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Purely gravitational generalization of spin-rotation couplings

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The nature of Mashhoon's spin-rotation coupling is the interaction between a particle spin (gravitomagnetic moment) and a gravitomagnetic field. Here we will consider the coupling of graviton spin to the weak gravitomagnetic fields by analyzing the Lagrangian density of weak gravitational field, and hence study the purely gravitational generalization of Mashhoon's spin-rotation couplings.

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With the development of laser technology and its application to the gravitational interferometry experiments [1], some weak gravitational effects [2–5] associated with gravitomagnetic fields have become increasingly important and therefore captured many authors both theoretically and experimentally. During the past 20 years, neutron interferometry was developed with increasing accuracy. By using these technologies, Werner *et al.* investigated the neutron analog of the Foucault-Michelson-Gale effect in 1979 [2] and Atwood *et al.* found the neutron Sagnac effect in 1984 [3]. Aharonov, Carmi *et al.* [4–6] proposed a gravitational analog to the Aharonov-Bohm effect [7], which is a geometric effect of vector potential of gravity: specifically, in a rotating frame the matter wave propagating along a closed path will acquire a nonintegral phase factor (geometric phase factor). This phenomenon has now been called the Aharonov-Carmi effect, or the gravitational Aharonov-Bohm effect. Overhauser, Colella [8], Werner and Standenmann *et al.* [2] have proven the existence of the Aharonov-Carmi effect by means of the neutron-gravity interferometry experiment. Note that here the Aharonov-Carmi effect results from the interaction between the momentum of a moving particle and the rotating frame. Even though the interaction of a spinning particle such as neutron with the rotating frame has the same origin of Aharonov-Carmi effect, i.e., both arise from the presence of the inertial force, the Aharonov-Carmi effect mentioned above does not contain the spin-rotation coupling [9].

It has become feasible to use polarized neutrons in the interferometer experiments [10]. Since a particle with an intrinsic spin possesses a gravitomagnetic moment, Mashhoon considered the interaction of the particle spin with the rotation of a noninertial reference frame, which was referred to as the *spin-rotation coupling* [9]. Recently, Mashhoon analyzed the Doppler effect of wavelight in a rotating frame with respect to the fixing frame [11,12] and derived the photon spin-rotation coupling effect. We considered the coordinate transformation (from the fixing frame to the rotating frame) of gravitomagnetic potential $g_{0\varphi}$ of Kerr metric and then obtained the Hamiltonian of neutron spin-rotation coupling [13]. A straightforward and unified way within the framework of special relativity to derive the inertial effects (including spin-rotation coupling, Bonse-Wroblewski and Page-Werner effects) of a Dirac particle was proposed by Hehl *et al.*, where they put the special-relativistic Dirac equation into a noninertial reference frame by standard methods [10]. Mashhoon's spin-rotation coupling has some interesting applications, e.g., the spin-rotation coupling experienced by valency electrons in rapidly rotating C_{60} molecules (with the rotating frequency ranges from 10^{10} to 10^{12} rad/s in the orientationally disordered phase) can be applied to the investigations of photoelectron spectroscopy (and hence to the molecular dynamics of C_{60} solid) [14].

Basically, the spin-rotation coupling considered above is just one of the gravitomagnetic effects, for the rotating frequency of the noninertial frame can be viewed as a gravitomagnetic field (or a piece of the gravitomagnetic field) [15] according to the principle of equivalence. To the best of our knowledge, the spin-rotation coupling of the photon, the electron and the neutron has been taken into account in the literature [12–14]. However, the gravitational coupling of graviton spin to the gravitomagnetic fields, which may also be of physical interest, has not yet been considered. Here we will extend Mashhoon's spin-rotation coupling to a purely gravitational case, where the graviton spin will be coupled to the gravitomagnetic fields. It will be shown that by analyzing the third-order terms in Lagrangian density of weak gravitational field, one can obtain the expression of the Lagrangian density for the interaction between the graviton spin and gravitomagnetic fields.

