PHYSICAL REVIEW D, VOLUME 70, 065011

Gauge dependence of the fermion quasiparticle poles in hot gauge theories

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(Received 31 May 2004; published 15 September 2004)

The gauge dependence of the complex fermion quasiparticle poles corresponding to soft collective excitations is studied in hot gauge theories at one-loop order and next-to-leading order in the high-temperature expansion, with a view towards going beyond the leading order hard thermal loops and resummations thereof. We find that for collective excitations of momenta $k \sim eT$ the dispersion relations are gauge independent, but the corresponding damping rates are gauge dependent. For $k \ll eT$ and in the $k \to 0$ limit, both the dispersion relations and the damping rates are found to be gauge dependent. The gauge dependence of the position of the complex quasiparticle poles signals the need for resummation. Possible cancellation of the leading gauge dependence at two-loop order in the case of QED is briefly discussed.

DOI: 10.1103/PhysRevD.70.065011 PACS numbers: 11.10.Wx, 11.15.–q, 12.38.Cy

I. INTRODUCTION

The demonstration of gauge independence of physical quantities is of fundamental importance in gauge theories at finite temperature. Interestingly, the main breakthrough in perturbative gauge theories at high temperature was in fact motivated by the quest for gauge-independent damping rates of plasma excitations. It was realized [1] that in hot gauge theories the usual connection between the order of the loop expansion and powers of the gauge coupling constant is lost, hence contributions of leading order in the gauge coupling constant arise from every order in the loop expansion. Resummation to all orders of these leading, gauge-independent hard thermal loops (HTLs) [1–6] is necessary for a consistent calculation. The HTL resummation program [1,2] leads to an effective theory that systematically includes contributions of different momentum scales [7].

It was proved in Ref. [8] that the singularity structure (i.e., the position of poles and branch singularities) of gauge boson and fermion propagators is gauge independent when all contributions of a given order of a systematic expansion scheme are accounted for. While the position of quasiparticle poles in the leading order HTL approximation are completely gauge independent [9,10], nevertheless gauge dependence will enter at subleading order and gauge-independent extensions beyond the leading order HTL results are not yet available. More recently, several authors [11–13] have shown that the truncated on-shell two-particle-irreducible (2PI) effective action has a controlled gauge dependence, with the explicit gauge-dependent terms always appearing at higher order. It would be interesting to study possible cancellation of the leading gauge dependence in the singularity structure of gauge boson and fermion propagators beyond the leading order HTL approximation.

The propagation of fermions in hot and dense matter is of interest in a wide variety of physically relevant situations. In ultrarelativistic heavy ion collisions and the formation and evolution of the quark-gluon plasma, lepton pairs play a very important role as clean probes of the early, hot stage of the new state of matter [14]. The propagation of quarks during the nonequilibrium stages of the electroweak phase transition is conjectured to be an essential ingredient for baryogenesis at the electroweak scale in (non)supersymmetric extensions of the standard model [15]. In stellar astrophysics, electrons and neutrinos play a major role in the evolution of dense stars such as white dwarfs, neutron stars, and supernovae [16].

The goal of this article is to investigate the gauge dependence of the *complex* fermion quasiparticle poles corresponding to *soft* collective excitations at one-loop order and next-to-leading order in the high-temperature expansion. There have been investigations of how the leading HTL dispersion relations of fermions (and the gauge independence thereof) are affected by retaining nonleading powers of temperature [17,18]. While it is known that quasiparticles at finite temperature will in general acquire thermal widths due to collisional broadening rendering the quasiparticle poles complex, the previous authors considered mainly the real parts of the complex quasiparticle poles (dispersion relations) without discussing the corresponding imaginary parts (damping rates). In this article, we study the gauge dependence of both the real and imaginary parts of the complex quasiparticle poles on equal footing. Furthermore, we focus on the soft fermion collective excitations of momenta $k \lesssim eT$, where $e \ll 1$ is the gauge coupling constant and T is the temperature. On the one hand, it allows us to fill a gap in the literature where either a numerical analysis with $k \sim T$ was carried out [17] or the limiting cases of $eT \ll k \ll T$ and $k = 0$ were considered [18]. On the other hand, $k \lesssim eT$ is the relevant momentum region

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at the heart of the HTL resummation program [1,2]. Progress made in the soft momentum region will certainly shed light on the issue of going beyond the HTL resummation.

The rest of this article is organized as follows. In Sec. II, we calculate the real and imaginary parts of the one-loop fermion self-energy in general covariant gauges and up to next-to-leading order in the high-temperature expansion. In Sec. III, we study the gauge dependence of the complex fermion quasiparticle poles corresponding to soft collective excitations of momenta $k \lesssim eT$. Possible cancellation of the leading gauge dependence at two-loop order in the case of QED is briefly discussed. Section IV presents our conclusions. In the appendix, we calculate the vacuum contribution to the fermion self-energy that is neglected in the main text.

II. ONE-LOOP FERMION SELF-ENERGY IN COVARIANT GAUGES

We will carry out our perturbative calculations in the imaginary-time (Matsubara) formalism (ITF) of finite-temperature field theory [9,10]. The continuation to imaginary time that describes the theory at finite temperature T is obtained by replacing $it \to \tau$ with $0 \le \tau \le \beta \equiv 1/T$. Contrary to the usual ITF, however, we will *not* work in Euclidean spacetime but keep the metric tensor and the Dirac gamma matrices the same as in Minkowski spacetime.

