PHYSICAL REVIEW D, VOLUME 70, 066007

Deforming baryons into confining strings

Sean A. Hartnoll[1,*] and Rubén Portugues[2,†]
[1] DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 OWA, United Kingdom
[2]Centro de Estudios Cientificos (CECS), Avenida Arturo Prat 514, Casilla 1469, Valdivia, Chile
(Received 1 June 2004; published 15 September 2004)

We find explicit probe D3-brane solutions in the infrared of the Maldacena-Nuñez background. The solutions describe deformed baryon vertices: q external quarks are separated in spacetime from the remaining $N - q$. As the separation is taken to infinity we recover known solutions describing infinite confining strings in $\mathcal{N} = 1$ gauge theory. We present results for the mass of finite confining strings as a function of length. We also find probe D2-brane solutions in a confining type IIA geometry, the reduction of a G_2 holonomy M theory background. The relation between these deformed baryons and confining strings is not as straightforward.

DOI: 10.1103/PhysRevD.70.066007 PACS numbers: 11.25.Tq, 11.15.–q, 11.27.+d

I. INTRODUCTION

The advent of the gravity/gauge theory correspondence [1–3] has opened a new window into strongly coupled field theories. Consequently, a lot of effort has been invested in finding supergravity duals of $\mathcal{N} = 1$ theories in four dimensions. Two of the most studied backgrounds are [4,5]. Such solutions allow the use of supergravity analysis to study strong coupling properties of the dual QCD-like field theories.

Within the dual backgrounds, probe brane analysis has been used extensively to understand the gravity counterparts of nonperturbative field theory objects such as baryons, mesons and confining strings. The initial studies considered brane probes in $AdS_5 \times S^5$, the dual geometry to $\mathcal{N} = 4$ Yang-Mills theory. Following [6,7], D5-brane probes wrapping the S^5 were used to construct explicitly the baryon vertex [8–10]. The wrapped brane was identified as the baryon vertex because the background has N units of Ramond-Ramond (RR) fiveform flux on the S^5: through the Wess-Zumino term of the probe D5-brane action the flux acts as a source for N fundamental strings on the worldvolume. The strings then join the vertex to external quarks. The work of [8] was extended in [11] to find numerically solutions describing baryons in which a fraction of the N quarks were pulled apart from the rest in spacetime. When the quarks are far apart, the branes were found to describe the confining strings of the gauge theory. The probe brane description of confining strings was further considered in [12], using results from [13–15].

In this work we find explicit analytic solutions of the full second order Dirac-Born-Infeld (DBI) equations of motion. These will be nonsupersymmetric probe branes in the infrared of supergravity geometries which are dual to $\mathcal{N} = 1$ confining gauge theories. The solutions describe baryon vertices where q quarks are being pulled apart from the remaining $N - q$. In the limit of infinite separation, the solutions recover known infinite confining string solutions [11,12]. The fact that our solutions are very explicit will enable us to calculate analytically various properties of the deformed baryons. For instance, we calculate the mass of finite confining strings as a function of length.

A schematic illustration of our solutions is given in Fig. 1 in Section IIB below. Figure 2 shows a couple of solutions more explicitly. The solutions are one dimensional from a spacetime perspective. At each point the remaining probe brane directions partially wrap an internal sphere of the background.

We focus on two background geometries in particular: the Maldacena-Nuñez solution [5] of IIB supergravity and a IIA solution which results from dimensional reduction of M theory on a manifold of G_2 holonomy [16–19]. Both of these backgrounds describe the near horizon geometry of wrapped branes; some comparisons are made in [20].

The baryon vertex of $\mathcal{N} = 1$ Yang-Mills theory in these backgrounds is given by a wrapped probe brane in an entirely analogous way to the $AdS_5 \times S^5$ case. In the Maldacena-Nuñez solution the baryon vertex is a D3-brane wrapping a nontrivial S^3 of the background [21]. For the IIA background it has been proposed that the baryon vertex is a D2-brane wrapping an S^2 [22].

Amongst the deformed baryons we consider, in the Maldacena-Nuñez case it is easy to identify the finite length confining strings. In the IIA background, there does not seem to be a limit of the deformed baryons that is immediately connected with confining strings.

II. IIB BACKGROUND: THE MALDACENA-NUÑEZ SOLUTION

A. The Infrared Background: $\mathcal{M}^4 \times S^3$

The Maldacena-Nuñez background [5] is a solution of type IIB string theory describing the result of the geometric transition induced by D5-branes wrapping an S^2 in the resolved conifold. The background preserves four

* E-mail: s.a.hartnoll@damtp.cam.ac.uk
†E-mail: rp@cecs.cl

supercharges and is dual to $\mathcal{N} = 1$ super Yang-Mills theory, modulo issues of decoupling of Kaluza-Klein and little string theory modes.

The far infrared of the field theory is described by the $r \to 0$ region of the background. The background collapses at $r = 0$ to $\mathcal{M}^4 \times S^3$ with N units of RR flux through the sphere

$$\begin{aligned} ds^2_{\rm IIB} &= e^{\Phi_0}[dx^2_{1,3} + \alpha' N(d\psi^2 \\ &\quad + \sin^2\psi[d\theta^2 + \sin^2\theta d\phi^2])] \\ C_2^{RR} &= -\alpha' N\left(\psi - \frac{1}{2}\sin 2\psi\right)\sin\theta d\theta \wedge d\phi, \end{aligned} \tag{1}$$

where Φ_0 is the value of the dilaton at the origin. The RR field strength is thus

$$\begin{aligned} F_3^{RR} = dC_2^{RR} &= -2\alpha' N \sin^2\psi \sin\theta d\psi \wedge d\theta \wedge d\phi \\ &= -2\alpha' N \mathrm{vol}_{S^3}. \end{aligned} \tag{2}$$

The ranges of the angles are $0 \le \psi \le \pi$, $0 \le \theta \le \pi$, $0 \le \phi < 2\pi$. We are working in the string frame.

