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Neutrino mixing and quark-lepton complementarity

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As a result of the identification of the solution to the solar neutrino problem, a rather precise relation $\theta_{\rm sun} + \theta_C = \pi/4$ between the leptonic 1-2 mixing angle $\theta_{\rm sun}$ and the Cabibbo angle has emerged. It would mean that the lepton and the quark mixing angles add up to the maximal, suggesting a deep structure by which quarks and leptons are interrelated. We refer to the relation as ''quark-lepton complementarity'' (QLC) in this paper. We formulate general conditions under which the QLC relation is realized. We then present several scenarios which lead to the relation and elaborate on phenomenological consequences which can be tested by the future experiments. We also discuss implications of the QLC relation for the quark-lepton symmetry and the mechanism of neutrino mass generation.

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I. INTRODUCTION

The most distinct feature of the lepton flavor mixing is the existence of two large mixing angles in the Maki-Nakagawa-Sakata (MNS) matrix [1], which is in sharp contrast to the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing [2]. One of the large angles comes from the atmospheric neutrino experiments [3] which have discovered the neutrino oscillation [1,4], whereas the other one comes from the solar [5] and the reactor neutrino observations [6]. The atmospheric mixing is suspected to be maximal or close to the maximal, though the experiment gives only a mild constraint $36° \leq \theta_{23} \leq 54°$ [7]. On the other hand, the solar angle θ_{12} is known to be away from the maximal mixing value [8,9].

It was realized a long time ago that the large mixing angle required for a solution of the solar neutrino problem may appear as a difference between the maximal mixing angle $\pi/4$ and the Cabibbo angle θ_C, so that

$$\theta_{\rm sun} + \theta_C = \frac{\pi}{4}, \tag{1}$$

or $\tan 2\theta_{\rm sun} = 1/\tan 2\theta_C$ [10]. The equality holds with a rather high accuracy as became clear by accumulating data of the solar neutrino experiments [11]. Indeed, the global fit of the solar neutrino and KamLAND results gives [8,9,12,13]

$$\theta_{\rm sun} = 32.3° \pm 2.4° \quad (1\sigma). \tag{2}$$

Taking the Cabibbo angle at the Z^0 pole

$$\theta_C = 12.8° \pm 0.15°, \tag{3}$$

we obtain

$$\theta_{\rm sun} + \theta_C = 45.1° \pm 2.4° \quad (1\sigma). \tag{4}$$

In terms of the oscillation observable the relation can be expressed as

$$\sin^2\left(\frac{\pi}{4} - \theta_C\right) = 0.284 \pm 0.002, \qquad \sin^2\theta_{\rm sun} = 0.286 \pm 0.038, \tag{5}$$

so that

$$\Delta\sin^2\theta_{12} \equiv \sin^2\theta_{\rm sun} - \sin^2\left(\frac{\pi}{4} - \theta_C\right) = 0.002 \pm 0.040. \tag{6}$$

The deviation of the central value is well within the present experimental errors at 1σ confidence level (CL). Notice that the best fit values of the solar angle from analyses of different groups have a very small spread: $\theta_{\rm sun} = 32.0°–33.2°$. This shows stability of the result and may indicate that the true value of $\theta_{\rm sun}$ is indeed in this narrow interval, unless some systematic shift in the experimental data is found. With this interval we obtain for the sum of the best fit angles

$$\theta_{\rm sun} + \theta_C = 44.8°–46.0°. \tag{7}$$

The equality (1) relates the 1-2 mixing angles in quark and lepton sectors, and if not accidental, implies certain a relation between quarks and leptons. It is very suggestive of a bigger structure in which quarks and leptons are complementary. The equality probably means a quark-lepton symmetry or quark-lepton unification [14] in some form. It may be considered as evidence of the grand unification, and/or certain flavor symmetry [15]. If not accidental, it can give clues to understand the fermion masses in a general context. In what follows we will call the equality (1) the quark-lepton complementarity (QLC) relation.

In this paper, we try to answer the following questions: Can the QLC relation not be accidental? What are the general conditions for the QLC relation? What is the underlying physical structure and the resultant scenarios that satisfy the conditions? What are the experimental predictions of these scenarios and how can they be tested?

As a whole, we explore experimental consequences and theoretical implications of the QLC relation.

The paper is organized as follows. In Sec. II we formulate general conditions for the QLC relation. In Secs. III and IV we elaborate on various scenarios which realize the relation (1). In Sec. III a possibility of "bimaximal minus CKM mixing" is studied. In Sec. IV we consider single-maximal mixing scenarios. In Sec. V the predictions of various scenarios are summarized. In Sec. VI we give a summary with brief comment on how to test them experimentally. Some theoretical implications of the QLC relation and heuristic remarks are also presented.

In Secs. III and IV we give a detailed and comprehensive description of possible phenomenological scenarios providing for each case with comments on implications for neutrino mass matrix and quark-lepton symmetry. For those who want to avoid these details we recommend, after reading Sec. II, to go directly to Sec. V in which an overview of phenomenological aspects of our results are summarized, in particular, in Table I. One can go back to Secs. III and IV for details of particular scenarios.

II. GENERAL CONDITIONS FOR THE QUARK-LEPTON COMPLEMENTARITY RELATION

The lepton mixing matrix $U_{\rm MNS}$ is defined as

$$U_{\rm MNS} = U_e^\dagger U_\nu, \tag{8}$$

where U_e and U_ν are the transformations of the left-handed components which diagonalize the mass matrices of the charged leptons and neutrinos, respectively. In the standard parametrization [16] the MNS matrix reads[1]

$$U_{\rm MNS} = R_{23}\Gamma_{\delta_l}R_{13}R_{12}, \tag{9}$$

where R_{ij} is the matrix of rotation in the ij plane. In this form, the angle of 1-2 rotation is identified with the solar angle, $\theta_{12} = \theta_{\rm sun}$, the angle of 2-3 rotation—with the atmospheric angle, $\theta_{23} = \theta_{\rm atm}$, and θ_{13}—with the angle restricted by the CHOOZ experiment [18]. The matrix with the CP-violating phase is parametrized as

$$\Gamma_\delta \equiv {\rm diag}(1, 1, e^{i\delta_l}).$$

To identify the mixing angles with those measured in experiments one should reduce a given mixing matrix to the form (9).

Let us formulate general conditions which lead to the QLC relation.

[1] While the form in (9) utilizes a slightly nonstandard way of introducing a CP-violating phase into the MNS matrix [17], it can be shown that the correspondence of the angles with the experimental observable is the same as those of the standard parametrization [16].

A. Single maximal or bimaximal

In principle, it is enough to have a single-maximal mixing, that is $R_{12}^m \equiv R_{12}(\pi/4)$, to realize relation (1). However, the existence of maximal or near maximal 2-3 leptonic mixing hints that the whole pattern of fermion mixings may be generated as a combination of no mixing, a maximal, and the CKM mixings. Namely, we comment on the scenario characterized by

$$\text{"bimaximal minus CKM mixing."} \tag{10}$$

Because it is very predictive and the easiest to test experimentally, it deserves a separate description from more general cases. A possibility of the lepton mixing as a small deviation from the bimaximal mixing [19] has been extensively discussed recently [20] but without identification of a small deviation with the quark mixing. See, however, the first reference in [20]. Relation (1) allows one to restore the bimaximal mixing [19] as the element of underlying theory [15].

It should be stressed [21] that the present data do not yet give a strong bound on deviation of 2-3 mixing from the maximal, which can be characterized by

$$D_{23} \equiv 0.5 - \sin^2\theta_{23}. \tag{11}$$

It is constrained by $|D_{23}| \leq 0.16$ or $|D_{23}|/\sin^2\theta_{23} \leq 0.47$ at 90% CL [7]. Furthermore, the latest analysis (without renormalization of the original fluxes) shows some excess of the e-like events at sub-GeV energies and the absence of excess in the multi-GeV sample, thus giving a hint to nonzero D_{23} [22].

