Large neutrino mixing and normal mass hierarchy: A discrete understanding

Shao-Long Chen,* Michele Frigerio,† and Ernest Ma‡

Department of Physics, University of California, Riverside, California 92521, USA

(Received 19 April 2004; published 12 October 2004; publisher error corrected 15 October 2004)

We discuss the possibility of flavor symmetries to explain the pattern of charged lepton and neutrino masses and mixing angles. We emphasize what are the obstacles for the generation of an almost maximal atmospheric mixing and what are the minimal ingredients to obtain it. A model based on the discrete symmetry S_3 is constructed, which leads to the dominant $\mu\tau$-block in the neutrino mass matrix, thus predicting normal hierarchy. This symmetry makes it possible to reproduce current data and predicts $0.01 \lesssim \theta_{13} \lesssim 0.03$ and strongly suppressed neutrinoless 2β-decay. Moreover, it implies a relation between lepton and quark mixing angles: $\theta_{23}^q \approx 2(\pi/4 - \theta_{23})$. The Cabibbo mixing can also be reproduced and $\theta_{13}^q \sim \theta_{12}^q \theta_{23}^q$. S_3 is thus a candidate to describe all the basic features of standard model fermion masses and mixing.

DOI: 10.1103/PhysRevD.70.073008 PACS numbers: 14.60.Pq, 11.30.Hv, 12.15.Ff, 12.60.Fr

I. INTRODUCTION

In the last few years our knowledge of the flavor structure of leptons has been strongly improved, thanks to neutrino oscillation experiments [1–6]. The hierarchy between the solar and atmospheric mass squared differences is given by a factor $\Delta m_{sol}^2/\Delta m_{atm}^2 \approx 0.035$. This translates into a very mild hierarchy between two mass eigenstates, $m_2/m_3 \gtrsim 0.15$. The mass m_1 can be much smaller or almost equal to the other two, depending on the degree of degeneracy of the spectrum. The corresponding hierarchy parameters for charged leptons are $m_\mu/m_\tau \approx 1/20$ and $m_e/m_\mu \approx 1/200$. The mixing between second and third generation is almost maximal ($\sin^2 2\theta_{23} \gtrsim 0.9$), that between the first and the second is large but nonmaximal ($\sin^2 2\theta_{12} \approx 0.8$) and finally $\sin^2 2\theta_{13} \lesssim 0.15$. Also in the quark sector the hierarchy between first and second generation masses is stronger than between second and third, but the mixing pattern is reversed: θ_{23}^q is much smaller than the Cabibbo angle θ_{12}^q.

In the search for the underlying flavor symmetry dictating the relations among fermion masses and mixing, one has to understand what features of the data are directly connected with the symmetries of the mass matrices and what are second order effects. In this paper we take the point of view that the first step should be to explain the almost maximal atmospheric mixing. This angle is at present poorly constrained ($37° \lesssim \theta_{23} \lesssim 53°$ at 90% C.L.) and next generation experiments will not improve this bound significantly [7]. However, there are already many indications that such a large mixing requires a specific mechanism for its generation, being θ_{23} exactly maximal or not:

(i) Present data allow for several structures of the Majorana neutrino mass matrix M_ν, depending on the mass spectrum and CP-violating phases. In most cases, the large atmospheric mixing is imprinted in the dominant structure of the matrix, the unique exception being $M_\nu \approx m_o 1$ [8,9].

(ii) In the case of normal mass hierarchy, which is the one closer to other fermion mass spectra, the neutrino mass matrix is dominated by the $\mu\mu$, $\mu\tau$ and $\tau\tau$ entries [10], because the heaviest mass eigenstate is almost equally shared between ν_μ and ν_τ.

(iii) Since neutrinos and charged leptons belong to the same $SU(2)_L$ representation, one should naively expect a flavor alignment between them (that is a cancellation between the left-handed mixing in the two mass matrices). The alignment is observed, in fact, between down and up quarks ($\theta_{23}^q \approx 2°$). Notice that this argument is roughly independent from the Majorana nature of the neutrino mass matrix.

(iv) The mixing can be enhanced through Renormalization Group running from high energy to electroweak scale (see [11,12] and references therein). This works only for quasidegenerate neutrino masses and in general the enhancement is efficient only for the solar mixing. The unique exception is, once again, the case $M_\nu \approx m_0 1$, but the pattern of radiative corrections has to be chosen *ad hoc* [13] or the initial conditions at the high scale have to be fine-tuned [14,15].

In the context of the seesaw mechanism for the generation of small neutrino masses, the possibility of a dynamical origin of the large mixing has been investigated. In the case of type I seesaw [16–19], conditions have been found for a 'seesaw enhancement' of lepton mixing [20,21]. Specific correlations between the neutrino Dirac mass matrix and the associated right-handed Majorana mass matrix are required. Recently it has been pointed out [22] that, in unified models with dominant

*Electronic address: shaolong.chen@email.ucr.edu
†Electronic address: frigerio@physics.ucr.edu
‡Electronic address: ma@physics.ucr.edu

type II seesaw [23–26], maximal atmospheric mixing can be related phenomenologically with $b - \tau$ Yukawa unification. In both cases, it seems to us that an underlying symmetry is still required to enforce θ_{23} to be almost maximal. In particular, we will show that the resulting value of the mixing at low energy depends crucially on which heavy fields give the dominant contribution to M_ν and what their flavor symmetries are.

Many flavor models for leptons have been developed based on discrete or continuous symmetries, both Abelian and non-Abelian. The most popular class of models is based on $U(1)$ flavor symmetries and the Froggatt-Nielsen mechanism [27]. A review of $U(1)$ flavor symmetries in the lepton sector and references can be found in [28]. Other models make use of the Abelian discrete symmetries Z_n [29–33]. However, minimal realizations of Abelian symmetries encounter some difficulties in reproducing data. In section II we will comment on the present status of $U(1)$ and Z_n flavor symmetries in confronting neutrino oscillation data and, in particular, the large atmospheric mixing (see also [34]). To overcome at least some of the problems of Abelian symmetries and obtain greater predictability, a variety of non-Abelian symmetries have been used, both discrete [35–38] and continuous [39–42].

In section III we will show that the basic features of lepton masses and mixings (and also those of quarks) can be traced back to a minimal realization of the smallest non-Abelian group, S_3, i.e., the permutations of three objects. This group was first used for flavor physics in [43] and it has been analyzed, for example, in few other papers [44–48]. The less minimal possibility of $S_{3L} \times S_{3R}$ symmetry was first considered in [49] and subsequently exploited in [50–54]. Motivations for the use of S_3 flavor symmetries in supersymmetric models can be found in [55]: in particular, the SUSY flavor problem can be relaxed.