The Dirac-Born-Infeld action for a probe D3-brane with these background fluxes is

$$\begin{aligned} S_{\rm DBI} &= \int d^4\xi \mathcal{L} \\ &= -T_3 \int d^4\xi e^{-\Phi}\sqrt{-\det(^\star G + \mathcal{F})} \\ &\quad + T_3 \int \mathcal{F} \wedge ^\star C_2^{RR}, \end{aligned} \tag{3}$$

where as usual $T_p = 1/[(2\pi)^p \alpha'^{(p+1)/2}]$ and here $\mathcal{F} = 2\pi\alpha' F$. We use $^\star G$ and $^\star C_2^{RR}$ to denote the pullback onto the worldvolume of the metric and the RR potential.

We will find solutions to the DBI equations of motion in the background at $r = 0$, given in (1). Setting $r(\xi) = 0$ is a consistent truncation of the full equations of motion.

B. Probe D3-brane Solutions

We are looking for solutions describing deformed baryons. To this end, we look for probe D3-brane solutions with the following ansatz. The ansatz can be thought of as describing fundamental strings extended in the x direction that the RR flux has blown up at each point, in an Emparan-Myers effect [23,24], to a D3-brane "wrapping" an S^2 in the S^3 of the background. See Figs. 1 and 2.

$$\begin{aligned} &t = \alpha'^{1/2}\xi_0, \quad x = \alpha'^{1/2}\xi_1, \quad \psi = \psi(\xi_1), \quad \theta = \xi_2, \quad \phi = \xi_3, \\ &A = k(\xi_1)\xi_0 d\xi_1 \quad \Rightarrow \quad F = k(\xi_1) d\xi_0 \wedge d\xi_1. \end{aligned} \tag{4}$$

Plugging this ansatz into the full DBI equations of motion, one finds that the equations are solved if the functions $k(\xi_1)$ and $\psi(\xi_1)$ satisfy the following relations

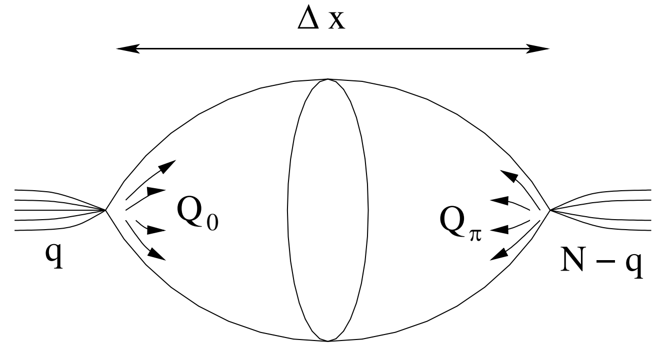

FIG. 1. D3-brane pulled apart by $N - q$ and q fundamental strings.

$$\begin{aligned} \bar{k}^2 &= (\psi - \sin\psi\cos\psi)^2, \\ \left[\frac{d\psi}{d\xi_1}\right]^2 &= \frac{e^{-2\Phi_0}}{N}\left[-e^{2\Phi_0} + C^4\sin^4\psi + C^4\left(\bar{k} + \frac{2\pi k_0}{C^2}\right)^2\right], \end{aligned} \tag{5}$$

where we have introduced

$$\bar{k} = \frac{2\pi}{C^2}(k - k_0). \tag{6}$$

Note that the solution has two dimensionless constants C, k_0. The range of k_0 when positive is restricted to $[e^{\Phi_0}/2\pi, \infty)$. For negative k_0 the allowed range is more complicated and will be discussed below. One may take either the positive or negative square roots in (5) to obtain the same solutions.

It is perhaps surprising that we can obtain such an explicit form for the solutions. The solutions are given precisely, up to an integral. We have found a two parameter family of solutions to the full nonlinear second order DBI equations of motion in the infrared background. We have not restricted to supersymmetric solutions. Indeed in the Appendix A we show that none of our solutions are supersymmetric. The first equation in (5) has appeared before in similar configurations [25,26] and is essentially Gauss's law for the electric field.

The Eqs. (5) have solutions where ψ goes from 0 to π. The spatial sections of these solutions are topologically S^3, with the worldvolume S^3 in the nontrivial homology class of the background $H_3(\mathcal{M}^4 \times S^3)$. However, the solutions are not just sitting at a point in $\mathcal{M}^4$ wrapping the S^3, but have an extension along the x direction given by

$$\Delta x = \alpha'^{1/2}\int_0^\pi \frac{d\xi_1}{d\psi} d\psi. \tag{7}$$

Two limits at fixed C illustrate the possible behaviors:

$$\Delta x \sim \left(\frac{\alpha' N e^{2\Phi_0}}{4}\right)^{1/2} \frac{1}{k_0} + \cdots, \quad \text{as} \quad k_0 \to \infty,$$
$$\Delta x \sim 2.8044\left[\frac{\alpha' N e^{\Phi_0}}{4}\right]^{1/2}\left[\frac{9\pi}{8C^4}\right]^{1/6} \frac{1}{(2\pi k_0 - e^{\Phi_0})^{1/6}} + \cdots, \quad \text{as} \quad k_0 \to \frac{e^{\Phi_0}}{2\pi}, \tag{8}$$

where the numerical factor comes from an elliptic integral. Therefore the limits correspond to short and large extensions in spacetime. Note that in the large length limit only the $\psi = 0$ end goes to infinity, due to a pole in $d\xi_1/d\psi$ from (5) at $\psi = 0$. The $\psi = \pi$ end remains at a finite position. We can obtain a pole at $\psi = \pi$ rather than $\psi = 0$ by noting that the transformation $\{\psi \to \pi - \psi, k_0 \to -C^2/2 - k_0\}$ leaves the solutions (5) invariant. This symmetry will appear later as the symmetry $q \leftrightarrow N - q$. We will see below that it is a different large length limit that is related to confining strings.