In ITF it proves convenient to work in the spectral representation of the propagators. The fermion propagator is given by

$$s(i\nu_n, \mathbf{k}) = \int_{-\infty}^{\infty} dk_0 \frac{\rho_f(k_0, \mathbf{k})}{k_0 - i\nu_n}, \qquad \nu_n = (2n+1)\pi T, \tag{1}$$

where ρ_f is the free Dirac fermion spectral function (zero chemical potential) [9]

$$\rho_f(k_0, \mathbf{k}) = \not{K}\, \mathrm{sgn}(k_0)\delta(K^2), \qquad K = (k_0, \mathbf{k}), \tag{2}$$

with $\mathrm{sgn}(x)$ being the sign function. The gauge boson propagator is given by

$$d^{\mu\nu}(i\omega_n, \mathbf{p}) = \int_{-\infty}^{\infty} dp_0 \frac{\rho^{\mu\nu}(p_0, \mathbf{p})}{p_0 - i\omega_n}, \qquad \omega_n = 2n\pi T, \tag{3}$$

where $\rho^{\mu\nu}$ is the free gauge boson spectral function in general covariant gauges [19]

$$\rho^{\mu\nu}(p_0, \mathbf{p}) = \mathrm{sgn}(p_0)\left[-g^{\mu\nu} + (\xi - 1)P^\mu P^\nu \frac{\partial}{\partial p_0^2}\right]\delta(P^2), \tag{4}$$

with $P = (p_0, \mathbf{p})$. The covariant gauge parameter ξ is defined in such a way that $\xi = 1$ is the Feynman gauge.

Since at one-loop order the fermion self-energy has the same structure (up to gauge group factors) in both Abelian and non-Abelian gauge theories, for notational simplicity we will consider the Abelian case in what follows. The non-Abelian case can be obtained through the replacement $e^2 \to g^2 C_F$, where $C_F = (N^2 - 1)/2N$ is the Casimir invariant of the fundamental representation in SU(N) gauge theories.

In QED the one-loop fermion self-energy in ITF is given by

$$\Sigma(i\nu_n, \mathbf{k}) = e^2 T \sum_{i\omega_m} \int \frac{d^3p}{(2\pi)^3} \times \gamma^\mu s(i\nu_n + i\omega_m, \mathbf{q})\gamma^\nu d_{\mu\nu}(i\omega_m, \mathbf{p}), \tag{5}$$

where $\mathbf{q} = \mathbf{k} + \mathbf{p}$. Upon substituting the fermion and gauge boson propagators into (5), the sum over the bosonic Matsubara frequency can be done easily [9] leading to

$$\Sigma(i\nu_n, \mathbf{k}) = e^2 \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\infty} dp_0 \times \int_{-\infty}^{\infty} dq_0\, \rho_{\mu\nu}(p_0, \mathbf{p})\gamma^\mu \rho_f(q_0, \mathbf{q})\gamma^\nu \times \frac{n_B(p_0) + n_F(q_0)}{q_0 - p_0 - i\nu_n}, \tag{6}$$

where $n_{B,F}(x) = 1/(e^{\beta x} \mp 1)$ are the Bose and Fermi distribution functions, respectively.

After the analytic continuation $i\nu_n \to \omega + 0^+$ to arbitrary frequency ω, the imaginary part of the (retarded) self-energy can be readily found to be given by

$$\mathrm{Im}\,\Sigma(\omega, \mathbf{k}) = e^2\pi \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\infty} dp_0 \times \int_{-\infty}^{\infty} dq_0\, \rho_{\mu\nu}(p_0, \mathbf{p})\gamma^\mu \rho_f(q_0, \mathbf{q})\gamma^\nu \times [n_B(p_0) + n_F(q_0)]\delta(\omega - q_0 + p_0). \tag{7}$$

The real part of the self-energy is obtained from the imaginary one through the dispersive representation

$$\mathrm{Re}\,\Sigma(\omega, \mathbf{k}) = \mathrm{PV}\int_{-\infty}^{\infty} \frac{dk_0}{\pi} \frac{\mathrm{Im}\,\Sigma(k_0, \mathbf{k})}{k_0 - \omega}, \tag{8}$$

where PV denotes the principal value.

For a massless fermion, rotational invariance and chiral symmetry entail that the fermion self-energy in equilibrium can be parametrized by [9,10]

$$\Sigma(\omega, \mathbf{k}) = \Sigma^{(0)}(\omega, k)\gamma^0 + \Sigma^{(1)}(\omega, k)\boldsymbol{\gamma} \cdot \hat{\mathbf{k}}, \tag{9}$$

where $k \equiv |k|$, $\hat{\mathbf{k}} = \mathbf{k}/k$, and $\Sigma^{(0)}$ and $\Sigma^{(1)}$ are scalar functions

$$\Sigma^{(0)}(\omega, k) = \tfrac{1}{4}\mathrm{tr}[\Sigma(\omega, \mathbf{k})\gamma^0],$$
$$\Sigma^{(1)}(\omega, k) = -\tfrac{1}{4}\mathrm{tr}[\Sigma(\omega, \mathbf{k})\boldsymbol{\gamma}\cdot\hat{\mathbf{k}}]. \tag{10}$$

Since we are interested in the self-energy calculated in general covariant gauges, it is convenient to decompose the former into gauge-independent and -dependent contributions. We write

$$\Sigma(\omega, \mathbf{k}) = \Sigma_{\rm FG}(\omega, \mathbf{k}) + \Sigma_\xi(\omega, \mathbf{k}), \tag{11}$$

where $\Sigma_{\rm FG}$ is the gauge-independent part calculated in the Feynman gauge and Σ_ξ is the remaining gauge-dependent part.