In this report, we deal with the weak gravitational fields only, which are described by the linearized gravity theory. One speaks of a linearized theory when the metric deviates only slightly from that of flat space. A weak gravitational field (in which spacetime is nearly flat) is defined as a manifold on which coordinates exist, where the metric has components $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ with

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$|h_{\alpha\beta}| \ll 1$. Such coordinates are called nearly Lorentz coordinates, where the indices of tensors are raised and lowered with the flat-Minkowski metric. First, we consider the expression for the spin of weak gravitational fields in the linearized gravity theory. In the nearly Lorentz coordinate system, to the first-order in $h_{\mu\nu}$, the Lagrangian density of the gravitational field takes the form

$$\mathcal{L} = -2\left(h^{\beta\nu,\mu}h_{\mu\nu,\beta} - \frac{1}{2}h^{\beta\nu,\mu}h_{\beta\nu,\mu} - \frac{1}{2}h^{\beta\nu}_{,\nu}h^{\mu}_{\mu,\beta}\right). \quad (1)$$

According to the canonical procedure, the canonical momentum of the linearized gravitational field is

$$\pi^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial\dot{h}_{\mu\nu}} = -4\left(h^{0\nu,\mu} - \frac{1}{2}h^{\mu\nu,0}\right) + \eta^{\mu\nu}h^{0\lambda}_{,\lambda} + \eta^{\nu0}h^{,\mu}, \quad (2)$$

where dot denotes the derivative with respect to time, and $h = \eta^{\mu\nu}h_{\mu\nu}$. In accordance with the Noether theorem, the spin of a field that is characterized by the Lagrangian density (1) is of the form $S^{\theta\tau} = \int d^3x(\pi^{\mu\nu}\sum^{\theta\tau}_{\nu\eta}h^{\eta}_{\mu})$, where $\sum^{\theta\tau}_{\nu\eta} = \delta^{\theta}_{\nu}\delta^{\tau}_{\eta} - \delta^{\theta}_{\eta}\delta^{\tau}_{\nu}$. It is well known that in the linearized gravity theory, there are two fundamental types of coordinate transformations, which take one nearly Lorentz coordinate system into another. The two coordinate transformations are the background Lorentz transformation, where the background metric takes the form of simple diagonal $(+1, -1, -1, -1)$, and the gauge transformation (for $h_{\mu\nu}$). To the first-order in small quantities, such a gauge transformation for $h_{\mu\nu}$ is

$$h'_{\alpha\beta} = h_{\alpha\beta} - \zeta_{\alpha,\beta} - \zeta_{\beta,\alpha}, \quad (3)$$

where ζ_α is an infinitesimal variation in the coordinate transformation, i.e., $x'_\alpha = x_\alpha + \zeta_\alpha(x_\beta)$. It is readily verified that by using the appropriate gauge conditions [15], the integrand (the density of spin, $s^{\theta\tau}$, of gravitational field) in $S^{\theta\tau}$ can be rewritten as [15]

$$s^{\theta\tau} = -2(h^{0\tau}h^{0\theta,0} - h^{0\theta}h^{0\tau,0}), \quad (4)$$

where $h^{0\theta}$ $(\theta = 1, 2, 3)$ can be regarded as the gravitomagnetic vector potentials.

Since in a rotating frame of reference, the expression $\partial_m h_{0n} - \partial_n h_{0m}$ for the gravitomagnetic field is such a quantity, the magnitude of which is proportional to the angular frequency, ω, of the rotating frame [15]. Thus, it is believed that the expression associated with $(h^{0m}\dot{h}^{0n} - h^{0n}\dot{h}^{0m})(\partial_m h_{0n} - \partial_n h_{0m})$ in the gravitational Lagrangian density (containing the nonlinear terms) can be viewed as an interaction term of the generalized Mashhoon's spin-rotation coupling, i.e., the interaction of the graviton spin with the gravitomagnetic fields. Here the repeated indices imply the summation carried out over 1, 2, 3.

Consider the Christoffel symbol of the weak gravitational field, the first and second-order terms of which may be expressed as follows [16,17]:

$$\Gamma^{\alpha}_{\beta\gamma} = -\frac{1}{2}K\left(h^{\alpha}_{\beta,\gamma} + h^{\alpha}_{\gamma,\beta} - h^{,\alpha}_{\beta\gamma} - \frac{1}{2}\delta^{\alpha}_{\beta}h^{\lambda}_{\lambda,\gamma} - \frac{1}{2}\delta^{\alpha}_{\gamma}h^{\lambda}_{\lambda,\beta} + \frac{1}{2}\eta_{\beta\gamma}h^{\lambda,\alpha}_{\lambda}\right) + \frac{1}{2}K^2(h_{\beta\lambda}h^{\lambda\alpha}_{,\gamma} + h_{\gamma\lambda}h^{\lambda\alpha}_{,\beta} + h^{\alpha\lambda}h_{\beta\gamma,\lambda} - h_{\beta\lambda}h^{\lambda,\alpha}_{\gamma} - h_{\gamma\lambda}h^{\lambda,\alpha}_{\beta}) + \frac{1}{2}K^2\left(-\frac{1}{2}\delta^{\alpha}_{\beta}h_{\lambda\tau}h^{\lambda\tau}_{,\gamma} - \frac{1}{2}\delta^{\alpha}_{\gamma}h_{\lambda\tau}h^{\lambda\tau}_{,\beta} - \frac{1}{2}\eta_{\beta\gamma}h^{\alpha\lambda}h^{\tau}_{\tau,\lambda} + \frac{1}{2}h_{\beta\gamma}h^{\lambda,\alpha}_{\lambda} + \frac{1}{2}\eta_{\beta\gamma}h_{\lambda\tau}h^{\lambda\tau,\alpha}\right) + \mathcal{O}(h^3), \quad (5)$$

where K and $h^{\mu\nu}$ are so defined that $\sqrt{-g}g^{\mu\nu} = \eta^{\mu\nu} + Kh^{\mu\nu}$ is satisfied. Because of $g^{\mu\nu}R_{\mu\nu}$ containing $g^{0m}R_{0m} + g^{m0}R_{m0}$ (i.e., $2g^{0m}R_{0m}$), we will analyze only the $0m$ component of the Ricci tensor $R_{\mu\nu}$ in the following. Note that the expression for $R_{\mu\nu}$ is written as $R_{\mu\nu} = \Gamma^{\rho}_{\mu\rho,\nu} - \Gamma^{\rho}_{\mu\nu,\rho} - \Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho} + \Gamma^{\rho}_{\sigma\nu}\Gamma^{\sigma}_{\mu\rho}$. In what follows, for brevity, the above generalized interaction will be referred to as the *graviton S-G coupling* (the coupling of graviton spin to the gravitomagnetic fields). Now we will extract the expression $\sim\dot{g}^{0n}(\partial_m g_{0n} - \partial_n g_{0m})$ from the Ricci tensor $R_{\mu\nu}$.

$\Gamma^{\rho}_{\mu\rho,\nu} - \Gamma^{\rho}_{\mu\nu,\rho}$.—It follows from the expression (5) that the second-order terms (with the coefficient being $\frac{1}{2}K^2$), which will probably contribute to the graviton S-G coupling, in the derivative $\Gamma^{\rho}_{\mu\rho,\nu}$, are given as follows:

$$h_{\mu\lambda,\nu}h^{\lambda\rho}_{,\rho} + h_{\rho\lambda,\nu}h^{\lambda\rho}_{,\mu} + h^{\rho\lambda}_{,\nu}h_{\mu\rho,\lambda} - h_{\mu\lambda,\nu}h^{\lambda,\rho}_{\rho} - h_{\rho\lambda,\nu}h^{\lambda,\rho}_{\mu} - \frac{5}{2}h_{\lambda\tau,\nu}h^{\lambda\tau}_{,\mu} - \frac{1}{2}h^{\lambda}_{\mu,\nu}h^{\tau}_{\tau,\lambda} + \frac{1}{2}h_{\mu\rho,\nu}h^{\lambda,\rho}_{\lambda} + \frac{1}{2}h_{\lambda\tau,\nu}h^{\lambda\tau}_{,\mu}.$$

But the detailed analysis shows that the terms that truly gives contribution to the graviton S-G coupling are $-h_{\rho\lambda,\nu}h^{\lambda\rho}_{,\mu}$ and $h_{\rho\lambda,\nu}(h^{\rho,\lambda}_{\mu} - h^{\lambda,\rho}_{\mu})$ only, the latter of which, however, vanishes. So, when taking into account the Lagrangian density $\sqrt{-g}g^{\mu\nu}R_{\mu\nu}$ of gravitational field, it is clearly seen that $-\sqrt{-g}g^{\mu\nu}h_{\rho\lambda,\nu}h^{\lambda\rho}_{,\mu}$ contains the expression $\sqrt{-g}g^{0m}[-h_{\rho\lambda,0}h^{\lambda\rho}_{,m} - h_{\rho\lambda,m}h^{\lambda\rho}_{,0}]$ (i.e., $-2\sqrt{-g}g^{0m}h_{\rho\lambda,0}h^{\lambda\rho}_{,m}$), which contains