The induced spatial metric of the D3-branes is given by

$$ds^2_{\rm D3} = \alpha' N e^{\Phi}\left[\left(\left[\frac{d\psi}{d\xi_1}\right]^2 + \frac{1}{N}\right)d\xi_1^2 + \sin^2\psi\left(d\xi_2^2 + \sin^2\xi_2 d\xi_3^2\right)\right]. \tag{9}$$

As $\psi \to 0, \pi$, the induced geometry has a conical singularity. Far from being a problem, this is exactly what one should expect. The spatial sections of the D3-branes are compact, yet the Wess-Zumino term in the DBI action is a source for the abelian gauge field on the brane. The total charge on the compact sections must be zero, so there must be more sources. These will be fundamental strings attached at the conical points.

We will now understand the conical singularities as being due to attached fundamental strings. The endpoints of fundamental strings are electric sources for the worldvolume gauge field on the D3-branes [27,28]. The equations of motion in the presence of external sources at $\psi = 0$ and $\psi = \pi$ become

$$\partial_i \frac{\partial \mathcal{L}}{\partial \partial_i A_j} = \frac{\partial \mathcal{L}}{\partial A_j} + Q_0^j \delta^{(3)}(x + \Delta x) + Q_\pi^j \delta^{(3)}(x). \tag{10}$$

In this expression, we should integrate the Wess-Zumino term in the action by parts, so that it contributes to the equations of motion as part of the $\frac{\partial \mathcal{L}}{\partial A_j}$ term. The electric charges may be read off the solutions as

$$Q_0^0 = \lim_{\psi\to 0}\int_{\xi_2=0}^{\pi}\int_{\xi_3=0}^{2\pi}\frac{\partial \mathcal{L}}{\partial\partial_1 A_0}d\xi_2 d\xi_3,$$
$$Q_\pi^0 = -\lim_{\psi\to \pi}\int_{\xi_2=0}^{\pi}\int_{\xi_3=0}^{2\pi}\frac{\partial \mathcal{L}}{\partial\partial_1 A_0}d\xi_2 d\xi_3. \tag{11}$$

This is just Gauss's law on the brane worldvolume. Applying the formulas to our solutions

$$Q_0^0 = -\frac{2Nk_0}{C^2}, \qquad Q_\pi^0 = N + \frac{2Nk_0}{C^2}. \tag{12}$$

The total charge due to sources at the conical points is N. One can check that this precisely cancels the contribution from the Wess-Zumino term. Let us write $Q_\pi^0 = N - q$ and $Q_0^0 = q$. In the full string theory $q \in \mathbb{Z}$, corresponding to q string endpoints.

The interpretation of the solutions now becomes clear. The baryon vertex with N external quarks is being pulled apart along the x axis with $N - q$ quarks at one end and q at the other. The configuration is illustrated in Fig. 1.

The next step is to understand the energetics of the deformation process. The energy density of the solution is given by

$$\mathcal{E} = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu}\dot{X}^\mu + \frac{\partial \mathcal{L}}{\partial \dot{A}}\cdot \dot{A} - \mathcal{L}, \tag{13}$$

where the X^μ denote the spatial coordinates of the background and $\dot{}$ denotes differentiation with respect to ξ_0. Considered as an object in four dimensional spacetime, the tension is

$$T(\xi_1) = \int d\xi_2 d\xi_3 \mathcal{E} = \frac{N}{\alpha'^{1/2}2\pi^2C^2}[C^4\sin^2\psi - (C^2\psi + 2\pi k_0)(2C^2\sin\psi\cos\psi - C^2\psi - 2\pi k_0)]. \tag{14}$$

The mass of the solution is $M = \int d\xi_1 T(\xi_1)$. We can calculate the masses in the two limits considered previously in (8)

$$M \sim \frac{N^2 e^{2\Phi_0}}{2C^2}\frac{1}{\Delta x} + \cdots, \quad \text{as} \quad \Delta x \to 0 \quad [k_0 \to \infty],$$
$$M \sim \frac{e^{\Phi_0}}{2\pi\alpha'}|q|\Delta x + \cdots, \quad \text{as} \quad \Delta x \to \infty \quad \left[k_0 \to \frac{e^{\Phi_0}}{2\pi}\right]. \tag{15}$$

The short length limit takes us outside the regime of validity of the DBI effective action, because $\partial_1 F \sim \alpha'^{1/2}/\Delta x$ becomes large and α' corrections are important. Therefore the mass divergence for short extension should probably not be taken seriously, although it is consistent with a $1/\Delta x$ Coulomb potential. The large separation solution has energy linear in separation, as expected for a confining theory. The large length expression has a simple interpretation. It is the mass of q fundamental strings in the background (1)! This result is as we should expect in this limit, where almost all the mass is coming from near the $\psi = 0$ endpoint. Most of the D3-brane has collapsed to form fundamental strings [23].