In our approach, the choice of S_3 is minimal since we have to deal with three generations of fermions and we need at least one 2-dimensional irreducible representation (irrep), in order to connect the two generations which maximally mix. The group S_3 has, in fact, three irreps: **1**, **1′** and **2**. It turns out that the existence of two inequivalent 1-dimensional representations is crucial to reproduce fermion masses and mixing. We will analyze first the lepton $2 - 3$ sector (section IIIA), then we will extend our model to include the first generation (section IIIB), finally we will consider the quark sector (section IIIC). In section IV we summarize our results and discuss merits and limits of the model.

II. A CRITICAL VIEW OF ABELIAN FLAVOR SYMMETRIES

Let us discuss first continuous Abelian symmetries. The most often considered $U(1)$ charge in the lepton sector is $L_e - L_\mu - L_\tau$, introduced long ago in connection with pseudo-Dirac neutrinos [56]. In fact, this nonstandard lepton charge induces naturally large mixing. However, the predicted phenomenology is no longer compatible with present data, at least in minimal realizations of the flavor symmetry. Let us review the basic reasons for the tension between $L_e - L_\mu - L_\tau$ and experiments.

Let us call q_X the $U(1)$-charge of the field X. In leading order, only the couplings with field charges adding up to zero are allowed [27]. We denote with $(\nu_\alpha l_\alpha)^T$ the isodoublet of left-handed Weyl spinors with charge 0 and -1, respectively, and with l^c_α the isosinglet partners, which are left-handed and have charge $+1$. Since $q_{e,\nu_e} = 1$ and $q_{\mu,\nu_\mu,\tau,\nu_\tau} = -1$, the matrices of $U(1)$-charges relevant for the neutrino and charged lepton mass matrices M_ν and M_l are

$$Q_{\nu\nu} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{pmatrix}, \quad Q_{ll^c} = \begin{pmatrix} 1+q_{e^c} & 1+q_{\mu^c} & 1+q_{\tau^c} \\ -1+q_{e^c} & -1+q_{\mu^c} & -1+q_{\tau^c} \\ -1+q_{e^c} & -1+q_{\mu^c} & -1+q_{\tau^c} \end{pmatrix}, \tag{1}$$

where the charges of e^c, μ^c, τ^c are not yet assigned.

The structure of the mass matrices depends on q_ϕ, where ϕ is the standard model Higgs isodoublet. Only two viable neutrino mass matrices can be obtained via the usual five-dimensional operator $\nu\nu\phi\phi$, for $q_\phi = 0$ and $q_\phi = 1$ respectively:

$$M^I_\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}, \qquad M^N_\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c & d \\ 0 & d & e \end{pmatrix}, \tag{2}$$

where a and b (c, d and e) are of the same order. It is easy to check that M^I_ν corresponds to inverted mass hierarchy with eigenvalues $0, \pm\sqrt{a^2+b^2}$, with an order one 23-mixing ($\tan\theta = a/b$) and a maximal mixing between the two mass degenerate states. The matrix M^N_ν corresponds to normal mass hierarchy with one zero eigenvalue and order one mixing between the two massive states.

As far as charged leptons are concerned, q_{τ^c} should be chosen to allow a nonzero 33-entry in M_l, in order to generate the dominant τ mass. Then, it is straightforward to show that the structure of $M_l M_l^\dagger$ is given by

$$M_l M_l^\dagger = \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C^* & D \end{pmatrix}. \tag{3}$$

The parameters B, C, D are of the same order, while A can be suppressed by the choice of q_{e^c} and q_{μ^c}. In any case, the contribution of charged leptons to the mixing amounts only to an order one 23-mixing.

Therefore, in both normal and inverted hierarchy cases, the 23-mixing can be large but it is not naturally maximal. Moreover, if all order one parameters are taken to be really close to 1 (thus leading to $m_\mu^2 \ll m_\tau^2$ in $M_l M_l^\dagger$ and to $\Delta m_{sol}^2 \ll \Delta m_{atm}^2$ in M_ν^N), the $2-3$ mixings are almost maximal in the two sectors, but they cancel each other almost completely! The 12-mixing is maximal in the case of inverted hierarchy and zero in the normal hierarchy case, both in disagreement with experiment. Symmetry breaking corrections to these predictions are usually small and do not reproduce data easily.

The problem to generate maximal atmospheric mixing is common to all minimal models with $U(1)$ flavor symmetry. The assignment of the same charge to μ and τ isodoublets leads to a cancellation between large mixing in neutrino and charged lepton mass matrices. Models with inverted hierarchy can hardly explain the deviation of solar mixing from maximal. Models embedded in Grand Unification theories usually favor normal hierarchy, but the smallness of first generation masses tend to prevent a large mixing in the 12-sector.

Let us discuss now the Abelian discrete symmetries Z_n. In this type of models the fields transform under Z_n via discrete rotations, given by $1, \omega, \omega^2, \ldots, \omega^{n-1}$, where $\omega \equiv e^{2\pi i/n}$ is the n-th root of unity. A coupling among a given set of fields is allowed by the symmetry only if the product of the corresponding rotation phases is equal to one.

In the case of Z_2, one can assign muon and tau leptons to the representation with phase $\omega \equiv -1$ and assume that the electron leptons are Z_2 invariant. It is easy to check that the same problems are found as in the case of $L_e - L_\mu - L_\tau$ symmetry: order one $2-3$ mixings are generated in both neutrino and charged lepton sector and they tend to cancel each other; moreover the other two mixings are zero in the limit of exact Z_2 symmetry.

The situation is better in the case of Z_3. One can assume e, μ, τ isodoublets to transform as $1, \omega, \omega^2$ respectively, where $\omega \equiv e^{2\pi i/3}$. Then the analog of Eq. (1) is

$$\begin{aligned} \nu\nu\phi\phi &\sim \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{pmatrix} a_\phi^2, \\ ll^c\phi &\sim \begin{pmatrix} a_{e^c} & a_{\mu^c} & a_{\tau^c} \\ \omega a_{e^c} & \omega a_{\mu^c} & \omega a_{\tau^c} \\ \omega^2 a_{e^c} & \omega^2 a_{\mu^c} & \omega^2 a_{\tau^c} \end{pmatrix} a_\phi, \end{aligned} \tag{4}$$

where $a_{e^c,\mu^c,\tau^c,\phi}$ are $1, \omega$ or ω^2 depending on the Z_3 assignment of e^c, μ^c, τ^c and ϕ. Whatever the assignment of these fields, it is clear that only one element in each column of the charged lepton mass matrix is allowed. This means that this matrix is diagonal up to an unobservable permutation of the fields e^c, μ^c, τ^c. Therefore all the mixing comes from the neutrino sector and the only viable dominant structure is

$$M_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}, \tag{5}$$

where a and b are of the same order. A maximal mixing in the atmospheric sector is generated and a large solar mixing can appear easily from subleading corrections. However, the neutrino spectrum has a very unpleasant feature: in leading order the atmospheric mass difference is zero while the solar one is not.

