k_\nu v_{Lk} - \frac{\sqrt{2}}{2} h M_R \frac{v_S^2 + v_R^2}{v_\Phi}, \quad \text{(B18)}$$

$$M^{S^2}_{SS_{12}} = \frac{\sqrt{2}}{2} h (A_h + \hat{M}_\Phi) v_R + \sqrt{2} h \hat{M}_R v_S + \frac{1}{2} h v_u \sum_{k=1}^{3} h^k_\nu v_{Lk}, \quad \text{(B19)}$$

$$M^{S^2}_{SS_{13}} = \frac{\sqrt{2}}{2} h (A_h + \hat{M}_\Phi) v_S - \frac{1}{2} h_0 v_d \sum_{k=1}^{3} h^k_\nu v_{Lk} + \sqrt{2} h \hat{M}_R v_R, \quad \text{(B20)}$$

$$M^{S^2}_{SS_{22}} = -\Gamma \frac{v_R}{v_S} - \frac{\sqrt{2}}{2}\frac{v_u}{v_S}\hat{M}_R \sum_{k=1}^{3} h^k_\nu v_{Lk}, \quad \text{(B21)}$$

$$M^{S^2}_{SS_{23}} = \Gamma + h^2 v_R v_S, \quad \text{(B22)}$$

$$M^{S^2}_{SS_{33}} = -\Gamma \frac{v_S}{v_R} + \frac{\sqrt{2}}{2}\frac{\mu v_d}{v_R}\sum_{k=1}^{3} h^k_\nu v_{Lk} - \frac{\sqrt{2}}{2}\frac{v_u}{v_R} \times \sum_{k=1}^{3} A_{h_\nu} h^k_\nu v_{Lk}, \quad \text{(B23)}$$

where

$$\Gamma = B_{M_R} M_R - \delta^2 h + \frac{1}{4} h \lambda v_\Phi^2 - \frac{1}{2} h h_0 v_u v_d + \frac{\sqrt{2}}{2} h (A_h + M_\Phi) v_\Phi. \quad \text{(B24)}$$

APPENDIX C: THE NEUTRAL PSEUDOSCALAR MASS MATRIX

The 8×8 pseudoscalar mass matrix is a symmetric matrix that can be written in the form

$$M^{P^2} = \begin{bmatrix} M^{P^2}_{HH} & M^{P^2}_{H\tilde{L}} & M^{P^2}_{HS} \\ M^{P^2 T}_{H\tilde{L}} & M^{P^2}_{\tilde{L}\tilde{L}} & M^{P^2}_{\tilde{L}S} \\ M^{P^2 T}_{HS} & M^{P^2 T}_{\tilde{L}S} & M^{P^2}_{SS} \end{bmatrix}, \quad \text{(C1)}$$

where the blocks have the same structure as before. We can write the mass matrix by giving the components of the various blocks. We get

(i) $M^{P^2}_{HH}$

$$M^{P^2}_{HH_{11}} = \Omega \tan\beta + \frac{\sqrt{2}}{2}\mu \frac{v_R}{v_d}\sum_{i=1}^{3} h^i_\nu v_{Li}, \quad \text{(C2)}$$

$$M^{P^2}_{HH_{12}} = \Omega, \quad \text{(C3)}$$

$$M^{P^2}_{HH_{22}} = \Omega\cot\beta - \frac{\sqrt{2}}{2}\frac{v_R}{v_u}\sum_{i=1}^{3} A_{h_\nu} h^i_\nu v_{Li} - \frac{\sqrt{2}}{2}\hat{M}_R\frac{v_S}{v_u}\sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C4}$$

where Ω and μ are given in Eqs. (B5) and (15).

(ii) $M^{P^2}_{\tilde{L}\tilde{L}}$

$$M^{P^2}_{\tilde{L}\tilde{L}_{ij}} = \frac{1}{2}(v_R^2 + v_u^2)h^i_\nu h^j_\nu + \delta_{ij}\Big(-\frac{\sqrt{2}}{2}\frac{v_u v_R}{v_{Li}} \times A_{h_\nu} h^i_\nu + \frac{\sqrt{2}}{2}\frac{v_d v_R}{v_{Li}} h^i_\nu \mu - \frac{1}{2}\frac{v_R^2 + v_u^2}{v_{Li}} h^i_\nu \times \sum_{k=1}^{3} h^k_\nu v_{Lk} - \frac{\sqrt{2}}{2}\hat{M}_R\frac{v_S v_u}{v_{Li}} h^i_\nu\Big), \tag{C5}$$