There is a different large separation limit that has a more interesting physical interpretation. An infinite length occurs whenever $d\xi_1/d\psi$ has a pole at some $\psi_0 \in [0, \pi]$. In the large length limit we have just considered, the pole is at the endpoint $\psi_0 = 0$ when $k_0 = e^{\Phi_0}/2\pi$. From Eq. (5) one can see that there is another possibility. This is the interior point $\psi_0 = -2\pi k_0/C^2 = \pi q/N$ when $C^2 \sin\psi_0 = e^{\Phi_0}$. In this case one finds the mass

$$M \sim \frac{e^{\Phi_0} N}{2\pi^2 \alpha'} \sin\frac{\pi q}{N} \Delta x + \cdots, \quad \text{as } \Delta x \to \infty \quad \left[k_0 \to \frac{-C^2}{2\pi} \sin^{-1} \frac{e^{\Phi_0}}{C^2}\right]. \tag{16}$$

Note that in this limit k_0 is negative and hence q is positive. The expression (16) is immediately recognized as the sine formula for the mass of the confining strings of the theory [12,29,30]. The mass per unit length of the qth confining string in the infinite length limit becomes

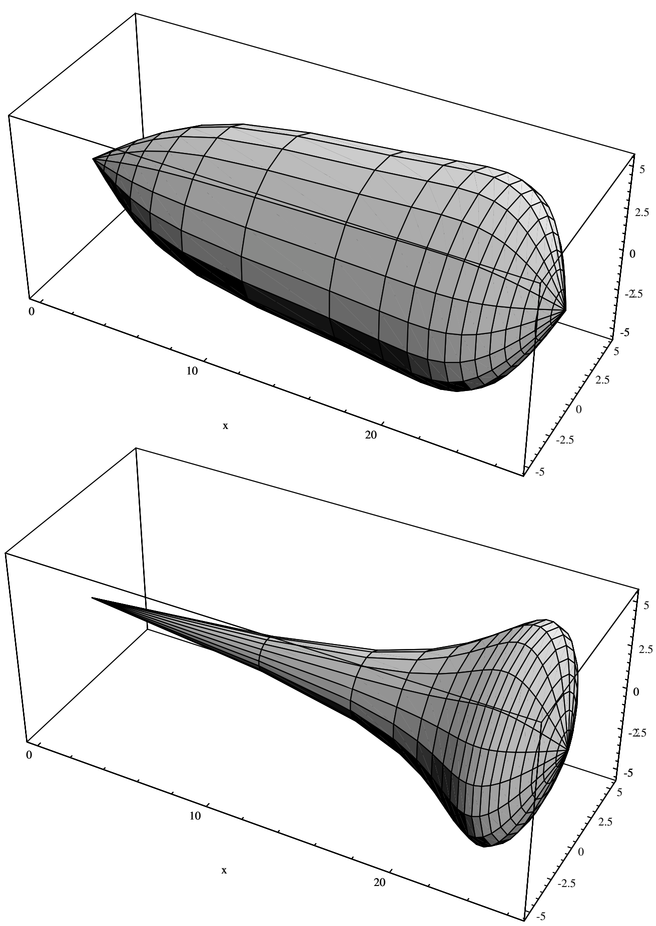

FIG. 2. Probe D3-brane solutions with a large but finite extension in the x direction. The background has $N = 30$ and $\Phi_0 = 1$. The top configuration has $q = -10$ while the bottom configuration has $q = 10$. One angular direction has been suppressed.

$$T = \frac{e^{\Phi_0} N}{2\pi^2 \alpha'^{1/2}} \sin\frac{\pi q}{N}, \tag{17}$$

recovering the result obtained in [12]. Our solutions place the confining strings in a larger context. The confining strings arise as an infinite length limit of a two parameter family of explicit probe brane solutions describing deformed baryon vertices.

The difference between the two different long length limits, ([15,16]), is illustrated in Fig. 2. We see how in the former case the brane collapses at one end, as we noted following the tension calculation ([15]).

The physical reason for the difference between the two cases is as follows. In the confining string case $q \leq N$ is positive and hence the charges at each end, Q_0^0 and Q_π^0, are both positive. The brane expands in a dielectric effect [23,24] due to the RR flux on the S^3 of the background, with slightly more expansion at the end with more charge. However, in the collapsing case q is negative. Therefore the charge Q_0^0 is negative while Q_π^0 is positive. The result is that the brane expands near the end with positive charge but the negatively charged end collapses into fundamental strings.

The allowed range of negative k_0 with C^2 fixed is complicated and we see in the next section that it is more natural to parameterize in terms of C with q fixed. Suffice to note that if $e^{\Phi_0} < C^2$ then one allowed range is $k_0 \in (-C^2/2 - e^{\Phi_0}/(2\pi), -C^2 \sin^{-1}(e^{\Phi_0}/C^2)/(2\pi)]$. In this case $-C^2 \sin^{-1}(e^{\Phi_0}/C^2)/(2\pi) \leq -e^{\Phi_0}/(2\pi)$ so the pole at $k_0 = -e^{\Phi_0}/(2\pi)$ is not reached.