$$\sqrt{-g}g^{0m}[-2h_{0n,0}h^{n0}_{,m} - 2h_{n0,0}h^{0n}_{,m}] = -4\sqrt{-g}g^{0m}\dot{g}_n h^{n0}_{,m} = 4\sqrt{-g}g^{0m}\dot{g}_n\partial_m g_n. \quad (6)$$

Here $h^{0n} = g^n = -g_n$. The flat-Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}[+1, -1, -1, -1]$. Clearly, the relation between the metric g^{0m} (with $m = 1, 2, 3$) and the gravitomagnetic vector potential g^m is that $\sqrt{-g}g^{0m} = Kh^{0m} = Kg^m$.

It is readily verified that the second-order terms in $\Gamma^{\rho}_{\mu\nu,\rho}$ which may have effect on the graviton S-G coupling are $(h_{\mu\lambda,\rho}h^{\lambda\rho}_{,\nu} + h_{\nu\lambda,\rho}h^{\lambda\rho}_{,\mu}) + h^{\rho\lambda}_{,\rho}h_{\mu\nu,\lambda} - (h_{\mu\lambda,\rho}h^{\lambda,\rho}_{\nu} + h_{\nu\lambda,\rho}h^{\lambda,\rho}_{\mu}) - \frac{1}{2}(h_{\lambda\tau,\mu}h^{\lambda\tau}_{,\nu} + h_{\lambda\tau,\nu}h^{\lambda\tau}_{,\mu}) - \frac{1}{2}\eta_{\mu\nu}h^{\rho\lambda}_{,\nu}h^{\tau}_{\tau,\lambda} + \frac{1}{2}h_{\mu\nu,\rho}h^{\lambda,\rho}_{\lambda} + \frac{1}{2}\eta_{\mu\nu}h_{\lambda\tau,\rho}h^{\lambda\tau,\rho}$. Further analysis demonstrates that the terms which truly give contribution to the graviton S-G coupling are the following four terms:

(i) $\sqrt{-g}g^{\mu\nu}(h_{\mu\lambda,\rho}h^{\lambda\rho}_{,\nu} + h_{\nu\lambda,\rho}h^{\lambda\rho}_{,\mu})$ contains $\sqrt{-g}g^{0m}\times (h_{0\lambda,\rho}h^{\lambda\rho}_{,m} + h_{m\lambda,\rho}h^{\lambda\rho}_{,0} + h_{0\lambda,\rho}h^{\lambda\rho}_{,m} + h_{m\lambda,\rho}h^{\lambda\rho}_{,0})$, i.e., $2\sqrt{-g}g^{0m}(h_{0\lambda,\rho}h^{\lambda\rho}_{,m} + h_{m\lambda,\rho}h^{\lambda\rho}_{,0})$, which includes the following terms:

$$\begin{aligned}&2\sqrt{-g}g^{0m}(h_{0n,0}h^{n0}_{,m} + h_{m0,n}h^{0n}_{,0})\\&\quad = -2\sqrt{-g}g^{0m}\dot{g}_n(\partial_m g_n + \partial_n g_m).\end{aligned} \tag{7}$$

(ii) $-\sqrt{-g}g^{\mu\nu}(h_{\mu\lambda,\rho}h^{\lambda,\rho}_{\nu} + h_{\nu\lambda,\rho}h^{\lambda,\rho}_{\mu})$ contains $-\sqrt{-g}g^{0m}(h_{0\lambda,\rho}h^{\lambda,\rho}_{m} + h_{m\lambda,\rho}h^{\lambda,\rho}_{0} + h_{0\lambda,\rho}h^{\lambda,\rho}_{m} + h_{m\lambda,\rho}h^{\lambda,\rho}_{0})$, i.e., $-2\sqrt{-g}g^{0m}(h_{0\lambda,\rho}h^{\lambda,\rho}_{m} + h_{m\lambda,\rho}h^{\lambda,\rho}_{0})$, which will give no contribution to the graviton S-G coupling. So here we will not further consider it.