A. The imaginary part

We first study the gauge-independent contribution $\mathrm{Im}\,\Sigma_{\rm FG}$. Evaluating the Dirac traces and performing the trivial integrals over p_0 and q_0 in (7), we obtain

$$\begin{aligned}
\mathrm{Im}\,\Sigma^{(0)}_{\rm FG}(\omega, k) &= e^2\pi\int\frac{d^3p}{(2\pi)^3}\frac{1}{2p}\{[1+n_B(p)-n_F(q)]\delta(p+q-\omega)+[n_B(p)+n_F(q)]\delta(p-q+\omega)\}+(\omega\to-\omega),\\
\mathrm{Im}\,\Sigma^{(1)}_{\rm FG}(\omega, k) &= -e^2\pi\int\frac{d^3p}{(2\pi)^3}\frac{\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}}{2p}\{[1+n_B(p)-n_F(q)]\delta(p+q-\omega)+[n_B(p)+n_F(q)]\delta(p-q+\omega)\}\\
&\quad -(\omega\to-\omega).
\end{aligned} \tag{12}$$

In the above expressions, the support of the energy-conserving delta functions $\delta(p+q-\omega)$ and $\delta(p+q+\omega)$ is $\omega > k$ and $\omega < k$, respectively, corresponding to the usual two-particle cuts, while that of $\delta(p-q\mp\omega)$ is $\omega^2 < k^2$ corresponding to the Landau damping cut which is purely a medium effect [1,9].

The different contributions in (12) have a physical interpretation in terms of (off-shell) scattering processes taking place in the thermal medium. The terms proportional to $\delta(p+q-\omega)$ arise from the processes in which a (timelike) fermion decays into a fermion and a gauge boson $f^*\to f+\gamma$, and those proportional to $\delta(p-q+\omega)$ originate in the Landau damping process in which a (spacelike) fermion scatters off a gauge boson in the medium $f^*+\gamma\to f$. Here the off-shell fermion is denoted by a superscript "$*$."

In what follows we will neglect the vacuum contribution, i.e., the terms which do not contain any thermal distribution functions, as we are interested mainly in the finite-temperature medium effects. Interested readers can find in the appendix the calculation for the vacuum contribution.

The angular integration over $\eta = \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}$ in (12) can be performed analytically by using change of variables $\eta\to z = q(\eta)$ and the energy-conserving delta functions. The requirement that the energy-conserving delta functions must have a nonempty support restricts the range of the radial integration over p for fixed k and ω. The remaining integration over p for arbitrary k and ω is an involved numerical task [17] which, however, is not very useful for the purpose of studying next-to-leading order corrections. In order to compare with the leading order HTL results, we here focus on a high-temperature expansion for which $k, \omega \ll T$ and keep terms in $\mathrm{Im}\,\Sigma$ to $\mathcal{O}(T)$ in the Landau damping cut contribution but to $\mathcal{O}(T^0)$ in the two-particle cut contribution.

A comment here is in order. One would presumably expect that it is sufficient to keep terms in $\mathrm{Im}\,\Sigma$ uniformly up to next-to-leading order in the high-temperature expansion, namely, terms of order $\mathcal{O}(T)$. Whereas this is correct for generic $k\sim\omega\ll T$, it is not correct in the limit $k\ll\omega\ll T$ which is in turn relevant to the effective fermion mass and the damping rate at rest that is obtained in the long-wavelength limit ($k\to 0$). As will be seen below, the $\mathcal{O}(T^0)$ two-particle cut contribution in $\mathrm{Im}\,\Sigma$ will give rise to the $\mathcal{O}(\ln T)$ term in $\mathrm{Re}\,\Sigma$ which will become next-to-leading order in T in the limit $k\ll\omega\ll T$.

Separating the leading HTL contributions that can be calculated analytically, then in the remaining contributions expanding the distribution functions in the high-temperature limit and cutting off the potentially divergent momentum integrations at T, we obtain after some algebra ($k, \omega \ll T$)

$$\begin{aligned}
\mathrm{Im}\,\Sigma^{(0)}_{\rm FG}(\omega, k) &\simeq \frac{\pi e^2T^2}{16k}\left[1+\frac{2\omega}{\pi^2T}\ln\left|\frac{\omega+k}{\omega-k}\right|\right]\theta(k^2-\omega^2)+\frac{e^2T}{8\pi}\left[\frac{\omega}{k}\ln\left|\frac{\omega+k}{\omega-k}\right|-1-\frac{|\omega|}{2T}\right]\theta(\omega^2-k^2),\\
\mathrm{Im}\,\Sigma^{(1)}_{\rm FG}(\omega, k) &\simeq -\frac{\pi e^2T^2}{16k}\left[\frac{\omega}{k}+\frac{k}{\pi^2T}\left(1+\frac{\omega^2}{k^2}\right)\ln\left|\frac{\omega+k}{\omega-k}\right|\right]\theta(k^2-\omega^2)\\
&\quad+\frac{e^2T}{8\pi}\left[\frac{\omega}{k}-\frac{1}{2}\left(1+\frac{\omega^2}{k^2}\right)\ln\left|\frac{\omega+k}{\omega-k}\right|+\mathrm{sgn}(\omega)\frac{k}{2T}\right]\theta(\omega^2-k^2),
\end{aligned} \tag{13}$$

where $\theta(x)$ is the Heaviside step function. Three important features are gleaned from the above expressions: (i) The gauge-independent part $\mathrm{Im}\,\Sigma_{\mathrm{FG}}$ receives contributions both from above and below the light cone, corresponding, respectively, to the two-particle and Landau damping cuts. (ii) The leading $\mathcal{O}(T^2)$ terms in $\mathrm{Im}\,\Sigma_{\mathrm{FG}}$ are recognized as the HTL results, which arise solely from the Landau damping process [1,5,6,9]. (iii) The subleading terms are suppressed by inverse powers of T and originate in the Landau damping as well as in the fermion decay processes.

Next, we calculate the gauge-dependent contribution $\mathrm{Im}\,\Sigma_\xi$. The gauge-dependent part of the free gauge boson spectral function contains the derivative of the on-shell delta function $\partial\delta(p_0^2 - p^2)/\partial p_0^2$, thus the integral over p_0 can be done using integration by parts which in turn gives rise to derivatives of the Bose distribution function $dn_B(p)/dp$ as well as of the energy-conserving delta functions $\partial\delta(p - q - \omega)/\partial\omega$, etc. Such structures are expected to be generic to the gauge-dependent higher loop contributions linear in $(\xi - 1)$.