C. Energetics of Finite Length Confining Strings

We can use the explicit expressions for the mass (14) and length (7) to examine the energetics of the full space of probe D3-brane solutions. We would like to calculate the mass of the D3-brane with a fixed length in spacetime,

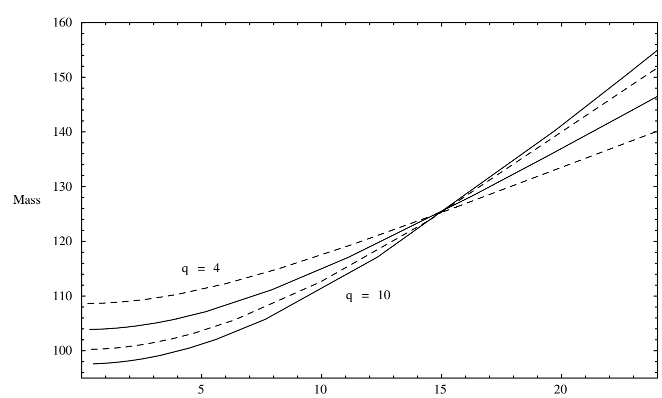

FIG. 3. Mass against length for confining strings with $q = 4, 6, 8, 10$. The background has $N = 30$ and $\Phi_0 = 1$.

Δx, and a fixed charge imbalance q. There is a unique solution associated with a pair $(\Delta x, q)$, providing a more physical parameterization of the space of solutions than (C, k_0).

Figure 3 is a plot of mass against length for finite length confining strings at four values of q.

As the length goes to infinity, the tension becomes the known result for infinite confining strings (17). It is curious that the curves almost intersect at a single point, call it (L_0, M_0), although they do not seem to go through precisely the same point. It seems plausible that the intersection is exact to subleading order at large Δx. Mathematically, this fact requires that the subleading correction to (16) take the following form

$$M(q) \sim \frac{e^{\Phi_0}N}{2\pi^2\alpha'}\sin\frac{\pi q}{N}[\Delta x - L_0] + M_0 + \cdots, \quad \text{as } \Delta x \to \infty. \tag{18}$$

This is an interesting result that completely determines the q dependence of the subleading term. It seems difficult to derive (18) directly from the integral for the mass.

Note that in the other limit, $\Delta x \to 0$, there is no mass divergence in these cases. This limit is $C \to \infty$ at fixed positive q.

It is useful to write the expression for mass (14) in terms of q

$$M = \frac{N^{3/2}e^{\Phi_0}}{2\pi^2\alpha'^{1/2}}\int_0^\pi \frac{\sin^2\psi - (\psi - \pi q/N)(2\sin\psi\cos\psi - \psi + \pi q/N)}{\sqrt{-e^{2\Phi_0}/C^4 + \sin^4\psi + (\sin\psi\cos\psi - \psi + \pi q/N)^2}}d\psi. \tag{19}$$

It is simple to check that this integral is invariant under $q \leftrightarrow N - q$, as we should expect. If q is positive, then the allowed range of C^2 is $[e^{\Phi_0}/\sin(\pi q/N), \infty)$.

Figure 3 and Eq. (19) constitute concrete predictions for the mass of confining strings with finite length. The subleading, constant, contribution to the mass in (18)

$$M_0 - \frac{e^{\Phi_0}N}{2\pi^2\alpha'}\sin\frac{\pi q}{N}L_0, \tag{20}$$

is presumably concentrated near the endpoints of the confining string.

III. IIA BACKGROUND: G_2 MANIFOLDS

A. The Infrared Background: $\mathcal{M}^4 \times S^2$

The $\mathbb{D}_7$ family of G_2 holonomy manifolds are classical solutions of M theory [16–19]. From a IIA perspective, they describe the result of the geometric transition induced by D6-branes wrapping an S^3 in the deformed conifold. Like the Maldacena-Nuñez background, the solution preserves four supercharges and is thought to be dual to $\mathcal{N} = 1$ super Yang-Mills theory, modulo issues of decoupling of Kaluza-Klein and gravitational modes [20].

In the infrared regime, $r \to 0$, the background collapses to $\mathcal{M}^4 \times S^2$ with N units of RR flux through the sphere[1]

$$ds^2_{\rm IIA} = e^{2\Phi_0}\left(dx^2_{1,3} + \alpha' N^2\frac{1}{4}[d\theta^2 + \sin^2\theta d\phi^2]\right)$$
$$C_1^{RR} = \alpha'^{1/2}N\frac{1}{2}\cos\theta d\phi. \tag{21}$$

[1]There is a factor of 2 missing in the IIA expressions of [18]. The angle of the M theory circle should be rescaled to have range 2π.

The RR twoform flux is

$$G_2^{RR} = dC_1^{RR} = -\alpha'^{1/2}N\frac{1}{2}\sin\theta d\theta \wedge d\phi = -\alpha'^{1/2}N\frac{1}{2}\mathrm{vol}_{S^2}. \tag{22}$$

The ranges of the angles are $0 \le \theta \le \pi$ and $0 \le \phi < 2\pi$. The general $\mathbb{D}_7$ solution has another free parameter at $r = 0$ which determines the squashing of an S^3 in the G_2 geometry. We have set this parameter to one for simplicity. Further, we have rescaled the Minkowski coordinates to emphasize the similarity with the infrared of the Maldacena-Nuñez solution (1).

The action for a probe D2-brane with these fluxes is

$$S_{\rm DBI} = -T_2\int d^3\xi e^{-\Phi}\sqrt{-\det({}^\star G + \mathcal{F})} + T_2\int \mathcal{F} \wedge^\star C_1^{RR}. \tag{23}$$

As before, setting $r(\xi) = 0$ is a consistent truncation of the full probe brane equations of motion. Therefore we may use the DBI action in the background at $r = 0$ given in (21).