(iii) $\sqrt{-g}g^{\mu\nu}h^{\rho\lambda}_{,\rho}h_{\mu\nu,\lambda}$ contains the components $\sqrt{-g}g^{0m}(h^{\rho\lambda}_{,\rho}h_{0m,\lambda} + h^{\rho\lambda}_{,\rho}h_{m0,\lambda})$, i.e., $2\sqrt{-g}g^{0m}\times h^{\rho\lambda}_{,\rho}h_{0m,\lambda}$, which includes

$$2\sqrt{-g}g^{0m}h^{0n}_{,0}h_{0m,n} = -2\sqrt{-g}g^{0m}\dot{g}_n\partial_n g_m. \tag{8}$$

(iv) $-\frac{1}{2}\sqrt{-g}g^{\mu\nu}(h_{\lambda\tau,\mu}h^{\lambda\tau}_{,\nu} + h_{\lambda\tau,\nu}h^{\lambda\tau}_{,\mu})$ includes $\sqrt{-g}g^{0m}[-h_{\lambda\tau,0}h^{\lambda\tau}_{,m} - h_{\lambda\tau,m}h^{\lambda\tau}_{,0}]$, which can be rewritten as $-2\sqrt{-g}g^{0m}h_{\lambda\tau,0}h^{\lambda\tau}_{,m}$, and contains

$$\begin{aligned}&\sqrt{-g}g^{0m}[-2h_{0n,0}h^{0n}_{,m} - 2h_{n0,0}h^{n0}_{,m}]\\&\quad = -4\sqrt{-g}g^{0m}h_{0n,0}h^{0n}_{,m} = 4\sqrt{-g}g^{0m}\dot{g}_n\partial_m g_n.\end{aligned} \tag{9}$$

Thus it follows from Eq. (7)–(9) that the terms in $\sqrt{-g}g^{\mu\nu}\Gamma^{\rho}_{\mu\nu,\rho}$, which contribute to the graviton S-G coupling, are given as follows:

$$\sqrt{-g}g^{\mu\nu}[-2\dot{g}_n(\partial_m g_n + \partial_n g_m) - 2\dot{g}_n\partial_n g_m + 4\dot{g}_n\partial_m g_n] = \sqrt{-g}g^{\mu\nu}[2\dot{g}_n\partial_m g_n - 4\dot{g}_n\partial_n g_m]. \tag{10}$$

Hence it follows from (6) and (10) that the terms in $\sqrt{-g}g^{\mu\nu}(\Gamma^{\rho}_{\mu\rho,\nu} - \Gamma^{\rho}_{\mu\nu,\rho})$ which contribute to the graviton S-G coupling are written in the form

$$\sqrt{-g}g^{0m}[4\dot{g}_n\partial_m g_n - (2\dot{g}_n\partial_m g_n - 4\dot{g}_n\partial_n g_m)] = 2\sqrt{-g}g^{0m}(\dot{g}_n\partial_m g_n + 2\dot{g}_n\partial_n g_m). \tag{11}$$

$-\Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho}$.—The first-order terms (proportional to $-\frac{1}{2}K$) in $\Gamma^{\sigma}_{\mu\nu}$ are of the form $\Gamma^{\sigma}_{\mu\nu}(\propto -\frac{1}{2}K) = h^{\sigma}_{\mu,\nu} + h^{\sigma}_{\nu,\mu} - h^{,\sigma}_{\mu\nu} - \frac{1}{2}\delta^{\sigma}_{\mu}h^{\lambda}_{\lambda,\nu} - \frac{1}{2}\delta^{\sigma}_{\nu}h^{\lambda}_{\lambda,\mu} + \frac{1}{2}\eta_{\mu\nu}h^{\lambda,\sigma}_{\lambda}$, which includes the *valuable* terms $-\frac{1}{2}K(h^{\sigma}_{\mu,\nu} + h^{\sigma}_{\nu,\mu} - h^{,\sigma}_{\mu\nu})$. Meanwhile, the terms proportional to $-\frac{1}{2}K$ in $\Gamma^{\rho}_{\sigma\rho}$ take the form $\Gamma^{\rho}_{\sigma\rho}(\propto -\frac{1}{2}K) = -\frac{1}{2}K(h^{\rho}_{\sigma,\rho} + h^{\rho}_{\rho,\sigma} - h^{,}_{\sigma\rho}\rho + \ldots) = -\frac{1}{2}Kh^{\rho}_{\rho,\sigma}$.