This drawback is generic to all models where the large atmospheric mixing is obtained via dominant off-diagonal entries in the $2-3$ sector of the neutrino mass matrix. In fact, phenomenology tells us that the two maximally mixed states are associated with the largest mass splitting. There are two ways to go around this difficulty. The first is to consider models predicting at leading order three degenerate neutrinos. In this case both mass splittings are considered small perturbations while the maximal $2-3$ mixing is an outcome of the symmetry. A good example is the A_4 model [37], which predicts the matrix (5) with $a = b$. The second way is to construct the maximal mixing without dominant off-diagonal entries in the $2-3$ sector of M_ν. The advantage is that both mass differences are naturally nonzero. In the following, we will pursue this second way.

III. THE S_3 MODEL

The S_3 group has six elements divided in three conjugacy classes: the identity (e), the cyclic and anticyclic permutations of three objects (g_c and g_a), the three interchanges of two objects leaving the third fixed (g_1, g_2, g_3). Two independent 1-dimensional irreps are possible, depending if the action of all six elements is trivial (**1**) or if $g_i (i = 1, 2, 3)$ act with a change of sign (**1′**). We will call "odd" S_3 singlets the fields transforming in the **1′** representation. The third and last irrep is 2-dimensional (**2**). Since we deal only with complex fields, we have the freedom to choose a complex realization of the **2** [57], which leads to very convenient tensor product rules:

$$\begin{aligned} R_2(e) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & R_2(g_c) &= \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \\ R_2(g_a) &= \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, & R_2(g_1) &= \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \\ R_2(g_2) &= \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}, & R_2(g_3) &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \end{aligned} \tag{6}$$

where $\omega \equiv e^{2i\pi/3}$. Notice that, in this realization,

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \in \mathbf{2} \Rightarrow \begin{pmatrix} \psi_2^\dagger \\ \psi_1^\dagger \end{pmatrix} \in \mathbf{2}. \tag{7}$$

It is trivial to show that $\mathbf{1} \times \mathbf{1}' = \mathbf{1}'$, $\mathbf{1}' \times \mathbf{1}' = \mathbf{1}$, $\mathbf{2} \times \mathbf{1} = \mathbf{2}$ and $\mathbf{2} \times \mathbf{1}' = \mathbf{2}$. Finally and most importantly, one has $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{1}' + \mathbf{2}$, where, if $(\psi_1 \psi_2)^T$ and $(\varphi_1 \varphi_2)^T$ are S_3 doublets, then

$$\begin{aligned} &\begin{pmatrix} \psi_2 \varphi_2 \\ \psi_1 \varphi_1 \end{pmatrix}, \qquad \begin{pmatrix} \psi_1^\dagger \varphi_2 \\ \psi_2^\dagger \varphi_2 \end{pmatrix} \in \mathbf{2}, \\ &\psi_1 \varphi_2 + \psi_2 \varphi_1, \qquad \psi_1^\dagger \varphi_1 + \psi_2^\dagger \varphi_2 \in \mathbf{1}, \\ &\psi_1 \varphi_2 - \psi_2 \varphi_1, \qquad \psi_1^\dagger \varphi_1 - \psi_2^\dagger \varphi_2 \in \mathbf{1}'. \end{aligned} \tag{8}$$

With these few ingredients one can construct easily S_3 invariants, once the assignment of standard model fields to S_3 irreps is given.

A. The $\mu\tau$ Sector

Let the following fields transform under the **2** irrep of S_3:

$$\begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix}, \qquad \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \qquad \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}, \tag{9}$$

where $L_\alpha = (\nu_\alpha l_\alpha)^T$, $\Phi_i = (\phi_i^0 \phi_i^-)^T$ and $\xi_i = (\xi_i^{++} \xi_i^+ \xi_i^0)^T$ are scalar isotriplets. Let us assign also

$$\mu^c \in \mathbf{1}, \qquad \tau^c \in \mathbf{1}'. \tag{10}$$

The invariants relevant for lepton masses are

$$\begin{aligned} (\tau \phi_1^0 + \mu \phi_2^0)\mu^c, \qquad (\tau \phi_1^0 - \mu \phi_2^0)\tau^c, \\ \nu_\mu \nu_\mu \xi_1^0 + \nu_\tau \nu_\tau \xi_2^0. \end{aligned} \tag{11}$$

After electroweak symmetry breaking, the neutral scalar fields take VEVs $<\phi_i^0> = v_i$ and $<\xi_i^0> = u_i$, so that

$$\begin{aligned} M_l &= \begin{pmatrix} f_1 v_2 & -f_2 v_2 \\ f_1 v_1 & f_2 v_1 \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} f_1(v_2 + v_1) & f_2(v_1 - v_2) \\ f_1(v_1 - v_2) & f_2(v_2 + v_1) \end{pmatrix}, \\ M_\nu &= \begin{pmatrix} f_3 u_1 & 0 \\ 0 & f_3 u_2 \end{pmatrix}, \end{aligned} \tag{12}$$

where f_i are dimensionless coupling constants.

It is apparent that, if $v_1 = v_2 = v$ (and $u_1 \neq u_2$), maximal mixing is generated. Indeed the condition $v_1 = v_2$ minimizes the S_3 invariant scalar potential (see section IIIA). The lepton masses are given by $m_\mu = \sqrt{2} f_1 v$, $m_\tau = \sqrt{2} f_2 v$, $\Delta m_{atm}^2 = f_3^2(u_2^2 - |u_1|^2)$ (notice that all complex phases can be rotated away, except a Majorana phase in the neutrino sector, which we can think as associated to u_1). The deviation from maximal mixing can be easily computed as

$$\theta_{23} - \frac{\pi}{4} \approx \frac{v_1 - v_2}{v_1 + v_2}. \tag{13}$$

A comment is in order about different contributions to the neutrino mass matrix. Since the VEVs of ξ_i have to be seesaw suppressed (see section IIIA), in general a comparable contribution to neutrino masses can come from the nonrenormalizable operator $L_\alpha L_\beta \bar{\Phi}_i \bar{\Phi}_j / M_R$, where M_R is the seesaw scale and $\bar{\Phi}_i \equiv i\sigma_2 \Phi_i^* = (\phi_i^+ - \phi_i^{0*})^T$. Taking into account that $(\bar{\Phi}_2 \bar{\Phi}_1)^T \in \mathbf{2}$, one finds the following S_3 invariants:

$$\begin{aligned} \nu_\tau \nu_\tau (\phi_2^{0*})^2 + \nu_\mu \nu_\mu (\phi_1^{0*})^2, \\ (\nu_\mu \nu_\tau + \nu_\tau \nu_\mu)(\phi_1^0 \phi_2^0 + \phi_2^0 \phi_1^0)^*. \end{aligned} \tag{14}$$

The first invariant can be mediated, at the seesaw scale, by the triplets ξ_i and its contribution to neutrino masses modifies the values of u_i but does not affect maximal mixing. The second invariant, on the contrary, generates a nonzero off-diagonal entry in the neutrino sector which modifies the resulting value of θ_{23}, potentially preventing an almost maximal mixing. However, this invariant cannot be mediated by ξ_i, so in our minimal model it does not contribute. One can check that it is mediated by scalar isotriplets $\xi \in \mathbf{1}, \mathbf{1}'$ and/or by heavy neutrino states $\nu^c \in \mathbf{2}, \mathbf{1}, \mathbf{1}'$. Maximal mixing indicates that all these fields are absent or their contribution to M_ν is suppressed.