B. Probe D2-brane Solutions

We will find probe D2-brane solutions similar to the D3-brane solutions of the previous section. The ansatz we take describes fundamental strings extended in the x direction blown up to a D2-brane by the Emparan-Myers effect [23,24]. At each value of x the D2-brane is wrapping an S^1 in the S^2 of the background.

$$t = \alpha'^{1/2}\xi_0, \quad x = \alpha'^{1/2}\xi_1, \quad \theta = \theta(\xi_1), \quad \phi = \xi_2,$$
$$A = k(\xi_1)\xi_0 d\xi_1 \quad \Rightarrow \quad F = k(\xi_1)d\xi_0 \wedge d\xi_1. \tag{24}$$

One finds that the DBI equations of motion are solved if the functions $k(\xi_1)$ and $\theta(\xi_1)$ satisfy the following relations

$$\begin{aligned} \cos^2\theta &= \bar{k}^2, \\ \left[\frac{d\bar{k}}{d\xi_1}\right]^2 &= C^2[D+\bar{k}][1-\bar{k}^2] = C^2[D+\bar{k}]\sin^2\theta, \end{aligned} \tag{25}$$

where we have introduced

$$\bar{k} = \frac{32\pi^2 k_0}{N^2 e^{4\Phi_0} C^2}(k-k_0), \tag{26}$$

and the three dimensionless constants C, D, k_0 are related by

$$D = \frac{4}{N^2 e^{4\Phi_0} C^2}\left[\frac{N^4 e^{8\Phi_0} C^4}{256\pi^2 k_0^2} - e^{4\Phi_0} + 4\pi^2 k_0^2\right]. \tag{27}$$

The solution therefore has two arbitrary constants. The range of D is restricted to $[1, \infty)$.

We have found a two parameter family of solutions to the full second order equations of motion. It seems very likely that these solutions are not supersymmetric, as we showed for the IIB solutions in the Appendix A. However, we have not explicitly checked nonsupersymmetry in the present IIA case.

From the Eqs. (25), we see that there will be solutions where θ runs from 0 to π. This corresponds to $\bar{k}$ running between -1 and 1. The spatial sections of these solutions are topologically S^2, with the worldvolume S^2 in the nontrivial homology class of the background $H_2(\mathcal{M}\times S^2)$.

The length of the solutions in the x direction is given by an elliptic integral

$$\Delta x = \frac{\alpha'^{1/2}}{C}\int_{-1}^{1}\frac{d\bar{k}}{\sqrt{(1-\bar{k}^2)(\bar{k}+D)}}. \tag{28}$$

It is clear that the length is inversely proportional to C. We may use asymptotic properties of elliptic integrals to calculate the length as $D \to 1$ and as $D \to \infty$

$$\begin{aligned} \Delta x &\sim \frac{\alpha'^{1/2}\sqrt{2}}{2C}\ln(D-1)+\cdots, \quad \text{as} \quad D\to 1, \\ \Delta x &\sim \frac{\alpha'^{1/2}\pi}{C}\frac{1}{D^{1/2}}+\cdots, \quad \text{as} \quad D\to\infty, \end{aligned} \tag{29}$$

corresponding to long and short extensions, respectively. In the long solutions, only the $\bar{k}=-1$ end goes to infinity, the $\bar{k}=1$ point remains at a finite position.

The induced spatial metric of the solutions is given by

$$ds^2_{\mathrm{D2}} = \frac{\alpha' N^2}{4}e^{2\Phi_0}\left(\left\{\frac{4}{N^2}+C^2[D+\bar{k}(\xi_1)]\right\}d\xi_1^2 + [1-\bar{k}(\xi_1)^2]d\xi_2^2\right). \tag{30}$$

As $\bar{k}\to\pm 1$ one can see that the induced geometry has a conical singularity due to an angular deficit. As previously, we interpret the singularities as due to the presence of fundamental string sources. In this case, the electric charges at the conical points are

$$\begin{aligned} Q_0^0 &= \lim_{\theta\to 0}\int_{\xi_2=0}^{2\pi}\frac{\partial\mathcal{L}}{\partial\partial_1 A_0}d\xi_2, \\ Q_\pi^0 &= -\lim_{\theta\to\pi}\int_{\xi_2=0}^{2\pi}\frac{\partial\mathcal{L}}{\partial\partial_1 A_0}d\xi_2. \end{aligned} \tag{31}$$

Applying this formula to our solutions, we find

$$Q_0^0 = \frac{N}{2} - \frac{16\pi^2 k_0^2}{NC^2 e^{4\Phi_0}}, \qquad Q_\pi^0 = \frac{N}{2} + \frac{16\pi^2 k_0^2}{NC^2 e^{4\Phi_0}}. \tag{32}$$

The total charge of the sources is N, which again precisely cancels the contribution of the Wess-Zumino term as required. It will be useful to write $Q_\pi^0 = N/2 + q$ and $Q_0^0 = N/2 - q$ with $N/2 \pm q \in \mathbb{Z}$. The total number of fundamental strings ending on the probe brane is again N, so the solutions may be interpreted as deformed baryon vertices.