After some lengthy but straightforward algebra, we obtain

$$\begin{aligned}
\mathrm{Im}\,\Sigma_\xi^{(0)}(\omega, k) &= (\xi - 1)e^2\pi \int \frac{d^3p}{4(2\pi)^3}\bigg\{(1 + \hat{\mathbf{p}}\cdot\hat{\mathbf{q}})\bigg[[1 + n_B(p) - n_F(q)]\frac{\partial}{\partial\omega}\delta(p + q - \omega) - \frac{dn_B(p)}{dp}\delta(p + q - \omega)\bigg] \\
&\quad -(1 - \hat{\mathbf{p}}\cdot\hat{\mathbf{q}})\bigg[[n_B(p) + n_F(q)]\frac{\partial}{\partial\omega}\delta(p - q + \omega) + \frac{dn_B(p)}{dp}\delta(p - q + \omega)\bigg]\bigg\} + (\omega \to -\omega), \\
\mathrm{Im}\,\Sigma_\xi^{(1)}(\omega, k) &= (\xi - 1)e^2\pi \int \frac{d^3p}{4(2\pi)^3}\bigg\{\frac{(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}) - \hat{\mathbf{k}}\cdot\hat{\mathbf{q}}}{p}[[1 + n_B(p) - n_F(q)]\delta(p + q - \omega) + [n_B(p) + n_F(q)] \\
&\quad \times\delta(p - q + \omega)] + \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}\bigg[(1 + \hat{\mathbf{p}}\cdot\hat{\mathbf{q}})\bigg([1 + n_B(p) - n_F(q)]\frac{\partial}{\partial\omega}\delta(p + q - \omega) - \frac{dn_B(p)}{dp} \\
&\quad \times\delta(p + q - \omega)\bigg) + (1 - \hat{\mathbf{p}}\cdot\hat{\mathbf{q}})\bigg([n_B(p) + n_F(q)]\frac{\partial}{\partial\omega}\delta(p - q + \omega) + \frac{dn_B(p)}{dp}\delta(p - q + \omega)\bigg)\bigg]\bigg\} \\
&\quad -(\omega \to -\omega).
\end{aligned} \tag{14}$$

It is noted that the various contributions in (14), being gauge dependent and hence in contrast to those in (12), will not have a physical interpretation in terms of the scattering processes taking place in the medium. The gauge-dependent contributions are expected to cancel in physical quantities in a consistent calculation that generally requires resummation of perturbation theory. Clearly, such a task is beyond the scope of this article.

The angular integration in (14) can be done analytically as in the gauge-independent contribution. Again neglecting the vacuum contribution, we find in the high-temperature limit the following rather compact expressions ($k, \omega \ll T$):

$$\begin{aligned}
\mathrm{Im}\,\Sigma_\xi^{(0)}(\omega, k) &\simeq (\xi - 1)\frac{e^2T}{16\pi}\bigg[1 + \frac{\omega}{k}\ln\bigg|\frac{\omega + k}{\omega - k}\bigg| - \frac{|\omega|}{T}\theta(\omega^2 - k^2)\bigg], \\
\mathrm{Im}\,\Sigma_\xi^{(1)}(\omega, k) &\simeq (\xi - 1)\frac{e^2T}{16\pi}\bigg[\frac{\omega}{k} - \frac{1}{2}\bigg(1 + \frac{\omega^2}{k^2}\bigg) \times\ln\bigg|\frac{\omega + k}{\omega - k}\bigg| + \mathrm{sgn}(\omega)\frac{k}{T}\theta(\omega^2 - k^2)\bigg],
\end{aligned} \tag{15}$$

where we have combined part of the contributions from above and below the light cone by using the identity $\theta(x) + \theta(-x) = 1$. The leading gauge-dependent terms in $\mathrm{Im}\,\Sigma_\xi$ are of order $\mathcal{O}(T)$ and the subleading ones are suppressed by inverse powers of T as is the case for $\mathrm{Im}\,\Sigma_{\mathrm{FG}}$.

B. The real part

From the above results for the imaginary part of the fermion self-energy, the corresponding real part can be obtained using (8). In the high-temperature limit that we are interested in, the upper and lower limits of the integral over k_0 in (8) can be cut off at T and $-T$, respectively. The contributions to $\mathrm{Re}\,\Sigma$ from the ignored regions of integration are at most of order $\mathcal{O}(T^0)$, hence are negligible at the next-to-leading order under consideration. It is worthy noting that, since $\mathrm{Im}\,\Sigma^{(0)}$ [$\mathrm{Im}\,\Sigma^{(1)}$] is an even (odd) function of ω therefore, as a result of (8), $\mathrm{Re}\,\Sigma^{(0)}$ [$\mathrm{Re}\,\Sigma^{(1)}$] is an odd (even) function of ω.

In the high-temperature limit we obtain for the gauge-independent part ($k, \omega \ll T$)

$$\mathrm{Re}\,\Sigma^{(0)}_{\mathrm{FG}}(\omega,k) \simeq -\frac{e^2T^2}{16k}\left(1+\frac{2k}{\pi^2 T}\right)\ln\left|\frac{\omega+k}{\omega-k}\right| - \frac{e^2\omega}{16\pi^2} \times \ln\frac{T^2}{|\omega^2-k^2|}, \tag{16}$$
$$\mathrm{Re}\,\Sigma^{(1)}_{\mathrm{FG}}(\omega,k) \simeq -\frac{e^2T^2}{8k}\left(1+\frac{2k}{\pi^2 T}\right)\left[1-\frac{\omega}{2k}\ln\left|\frac{\omega+k}{\omega-k}\right|\right] + \frac{e^2k}{16\pi^2}\ln\frac{T^2}{|\omega^2-k^2|},$$

where we have kept terms up to $\mathcal{O}(\ln T)$ as remarked above. This is because the $\mathcal{O}(T)$ term in $\mathrm{Re}\,\Sigma^{(0)}$ [$\mathrm{Re}\,\Sigma^{(1)}$] is proportional to k (k^2) in the limit $k \ll \omega \ll T$, therefore the putative subleading $\mathcal{O}(\ln T)$ term becomes next-to-leading order in T in this limit [note that $\mathrm{Re}\,\Sigma^{(1)}(\omega,k)$ vanishes identically in the limit $k \to 0$ due to rotational symmetry].