In the scenario (10), one expects the deviation to be small: $\pi/4 - \theta_{23} \lesssim \theta_{23}^{\rm CKM}$, or

$$|D_{23}| \lesssim \sin\theta_{23}^{\rm CKM} \approx V_{cb} \simeq \sin^2\theta_C \simeq 0.04. \tag{12}$$

For specific scenarios see Sec. III. The next generation long-baseline experiments, in particular, the J-PARC-SK, will be sensitive to $|D_{23}| \sim 0.05$ [23–25]. Also it would be a challenge for the future atmospheric neutrino experiments to achieve the required sensitivity. Establishing the deviation from the maximal mixing to be more significant than the one in (12) will exclude the scenario (10).

If the bimaximal scenario is not realized and D_{23} is large, an additional 1-3 rotation (apart from 1-3 CKM rotation) should be considered. Indeed, generically, the same symmetry (e.g., Z_2) leads to the maximal 2-3 mixing and simultaneously vanishing 1-3 mixing [26]. Therefore, the deviation from the maximal 2-3 angle, D_{23}, which implies violation of the symmetry, should also be accompanied by a nonzero 1-3 mixing. In this case, predictability will be lost unless one imposes the condition that such an additional 1-3 rotation is very small.

B. Order of rotations

To reproduce the equality (1) exactly one needs to have the following order of rotations:

$$U_{\rm MNS} = \cdots R_{23}^m \cdots R_{12}^{\rm CKM\dagger} R_{12}^m \quad \text{or}$$
$$U_{\rm MNS} = \cdots R_{23}^m \cdots R_{12}^m R_{12}^{\rm CKM\dagger}. \tag{13}$$

That is, the maximal and the CKM rotations must be attached to each other. Here, $R_{ij}^{\rm CKM} \equiv R_{ij}(\theta_{ij}^{\rm CKM})$ describes the CKM rotation in the ij plane, and R_{ij}^m denotes the maximal mixing rotations, $R_{ij}^m \equiv R_{ij}(\pi/4)$. In (13) "$\cdots$" denotes possible insertion of the CKM rotations, $R_{23}^{\rm CKM}$ and $R_{13}^{\rm CKM}$. (A similar structure holds also in the case that R_{23} is not maximal.) The complete CKM matrix is parametrized as

$$V^{\rm CKM} = R_{23}^{\rm CKM} \Gamma_{\delta_q} R_{13}^{\rm CKM} R_{12}^{\rm CKM}. \tag{14}$$

The reversed ordering of maximal mixing rotations in (13), namely $R_{12}^m \cdots R_{23}^m$, would lead to an unacceptably large 1-3 mixing: $\sin\theta_{13} = 0.5$ and incorrect 1-2 mixing, $\theta_{\rm sun} \sim \pi/6 \pm \theta_C$, after reducing the mixing matrix to the form (9).

Two other CKM rotations, $R_{23}^{\rm CKM}$ and $R_{13}^{\rm CKM}$, can be located in any place indicated by dots. Their effect on the relation (1) is negligible even if they are situated on the right-hand side (RHS) of the combinations in (13) or between two 1-2 rotations. The largest possible deviation appears for the case $R_{12}^m R_{12}^{\rm CKM\dagger} R_{23}^{\rm CKM}$ which, however, reduces to a small unobservable correction:

$$\sin^2\theta_{\rm sun} \to \sin^2\theta_{\rm sun}(1 - V_{cb}^2), \tag{15}$$

where $\sin\theta_{23}^{\rm CKM} \approx V_{cb} = 0.04$ ($\theta_{23}^{\rm CKM} = 2.3°$). In what follows we will neglect these types of corrections to the 1-2 mixing. However, the position of small CKM rotations can become important for other observables such as U_{e3} or the deviation of the 2-3 mixing from the maximal one.

We will also consider the combination

$$U_{\rm MNS} = \cdots R_{12}^{\rm CKM\dagger} R_{23}^m \cdots R_{12}^m \tag{16}$$

which is not excluded experimentally, though leading to the QLC relation (1) only in an approximate way.

C. CKM matrix and the quark-lepton symmetry

The natural framework in which the CKM angles appear in the lepton mixing is the quark-lepton symmetry [14] according to which in a certain basis

$$V_\nu = V_u = V^{\rm CKM\dagger} \quad \text{or} \quad V_l = V_d = V^{\rm CKM}. \tag{17}$$

Then according to the definition (8) in both cases the CKM matrix will appear in the leptonic matrix as a Hermitian conjugate,

$$U_{\rm MNS} \propto \cdots V^{\rm CKM\dagger} \cdots = \cdots R_{12}^{\rm CKM\dagger} R_{13}^{\rm CKM\dagger} R_{23}^{\rm CKM\dagger} \cdots. \tag{18}$$

Therefore, some permutations of $R_{12}^{\rm CKM\dagger}$ and other matrices are necessary which lead to a violation of the exact relation (1). The smallest corrections are produced when only R_{12}^m appears right next to $V^{\rm CKM\dagger}$ on the RHS of the mixing matrix (13). In this case $\Delta\sin^2\theta_{12} \sim \sin\theta_C V_{cb}^2$.

It is possible that the quark-lepton connection is not realized in a straightforward way as in (17). The Cabibbo angle could be the universal parameter which controls the whole structure of fermion masses and therefore appears in many places such as mass ratios and mixing parameters (see Sec. VI).

D. Naturalness

In underlying models one expects that some deviation from the exact QLC relation always exists. It can be parametrized as

$$\theta_{\rm sun} - \frac{\pi}{4} + \theta_C = \Delta\theta_{12}(X_i), \tag{19}$$

where X_i denote parameters of a model. Note that $\Delta\sin^2\theta_{12} = \sin 2\theta_{\rm sun}\Delta\theta_{12}$. Then, one should require that $\Delta\theta_{12}(X_i)$ is very small in whole allowed ranges of the parameters X_i. Otherwise, the QLC relation appears as a result of fine-tuning of several parameters and in this sense turns out to be *unnatural* or *accidental*.

This leads to immediate and nontrivial conditions: $\Delta\theta_{12}(X_i)$ should not depend on the masses of quarks and leptons or the dependence must be weak. Indeed, masses of down quarks and charged leptons for the first and the second generations (which are relevant here) are substantially different. Therefore, one would not expect an appearance of the same mixing angle θ_C in the quark and the lepton sector. The quark-lepton symmetry should be realized in terms of mixings and not masses.

E. Effect of *CP* violation

Diagonalization of the neutrino and charge lepton mass matrices can lead to the *CP*-violating phases in U_l and U_ν (which eventually will be reduced to the unique phase δ_l in $U_{\rm MNS}$). This can be described by the phase matrices

$$\Gamma_{\delta'\delta} = {\rm diag}(e^{i\delta'}, 1, e^{i\delta})$$

which appear in various places of the products (13). To keep the equality (1), the matrices $\Gamma_{\delta,\delta'}$ should not be between $R_{12}^{\rm CKM}$ and R_{12}^m, or the corresponding phases should be small enough. Indeed, the structure $R_{12}^m \Gamma_{\delta' 0} R_{12}^{\rm CKM}$ leads to

$$\Delta\sin^2\theta_{12} = \tfrac{1}{2}\sin 2\theta_C(1 - \cos\delta'). \tag{20}$$

We find that the QLC relation (1) is satisfied within 1σ, provided that $\delta' < 34°$.