1. Remarks On the Scalar VEVs (I)

The most general S_3 invariant scalar potential for $\Phi_{1,2}$ is given by

$$\begin{aligned} V_\Phi &= m^2(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) + \tfrac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)^2 \\ &\quad + \tfrac{\lambda_2}{2}(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2)^2 + \lambda_3 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1. \end{aligned} \tag{15}$$

Replacing $\Phi_{1,2}^{(\dagger)}$ with $v_{1,2}^{(*)}$, one can check that $V_\Phi(v_1, v_2)$ is bounded from below if and only if $\lambda_1 + \lambda_2 > 0$ and $-2\lambda_1 < \lambda_3 < 2\lambda_2$. In this region of parameters, the absolute minimum, in the case $m^2 < 0$, is given by $v_1^2 = v_2^2 = -m^2/(2\lambda_1 + \lambda_3)$. This justifies the assumption $v_1 = v_2$ made in the previous section.

Notice that V_Φ is invariant under two independent $U(1)$ transformations: $\Phi_{1,2} \to e^{i\theta_{1,2}} \Phi_{1,2}$. As a consequence, electroweak symmetry breaking leaves us with an undesired real massless scalar, the residual Goldstone boson. However, since the S_3 symmetry is broken at electroweak scale, one can allow in V_Φ soft breaking terms with size comparable to the quadratic term in Eq. (15). This S_3 explicit breaking can originate from the VEVs of extra fields belonging to a hidden sector of the theory, similarly to the soft breaking terms in supersymmetric models. Let us consider the extra term

$$\Delta V_\Phi = \eta^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1). \tag{16}$$

It breaks a $U(1)$ symmetry since it enforces the relation $\theta_1 = \theta_2$. In fact, the term in Eq. (16) respects the discrete symmetry $\Phi_1 \leftrightarrow \Phi_2$, which is also a symmetry of the S_3 invariant potential V_Φ. Therefore the potential obtained adding Eq. (15) and Eq. (16) can be still minimized by $v_1 = v_2 = v$, where now $v^2 = -(m^2 + \eta^2)/(2\lambda_1 + \lambda_3)$. We assume that this "custodial" symmetry is preserved till scales much smaller than electroweak. Computing the quadratic part of $V_\Phi + \Delta V_\Phi$ after electroweak symmetry breaking, one can check that all physical scalar fields take a mass of the order of the electroweak scale.

The VEVs of ξ_i are induced by the Φ_i VEVs via the following scalar potential:

$$V_\xi = \frac{1}{2}(M_\xi^2)^{ij}\xi_i^\dagger\xi_j + (M_{\xi\phi\phi})^{ijk}\xi_i\Phi_j\Phi_k + \text{h.c.}. \tag{17}$$

The S_3 invariant mass term, $M_\xi^2(\xi_1^\dagger\xi_1 + \xi_2^\dagger\xi_2)$, is supposed to be very heavy, $M_\xi^2 \gg v^2$, in order to suppress the triplet VEVs u_i via the usual type II seesaw mechanism [23–26]. The actual values of u_1 and u_2 depend on the scale of different trilinear couplings. The S_3 invariant trilinear term is given by

$$M_{\xi\phi\phi}(\xi_1\Phi_1\Phi_1 + \xi_2\Phi_2\Phi_2) + \text{h.c.}. \tag{18}$$

If $M_{\xi\phi\phi}$ is the dominant trilinear coupling, then integrating out the heavy fields ξ_i one obtains $u_i^* = -v_i^2 M_{\xi\phi\phi}/M_\xi^2$, so that $v_1 = v_2$ implies $u_1 = u_2$. However, if we assume that all the soft breaking term couplings are smaller than or equal to the electroweak scale, where S_3 is spontaneously broken, than there is no reason to expect that the S_3 invariant trilinear coupling is dominating over the others. For example, if the dominant trilinear term is

$$\sum_{i=1,2} M_{\xi\phi\phi}^i(\xi_i\Phi_1\Phi_2) + \text{h.c.}, \tag{19}$$

than one obtains $u_i^* = -v_1 v_2 M_{\xi\phi\phi}^i/M_\xi^2$, so that $u_1 \neq u_2$ is naturally induced. Notice that Eq. (19) preserves the discrete symmetry $\Phi_1 \leftrightarrow \Phi_2$. If this custodial symmetry is broken only at scales much smaller than v, than it is natural to take $M_{\xi\Phi\Phi}^i \sim v \gg M_{\xi\Phi\Phi}$.

Notice that the most general V_ξ breaks S_3 only softly, thus not affecting the mass matrices found in the previous section and allowing, at the same time, $u_1 \neq u_2$. The symmetry $\Phi_1 \leftrightarrow \Phi_2$, that preserves $v_1 = v_2$, is somewhat analogue to the strong isospin, which is a good approximate symmetry at the scale Λ_{QCD} ($m_p \approx m_n$), not because $m_u \approx m_d$, but because $m_u, m_d \ll \Lambda_{QCD}$.