Again the leading $\mathcal{O}(T^2)$ terms in $\mathrm{Re}\,\Sigma_{\mathrm{FG}}$ are the HTL results [3,5,6,9,10]. The $\mathcal{O}(\ln T)$ terms in (16) arise from the $\mathcal{O}(T^0)$ terms *above* the light cone in (13), thus originating in the region of hard loop momentum. Such $\ln T$ contributions are not unique in the one-loop fermion self-energy. Indeed, similar $\ln T$ behavior that originates also in the hard loop momentum region has been found in the gauge boson polarization [4,20] as well as, in general, the n-point vertex function [21] at one-loop order in high-temperature non-Abelian gauge theories.

Similarly we find for the gauge-dependent part ($k, \omega \ll T$)

$$\begin{aligned}\mathrm{Re}\,\Sigma^{(0)}_{\xi}(\omega,k) &\simeq -(\xi-1)\frac{e^2\omega}{16\pi^2}\ln\frac{T^2}{|\omega^2-k^2|},\\ \mathrm{Re}\,\Sigma^{(1)}_{\xi}(\omega,k) &\simeq (\xi-1)\frac{e^2k}{16\pi^2}\ln\frac{T^2}{|\omega^2-k^2|}.\end{aligned} \tag{17}$$

The $\ln T$ contributions in (17), like the gauge-independent ones in (16), arise from the region of hard loop momentum as well. Following the argument in Ref. [5] based on the ultraviolet divergences in the absence of distribution functions, one might expect the leading term in $\mathrm{Re}\,\Sigma_\xi$ to be linear in T. Nevertheless, explicit calculation shows that the leading term actually goes like $\ln T$ at high temperature. As we will see momentarily, the absence of the $\mathcal{O}(T)$ term in $\mathrm{Re}\Sigma_\xi$ has an important consequence in the gauge dependence of fermion dispersion relations.

Furthermore, upon comparing (16) and (17), we find that the $\mathcal{O}(\ln T)$ term in $\mathrm{Re}\,\Sigma_\xi$ has the same prefactor as that in the corresponding $\mathrm{Re}\,\Sigma_{\mathrm{FG}}$, hence they can be combined together to yield a single gauge-dependent term proportional to ξ. This, however, is not the case for the imaginary part of the self-energy, where only some of the terms in $\mathrm{Im}\,\Sigma_\xi$ and $\mathrm{Im}\,\Sigma_{\mathrm{FG}}$ can be combined together.

III. GAUGE DEPENDENCE OF THE FERMION QUASIPARTICLE POLES

Having calculated the one-loop fermion self-energy at next-to-leading order in T in general covariant gauges, we now proceed to study gauge dependence of the complex fermion quasiparticle poles.

The full (retarded) inverse fermion propagator is given by the Dyson-Schwinger equation

$$iS^{-1}(\omega,\mathbf{k}) = \omega\gamma^0 - k\boldsymbol{\gamma}\cdot\hat{\mathbf{k}} + \Sigma(\omega,\mathbf{k}), \tag{18}$$

where $\Sigma(\omega,\mathbf{k})$ is the (retarded) fermion self-energy. Equation (18) can be inverted to yield [9,10]

$$S(\omega,\mathbf{k}) = \frac{i}{2}\left[\frac{\gamma^0-\boldsymbol{\gamma}\cdot\hat{\mathbf{k}}}{\Delta_+(\omega,k)} + \frac{\gamma^0+\boldsymbol{\gamma}\cdot\hat{\mathbf{k}}}{\Delta_-(\omega,k)}\right], \tag{19}$$

where

$$\Delta_\pm(\omega,k) = \omega \mp k + \Sigma_\pm(\omega,k), \tag{20}$$

with

$$\Sigma_\pm(\omega,k) = \Sigma^{(0)}(\omega,k) \pm \Sigma^{(1)}(\omega,k). \tag{21}$$

The analytic continuation of the fermion propagator to complex frequency features the following singularities:

(i) *Isolated poles.* Isolated *real* poles of the fermion propagator correspond to stable quasiparticle excitations, whereas *complex* poles to unstable excitations (resonances) with finite widths.

(ii) *Branch cuts.* Branch cuts of the fermion propagator correspond to multiparticle states.

As we are interested in the collective excitation in the medium, we will consider isolated poles in the rest of this section.

Write $\omega = E - i\gamma$ with E and γ real. In the narrow width approximation for which $\gamma \ll E$, the equations that determine the position of the complex poles are given by [22]

$$\begin{aligned} E \mp k + \mathrm{Re}\,\Sigma_\pm(E,k) &= 0,\\ \gamma + \mathrm{sgn}(\gamma) Z_\pm(k)\,\mathrm{Im}\,\Sigma_\pm(E,k) &= 0,\end{aligned} \tag{22}$$

where $Z_\pm(k)$ in the second equation are the residues at the poles (wave function renormalizations)

$$Z_\pm(k) = \left[1 + \frac{\partial\,\mathrm{Re}\,\Sigma_\pm(\omega,k)}{\partial\omega}\right]^{-1}_{\omega=E}. \tag{23}$$

If the product of the residue at the pole and the imaginary part of the self-energy on the quasiparticle mass shell $Z_\pm(k)\,\mathrm{Im}\,\Sigma_\pm(E,k)$ is negative, there are two complex poles that conjugate to each other in the physical sheet corresponding to a growing and a decaying solution, i.e., an instability. On the other hand, if the product is positive there is no complex pole in the physical sheet; the pole has moved off into the unphysical (second or higher) sheet. In this case the spectral function features a Breit-Wigner shape resonance with a width given by the damping rates

$$\gamma(k) = Z_{\pm}(k)\,\mathrm{Im}\,\Sigma_{\pm}(E, k), \tag{24}$$

which in general determines an exponential falloff $e^{-\gamma(k)t}$ of the fermion propagator in real time. It is worth noting that numerical results reveal that at next-to-leading order the product $Z_{\pm}(k)\,\mathrm{Im}\,\Sigma_{\pm}(E, k)$ is positive in the Feynman gauge.