With the additional phase δ', the QLC relation (1) appears as a result of fine-tuning of the parameters and therefore is not natural. Hence, we restrict ourselves into the choice $\Gamma_\delta \equiv \mathrm{diag}(1, 1, e^{i\delta})$ in the rest of the paper. Then, the place where we can insert the phase matrix is unique: it can be easily checked that all other possible insertions either can be reduced to this possibility or lead to zero CP violation.

Furthermore, the δ dependence comes into expressions of the various mixing matrix elements and the Jarlskog invariant only together with $|V_{cb}| \simeq 0.04$. Indeed, in the limit of zero rotation $R_{23}^{\mathrm{CKM}} = 1$ (and $R_{13}^{\mathrm{CKM}} = 1$) the mixing matrices U_{MNS} (13) and (16) are reduced to

$$R_{23}^m R_{12}^m R_{12}^{\mathrm{CKM}\dagger} \quad \text{or} \quad R_{12}^{\mathrm{CKM}\dagger} R_{23}^m R_{12}^m. \tag{21}$$

In both cases any insertions of the phase matrices Γ_δ will not lead to the physical CP violation phase. Therefore, in the limit $V_{ub} = 0$ the CP-violation effects (Jarlskog invariant) are proportional to V_{cb}:

$$J_{\mathrm{lep}} \equiv \mathrm{Im}[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \propto V_{cb}. \tag{22}$$

We note in passing that if V^{CKM} is the only origin of the CP violation, namely, if $\delta = 0$, we obtain generically

$$\sin\delta_l = \frac{V_{ub}}{U_{e3}} \sin\delta_q, \tag{23}$$

where δ_q is the phase in the CKM matrix. Since U_{e3} can be larger than V_{ub} due to the contribution induced by "permutations," the leptonic CP violation phase is strongly suppressed in this case. Induced CP violation associated with δ can be much larger.

F. Renormalization group effect

The QLC relation (1) holds at low energies. However, the quark-lepton symmetry (unification) which leads to (1) is realized most probably at some high-energy scales, e.g., the grand unification scale. To guarantee the QLC relation at high energies one should require that the renormalization group effects on the equality from this high scale to the low energy scale are small. In the standard model (SM) or minimal supersymmetric standard model (MSSM) the renormalization of the Cabibbo angle is indeed small. For instance, in MSSM with $\tan\beta = 50$ the parameter $\sin\theta_C$ decreases from 0.2225 at the m_Z down to 0.2224 at the 10^{16} GeV [27].

The renormalization effect on the leptonic θ_{12} depends on the type of mass spectrum of light neutrinos. For the spectrum with normal mass hierarchy, $m_1 < m_2 \ll m_3$, the effect is negligible. In contrast, in the case of the quasidegenerate spectrum, $m_1 \approx m_2 \approx m_3 = m_0$, or the spectrum with inverted mass hierarchy the effects can be large [28].

In the limit of small 1-3 mixing $\theta_{13} \ll 10°$, the running is determined by [29]

$$\frac{d\theta_{12}}{dt} \approx -\frac{Cy_\tau^2}{32\pi^2} \sin2\theta_{12}\sin^2\theta_{23} \frac{|m_1 e^{i\phi_1} + m_2 e^{i\phi_2}|^2}{\Delta m_{\mathrm{sun}}^2}, \tag{24}$$

where $t \equiv ln(\mu/\mu_0)$, μ is the renormalization scale, $C = 1$ in the MSSM, and $C = -3/2$ in the SM; y_τ is the Yukawa coupling of the tau lepton:

$$\frac{Cy_\tau^2}{32\pi^2} \approx \begin{cases} 0.3 \times 10^{-6}, & \text{SM}, \\ 0.3 \times 10^{-6}(1 + \tan^2\beta), & \text{MSSM}, \end{cases} \tag{25}$$

and $\tan\beta$ is the usual ratio of the vacuum expectation value (VEV)'s. In Eq. (24) ϕ_1 and ϕ_2 are the Majorana phases of the eigenstates ν_1 and ν_2. According to (24), the running effect is proportional to the absolute mass scale squared and the relative phase difference: $\dot{\theta}_{12} \sim m_0^2 \cos(\phi_2 - \phi_1)/2$. In SM and in MSSM with $\tan\beta <$ 10 the corrections are small even for the quasidegenerate mass spectrum. In MSSM with large $\tan\beta$ ($\tan\beta = 50$) one finds that $\Delta\theta_{12} \sim \theta_{12}$ even for the common scale $m_0 \sim 0.1$ eV [29] as a result of running from the scale of the RH neutrinos (10^{10}–10^{12} GeV) or the grand unified theory (GUT) scale. Clearly, such a large correction destroys the QLC relation, which leads us to the following conclusions:

(1) The QLC relation is not violated by the renormalization effect in the SM and in the MSSM with small $\tan\beta$ even for the quasidegenerate mass spectrum of neutrinos.

(2) In the MSSM with large $\tan\beta$ and the quasidegenerate mass spectrum the corrections are in general large. Furthermore, the corrections depend on other continuous (and presently unknown) parameters: ϕ_i, m_0 (and also θ_{13}), so that the QLC relation would require fine-tuning of several parameters. Therefore, the QLC relation, once it is established with a good accuracy, testifies against such models, unless the required tuning is a natural outcome of an additional symmetry. Notice that according to (24), the corrections can be strongly suppressed if the quasidegenerate mass eigenstates ν_1 and ν_2 have opposite CP parities: $\phi_2 - \phi_1 \approx \pi$ [28].

(3) In some cases the renormalization effect can help to reproduce the QLC relation (see Sec. III A).

G. Basis dependence

The form of the mass matrices and diagonalizing rotations depend on the basis of the quark and lepton states. Let us introduce a basis called the symmetry basis by which a symmetry that determines the structure of mass matrices is defined. (In some publications this basis is named as the Lagrangian basis.)

In the symmetry basis, both the neutrino and the charged fermion mass matrices, in general, are not diagonal and therefore both produce rotations which make up the MNS matrix. In what follows we will consider several realizations of the structure of lepton mixing matrix, (13) and (16). They differ by the origin of the large (maximal) angle rotations: the neutrino or the charge lepton sectors. These different realizations have different theoretical and experimental implications.

III. BIMAXIMAL MINUS CKM MIXING

In this section we will consider different realizations of the possibility (10) in which only maximal mixings and the CKM rotations are involved in the formation of the fermion mixing matrices.

A. Bimaximal mixing from neutrinos

Let us assume that in the symmetry basis the bimaximal mixing originates from the neutrino mass matrix, whereas the charged lepton mixing matrix coincides with the CKM matrix:

$$U_\nu = R_{23}^m R_{12}^m, \qquad U_l = V^{\rm CKM}. \tag{26}$$

Then the lepton mixing matrix equals

$$U_{\rm MNS} = V^{\rm CKM\dagger}\Gamma_\delta R_{23}^m R_{12}^m = R_{12}^{\rm CKM\dagger} R_{13}^{\rm CKM\dagger} R_{23}^{\rm CKM\dagger}\Gamma_\delta R_{23}^m R_{12}^m, \tag{27}$$

where we have introduced the phase matrix Γ_δ following our general prescription described in Sec. II.

In the quark sector we have

$$V_u = I, \qquad V_d = V^{\rm CKM}, \tag{28}$$

so that the second equality in (26) implies the quark-lepton symmetry relation, $V_l = V_d$. We also assume that the neutrino Dirac matrix is diagonal due to the equality

$$m_\nu^D = m_u. \tag{29}$$

Then, the bimaximal rotation of neutrinos follows from the seesaw mechanism [30] and the specific structure of the mass matrix of RH neutrinos. Notice that the bimaximal mixing can be related to the quasidegenerate type mass spectrum of neutrinos. Such a possibility for the bimaximal neutrino mixing and general matrix U_l, not necessarily related to $V^{\rm CKM}$, has been discussed recently in [20].