It is readily verified that the contribution of $-\Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\sigma\rho}$ to the graviton S-G coupling is vanishing. So, here we will not consider it further.

$\Gamma^{\rho}_{\sigma\nu}\Gamma^{\sigma}_{\mu\rho}$.—It is apparently seen that $\sqrt{-g}g^{\mu\nu}\Gamma^{\rho}_{\sigma\nu}\Gamma^{\sigma}_{\mu\rho}$ includes the following terms $\sqrt{-g}g^{0m}(\Gamma^{\rho}_{\sigma0}\Gamma^{\sigma}_{m\rho} + \Gamma^{\rho}_{\sigma m}\Gamma^{\sigma}_{0\rho}) = \sqrt{-g}g^{0m}(\Gamma^{\rho}_{\sigma0}\Gamma^{\sigma}_{m\rho} + \Gamma^{\rho}_{0\sigma}\Gamma^{\sigma}_{\rho m}) = 2\sqrt{-g}g^{0m}\times \Gamma^{\rho}_{\sigma0}\Gamma^{\sigma}_{m\rho}$, where $\Gamma^{\rho}_{\sigma0}(\propto -\frac{1}{2}K) = h^{\rho}_{\sigma,0} + h^{\rho}_{0,\sigma} - h^{,\rho}_{\sigma0}$ and $\Gamma^{\sigma}_{m\rho}(\propto -\frac{1}{2}K) = h^{\sigma}_{m,\rho} + h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho}$. Thus, $\Gamma^{\rho}_{\sigma0}\Gamma^{\sigma}_{m\rho}$ contains $(h^{\rho}_{\sigma,0} + h^{\rho}_{0,\sigma} - h^{,\rho}_{\sigma0})(h^{\sigma}_{m,\rho} + h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho})$, which is rewritten as $[h^{\rho}_{\sigma,0}(h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho})] + h^{\rho}_{\sigma,0}h^{\sigma}_{m,\rho} + (h^{\rho}_{0,\sigma} - h^{,\rho}_{\sigma0})(h^{\sigma}_{m,\rho} + h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho})$. In the following discussions, for convenience, we classify the terms in $(h^{\rho}_{\sigma,0} + h^{\rho}_{0,\sigma} - h^{,\rho}_{\sigma0})(h^{\sigma}_{m,\rho} + h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho})$ into three categories:

(i) $h^{\rho}_{\sigma,0}(h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho})$ contains

$$h^{0}_{n,0}(h^{n}_{0,m} - h^{,n}_{m0}) = -\dot{g}_n(\partial_m g_n - \partial_n g_m). \tag{12}$$

(ii) $h^{\rho}_{\sigma,0}h^{\sigma}_{m,\rho}$ contains

$$h^{n}_{0,0}h^{0}_{m,n} = -\dot{g}_n\partial_n g_m. \tag{13}$$

(iii) $(h^{\rho}_{0,\sigma} - h^{,\rho}_{\sigma0})(h^{\sigma}_{m,\rho} + h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho})$ contains the following six terms:

$$\begin{aligned}h^{\rho}_{0,\sigma}h^{\sigma}_{m,\rho} &= h^{n}_{0,0}h^{0}_{m,n} + h^{0}_{0,0}h^{0}_{m,0} \Rightarrow -\dot{g}_n\partial_n g_m,\\ h^{\rho}_{0,\sigma}h^{\sigma}_{\rho,m} &= h^{n}_{0,0}h^{0}_{n,m} + h^{0}_{0,0}h^{0}_{0,m} \Rightarrow -\dot{g}_n\partial_m g_n,\\ &-h^{\rho}_{0,\sigma}h^{,\sigma}_{m\rho}\\ &\text{(giving no contribution to S} - \text{G coupling)},\\ &-h^{,\rho}_{\sigma0}h^{\sigma}_{m,\rho}\\ &\text{(giving no contribution to S} - \text{G coupling)},\\ -h^{,\rho}_{\sigma0}h^{\sigma}_{\rho,m} &= -h^{,0}_{n0}h^{n}_{0,m} - h^{,0}_{00}h^{0}_{0,m} \Rightarrow \dot{g}_n\partial_m g_n,\\ (-h^{,\rho}_{\sigma0})(-h^{,\sigma}_{m\rho}) &= h^{,0}_{n0}h^{,n}_{m0} + h^{,0}_{00}h^{,0}_{m0}\\ &\Rightarrow -\dot{g}_n\partial_n g_m.\end{aligned} \tag{14}$$