From the one-loop fermion self-energy calculated above, we recover at leading order in T [i.e., $\mathcal{O}(T^2)$] the celebrated HTL results [3,5,6,9,10]. There are two branches of *stable* fermion collective excitations (fermions and the so-called plasminos [9,10]) with positive and negative helicity to chirality ratios, respectively. Their dispersion relations are manifestly gauge independent and given by (for positive energy solutions)

$$E(k) = \begin{cases} \omega_+(k) & \text{fermion} \\ \omega_-(k) & \text{plasmino.} \end{cases} \tag{25}$$

A plot of the HTL dispersion relations $\omega_{\pm}(k)$ can be found in the literature (see, e.g., Refs. [9,10]). One of the main features is that the two branches of collective excitations develop a gap $m = eT/\sqrt{8}$, corresponding to an effective thermal mass that respects both gauge invariance and chiral symmetry [3,5,6,9,10].

The gauge dependence of the quasiparticle poles at next-to-leading order must be studied with care because the $\mathcal{O}(T)$ terms in $\mathrm{Re}\,\Sigma(\omega, k)$ become subleading in the limit $k \ll \omega \ll T$ as remarked above.

Anticipating that the next-to-leading order correction will not dramatically change the leading order HTL dispersion relations $\omega_{\pm}(k)$ which have a gap of order eT, we will first consider the case for which $k \sim eT$. As is clear from the above results, for generic $k, \omega \sim eT$ the next-to-leading order correction to $\mathrm{Re}\,\Sigma(\omega, k)$ is of order T and gauge independent. Upon substituting $\mathrm{Re}\,\Sigma$ at this order into (20), we find that for the collective excitations of momenta $k \sim eT$ the one-loop dispersion relations at next-to-leading order in T [i.e., $\mathcal{O}(T)$] are manifestly gauge independent. This is one of the novel contributions of this article. Numerical analysis shows that the dispersion relations at next-to-leading order have similar features but move slightly above the leading order HTL results in the same momentum region. The differences increase with increasing gauge coupling constant, in agreement with the full numerical result found in Ref. [17].

The next-to-leading order correction to $\mathrm{Im}\,\Sigma$ in the momentum region $k \sim eT$ is again of order T but gauge dependent. Whereas at next-to-leading order $\mathrm{Im}\,\Sigma$ has support *above* the light cone which presumably suggests finite damping rates $\gamma \sim e^2T$ (up to the logarithm of the gauge coupling constant) for the collective excitations at one-loop order, nevertheless the gauge-dependent contribution to $\mathrm{Im}\,\Sigma$ that first appears at this order makes such interpretations doubtful. This result is not unexpected, as it is well known that in high-temperature gauge theories a consistent, gauge-independent calculation of the quasiparticle damping rates requires a resummation of perturbation theory such as, e.g., the Braaten-Pisarski HTL resummation program [1].

We next consider the case for which $k \ll eT$. Expanding the fermion self-energy obtained above in the high-temperature limit [see (13) and (15)–(17)] in powers of k/ω and keeping terms leading in k/ω and up to next-to-leading order in T, we obtain ($k \ll \omega \sim eT$)

$$\begin{aligned} \mathrm{Re}\,\Sigma^{(0)}(\omega, k) &\simeq -\frac{e^2T^2}{8\omega} - \xi\frac{e^2\omega}{8\pi^2}\ln\frac{T}{|\omega|}, \\ \mathrm{Re}\,\Sigma^{(1)}(\omega, k) &\simeq \frac{k}{\omega}\left[\frac{e^2T^2}{24\omega} + \xi\frac{e^2\omega}{8\pi^2}\ln\frac{T}{|\omega|}\right], \end{aligned} \tag{26}$$

and

$$\begin{aligned} \mathrm{Im}\,\Sigma^{(0)}(\omega, k) &\simeq \frac{e^2T}{8\pi} + (\xi - 1)\frac{3e^2T}{16\pi}, \\ \mathrm{Im}\,\Sigma^{(1)}(\omega, k) &\simeq -\frac{k}{\omega}\left[\frac{e^2T}{6\pi} + (\xi - 1)\frac{e^2T}{12\pi}\right]. \end{aligned} \tag{27}$$

In the above expressions, we have combined the gauge-independent and -dependent contributions. The next-to-leading order corrections to $\mathrm{Re}\,\Sigma$ and $\mathrm{Im}\,\Sigma$ are of order $\ln T$ and T, respectively, and manifestly gauge dependent. Upon substituting the above results into (20), we find that for the collective excitations of momenta $k \ll eT$ the one-loop dispersion relations and damping rates at next-to-leading order in T are gauge dependent.

The effective fermion mass m as well as the damping rate of collective excitations at rest $\gamma(0)$ at next-to-leading order can be extracted by taking the long-wavelength limit $k \to 0$. The effective mass is determined by the following equation ($m > 0$):

$$m^2 = \frac{e^2T^2}{8} + \xi\frac{e^2m^2}{8\pi^2}\ln\frac{T}{m}, \tag{28}$$

which agrees with the result found in the real-time formalism [18]. Equation (28) indicates that the one-loop effective mass at next-to-leading order in T is gauge dependent and that the gauge-dependent correction to m is of order $e^4T^2\ln(1/e)$. The damping rate of collective excitations at rest is of order e^2T but with a gauge-dependent contribution of the same order.