B. The Electron Sector

Let us introduce the fields

$$L_e, e^c, \Phi_3 \in 1. \tag{20}$$

The S_3 singlet scalar isodoublet Φ_3 is necessary to provide a nonzero mass to the electron. The new invariants relevant for lepton masses are

$$(\tau\phi_1^0 + \mu\phi_2^0)e^c, \quad ee^c\phi_3^0, \quad e\mu^c\phi_3^0, \quad (\nu_\tau\xi_1^0 + \nu_\mu\xi_2^0)\nu_e. \tag{21}$$

Comparing the first invariant in Eq. (11) with the first in Eq. (21), one realizes that only one linear combination of μ^c and e^c is coupled to $(\tau\phi_1^0 + \mu\phi_2^0)$, while the orthogonal is not. Since a rotation of μ^c and e^c is unobservable (right-handed), we have the freedom to redefine e^c as the decoupled state. Then the charged lepton mass matrix takes the form

$$\begin{aligned} M_l &= \begin{pmatrix} f_4 v_3 & f_5 v_3 & 0 \\ 0 & f_1 v_2 & -f_2 v_2 \\ 0 & f_1 v_1 & f_2 v_1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \times \begin{pmatrix} f_4 v_3 & f_5 v_3 & 0 \\ 0 & \sqrt{2} f_1 v & 0 \\ 0 & 0 & \sqrt{2} f_2 v \end{pmatrix}, \end{aligned} \tag{22}$$

where $v_3 = <\phi_3^0>$ and in the last equality we have used $v_1 = v_2 = v$. Assuming $|v| \gg |v_3|$ (see section IIIB1), one gets

$$\frac{m_e}{m_\mu} \approx \frac{|f_4 v_3|}{\sqrt{2}|f_1 v|}, \qquad \theta_{12}^l \approx \frac{|f_5 v_3|}{\sqrt{2}|f_1 v|}. \tag{23}$$

If the coefficients f_i are of order one, then $\theta_{12}^l \sim m_e/m_\mu$.

The neutrino mass matrix has the following form:

$$\begin{aligned} M_\nu &= \begin{pmatrix} 0 & f_6 u_2 & f_6 u_1 \\ f_6 u_2 & f_3 u_1 & 0 \\ f_6 u_1 & 0 & f_3 u_2 \end{pmatrix} \\ &= f_3 u_2 \begin{pmatrix} 0 & \epsilon_f & \epsilon_f \epsilon_u \\ \epsilon_f & \epsilon_u & 0 \\ \epsilon_f \epsilon_u & 0 & 1 \end{pmatrix}, \end{aligned} \tag{24}$$

where $\epsilon_f \equiv f_6/f_3$ and $\epsilon_u \equiv u_1/u_2$. Neglecting for the moment the small angle θ_{12}^l, the symmetry basis is given by $[\nu_e, (\nu_\mu - \nu_\tau)/\sqrt{2}, (\nu_\mu + \nu_\tau)/\sqrt{2}]$. In flavor basis, one can write

$$M_\nu^{fl} = \frac{f_3 u_2}{2}\left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \sqrt{2}\epsilon_f(1+\epsilon_u) & -\sqrt{2}\epsilon_f(1-\epsilon_u) \\ \sqrt{2}\epsilon_f(1+\epsilon_u) & \epsilon_u & -\epsilon_u \\ -\sqrt{2}\epsilon_f(1-\epsilon_u) & -\epsilon_u & \epsilon_u \end{pmatrix}\right]. \qquad (25)$$

The 11-entry in M_ν^{fl} is zero, thus implying a strong suppression of neutrinoless 2β-decay. It is well known that this can happen only in the case of normal hierarchical neutrino spectrum. Therefore the parameters $\epsilon_{f,u}$ have to be taken small and the first term in Eq. (25) is the dominant $\mu\tau$-block [10]. This dominant structure of M_ν^{fl} is the unique one allowed by data in the case of normal mass hierarchy [8]. We have shown that an S_3 symmetry is suitable to generate simply the $\mu\tau$-block.

Diagonalizing Eq. (24), one finds

$$m_3 \approx |f_3 u_2| \approx \sqrt{\Delta m^2_{atm}}, \quad |\theta_{23} - \tfrac{\pi}{4}| \approx |\epsilon_f^2 \epsilon_u|,$$
$$\theta_{13} \approx |\epsilon_f \epsilon_u|, \qquad \delta \approx \arg\epsilon_u, \qquad \tan 2\theta_{12} \approx 2\left|\frac{\epsilon_f}{\epsilon_u}\right|, \qquad (26)$$
$$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \sqrt{|\epsilon_u|^4 + 4|\epsilon_u \epsilon_f|^2},$$

where δ is the Dirac-type *CP*-violating phase in the standard parametrization of the lepton mixing matrix [58]. The correlations among different observables are in agreement with present data and can be tested in future precision measurements. Using the best fit values $\tan 2\theta_{12} = 2.1$ and $\Delta m^2_{sol}/\Delta m^2_{atm} = 0.035$ [1–5], we find $|\epsilon_u| \approx 0.12$ and $|\epsilon_f| \approx 0.13$, which imply $|\theta_{23} - \pi/4| \approx 0.002$ and $\theta_{13} \approx 0.016$.

The allowed values of θ_{13} can be better evaluated noticing that Eq. (26) implies

$$\theta_{13} \approx \frac{1}{2}\sin 2\theta_{12}\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}. \qquad (27)$$

Using 90% C.L. allowed ranges, we obtain the prediction

$$0.008 \lesssim \theta_{13} \lesssim 0.032. \qquad (28)$$

Correspondingly, the parameters $|\epsilon_f|$ and $|\epsilon_u|$ are constrained in the range $0.1 \div 0.2$. A numerical diagonalization of the matrix (24) has been performed and the resulting predictions for θ_{13} are shown in Fig. 1, in good agreement with the approximations given in Eqs. (27) and (28).

We have neglected until now the contribution of the 12-mixing in the charged lepton sector (see Eq. (23)). Performing a careful commutation of rotation matrices, we find that θ^l_{12} affects all three observable mixing angles as follows:

$$\Delta^l \theta_{23} \approx \frac{(\theta^l_{12})^2}{4}, \qquad \Delta^l \theta_{13} \approx \frac{\theta^l_{12}}{\sqrt{2}}, \qquad \Delta^l \theta_{12} \approx \frac{\theta^l_{12}}{\sqrt{2}}. \qquad (29)$$

In the case of 13-mixing, this correction can be important even for θ^l_{12} as small as $m_e/m_\mu \approx 0.005$. The rotation θ^l_{12} generates also

$$m_{ee} \equiv |(M_\nu^{fl})_{11}| \approx \sqrt{2\Delta m^2_{atm}}|\epsilon_f|\theta^l_{12} \lesssim 10^{-2}\ \text{eV} \cdot \theta^l_{12}, \qquad (30)$$

which induces a nonzero (but still quite suppressed) neutrinoless 2β-decay.

It is worthwhile to give a look to the contribution of Φ_3 to the five-dimensional operator $L_\alpha L_\beta \bar{\Phi}_i \bar{\Phi}_j / M_R$, which can perturb the neutrino mass matrix (24). The possible S_3 invariants are

$$\nu_e \nu_e (\phi_3^{0*})^2, \qquad (\nu_\mu \nu_\tau + \nu_\tau \nu_\mu)(\phi_3^{0*})^2,$$
$$\nu_e(\nu_\mu \phi_1^{0*} - \nu_\tau \phi_2^{0*})\phi_3^{0*}, \qquad (\nu_\mu \nu_\mu \phi_2^{0*} + \nu_\tau \nu_\tau \phi_1^{0*})\phi_3^{0*}. \qquad (31)$$

The first two invariants are not mediated by $\xi_{1,2}$ and therefore are absent in the minimal model. The last two invariants can be mediated, but their contribution can be absorbed into a redefinition of $u_{1,2}$ and $f_{3,6}$ in Eq. (24).