The problem in this scenario is that in spite of the equality $V_d = V_l$ the mass eigenvalues are different: $m_d^{\rm diag} \neq m_l^{\rm diag}$, where $m_l^{\rm diag} \equiv {\rm diag}(m_e, m_\mu, m_\tau)$. Therefore, the mass matrices are also different. Some special conditions have to be met for the matrices such that they produce the same mixing despite the different eigenvalues. A possibility is the singular mass matrices for which different (strong) mass hierarchies can be reconciled with approximate equality of the mixing matrices [31].

Let us discuss the phenomenological consequences of this scenario.

(1) The mixing matrix (27) does not satisfy the conditions (13) and therefore the relation (1) receives corrections

$$\sin\theta_{\rm sun} = \sin\left(\frac{\pi}{4} - \theta_C\right) + \frac{\sin\theta_C}{2}(\sqrt{2} - 1 - V_{cb}\cos\delta). \tag{30}$$

Numerically, we obtain for $\theta_{\rm sun}$

$$\theta_{\rm sun} = 35.4° \pm 0.3°, \qquad \sin^2\theta_{\rm sun} = 0.335 \pm 0.005, \tag{31}$$

and for the deviation parameter

$$\Delta\sin^2\theta_{12} \approx \sin\theta_{\rm sun}\sin\theta_C(\sqrt{2} - 1 - |V_{cb}|\cos\delta) = 0.046 - 0.056, \tag{32}$$

where the intervals indicate uncertainty due to the unknown phase δ. The deviation in (32) is 15%–20%. It corresponds to $\theta_{\rm sun} + \theta_C - \frac{\pi}{4} \simeq 2.9°$–$3.6°$. Therefore, one needs to measure $\sin^2\theta_{\rm sun}$ with better than 10% accuracy to establish this difference. According to the estimations given in [32], the future solar neutrino and the KamLAND experiments may have a sensitivity of $\simeq$4% to $\sin^2\theta_{\rm sun}$, provided that θ_{13} is measured, or severely restricted. The sensitivity of a dedicated reactor θ_{12} experiment can reach $\simeq$ 2% [33]. The errors quoted are at the confidence level of 1σ. So with such an accuracy the equality (30) can be established at about $(4–5)\sigma$.

(2) For 1-3 mixing we obtain

$$\sin\theta_{13} = -\frac{1}{\sqrt{2}}\sin\theta_C(1 - |V_{cb}|\cos\delta) + V_{ub}, \tag{33}$$

where the first dominant term is induced by permutation of the Cabibbo rotation $R_{12}^{\rm CKM}$ with the nearly maximal 2-3 rotation.

The two elements of $U_{\rm MNS}$, $|U_{e3}|$ and $|U_{\mu 3}|$, are connected by a simple relation

$$|U_{e3}|^2 = \tan^2\theta_C |U_{\mu 3}|^2 \tag{34}$$

which does not depend on δ and θ_{23}^ν (the latter is taken to be $\pi/4$ in this section), and represents the characteristic feature of the scenario of bilarge mixing from neutrinos (see Sec. IV). Using the Super-Kamiokande bound [7] $0.34 \leq |U_{\mu 3}|^2 \leq 0.66$, we obtain the prediction for $|U_{e3}|^2$:

$$\sin^2\theta_{13} = 0.026 \pm 0.008 \tag{35}$$

which is just below the CHOOZ bound and falls into the region of sensitivity of the next generation accelerator [23,34–37] and the reactor experiments [38,39].

(3) The deviation of 2-3 mixing from the maximal can be written as

$$D_{23} = \tfrac{1}{2}\sin^2\theta_C + \cos^2\theta_C |V_{cb}|\cos\delta, \tag{36}$$

where the two terms are of the same order. Numerically it gives

$$D_{23} = 0.025 \pm 0.039, \tag{37}$$

and the interval is due to the unknown CP-violating phase. The maximal possible value of D_{23} is at the level of sensitivity of the J-PARC experiment [23].

(4) For the leptonic Jarlskog invariant we obtain

$$J_{\rm lep} = \frac{1}{8\sqrt{2}} \sin 2\theta_C |V_{cb}| \sin\delta \simeq 1.5 \times 10^{-3} \sin\delta. \quad (38)$$

It is a factor of $\simeq 30$ smaller than the maximal value of $J_{\rm lep}$ allowed by the CHOOZ constraint:

$$J_{\rm lep}^{\rm max} \simeq 0.04 \sin\delta. \quad (39)$$

We note that $J_{\rm lep}$ vanishes in the two-flavor limit $\theta_{13} \to 0$, as it should, because the limit implies $\theta_C \to 0$ (ignoring V_{ub}), as one can see from (33).

The smallness of $J_{\rm lep}$ in (38) despite the relatively large $\sin\theta_{13}$ means that the way of introducing the CP-violating phase δ in (27) is not quite general. As we have shown in Sec. II D the induced part is proportional to V_{cb} and if the CKM matrix is the only source of CP violation the resultant leptonic CP violation is extremely small.

Let us consider a possibility that the value of θ_{12} given in (31) is realized at high-energy scale, and it diminishes when running from high- to low- energy scales. So the better agreement with the QLC relation is achieved at the electroweak (EW) scale. As we have discussed in Sec. II E, a substantial effect due to renormalization can be obtained in the MSSM with large $\tan\beta$ and quasidegenerate neutrino mass spectrum. In this case, however, running toward low energies leads to an increase of θ_{12}, as follows from (24) for negligible $\sin\theta_{13}$. Therefore, to diminish θ_{12}, one needs (i) to suppress the main term given in (24), and (ii) to take into account the effect due to nonzero 1-3 mixing. The former can be reached in the case of opposite CP parities of ν_1 and ν_2. As far as the latter is concerned, it was shown in [29] that if $\phi_2 - \phi_1 \approx \pi$ the decrease of θ_{12} by 3°–5° can be easily achieved by running down from (10^{10}–10^{13}) GeV for $\theta_{13} = 5°$–$10°$.

B. Bimaximal mixing from charged leptons

Let us assume that the bimaximal mixing appears from diagonalization of the charged lepton mass matrix, whereas the CKM rotation originates from the neutrino sector:

$$V_\nu = V^{\rm CKM\dagger}, \qquad V_l = R_{12}^{m\dagger} R_{23}^{m\dagger}. \quad (40)$$

This possibility has been suggested in [15]. Our predictions, however, differ from those obtained in [15].

Notice that in U_l the 1-2 and 2-3 rotations need to be permuted in comparison with the standard definition of the bimaximal matrix to produce the correct order of rotations in $U_{\rm MNS}$. The lepton mixing matrix with the CP phase δ is given by

$$\begin{aligned} U_{\rm MNS} &= R_{23}^m \Gamma_\delta R_{12}^m V^{\rm CKM\dagger} \\ &= R_{23}^m \Gamma_\beta R_{12}(\pi/4 - \theta_{12}^{\rm CKM}) R_{13}^{\rm CKM\dagger} R_{23}^{\rm CKM\dagger}. \end{aligned} \quad (41)$$

In the quark sector we assume the left rotations

$$V_u = V^{\rm CKM\dagger}, \qquad V_d = I. \quad (42)$$

The former relations in (40) and (42) imply the quark-lepton symmetry, $V_\nu = V_u$. This in turn can originate from the equality of the up quark and the neutrino Dirac mass matrices, $m_u = m_\nu^D$ as in (29), under the assumption (in the seesaw context) that the Majorana mass matrix of the RH neutrinos does not produce any additional rotations [15]. However, the latter equalities in (40) and (42) require a departure from the simple quark-lepton symmetry. They can be easily accommodated in the "lopsided" schemes [40] of the SU(5) GUT. However, the relation (29) is not explained in SU(5). In SO(10) models which naturally lead to (29), on the other hand, the lopsided scenario requires further complications. The scenario does not appear to follow naturally from the grand unified models. Notice that the problem of equal mixings but different masses outlined in Sec. III A exists here also: In the basis where m_d and m_l are diagonal, that is $V_d = V_l = I$, the eigenvalues of the mass matrices are different. In other words the question is why m_d and m_l are diagonal in the same basis.