Before ending this section, we discuss briefly possible cancellation of the leading gauge dependence of the quasiparticle poles at two-loop order. For simplicity, we will consider the $k = 0$ case in QED. At two-loop order, there are three one-particle-irreducible (1PI) diagrams that contribute to the fermion self-energy, corresponding to corrections of the vertex, fermion self-energy, and gauge boson polarization. However, because of the Ward identity for the gauge boson polarization only the first two will give rise to gauge-dependent contributions. For the purpose of our discussion here, we only need the two-loop gauge-dependent contributions linear in $(\xi - 1)$. Presumably, the gauge-dependent contributions quadratic in $(\xi - 1)$ will be subleading in T.

To extract the leading gauge-dependent contribution at two-loop order, we focus on diagrams at the level of the propagator rather than the 1PI self-energy diagrams. Specifically, we consider the one-particle-reducible (1PR) two-loop self-energy diagram. Using $\Sigma^{(0)}$ given in (26) and (27), in the $k \to 0$ limit we find for the real part (i) the leading gauge-independent contribution is of order e^4T^4/ω^3 corresponding to what will be resummed by the HTLs at two-loop order, and (ii) the leading gauge-dependent contribution is of order $(e^4T^2/\omega) \times \ln(T/\omega)$ arising from taking the gauge-dependent contribution in one of the two loops. Similarly, for the imaginary part we find both the leading gauge-independent and -dependent contributions are of order e^4T^3/ω^2.

Assuming that the leading gauge-dependent contributions from the 1PI diagrams have the same behavior as those from the 1PR one, we obtain in the $k \to 0$ limit

$$\begin{aligned} m^2 &= \frac{e^2T^2}{8} + \xi \frac{e^2m^2}{8\pi^2} \ln\frac{T}{m} + a(\xi)e^4T^2 \ln\frac{T}{m}, \\ \gamma(0) &= Z(0)\left[\frac{e^2T}{8\pi} + (\xi - 1)\frac{3e^2T}{16\pi} + b(\xi)\frac{e^4T^3}{m^2}\right], \end{aligned} \tag{29}$$

where $a(\xi)$ and $b(\xi)$ are ξ-dependent coefficients yet to be determined by explicit calculations. To solve for m and $\gamma(0)$ at two-loop order, one can substitute into terms on the right-hand side in (29) the leading order result $m \sim \mathcal{O}(eT)$. *If* $a(\xi)$ and $b(\xi)$ are such that the gauge-dependent contributions in (29) cancel, one finds the following gauge-independent results:

$$m^2 \simeq \frac{e^2T^2}{8}\left[1 + \mathcal{O}\left(e^2 \ln\frac{1}{e}\right)\right], \qquad \gamma(0) \simeq \mathcal{O}(e^2T). \tag{30}$$

The above discussion seems to suggest that cancellation of the leading gauge dependence in m and $\gamma(0)$ at two-loop order and next-to-leading order in T is plausible. This situation is reminiscent of the recent proof that the truncated on-shell 2PI effective action has a controlled gauge dependence, with the explicit gauge-dependent terms always appearing at higher order [11–13].

IV. CONCLUSIONS

In this article we have studied the gauge dependence of the fermion quasiparticle poles in hot gauge theories at one-loop order and next-to-leading order in T. We focus on the quasiparticle poles corresponding to soft collective excitations of momenta $k \lesssim eT \ll T$, with a view towards going beyond the leading order HTL results and resummations thereof.

We have calculated the one-loop fermion self-energy (both the real and imaginary parts) in general covariant gauges up to next-to-leading order in the high-temperature expansion for which $k, \omega \ll T$. We find that, whereas the next-to-leading order contributions to the imaginary part behaves like T, the behavior of those to the real part depends on the range of ω as well as on their gauge dependence. The next-to-leading order gauge-independent contribution behaves like T for $k, \omega \sim eT$, but becomes $\ln T$ for $k \ll \omega \sim eT$. Nevertheless, the corresponding gauge-dependent contribution always behaves like $\ln T$ in the high-temperature limit. This analysis allows us to study in detail the gauge dependence of the complex quasiparticle poles corresponding to soft collective excitations.

For collective excitations of momenta $k \sim eT$, we find that the dispersion relations at next-to-leading order in T are gauge independent. Numerical results show that these next-to-leading order dispersion relations have similar features but move slightly above the leading order HTL ones in the same momentum region. However, the corresponding damping rates are gauge dependent, thus rendering the *complex* quasiparticle poles gauge dependent. For $k \ll eT$ and in the long-wavelength limit $k \to 0$, both the dispersion relations and the damping rates are gauge dependent. The gauge dependence of the position of the quasiparticle poles at one-loop order and next-to-leading order in T signals the need for resummations of perturbation theory.

We have discussed possible cancellation of gauge dependence at two-loop order in the case of the effective fermion mass and the damping rate for collective excitations at rest in QED. Our analysis suggests that cancellation of the leading gauge dependence at next-to-leading order in T is plausible. A detailed study at two-loop order that allows us to verify such cancellation in hot gauge theories is the subject of further investigations.

ACKNOWLEDGMENTS

The author thanks E. Mottola and G. C. Nayak for collaboration during the early stages of this work, illuminating discussions, and constant interest. He also thanks M. Carrington for discussions and D. Boyanovsky for discussions and carefully reading the manuscript. This work was supported by the U.S. Department of Energy under Contract No. W-7405-ENG-36.

APPENDIX: VACUUM CONTRIBUTION TO THE FERMION SELF-ENERGY

In this appendix, we calculate the vacuum contribution to the one-loop fermion self-energy for the sake of completeness. As a by-product, we show that the position of the singularities of the fermion propagator in vacuum is gauge independent at one-loop order.