The four dimensional tension of the solution is given by

$$T(\xi_1) = \int d\xi_2 \mathcal{E} = \frac{4|k_0|}{\alpha'^{1/2}N}\left[\frac{1}{C^2}+\frac{N^2 D}{4}+\frac{N^2\bar{k}}{4}\right]. \tag{33}$$

Integrating the tension over the spatial worldvolume gives the mass

$$\begin{aligned} M &= \int d\xi_1 T(\xi_1) \\ &= \frac{4|k_0|}{\alpha' NC^2}\Delta x + \frac{|k_0|N}{\alpha'^{1/2}C}\int_{-1}^{1}d\bar{k}\sqrt{\frac{D+\bar{k}}{(1-\bar{k}^2)}}. \end{aligned} \tag{34}$$

Considering the short and long limits, $D\to\infty$ and $D\to 1$ respectively, one obtains relations between the mass and the length. There are in fact two possible short limits

$$\begin{aligned} M &\sim \frac{\alpha'^{1/2}N^2 e^{2\Phi_0}\pi^2}{4C^2}\frac{1}{(\Delta x)^2}+\cdots, \\ &\quad \text{as } \Delta x\to 0 \quad [k_0\to\infty \Rightarrow D\to\infty], \\ M &\sim \frac{N^2 e^{2\Phi_0}}{8\alpha'^{1/2}}+\cdots, \\ &\quad \text{as } \Delta x\to 0 \quad [k_0\to 0 \Rightarrow D\to\infty]. \end{aligned} \tag{35}$$

As in the previously discussed IIB solutions, both of the short limits take us outside the regime for validity of the DBI action. The long limit is also precisely as for the IIB solutions. One may obtain the following relation

$$\begin{aligned} M &\sim \frac{e^{2\Phi_0}}{2\pi\alpha'}\left|\frac{N}{2}-q\right|\Delta x+\cdots, \\ &\quad \text{as } \Delta x\to\infty \quad [D\to 1], \end{aligned} \tag{36}$$

which is exactly the mass of $N/2-q$ fundamental strings in the background (21). Again this is consistent with the fact that most of the mass comes from the $\theta = 0$ region, where the D2-brane has collapsed to $N/2 - q$ fundamental strings.

Unlike in the Maldacena-Nuñez case, the large length limit which collapses at one end (36) is the only possible large length limit. This follows from the fact that in (25) we see that $d\xi_1/d\bar{k}$ can only have poles at the endpoints $\bar{k} = \pm 1$ and not in the interior. Therefore, there is no limit of the solution space analogous to that of the confining strings we found previously.

IV. DISCUSSION AND CONCLUSIONS

We have found explicit nonsupersymmetric probe D-brane solutions in the infrared of two $\mathcal{N} = 1$ confining geometries. There is a two parameter family of solutions which may be labeled by an extension in spacetime, Δx, and by a fraction of quarks, q/N, that is being pulled apart from the others in spacetime.

The solutions describe deformed baryon vertices. In the IIB case we considered, the Maldacena-Nuñez background, there was a limit at large Δx in which the solutions became the infinite confining strings of the dual theory. Away from the infinite length limit, the solutions give predictions for the mass of finite confining strings in the dual theory.

We found a similar two parameter family of solutions in a IIA geometry, obtained by dimensional reduction of a G_2 holonomy background of M theory. However, in the IIA solutions there is not a limit with the properties of infinite confining strings. The only large length limits involve collapse into fundamental strings at one end of the baryon vertex. There does exist a proposal for identifying the confining strings as membranes in the G_2 background [31]. Translating this idea into a formula for the string tensions with the required symmetry $q \leftrightarrow N - q$ remains an open problem.

Various possibilities for future research suggest themselves. It seems likely that solutions similar to those we have described will exist in other backgrounds, including $AdS_5 \times S^5$. In fact, presumably a systematic study of such DBI solutions in infrared geometries of the form $\mathcal{M}^4 \times S^{8-p}$ is possible, along the lines of [25]. The fact that the infrared geometry is independent of how the compact directions of the background D-brane are wrapped highlights the genericity of the solutions and of the dual field theory deformed baryons/confining strings.

It would be interesting to see if other appearances of confining strings in dualities admit similar energetics at finite length. Important examples are the confining strings of MQCD [29,30] and of the theory on nonextremal D4 branes at high temperature [11].

Ultimately, one would like to reproduce the properties of finite confining strings that we have described via a field theoretic calculation. Recent field theory work on confining strings in four dimensional $SU(N)$ theories has been both numerical, see for example [32], and analytical [33].

ACKNOWLEDGMENTS

The authors would like to thank David Berman, José Edelstein, Prem Kumar, Carlos Núñez, Alfonso Ramallo, Konstantin Savvidy, James Sparks and Steffan Theissen for interesting and helpful conversations. R. P. would like to acknowledge the generous support to Centro de Estudios Cientificos (CECS) by Empresas CMPC. CECS is a Millenium Science Institute and is funded in part by grants from Fundación Andes and the Tinker Foundation.

APPENDIX: CHECKING NON-SUPERSYMMETRY OF IIB SOLUTIONS

A probe brane is supersymmetric if at least one of the Killing spinors of the background, ϵ, satisfies

$$\Gamma_\kappa \epsilon = \epsilon, \tag{37}$$

where [34]

$$\Gamma_\kappa = \frac{i}{\sqrt{{}^\star G + F}} \sum_0^\infty \frac{1}{2^n n!} \gamma^{\mu_1 \nu_1 \ldots \mu_n \nu_n} F_{\mu_1 \nu_1} \ldots F_{\mu_n \nu_n} J_p^{(n)}. \tag{38}$$

In type IIB supergravity we have

$$J_p^{(n)} = (-1)^n [(\sigma_3)^{n + \frac{p-3}{2}} \sigma_2 \otimes \Gamma_{(0)}], \tag{39}$$

with

$$\Gamma_{(0)} = \frac{1}{(p+1)!} \epsilon^{\mu_1 \ldots \mu_{(p+1)}} \gamma_{\mu_1 \ldots \mu_{(p+1)}}. \tag{40}$$

We are using the standard notation in which we write the IIB spinors as an $SL(2, \mathbb{R})$ doublet of real spinors. The lower case gamma matrices are the pullback of the spacetime gamma matrices $\gamma_\mu = E^{\bar{a}}_\mu \Gamma_{\bar{a}}$.