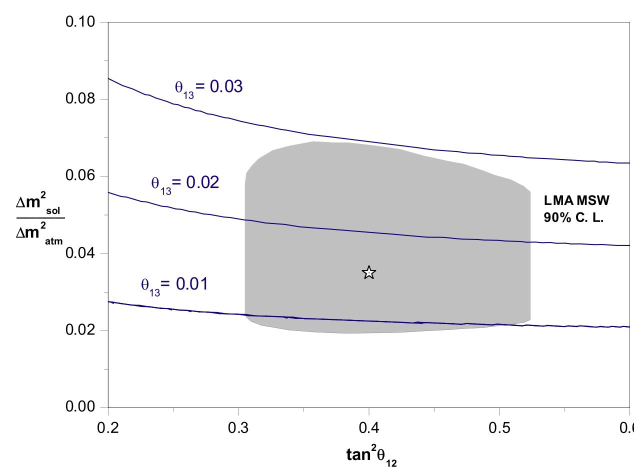

FIG. 1 (color online). Predictions of the S_3 model for the mixing angle θ_{13}. The gray area represents the Large Mixing Angle MSW allowed region for the parameters $\tan^2\theta_{12}$ and Δm^2_{sol}, stretched to include also the experimental uncertainty in Δm^2_{atm}. The best fit point is denoted with a star. The values of θ_{13} are determined by θ_{12} and $\Delta m^2_{sol}/\Delta m^2_{atm}$ as described approximately by Eq. (27). Possibly large corrections to these values of θ_{13} can come from charged lepton sector (see Eq. (29)).

1. Remarks On the Scalar VEVs (II)

Of course the introduction of the S_3 singlet Φ_3 in our model modifies the scalar potential discussed in section IIIA. The most general S_3 invariant potential is

$$\begin{aligned} V_{3\Phi} = V_{\Phi} &+ m_3^2\Phi_3^\dagger\Phi_3 + \tfrac{\lambda_4}{2}(\Phi_3^\dagger\Phi_3)^2 + \lambda_5(\Phi_3^\dagger\Phi_3) \\ &\times(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) + \lambda_6\Phi_3^\dagger(\Phi_1\Phi_1^\dagger \\ &+ \Phi_2\Phi_2^\dagger)\Phi_3 + [\lambda_7\Phi_3^\dagger\Phi_1\Phi_3^\dagger\Phi_2 \\ &+ \lambda_8\Phi_3^\dagger(\Phi_1\Phi_2^\dagger\Phi_1 + \Phi_2\Phi_1^\dagger\Phi_2) + \text{h.c.}] \end{aligned} \tag{32}$$

where V_ϕ is given in Eq. (15).

We do not discuss, here, the general minimization problem (see, for instance, [59]). For our purposes, it is enough to verify that a solution exists with $|v_1| = |v_2| \equiv |v| \gg |v_3|$. Assuming that all the couplings λ_i are of the same order and barring special cancellations among them, we find a minimum for

$$\begin{aligned} |v|^2 &\approx -\frac{m^2}{2\lambda_1 + \lambda_3}, \\ v_3 &\approx -\frac{2\tilde{\lambda}_8|v|^3}{m_3^2 + 2|v|^2(\lambda_5 + \lambda_6 + \tilde{\lambda}_7)}, \end{aligned} \tag{33}$$

where $\tilde{\lambda}_{7,8}$ are appropriate rephasings of $\lambda_{7,8}$, determined by the v_i complex phases. To satisfy the initial assumption $|v_3| \ll |v|$ is enough to choose $m_3^2 \gg |v|^2$, so that $|v_3| \approx 2|\tilde{\lambda}_8 v|(|v|^2/m_3^2)$.

The large parameter m_3^2 determines, in first approximation, the mass of the four real scalars contained in Φ_3. Since the effect of v_3 can be safely treated as a small perturbation, all further considerations made in section IIIA1 about $\Phi_{1,2}$ and $\xi_{1,2}$ masses and VEVs are still valid.

C. The Quark Sector

Let us assign quark fields to S_3 representations in exact analogy with leptons:

$$\begin{pmatrix} Q_2 \\ Q_3 \end{pmatrix} \in \mathbf{2}, \qquad Q_1, u^c, c^c, d^c, s^c \in \mathbf{1}, \qquad b^c, t^c \in \mathbf{1}', \tag{34}$$

where $Q_i = (u_i d_i)^T$. As for leptons, the third generation isosinglets are odd S_3 singlets while second generation ones are S_3 invariant. This turns out to be the origin of a sizable $1-2$ mixing and a suppressed $1-3$ mixing. It is straightforward to construct independent invariants contributing to quark mass matrices. The result is

$$\begin{aligned} M_u &= \begin{pmatrix} g_3^u v_3^* & g_4^u v_3^* & 0 \\ 0 & g_1^u v_1^* & -g_2^u v_1^* \\ 0 & g_1^u v_2^* & g_2^u v_2^* \end{pmatrix}, \\ M_d &= \begin{pmatrix} g_3^d v_3 & g_4^d v_3 & 0 \\ 0 & g_1^d v_2 & -g_2^d v_2 \\ 0 & g_1^d v_1 & g_2^d v_1 \end{pmatrix}. \end{aligned} \tag{35}$$

The S_3 assignment of the quarks parallels that of the leptons, as shown by the analogous structure of their mass matrices (22) and (35). This makes our model suitable for a possible embedding in a Grand Unification Theory. It is interesting that, in a class of $SO(10)$ inspired models known as "lopsided" [60,61], the almost maximal leptonic $2-3$ mixing originates in the charged lepton mass matrix as in the present case. However, the pattern is different in the quark sector: in our model the left-handed $2-3$ mixing cancels between down and up quark sectors; in lopsided models the large charged lepton mixing appears also in the down quark mass matrix, but on the right-handed side, therefore it does not show up in the CKM matrix.

In the limit $|v_1| = |v_2| = |v|$, the maximal $2-3$ mixing cancels exactly between up and down matrices and we get $m_c/m_t \approx |g_1^u/g_2^u|$, $m_s/m_b \approx |g_1^d/g_2^d|$. The experimental value, $\theta_{23}^q \approx 0.04$, can be explained by small corrections to $|v_1| = |v_2|$, due to soft breaking and/or Φ_3 contributions to the scalar potential in Eq. (32). In fact, one finds the interesting sum rule

$$\theta_{23}^q \approx 2\left(\frac{\pi}{4} - \theta_{23}\right) \approx 2\frac{|v_2| - |v_1|}{|v_2| + |v_1|}. \tag{36}$$

Larger deviations from maximal atmospheric mixing can be explained only by a nonzero contribution to the mixing coming from the neutrino sector (see discussion at the end of section IIIA).