Let us spell out the consequences of the lepton bimaximal scenario.

(1) The matrix (41) reproduces the relation (1) almost exactly,

$$\begin{aligned} \sin\theta_{\rm sun} = \sin\left(\frac{\pi}{4} - \theta_C\right) - \frac{1}{2}\sin\theta_{\rm sun}|V_{cb}|^2 \\ - \cos\theta_{\rm sun}|V_{cb}||V_{ub}|. \end{aligned} \quad (43)$$

Numerically we obtain

$$\Delta \sin^2\theta_{12} = -\sin^2\theta_{\rm sun}|V_{cb}|^2 \simeq -6 \times 10^{-4} \quad (44)$$

and $\Delta\theta_{12} = 0.04°$.

(2) For 1-3 mixing we have

$$\sin\theta_{13} = -\sin\theta_{\rm sun}|V_{cb}| - \cos\theta_{\rm sun}|V_{ub}| \approx -\sin\theta_{\rm sun}|V_{cb}|, \quad (45)$$

where the induced (by the permutation of matrices) first term dominates. Equation (45) leads to a very small value, $|U_{e3}|^2 \simeq 5 \times 10^{-4}$, or $\sin^2 2\theta_{13} = 1.9 \times 10^{-3}$ ($\theta_{13} = 1.2°$). It is beyond reach of the proposed superbeam experiments and may be reached only by the neutrino factory [41]. We note that U_{e3} being of the order λ^2 in the Wolfenstein parametrization [42], our result (45) differs from the estimation made in [15].

(3) The 2-3 mixing angle is determined, ignoring the terms of the order $|V_{cb}|^2$, by

$$\sin\theta_{23} = \sin\left(\frac{\pi}{4} - \theta_{23}^{\rm CKM}\right) + \frac{1}{\sqrt{2}}(1 - \cos\theta_{\rm sun}\cos\delta)|V_{cb}|. \tag{46}$$

The second term on the RHS of (46) is small, and the relation $\theta_{23} = \pi/2 - \theta_{23}^{\rm CKM}$ is satisfied with a good accuracy though it is not as precise as claimed in [15]. We find $0.995 \leq \sin^2 2\theta_{23} \leq 1.0$. The deviation from maximal mixing,

$$D_{23} = \cos\theta_{\rm sun}|V_{cb}|\cos\delta = 0.035\cos\delta, \tag{47}$$

is relatively large at $\delta \simeq 0$.

(4) The Jarlskog invariant equals

$$J_{\rm lep} = -\tfrac{1}{2}\cos\theta_{\rm sun}\sin^2\theta_{\rm sun}|V_{cb}|\sin\delta \sim -5 \times 10^{-3}\sin\delta. \tag{48}$$

Its absolute value is larger than that in the neutrino scenario of Sec. III A, but is an order of magnitude smaller than $J_{\rm lep}^{\rm max}$ (39).

C. Hybrid scenario

The maximal 1-2 and 2-3 mixings may come from different mass matrices. To keep the correct order of these rotations in the MNS matrix (13), we have to assume that in the symmetry basis the maximal 1-2 mixing originates from the neutrino mass matrix, whereas the maximal 2-3 mixing is generated by the charged lepton mass matrix.

The CKM rotation can come from neutrinos or charged leptons and also a mixed version is possible. We discuss only the former two cases. In the first case, we have the CKM mixing from the neutrino mass matrix:

$$U_\nu = V^{\rm CKM\dagger} R_{12}^m, \qquad U_l = R_{23}^{m\dagger}. \tag{49}$$

For quarks we take equalities (42) as in the "charged lepton" scenario.

This possibility looks more appealing than the second one. A realization can be as follows. In the symmetry basis due to the quark-lepton symmetry we have (29), $m_u = m_\nu^D$. This leads to the rotation which diagonalizes the neutrino Dirac mass matrix:

$$V_\nu^D = V_u = V^{\rm CKM\dagger}. \tag{50}$$

The maximal 1-2 rotation, R_{12}^m, is the outcome of the seesaw mechanism. It can be generated by the pseudo-Dirac (off-diagonal) 1-2 structure of the Majorana mass matrix of the RH neutrinos [10]. As a result, the rotation matrix (49) is reproduced. For the charged leptons and down quarks one should assume the lopsided scenario with a single-maximal mixing. Here, the quark-lepton symmetry is broken.

In the second case, the CKM mixing comes from the charged leptons:

$$U_\nu = R_{12}^m, \qquad U_l = V^{\rm CKM} R_{23}^{m\dagger}. \tag{51}$$

Both of the scenarios lead to the identical MNS matrix

$$U_{\rm MNS} = R_{23}^m V^{\rm CKM\dagger} R_{12}^m = R_{23}^m \Gamma_\delta R_{12}^{\rm CKM\dagger} R_{23}^{\rm CKM\dagger} R_{12}^m, \tag{52}$$

where we have ignored the $R_{13}^{\rm CKM}$ rotation.

Below we summarize the predictions of the hybrid scenario. The QLC relation (1) is satisfied to a good accuracy:

$$\sin\theta_{\rm sun} = \sin\left(\frac{\pi}{4} - \theta_C\right) + \frac{1}{2\sqrt{2}}\sin\theta_C|V_{cb}|^2, \tag{53}$$

$$\Delta\sin^2\theta_{12} = \frac{1}{\sqrt{2}}\sin\theta_{\rm sun}\sin\theta_C|V_{cb}|^2 \simeq 1.4 \times 10^{-4}. \tag{54}$$

The 1-3 mixing angle is very small:

$$\sin\theta_{13} = \sin\theta_C|V_{cb}| \simeq 9.1 \times 10^{-3} \tag{55}$$

which corresponds to $\sin^2 2\theta_{13} = 3.3 \times 10^{-4}$. The prediction for D_{23} reads

$$D_{23} = \cos\theta_C|V_{cb}|\cos\delta \simeq 0.04\cos\delta. \tag{56}$$

It is almost identical to the one in the lepton bimaximal scenario (47) but with replacing $\cos\theta_{\rm sun}$ by $\cos\theta_C$. For the Jarlskog invariant we obtain

$$J_{\rm lep} = \tfrac{1}{4}\sin\theta_C\cos 2\theta_C|V_{cb}|\sin\delta \simeq 2.1 \times 10^{-3}\sin\delta. \tag{57}$$

IV. SINGLE-MAXIMAL MIXING

To reproduce the QLC relation (1), it is sufficient to have a single-maximal mixing in 1-2 rotation (Sec. II). We discuss in this section the three scenarios which differ by the origin of large but not maximal atmospheric mixing.