If we use the following vielbein for the metric at the origin (1)

$$\begin{aligned} &E^i = e^{\Phi_0/2} dx^i \qquad (i = 0, 1, 2, 3), \\ &E^4 = 0, \qquad E^5 = 0, \qquad E^6 = 0, \\ &E^7 = e^{\Phi_0/2} d\psi, \qquad E^8 = e^{\Phi_0/2} \sin\psi d\theta, \\ &E^9 = e^{\Phi_0/2} \sin\psi \sin\theta d\phi, \end{aligned} \tag{41}$$

then the supersymmetry projector for the embedding (4) becomes

$$\begin{aligned} \Gamma_\kappa = &\frac{i}{\sqrt{k^2 - e^{2\Phi_0}[1 + (\partial_1 \psi)^2]}} \\ &\times [e^{\Phi_0} (\sigma_2 \otimes \Gamma_0 [\Gamma_1 + \partial_1 \psi \Gamma_7] \Gamma_{89}) \\ &+ k(\sigma_3 \sigma_2 \otimes \Gamma_{89})]. \end{aligned} \tag{42}$$

The Killing spinors of the Maldacena-Nuñez background are given in [35]. There are four real supercharges. However, the only property we shall need is that the spinors satisfy

$$(\sigma_1 \otimes 1)\epsilon = \epsilon. \quad (43)$$

If we use this property in (37) we find that a necessary condition for solutions is that $(\Gamma_1 + \partial_1\psi\Gamma_7)\epsilon = 0$. However this condition then requires $\partial_1\psi = \pm 1$, which is not consistent with the form of the solutions (5). Therefore, none of the solutions are supersymmetric.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999).
[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[4] I. R. Klebanov and M. J. Strassler, J. High Energy Phys. 0008, (2000) 052.
[5] J. M. Maldacena and C. Nunez, Phys. Rev. Lett. **86**, 588 (2001).
[6] E. Witten, J. High Energy Phys. 9807, (1998) 006.
[7] D. J. Gross and H. Ooguri, Phys. Rev. D **58**, 106002 (1998).
[8] C. G. Callan, A. Guijosa, and K. G. Savvidy, Nucl. Phys. B **547**, 127 (1999).
[9] Y. Imamura, Nucl. Phys. B **537**, 184 (1999).
[10] J. Gomis, A. V. Ramallo, J. Simon, and P. K. Townsend, J. High Energy Phys. 9911, (1999) 019.
[11] C. G. Callan, A. Guijosa, K. G. Savvidy, and O. Tafjord, Nucl. Phys. B **555**, 183 (1999).
[12] C. P. Herzog and I. R. Klebanov, Phys. Lett. B **526**, 388 (2002).
[13] J. Pawelczyk, J. High Energy Phys. 0008, (2000) 006.
[14] J. Pawelczyk and S. J. Rey, Phys. Lett. B **493**, 395 (2000).
[15] C. Bachas, M. R. Douglas, and C. Schweigert, J. High Energy Phys. 0005, (2000) 048.
[16] M. Cvetic, G. W. Gibbons, H. Lu, and C. N. Pope, Phys. Rev. Lett. **88**, 121602 (2002).
[17] M. Cvetic, G. W. Gibbons, H. Lu and C. N. Pope, Phys. Lett. B **534**, 172 (2002).
[18] A. Brandhuber, Nucl. Phys. B **629**, 393 (2002).
[19] J. D. Edelstein, A. Paredes, and A. V. Ramallo, J. High Energy Phys. 0301, (2003) 011.
[20] U. Gursoy, S. A. Hartnoll, and R. Portugues, Phys. Rev. D **69**, 086003 (2004).
[21] A. Loewy and J. Sonnenschein, J. High Energy Phys. 0108, (2001) 007.
[22] B. S. Acharya and C. Vafa, hep-th/0103011
[23] R. Emparan, Phys. Lett. B **423**, 71 (1998).
[24] R. C. Myers, J. High Energy Phys.9912, (1999) 022.
[25] J. M. Camino, A. V. Ramallo, and J. M. Sanchez de Santos, Nucl. Phys. B **562**, 103 (1999).
[26] J. M. Camino, A. Paredes, and A. V. Ramallo, J. High Energy Phys. 0105, (2001) 011.
[27] C. G. Callan and J. M. Maldacena, Nucl. Phys. B **513**, 198 (1998).
[28] G. W. Gibbons, Nucl. Phys. B **514**, 603 (1998).
[29] A. Hanany, M. J. Strassler, and A. Zaffaroni, Nucl. Phys. B **513**, 87 (1998).
[30] M. R. Douglas and S. H. Shenker, Nucl. Phys. B **447**, 271 (1995).
[31] B. S. Acharya, hep-th/0101206.
[32] B. Lucini, M. Teper, and U. Wenger, J. High Energy Phys. 0406, (2004) 012.
[33] A. I. Shoshi, F. D. Steffen, H. G. Dosch and H. J. Pirner, Phys. Rev. D **68**, 074004 (2003).
[34] E. Bergshoeff and P. K. Townsend, Nucl. Phys. B **490**, 145 (1997).
[35] C. Nunez, A. Paredes, and A. V. Ramallo, J. High Energy Phys. 0312, (2003) 024.