In analogy to the charged lepton sector, after the maximal $2-3$ rotation the structure of the $1-2$ blocks in M_d and in M_u implies

$$\begin{aligned} &\frac{m_d}{m_s} \approx \frac{|g_3^d v_3|}{\sqrt{2}|g_1^d v|} \sim \frac{1}{20}, \qquad \frac{m_u}{m_c} \approx \frac{|g_3^u v_3|}{\sqrt{2}|g_1^u v|} \sim \frac{1}{400}, \\ &\theta_{12}^{d,u} \approx \frac{|g_4^{d,u} v_3|}{\sqrt{2}|g_1^{d,u} v|}. \end{aligned} \tag{37}$$

The CKM mixing matrix is given by

$$\begin{aligned} U_{CKM} &\equiv U_{23}U_{13}U_{12} = (U_{23}^u U_{12}^u)^\dagger (U_{23}^d U_{12}^d) \\ &\equiv U_{12}^{u\dagger} U_{23}^q U_{12}^d, \end{aligned} \tag{38}$$

where $U_{23}^{u,d}$ are almost maximal 23-rotations that cancel up to the small angle given in Eq. (36), while $U_{12}^{u,d}$ are the 12-rotations quantified in Eq. (37). From the commutation of $U_{12}^{u\dagger}$ and U_{23}^q, a 13-mixing is generated: $\theta_{13}^q \approx \theta_{12}^u \theta_{23}^q$. One has to fit $\theta_{13}^q \approx 0.004$ and the Cabibbo angle $\theta_{12}^q \approx$

0.22, which results from the combination of $U^{u\dagger}_{12}$ and U^{d}_{12}. Looking at Eq. (37), one realizes that the fit is successful for $|v_3/v| \sim 0.1$ and the coefficients $g^{u,d}_i$ of order one. However, a significant suppression of g^u_3 is required to match the smallness of the up quark mass. We will suggest an explanation for this suppression in the next section.

IV. DISCUSSION AND CONCLUSIONS

We have analyzed the problem of constructing a maximal $2-3$ mixing in the lepton sector. As shown in section II, minimal models with Abelian flavor symmetries give a defective description of neutrino oscillation data. We have then constructed a model based on S_3 flavor symmetry which generates naturally maximal $2-3$ mixing. It contains only standard model particles plus an enlarged scalar sector, formed by three isodoublets at electroweak scale and two much heavier isotriplets.

Let us summarize the ingredients of the model:

(i) Second and third generation fermion isodoublets transform as an S_3 doublet.

(ii) Second generation fermion isosinglets are S_3 invariants while the third ones transform as odd S_3 singlets.

(iii) The two scalar isodoublets which generate the $2-3$ block of charged fermion mass matrices have the same VEV, while the one giving mass to first generation fermions takes a much smaller VEV.

(iv) The neutrino mass matrix is generated by an S_3 doublet of heavy scalar isotriplets, which have tiny and different VEVs.

As a consequence, maximal $2-3$ mixing is induced in the charged fermion mass matrices, while the $2-3$ block is diagonal in the Majorana mass matrix of neutrinos. Therefore, a maximal mixing results in the lepton sector, whereas complete cancellation takes place between up and down quark mixing. Small corrections in both sectors are allowed and they are correlated as in Eq. (36). Extra deviation from maximal atmospheric mixing can appear if heavy fields other than the two isotriplets give a subdominant contribution to the neutrino mass matrix. The ratio between second and third generation masses is not determined by the S_3 symmetry. However S_3 distinguishes the two corresponding sets of couplings, which are of the type $\mathbf{2}\times\mathbf{2}\times\mathbf{1}$ and $\mathbf{2}\times\mathbf{2}\times\mathbf{1}'$ for second and third generation, respectively. This suggests that the hierarchy between the two types can be induced by extra flavor structure to be added to our minimal model.

The spectrum of neutrinos is with normal hierarchy. In flavor basis the neutrino mass matrix has a dominant $\mu\tau$-block. The $1-2$ mixing can be naturally of order one, thus explaining the Large Mixing Angle MSW solution of the solar neutrino problem. The $1-3$ mixing is correlated with solar parameters by Eq. (27) and turns out to be about 0.02.

The neutrinoless 2β-decay is strongly suppressed. If the recent claim [62] of neutrinoless 2β-decay were confirmed, then $|(M_\nu)_{11}| \gtrsim 0.1$ eV and our model would be ruled out, unless the dominant mechanism of the decay is not the exchange of the light Majorana neutrinos [63]. Notice that in our model the suppression of $(M_\nu)_{11}$ is induced by the requirement to obtain an atmospheric mixing close to maximal. In fact, one can check that, adding $\xi \in \mathbf{1}, \mathbf{1}'$ and/or $\nu^c \in \mathbf{2}, \mathbf{1}, \mathbf{1}'$ to the model, a nonzero contribution to $(M_\nu)_{11}$ is accompanied by a contribution of the same order to $(M_\nu)_{23}$, which tends to cancel the maximal $2-3$ mixing coming from charged leptons.

In the quark sector, a small $1-2$ mixing is generated naturally because u and c^c (d and s^c) are both S_3 invariants, thus allowing a sizable 12-entry in the mass matrices. One can easily reproduce the Cabibbo angle. The different S_3 assignment of t^c and b^c (odd S_3 singlets) suppresses the $1-3$ mixing in M_d and M_u; the resulting CKM matrix contains $\theta^q_{13} \sim \theta^q_{12}\theta^q_{23}$, in agreement with data.

First generation masses are suppressed by the small ratio of scalar VEVs $|v_3/v|$. This ratio cannot be too small since the Cabibbo angle is correspondingly suppressed. In particular the smallness of the u quark (electron) mass indicates an extra source of suppression. This can be easily obtained, for example, introducing a Z_2 parity leaving all fields invariant but u^c (e^c), which is Z_2 odd. It is easy to check that, in the limit of exact Z_2 symmetry, m_u (m_e) is forbidden.

The model can be tested in the near future by

(i) precision measurements of neutrino oscillation parameters;

(ii) upper bounds on neutrino masses from cosmology, tritium β-decay and neutrinoless 2β-decay;

(iii) direct investigation of the scalar isodoublet sector at LHC;

(iv) flavor violating decays mediated by the scalars. A detailed study of the phenomenological implications of the model is left for future work.