A. Large 2-3 mixing from neutrinos

Here we relax the assumption of maximal 2-3 mixing in the neutrino scenario considered in Sec. III A. The lepton mixing matrix is given by (27) with the replacement $R_{23}^m \to R_{23}(\theta_{23}^\nu)$,

$$U_{\rm MNS} = V^{\rm CKM\dagger}\Gamma_\delta R_{23}(\theta_{23}^\nu) R_{12}^m. \tag{58}$$

Such a possibility can be realized in the following way. Suppose in the symmetry basis, (i) the up-quark mass matrix and the neutrino Dirac matrix are diagonal, (ii) the down-quark matrix generates the CKM mixing:

$$m_u = m_\nu^D = {\rm diag}, \qquad V_d = V_l = V^{\rm CKM}, \tag{59}$$

and (iii) the Majorana mass matrix of the right-handed neutrinos has the following form:

$$M_R \approx \begin{pmatrix} 0 & M_{12} & 0 \\ M_{12} & 0 & 0 \\ 0 & 0 & M_{33} \end{pmatrix}, \tag{60}$$

with $M_{12}/M_{33} \geq m_c^2/m_t^2$. Then, the seesaw mechanism leads to the maximal 1-2 mixing and enhancement of the 2-3 mixing [43] when also nonzero but small 2-3 entries are introduced in (60). Typically the 1-3 mixing turns out to be very small, and an additional 1-3 rotation in the neutrino mixing matrix (58) can be neglected.

We first discuss constraints on θ_{23}^{ν} from the CHOOZ and atmospheric neutrino data. Using $|U_{\mu 3}|^2 = \cos^2\theta_C(\sin^2\theta_{23}^{\nu} - \sin2\theta_{23}^{\nu}|V_{cb}|\cos\delta)$ and the Super-Kamiokande allowed range [7] gives a mild constraint $0.36 \leq \sin^2\theta_{23}^{\nu} \leq 0.69$, or $37° \leq \theta_{23}^{\nu} \leq 56°$. The CHOOZ constraint is satisfied due to the relation (34).

Because of the nonmaximal 2-3 mixing, the QLC relation is satisfied with slightly better accuracy as in the case of the bimaximal neutrino scenario of Sec. III A. The correction to this relation reads

$$\Delta\sin^2\theta_{12} = \sin2\theta_C\sin^2\left(\frac{\theta_{23}^{\nu}}{2}\right) - \frac{1}{2}\sin^2\theta_C\sin^2\theta_{23}^{\nu} - \sin\theta_C\sin\theta_{23}^{\nu}(\cos\theta_C - \sin\theta_C\cos\theta_{23}^{\nu})|V_{cb}|\cos\delta. \tag{61}$$

Neglecting the small δ-dependent term in (61) and using the bound on θ_{23}^{ν}, we obtain

$$0.034 \leq \Delta\sin^2\theta_{12} \leq 0.079 \tag{62}$$

which corresponds to $2.2° \leq \theta_{\rm sun} + \theta_C - \frac{\pi}{4} \leq 5.0°$.

Since the scenario can accommodate the whole region of $|U_{\mu 3}|^2$ allowed by the present data, the deviation from maximal θ_{23},

$$D_{23} = \tfrac{1}{2}\cos2\theta_{23}^{\nu} + \sin^2\theta_C\sin^2\theta_{23}^{\nu} - \cos^2\theta_C|V_{cb}|\cos\delta, \tag{63}$$

can be large, $|D_{23}| \leq 0.16$, which gives the opportunity for verification in the next generation experiments. The Jarlskog invariant is enhanced by a factor of $\simeq 4.6$ in comparison with the bimaximal case,

$$J_{\rm lep} = \tfrac{1}{4}\sin2\theta_C\sin^3\theta_{23}^{\nu}|V_{cb}|\sin\delta \leq 6.8\times10^{-3}\sin\delta \tag{64}$$

thanks to the mild constraint on θ_{23}^{ν}.

One can introduce small θ_{13}^{ν} rotation into the bilarge matrix (58) of the order of the CHOOZ limit. This gives an additional contribution to $\Delta\sin^2\theta_{12}$,

$$\sin\theta_{13}^{\nu}\sin\theta_C\sin\theta_{23}^{\nu}(\cos\theta_C - \sin\theta_C\cos\theta_{23}^{\nu}) \simeq 0.1\sin\theta_{13}^{\nu} \sim \pm0.016 \tag{65}$$

which can further reduce (for $\sin\theta_{13}^{\nu} < 0$) the deviation from the exact QLC relation. Within the same approximation, $|U_{e3}|^2$ obtains an additional term of the order $\sin\theta_{13}^{\nu}$:

$$-\tfrac{1}{2}\sin2\theta_C\sin\theta_{23}^{\nu}\sin2\theta_{13}^{\nu} \sim \mp0.05 \tag{66}$$

which mildly relaxes (tightens) the constraint on $\sin\theta_{23}^{\nu}$ for positive (negative) $\sin\theta_{13}^{\nu}$.

B. Large 2-3 mixing from charged leptons

One can relax the assumption of bimaximal mixing also in the case of the lepton scenario by introducing large but nonmaximal θ_{23}^{l}, so that the lepton mixing matrix takes the form

$$U_{\rm MNS} = R_{23}^{l}\Gamma_{\delta}R_{12}^{m}V^{\rm CKM\dagger}. \tag{67}$$

The QLC relation is satisfied almost exactly and the correction (43) remains unchanged.

Similar to the $|U_{e3}|$-$|U_{\mu 3}|$ relation in the neutrino-origin bilarge mixing scenario, there exists a relation

$$|U_{e3}|^2 = \tan^2\theta_{23}^{\rm CKM}|U_{e2}|^2 \simeq |V_{cb}|^2\sin^2\theta_{\rm sun} \tag{68}$$

independent of θ_{23}^{l} and δ. It immediately tells that $|U_{e3}|^2$ is small, $\simeq 5\times10^{-4}$.

Ignoring the small δ-dependent term, one can show that θ_{23}^{l} has a similar bound $36° \leq \theta_{23}^{l} \leq 54°$ as θ_{23}^{ν} from atmospheric neutrino data (see Sec. IV A). So apparently the deviation from maximal 2-3 mixing

$$D_{23} = \tfrac{1}{2}\cos2\theta_{23}^{l} + \cos\theta_{\rm sun}\sin2\theta_{23}^{l}|V_{cb}|\cos\delta \tag{69}$$

can cover the whole region allowed by the Super-Kamiokande data, $|D_{23}| \leq 0.16$. The Jarlskog invariant

$$J_{\rm lep} = -\tfrac{1}{2}\cos\theta_{\rm sun}\sin^2\theta_{\rm sun}\sin2\theta_{23}^{l}|V_{cb}|\sin\delta \tag{70}$$

being proportional to $\sin2\theta_{23}^{l}$ is bounded by $J_{\rm lep}$ (48) found for $\theta_{23}^{l} = \pi/4$.

One can introduce also small θ_{13} into the bilarge matrix (67), so that $|U_{e3}|$ saturates the CHOOZ limit. But, its effect to the QLC relation is $\sim$1%, and it produces an even smaller effect in $|U_{\mu 3}|$.

A nonmaximal 2-3 mixing can also be introduced into the hybrid scenario described in Sec. III C by replacing R_{23}^{m} by $R_{23}^{l} \equiv R_{23}(\theta_{23}^{l})$ in the MNS matrix in (52). In this case, the correction to the QLC relation, (54), and the result for U_{e3} in Eq. (55) are unchanged. The deviation parameter D_{23} is given by that in the lepton-origin single-maximal case (69), but with replacement $\theta_{\rm sun} \rightarrow \theta_C$. The upper bound on the deviation, $|D_{23}| \leq 0.16$, remains unchanged. The Jarlskog invariant gets an additional factor $\sin2\theta_{23}^{l}$ in comparison with (57).