So, the terms in $(h^{\rho}_{0,\sigma} - h^{,\rho}_{\sigma0})(h^{\sigma}_{m,\rho} + h^{\sigma}_{\rho,m} - h^{,\sigma}_{m\rho})$, which will have effect on the graviton S-G coupling, are

$$-2\dot{g}_n\partial_n g_m \tag{15}$$

only.

Thus it follows from (12), (13) and (15) that the total terms in $\sqrt{-g}g^{\mu\nu}\Gamma^{\rho}_{\sigma\nu}\Gamma^{\sigma}_{\mu\rho}$ which will give contribution to the graviton S-G coupling are expressed as follows:

$$2\sqrt{-g}g^{0m}[-\dot{g}_n(\partial_m g_n - \partial_n g_m) - \dot{g}_n\partial_n g_m - 2\dot{g}_n\partial_n g_m] = 2\sqrt{-g}g^{0m}[-\dot{g}_n(\partial_m g_n - \partial_n g_m) - 3\dot{g}_n\partial_n g_m]. \quad (16)$$

The final result.—Hence, according to the expressions (11) and (16), one can finally obtain the total contribution to the graviton S-G coupling as

$$\frac{1}{2}K^2 \cdot 2\sqrt{-g}g^{0m}(\dot{g}_n\partial_m g_n + 2\dot{g}_n\partial_n g_m) + \frac{1}{4}K^2 \cdot 2\sqrt{-g}g^{0m}[-\dot{g}_n(\partial_m g_n - \partial_n g_m) - 3\dot{g}_n\partial_n g_m]$$

i.e., $\frac{1}{2}K^2\sqrt{-g}g^{0m}[-2\dot{g}_n(\partial_m g_n - \partial_n g_m) + 3\dot{g}_n\partial_m g_n]$. Thus one can arrive at

$$\mathcal{L}_{s-g} = -K^2\sqrt{-g}g^{0m}\dot{g}_n(\partial_m g_n - \partial_n g_m) = \frac{1}{2}K^3(g_m\dot{g}_n - g_n\dot{g}_m)(\partial_m g_n - \partial_n g_m), \quad (17)$$

where the relation $\sqrt{-g}g^{0m} = Kh^{0m} = Kg^m = -Kg_m$ has been inserted, and the summation for the repeated indices is carried out over the values 1, 2, 3. Thus we present the Lagrangian density that describes the coupling of graviton spin (spinning moment) to gravitomagnetic fields, the generalized version of Mashhoon's spin-rotation couplings in the purely gravitational case. It may be believed that such a graviton S-G coupling deserves further detailed investigation, for the coupling of graviton spin to gravitomagnetic fields may provide us with a deep insight into Mashhoon's spin-rotation couplings [12] (and hence into the inertial effects of spinning particles) [10].

In addition, since the purely gravitational generalization of Mashhoon's spin-rotation coupling, i.e., the interaction of the graviton spin with the gravitomagnetic fields is actually a self-interaction of the spacetime (gravitational fields), here we will propose a definite prediction about such a purely gravitational interaction, which will arise in a noninertial frame of reference itself: specifically, a rotating frame that experiences a fluctuation of its rotational frequency will undergo such a weak self-interaction. In other words, such a self-interaction of the rotating frame, which can also be called the self-interaction of the spacetime of the rotating frame, is just a noninertial generalization of the interaction of the graviton spin with the gravitomagnetic fields. This can be understood as follows: in a rotating frame, the φ-component of the gravitomagnetic vector potential is $g_\varphi \simeq (2\omega r/c)\sin\theta$ [15]. It follows that if either the direction or the magnitude of the angular velocity of the rotating frame changes, then the time derivative of the gravitomagnetic vector potentials is nonvanishing, e.g., $|\dot{g}_\varphi| \simeq |(2\dot{\omega}r/c)\sin\theta| \neq 0$, and according to (17), such a rotating frame will thus be subjected to a self-interaction, the nature of which is just a nonlinear interaction of the spacetime of the noninertial frame of reference itself.

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