In conclusion, S_3 is the smallest flavor symmetry group which can explain in a minimal way the maximal atmospheric mixing. The required structure in the lepton $2-3$ sector enforces in a straightforward way the whole structure of three generation lepton and quark mass matrices. These matrices are suitable to explain all current data.

ACKNOWLEDGMENTS

This work has been partially supported by the U.S. Department of Energy under Grant No. DE-FG03-94ER40837.

[1] Super-Kamiokande and K2K, T. Nakaya, hep-ex/0209036.
[2] Super-Kamiokande, S. Fukuda *et al.*, Phys. Lett. B **539**, 179 (2002).
[3] SNO, Q. R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011302 (2002).
[4] SNO, S. N. Ahmed *et al.* Phys. Rev. Lett. **92**, 181301 (2004).
[5] KamLAND, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
[6] CHOOZ, M. Apollonio *et al.*, Phys. Lett. B **466**, 415 (1999).
[7] P. Huber, M. Lindner, M. Rolinec, T. Schwetz, and W. Winter, hep-ph/0403068.
[8] M. Frigerio and A. Y. Smirnov, Nucl. Phys. B **640**, 233 (2002).
[9] M. Frigerio and A. Y. Smirnov, Phys. Rev. D**67**, 013007 (2003).
[10] F. Vissani, J. High Energy Phys. **11**, 025 (1998).
[11] J. A. Casas, J. R. Espinosa, A. Ibarra, and I. Navarro, Nucl. Phys. B**573**, 652 (2000).
[12] S. Antusch, J. Kersten, M. Lindner, and M. Ratz, Nucl. Phys. B **674**, 401 (2003).
[13] M. Frigerio and A. Y. Smirnov, J. High Energy Phys. **02**, 004 (2003).
[14] R. N. Mohapatra, M. K. Parida, and G. Rajasekaran, Phys. Rev. D **69**, 053007 (2004).
[15] J. A. Casas, J. R. Espinosa, and I. Navarro, J. High Energy Phys. **0309**, 048(2003).
[16] T. Yanagida, *Workshop on the Unified Theory and Baryon Number in the Universe, Tsukuba, Japan*, edited by O.Sawada and A.Sugamoto [Prog. Theor. Phys. 64, 1103 (1980)].
[17] S. L. Glashow, *Quarks and Leptons*, 687-713 and (Harvard University, Cambridge, MA, 1979) (Report No. HUTP-79-A059 1979)
[18] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Z. Freedman (North Holland, Amsterdamn, 1979) (CERN Report No. 80-0576, 1979).
[19] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[20] A. Y. Smirnov, Phys. Rev. D**48**, 3264 (1993).
[21] I. Dorsner and A. Y. Smirnov, hep-ph/0403305.
[22] B. Bajc, G. Senjanovic, and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003).
[23] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. B **181**, 287 (1981).
[24] R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).
[25] J. Schechter and J.W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
[26] E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998).
[27] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147**, 277 (1979).
[28] G. Altarelli and F. Feruglio (2002), hep-ph/0206077.
[29] K. Fukuura, T. Miura, E. Takasugi, and M. Yoshimura, Phys. Rev. D **61**, 073002 (2000).
[30] W. Grimus and L. Lavoura, J. High Energy Phys. **07**, 045 (2001).
[31] P. F. Harrison and W. G. Scott, Phys. Lett. B **547**, 219 (2002).
[32] K. S. Babu, I. Gogoladze, and K. Wang, Nucl. Phys. B **660**, 322 (2003).
[33] E. Ma, Phys. Lett. B **583**, 157 (2004).
[34] C. I. Low, hep-ph/0404017 [Phys. Rev. D (to be published)].
[35] D. B. Kaplan and M. Schmaltz, Phys. Rev. D **49**, 3741 (1994).
[36] P. H. Frampton and A. Rasin, Phys. Lett. B **478**, 424 (2000).
[37] E. Ma and G. Rajasekaran, Phys. Rev. D **64**, 113012 (2001).
[38] W. Grimus and L. Lavoura, Phys. Lett. B **572**, 189 (2003).
[39] Z. Berezhiani and A. Rossi, J. High Energy Phys. **03**, 002 (1999).
[40] R. Barbieri, L. J. Hall, G. L. Kane, and G. G. Ross, hep-ph/9901228.
[41] M.-C. Chen and K. T. Mahanthappa, Phys. Rev. D **68**, 017301 (2003).
[42] S. F. King and G. G. Ross, Phys. Lett. B **574**, 239 (2003).
[43] S. Pakvasa and H. Sugawara, Phys. Lett. B **73**, 61 (1978).
[44] E. Ma, Phys. Rev. D **43**, 2761 (1991).
[45] E. Ma, Phys. Rev. D **61**, 033012 (2000).
[46] R. N. Mohapatra, A. Perez-Lorenzana, and C. A. de Sousa Pires, Phys. Lett. B **474**, 355 (2000).
[47] P. F. Harrison and W. G. Scott, Phys. Lett. B **557**, 76 (2003).
[48] J. Kubo, A. Mondragon, M. Mondragon, and E. Rodriguez-Jauregui, Prog. Theor. Phys. **109**, 795 (2003).
[49] H. Harari, H. Haut, and J. Weyers, Phys. Lett. B **78**, 459 (1978).
[50] Y. Koide, Phys. Rev. D **39**, 1391 (1989).
[51] H. Fritzsch and J. Plankl, Phys. Lett. B**237**, 451 (1990).
[52] C. D. Carone, L. J. Hall, and H. Murayama, Phys. Rev. D **53**, 6282 (1996).
[53] M. Fukugita, M. Tanimoto, and T. Yanagida, Phys. Rev. D **57**, 4429 (1998).
[54] M. Tanimoto, Phys. Lett. B **483**, 417 (2000).
[55] L. J. Hall and H. Murayama, Phys. Rev. Lett. **75**, 3985 (1995).
[56] S. T. Petcov, Phys. Lett. B **110**, 245 (1982).
[57] N. G. Deshpande, M. Gupta, and P. B. Pal, Phys. Rev. D **45**, 953 (1992).
[58] Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[59] J. Kubo, H. Okada, and F. Sakamaki Phys. Rev. D **70**, 036007 (2004).
[60] C. H. Albright, K. S. Babu, and S. M. Barr, Phys. Rev. Lett. **81**, 1167 (1998).
[61] K. S. Babu and S. M. Barr, Phys. Lett. B **525**, 289 (2002).
[62] H.V. Klapdor-Kleingrothaus, A. Dietz, I.V. Krivosheina, and O. Chkvorets, Nucl. Instrum. Methods Phys. Res., Sect. A **522**, 371 (2004).
[63] B. Brahmachari and E. Ma, Phys. Lett. B**536**, 259 (2002).