C. Large 2-3 mixing from neutrinos and charged leptons

The large 2-3 mixing can appear as a sum of contributions from the neutrinos and charged leptons. Let us assume that as a result of the seesaw mechanism, the neutrinos produce maximal 1-2 rotation and large but nonmaximal 2-3 rotation in a way described in Sec. IV A. (Note that it is easier to get a single-maximal mixing from the seesaw mechanism.) The charged leptons generate the CKM rotation and also relatively large (Cabibbo angle size) 2-3 rotation. So,

$$U_\nu = R_{23}^\nu R_{12}^m, \qquad U_l = V^{\mathrm{CKM}\dagger} R_{23}^{l\dagger}, \tag{71}$$

and consequently,

$$U_{\mathrm{MNS}} = R_{23}^l \Gamma_\delta V^{\mathrm{CKM}\dagger} R_{23}^\nu R_{12}^m. \tag{72}$$

The difference from the neutrino scenario (Sec. IVA) is that now the 2-3 rotation R_{23}^ν between R_{12}^{CKM} and R_{12}^m has the angle θ_{23}^ν which is smaller than θ_{atm}. Therefore, the correction to the QLC relation (1) is smaller. Instead of (30) we find, ignoring order $|V_{ub}|$ terms,

$$\sin\theta_{\mathrm{sun}} = \sin\left(\frac{\pi}{4} - \theta_C\right) + \frac{\sin\theta_C}{\sqrt{2}}[1 - \cos(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}})]. \tag{73}$$

For the purpose of estimations of numbers we take, throughout this subsection, $\theta_{23}^l = \theta_C = 13°$ and $\theta_{23}^\nu \simeq 2\theta_C = 27°$. The spirit behind the choice of these numbers is that we pursue the possibility that inherently there is no large mixing angle in building blocks of the MNS matrix. The latter choice is also motivated as the smallest choice consistent with the large atmospheric angle. Then, from (73) we obtain $\theta_{\mathrm{sun}} = 33°$, and $\sin^2\theta_{\mathrm{sun}} = 0.30$ which is substantially closer to the central experimental value than the oscillation parameter in the neutrino scenario.

The 1-3 mixing parameter determined now as

$$\sin\theta_{13} = \sin\theta_C \sin(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}}) \tag{74}$$

has the mildly suppressed value in comparison with the neutrino-origin single-maximal case (Sec. IVA): $\sin\theta_{13} = 0.093$, or $\sin^2 2\theta_{13} = 0.034$.

The 2-3 mixing matrix element is determined as

$$U_{\mu 3} = \sin(\theta_{23}^l + \theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}}) + 2\sin^2\left(\frac{\theta_C}{2}\right)\cos\theta_{23}^l \sin(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}}) + \sin\theta_{23}^l \cos(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}})(e^{i\delta} - 1). \tag{75}$$

A notable feature of (75) is that the argument of the sine function (the first term on the RHS) is the addition of modest size angles, which makes our "no inherent large angle" assumption in lepton mixing tenable. In fact, under the assumption $\theta_{23}^l = \theta_C$, the 2-3 mixing angle can be written as

$$\sin^2\theta_{23} = \sin^2(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}}) + \sin^2\theta_C[1 - 3\sin^2(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}})] - \sin\theta_C\cos^2\theta_C \sin 2(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}}) \times \cos\delta, \tag{76}$$

ignoring $\sin^4\theta_C$ terms. Numerically, for $\theta_{23}^\nu = 27°$, it gives $\sin^2\theta_{23} \simeq 0.28 - 0.16\cos\delta$. Therefore, the Super-Kamiokande bound is satisfied for $112° \leq \delta \leq 248°$.

The Jarlskog invariant can be written as

$$J_{\mathrm{lep}} = \tfrac{1}{4}\sin\theta_C \sin 2\theta_{23}^l \sin(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}})[\cos 2\theta_C + \sin^2\theta_C \sin^2(\theta_{23}^\nu - \theta_{23}^{\mathrm{CKM}})]\sin\delta. \tag{77}$$

Numerically, keeping the same numbers as above, we obtain $J_{\mathrm{lep}} = 9.1 \times 10^{-3}$, which is the largest among predictions from all the scenarios in this paper. It is because of this feature that some of the small angles in elements of the MNS matrix (72) are "absorbed" into the large angles, as in (74) and (75).

V. SUMMARY OF THE PREDICTIONS BY VARIOUS SCENARIOS

We compare predictions of different scenarios and discuss perspectives to disentangle them. In Table I we summarize predictions for observables obtained in the last two sections. One can see some typical features of the predictions from various scenarios. The lepton and the hybrid scenarios can be characterized by extremely small deviation from the QLC relation, which may be unobservable experimentally. They also have common features which predict small θ_{13} which probably requires facilities beyond the superbeam experiments. These statements apply not only to bimaximal scenarios but also to their variations with single-maximal mixing angle.

On the other hand, the predictions of the "neutrino" scenarios are markedly different. Both the bimaximal and the single-maximal cases predict a relatively large deviation from the exact QLC relation of $\Delta\sin^2\theta_{12}/\sin^2\theta_{12} \sim 17\%$. They lead to relatively large θ_{13} just below the CHOOZ limit which will be detected by the next generation long-baseline and reactor experiments.

The neutrino (lepton and the hybrid) bimaximal scenarios predict deviation from the maximal 2-3 mixing by 5%–7%. The prediction is lost when we modify the scenario by allowing the (2-3) mixing to be nonmaximal.

There exists a relation characteristic to the neutrino scenario, $|U_{e3}| = \tan\theta_C|U_{\mu 3}|$, which holds independently of δ and of whether the neutrino-origin 2-3 angle is maximal or not. Similarly, in the lepton scenario there exists an analogous relation $|U_{e3}| = \tan\theta_C|U_{e2}|$, which is again independent of whether the lepton-origin 2-3 angle is maximal or not. They represent general consequences of the neutrino- and lepton-origin bilarge mixing scenarios and can be tested by future measurement of θ_{13} as well as a more precise determination of θ_{23} and θ_{12}.

Throughout all scenarios, leptonic *CP* violation is small: the Jarlskog invariant is smaller than the presently allowed value by a factor of ~ 10.

There exist simple relations between predictions of the lepton and the hybrid scenarios. For the deviation from the exact QLC equality we find

$$\frac{(\Delta\sin^2\theta_{12})_l}{(\Delta\sin^2\theta_{12})_h} = \frac{\sqrt{2}\sin\theta_{\mathrm{sun}}}{\sin\theta_C} \simeq 3.4. \tag{78}$$

$\sin\theta_{13}$ and D_{23} are related by

TABLE I. Predictions to the deviation from the QLC relation $\Delta\sin^2\theta_{12}$, $\sin^2 2\theta_{13}$, the deviation parameter from the maximal 2-3 mixing D_{23}, and the leptonic Jarlskog factor $J_{\rm lep}$ for different scenarios. The numbers in parentheses in the first column indicate the equation numbers where the scenario is defined. The uncertainties indicated with $\pm$ come from the experimental uncertainty of the atmospheric mixing angle θ_{23}. Whenever there exists uncertainty due to the CP-violating phase δ we assume that $\cos\delta = 0$ to obtain an "average value." For the quantities which vanish at $\cos\delta = 0$ (indicated by $*$) the numbers are calculated by assuming $\cos\delta = 1$. "SK bound" implies the whole region allowed by the Super-Kamiokande: $|D_{23}| \leq 0.16$. The numbers for the last row (single-maximal case) are computed with the assumed values of $\theta_{23}^l = \theta_C$ and $\theta_{23}^\nu = 27°$.