(iii) $M^{P^2}_{\tilde{L}S}$

$$M^{P^2}_{\tilde{L}S_{i1}} = -\tfrac{1}{2}h_0 v_d v_R h^i_\nu + \tfrac{1}{2}h v_u v_S h^i_\nu, \tag{C6}$$

$$M^{P^2}_{\tilde{L}S_{i2}} = \frac{\sqrt{2}}{2}\hat{M}_R v_u h^i_\nu, \tag{C7}$$

$$M^{P^2}_{\tilde{L}S_{i3}} = -\frac{\sqrt{2}}{2} v_u A_{h_\nu} h^i_\nu + \frac{\sqrt{2}}{2} h^i_\nu \mu . v_d \tag{C8}$$

(iv) $M^{P^2}_{H\tilde{L}}$

$$M^{P^2}_{H\tilde{L}_{1i}} = -\frac{\sqrt{2}}{2}\mu v_R h^i_\nu,$$
$$M^{P^2}_{H\tilde{L}_{2i}} = -\frac{\sqrt{2}}{2} v_R A_{h_\nu} h^i_\nu - \frac{\sqrt{2}}{2} v_S \hat{M}_R h^i_\nu. \tag{C9}$$

(v) $M^{P^2}_{HS}$

$$M^{P^2}_{HS_{11}} = \frac{\sqrt{2}}{2} h_0 (A_{h_0} - \hat{M}_\Phi) v_u + \frac{1}{2} h_0 v_R \sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C10}$$

$$M^{P^2}_{HS_{12}} = -\tfrac{1}{2} h h_0 v_R v_u, \tag{C11}$$

$$M^{P^2}_{HS_{13}} = -\frac{1}{2} h h_0 v_S v_u - \frac{\sqrt{2}}{2}\mu\sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C12}$$

$$M^{P^2}_{HS_{21}} = \frac{\sqrt{2}}{2} h_0 (A_{h_0} - \hat{M}_\Phi) v_d + \frac{1}{2} h v_S \sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C13}$$

$$M^{P^2}_{HS_{22}} = -\frac{1}{2} h h_0 v_R v_d + \frac{\sqrt{2}}{2}\hat{M}_R\sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C14}$$

$$M^{P^2}_{HS_{23}} = -\frac{1}{2} h h_0 v_S v_d - \frac{\sqrt{2}}{2}\sum_{k=1}^{3} A_{h_\nu} h^k_\nu v_{Lk}, \tag{C15}$$

(vi) $M^{P^2}_{SS}$

$$M^{P^2}_{SS_{11}} = \delta^2 (C_\delta + M_\Phi)\frac{\sqrt{2}}{v_\Phi} - \frac{\sqrt{2}}{2}(v_d^2 + v_u^2)\frac{h_0\hat{\mu}}{v_\Phi} - \frac{\sqrt{2}}{4}\lambda(3A_\lambda + M_\Phi)v_\Phi - 2B_{M_\Phi}M_\Phi - \frac{\sqrt{2}}{2}h(A_h + M_\Phi)\frac{v_R v_S}{v_\Phi} + \frac{\sqrt{2}}{2}h_0(A_{h_0} + M_\Phi)\frac{v_u v_d}{v_\Phi} + \frac{1}{2}h_0\frac{v_d v_R}{v_\Phi}\sum_{k=1}^{3} h^k_\nu v_{Lk} + 2\delta^2\lambda + \lambda h_0 v_u v_d - \lambda h v_R v_S - \frac{1}{2}h\frac{v_u v_S}{v_\Phi}\sum_{k=1}^{3} h^k_\nu v_{Lk} - \frac{\sqrt{2}}{2} h M_R \frac{v_S^2 + v_R^2}{v_\Phi}, \tag{C16}$$

$$M^{P^2}_{SS_{12}} = -\frac{\sqrt{2}}{2} h (A_h - \hat{M}_\Phi) v_R - \frac{1}{2} h v_u \sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C17}$$

$$M^{P^2}_{SS_{13}} = -\frac{\sqrt{2}}{2} h (A_h - \hat{M}_\Phi) v_S - \frac{1}{2} h_0 v_d \sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C18}$$

$$M^{P^2}_{SS_{22}} = -\Gamma\frac{v_R}{v_S} - \frac{\sqrt{2}}{2}\hat{M}_R\frac{v_u}{v_S}\sum_{k=1}^{3} h^k_\nu v_{Lk}, \tag{C19}$$

$$M^{P^2}_{SS_{23}} = -\Gamma, \tag{C20}$$

$$M^{P^2}_{SS_{33}} = -\Gamma\frac{v_S}{v_R} + \frac{\sqrt{2}}{2}\frac{\mu v_d}{v_R}\sum_{k=1}^{3} h^k_\nu v_{Lk} - \frac{\sqrt{2}}{2}\frac{v_u}{v_R}\sum_{k=1}^{3} A_{h_\nu} h^k_\nu v_{Lk}, \tag{C21}$$

where Γ is given in Eq. (B24).

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