Scenarios	$\Delta\sin^2\theta_{12}$	$\sin^2 2\theta_{13}$	$D_{23} \equiv \frac{1}{2} - s_{23}^2$	$J_{\rm lep}/\sin\delta$
Neutrino bimaximal (27)	0.051	0.10 ± 0.032	0.025	1.5×10^{-3}
Lepton bimaximal (41)	-6×10^{-4}	2×10^{-3}	0.035^*	5×10^{-3}
Hybrid bimaximal (52)	1.4×10^{-4}	3.3×10^{-4}	0.04^*	2.1×10^{-3}
Neutrino max + large (58)	0.057 ± 0.023	0.10 ± 0.032	SK bound	$\leq 6.8 \times 10^{-3}$
Lepton max + large (67)	-6×10^{-4}	2×10^{-3}	SK bound	$\leq 5 \times 10^{-3}$
Hybrid max + large	1.4×10^{-4}	3.3×10^{-4}	SK bound	$\leq 2.1 \times 10^{-3}$
Single maximal (72)	0.015	0.034	0.06–0.16	9.1×10^{-3}

$$\frac{(\sin\theta_{13})_l}{(\sin\theta_{13})_h} = \frac{\sin\theta_{\rm sun}}{\sin\theta_C} \simeq 2.4, \qquad \frac{(D_{23})_l}{(D_{23})_h} = \frac{\cos\theta_{\rm sun}}{\cos\theta_C} \simeq 0.87. \tag{79}$$

However, it will be extremely difficult to measure the small values of θ_{13} and D_{23}, and consequently to check these relations. Therefore, distinguishing between these scenarios is an open question.

VI. DISCUSSION AND CONCLUSIONS

To summarize, the current solar neutrino data show a precise relation between the leptonic and the quark 1-2 mixing angles. The measured values of these angles sum up to $\pi/4$ in an accurate way such that the deviation of the central value is smaller than the experimental error at 1σ CL. The relation, which was referred as the QLC (quark-lepton complementarity) relation in this paper, seems indicative of a deeper connection between quarks and leptons, the most fundamental matter to date.

We have formulated general conditions under which the QLC relation is satisfied. They include (1) correct order of large rotations, which impose certain restrictions on the neutrino and charge lepton mass matrices, (2) certain restrictions of CP-violating phases in the mass matrices, and (3) the absence of large renormalization group effects. We require that no other free parameter enters the relation between these angles, otherwise the relation implies the tuning of parameters.

We explored, first, a possibility that lepton mixings appear as the combination of maximal mixing and the CKM rotations. This led to the bimaximal minus CKM mixing scenario which has several different realizations. These realizations differ by ways of how maximal mixings are generated. The generic prediction of all these realizations is a very small deviation of 2-3 mixing from maximal. So that if a large deviation is observed the scenario will be excluded.

A natural possibility would be the neutrino origin of the bimaximal structure. It leads to the QLC relation only at an approximate level, which is consistent with the current experimental data. This scenario can be identified by relatively large 1-3 mixing which is close to the present upper bound. In the (charged) lepton-origin and hybrid bimaximal scenarios deviation from the QLC relation, the 1-3 mixing angle, and deviation of the 2-3 mixing angle from the maximal one are predicted to all be very small. The former two features are shared by their bilarge extension, but the last one is not.

Let us make several theoretical and heuristic remarks:

(1) We have considered the origin of lepton mixing as the "maximal mixing minus Cabibbo mixing." There are two problems in this context:

- the origin of maximal (or bimaximal) mixing,
- propagation of the Cabibbo (or CKM) mixing to the leptonic sector.

The latter is rather nontrivial especially for the first and the second generation fermions in view of a large difference in mass hierarchies: $m_e/m_\mu = 0.0047$ and $m_d/m_s = 0.04$–0.06 as well as a difference in masses of the s quark and muon. The precise quark-lepton symmetry should show up in mixing and not in mass eigenvalues. This can be done rather easily in the two generation context but difficult to implement for the first and second families in the three generation case [44].

So, the main problem is propagation of the Cabibbo (or CKM) mixing from the quark sector to the lepton sector. Since the quark-lepton symmetry is broken by masses of quarks and lepton, one does not expect that the quark mixing is "transmitted" to the lepton sector exactly. On general ground one would get corrections to the mixing angle of the order

$$\Delta\theta_{12} \sim \theta_C \frac{m_d}{m_s} \sim 0.5°–1° \tag{80}$$

which, however, is below the present 1σ accuracy.

For illustration let us outline one possible scenario of such a propagation of mixing in the case of neutrino origin of maximal 1-2 mixing.

(i) The first and the second generations of fermions form the doublet of the flavor group and acquire masses independently of the third generation (singlet of the group). This is required to reconcile the propagation of the Cabibbo mixing with the $b - \tau$ unification.

(ii) The quark-lepton symmetry leads to the approximate equality of matrices of the Yukawa couplings for the first and the second generations. To explain the difference of masses of muon and s quark at GUT scale one needs to introduce two different Higgs doublets with different VEV's for quarks and for leptons. Notice that $m_s \approx m_\mu$ at the EW scale, so that if the flavor symmetry is realized at the EW scale one Higgs doublet is sufficient. In this case however the problem of flavor changing neutral currents both in the lepton and quark sectors becomes very severe.

(iii) In the basis where the Dirac mass matrices of up quarks and neutrinos are diagonal the matrices of the Yukawa couplings of the down quarks and charged leptons should be nearly equal and singular to reconcile equal mixings and different mass hierarchies of the quarks and leptons. The singularity and quark-lepton symmetry are broken by terms of the order m_d/m_s and this leads to the correction given in (80).

We emphasize that what is really needed for the QLC relation to hold is the single-maximal mixing in the 1-2 rotation either from neutrino or from lepton sectors. Theoretically, the single-maximal mixing can be realized much more easily. The mass matrix of the RH neutrinos can be the origin of the maximal mixing for the first and the second generations and it can lead to enhancement of the 2-3 mixing.

(2) It is not excluded that the quark-lepton connection, which leads to the relation between the angles, is not so direct. It may work for the Cabibbo angle only, since $\sin\theta_C$ may turn out to be a generic parameter of the whole theory of the fermion masses. Therefore, it may appear in various places as the mass ratios and the mixing angles. An empirical relation

$$\sin\theta_C \approx \sqrt{\frac{m_\mu}{m_\tau}} \tag{81}$$

is in favor of this point of view.

(3) One can consider some variations of the QLC equality (1). Noting that the 2-3 leptonic mixing angle measured with the atmospheric neutrinos is nearly maximal, $\theta_{atm} \equiv \theta_{23} \simeq \pi/4$, we may write instead of (1)

$$\theta_{sun} + \theta_C = \theta_{atm}, \tag{82}$$

allowing possible extension to the case of nonmaximal θ_{atm}.

(4) Still the QLC relation can be accidental. There is also another nontrivial coincidence:

$$\theta_{sun} + \theta_{\mu\tau} = \frac{\pi}{4}, \tag{83}$$

where the angle $\theta_{\mu\tau}$ is determined by the equality

$$\tan\theta_{\mu\tau} \approx \sqrt{\frac{m_\mu}{m_\tau}}. \tag{84}$$

Apparently, the equalities (82) and (83) have different interpretations from the QLC relation. In particular, (83) is a pure leptonic relation.

(5) The most important future measurements turn out to be

(i) Precise measurements of the 1-2 leptonic mixing and further checks of the QLC relation. The accuracy in $\sin^2\theta_{sun}$ determination must be better than 10% to discriminate the neutrino version of this scenario.

(ii) Searches for deviation of the 2-3 mixing from the maximal one which can discriminate whole bimaximal minus CKM approach.

(iii) Measurements of the 1-3 mixing angle.

In conclusion, it is possible that the equality (1) is not accidental, thus testifying for a certain quark-lepton relation. Implementation of the equality naturally involves the idea that the lepton mixing appears as maximal mixing minus the Cabibbo mixing. In this sense, the quark and lepton mixings are complementary. The approach leads to a number of interesting relations between the lepton and quark mixing parameters which can be tested in future precision measurements.

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