The vacuum contribution to the imaginary part of the self-energy $\mathrm{Im}\,\Sigma$ is extracted from (12) and (14) by neglecting the thermal distribution functions. The angular integration can be done analytically in the same manner as that in the finite-temperature contribution, and the remaining radial integration over momentum is elementary. We find

$$\mathrm{Im}\,\Sigma^{\mathrm{vac}}_{\mathrm{FG},\pm}(\omega, k) = \mathrm{sgn}(\omega)\frac{e^2}{16\pi}(\omega \mp k)\theta(\omega^2 - k^2),$$
$$\mathrm{Im}\,\Sigma^{\mathrm{vac}}_{\xi,\pm}(\omega, k) = (\xi - 1)\,\mathrm{sgn}(\omega)\frac{e^2}{16\pi}(\omega \mp k)\theta(\omega^2 - k^2), \tag{A1}$$

where $\Sigma_\pm$ are defined in (21). We note that the gauge-independent and -dependent contribution can be combined together to yield a single gauge-dependent term proportional to ξ. As expected, $\mathrm{Im}\,\Sigma$ has support only above the light cone corresponding to the usual two-particle cuts.

The real part obtained from the imaginary one through the dispersive representation (8) is ultraviolet divergent and, hence, has to be regularized by an ultraviolet frequency cutoff. Introducing counterterms with the renormalization condition $\mathrm{Re}\,\Sigma^{\mathrm{vac}}(\mu) = 0$ at some arbitrary renormalization point μ, we obtain

$$\mathrm{Re}\,\Sigma^{\mathrm{vac}}_{\pm}(\omega, k) = \frac{\xi e^2}{16\pi^2}(\omega \mp k)\ln\frac{\mu^2}{|\omega^2 - k^2|}, \tag{A2}$$

where the gauge-independent and -dependent contributions have been combined together.

Upon substituting the above results into (20), we find that the putative *real* poles of the fermion propagator in vacuum are determined by

$$(\omega \mp k)\left[1 + \frac{\xi e^2}{16\pi^2}\ln\frac{\mu^2}{|\omega^2 - k^2|}\right] = 0. \tag{A3}$$

Because of the infrared logarithmic divergence at threshold associated with the emission of soft gauge boson, $\omega \to \pm k$ are no longer isolated poles but the end points of logarithmic branch cuts. Nevertheless, we clearly see that the position of the singularities (in this case logarithmic branch points and branch cuts) of the fermion propagator in vacuum is gauge independent at one-loop order.

If one adds the vacuum and the finite-temperature contributions, one finds that the real part of the vacuum contribution can be exactly combined with the $\ln T$ term in the real part of the finite-temperature contribution to yield a single term proportional to $\ln(T/\mu)$. This generalizes the $k = 0$ case found in Ref. [18] to the case of $k, \omega \ll T$ and is the fermionic counterpart of the results found in Refs. [4,20,21]. However, no similar combination can be found in the corresponding imaginary part of the fermion self-energy.

[1] E. Braaten and R. D. Pisarski, Nucl. Phys. **B337**, 569 (1990); **B339**, 310 (1990); R. D. Pisarski, Physica A (Amsterdam) **158**, 146 (1989); Phys. Rev. Lett. **63**, 1129 (1989); Nucl. Phys. **A525**, 175c (1991).

[2] J. Frenkel and J. C. Taylor, Nucl. Phys. **B334**, 199 (1990); **B374**, 156 (1992).

[3] O. K. Kalashnikov and V. V. Klimov, Sov. J. Nucl. Phys. **31**, 699 (1980); V. V. Klimov, *ibid.* **33**, 934 (1981); Sov. Phys. JETP **55**, 199 (1982).

[4] H. A. Weldon, Phys. Rev. D **26**, 1394 (1982).

[5] H. A. Weldon, Phys. Rev. D **26**, 2789 (1982).

[6] H. A. Weldon, Phys. Rev. D **40**, 2410 (1989); Physica A (Amsterdam) **158**, 169 (1989).

[7] E. Braaten and A. Nieto, Phys. Rev. D **53**, 3421 (1996); C. Zhai and B. M. Kastening, *ibid.* **52**, 7232 (1995); P. Arnold and C. Zhai, *ibid.* **51**, 1906 (1995); **50**, 7603 (1994).

[8] R. Kobes, G. Kunstatter, and A. Rebhan, Phys. Rev. Lett. **64**, 2992 (1990); Nucl. Phys. **B355**, 1 (1991). For a recent review, see A. Rebhan, Lect. Notes Phys. **583**, 161 (2002).

[9] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996).

[10] U. Kraemmer and A. Rebhan, Rep. Prog. Phys. **67**, 351 (2004).

[11] A. Arrizabalaga and J. Smit, Phys. Rev. D **66**, 065014 (2002).

[12] E. Mottola, hep-ph/0304279 [Proceedings of SEWM 2002 (World Scientific, Singapore, to be published)].

[13] M. E. Carrington, G. Kunstatter, and H. Zaraket, hep-ph/0309084.

[14] J. W. Harris and B. Muller, Annu. Rev. Nucl. Part. Sci. **46**, 71 (1996); P. Jacobs and X. N. Wang, hep-ph/0405125 [Prog. Part. Nucl. Phys. (to be published)], and references therein.

[15] A. Riotto and M. Trodden, Annu. Rev. Nucl. Part. Sci. **49**, 35 (1999), and references therein.

[16] See, e.g., G. G. Raffelt, *Stars as Laboratories for Fundamental Physics* (University of Chicago Press, Chicago, Illinois, 1996).

[17] A. Peshier, K. Schertler, and M. H. Thoma, Ann. Phys. (N.Y.) **266**, 162 (1998).

[18] I. Mitra, Phys. Rev. D **62**, 045023 (2000).

[19] H. A. Weldon, Phys. Rev. D **62**, 056003 (2000).

[20] F. T. Brandt, J. Frenkel, and J. C. Taylor, Phys. Rev. D **44**, 1801 (1991).

[21] F. T. Brandt and J. Frenkel, Phys. Rev. D **55**, 7808 (1997).

[22] S.-Y. Wang, D. Boyanovsky, H. J. de Vega, D.-S. Lee, and Y. J. Ng, Phys. Rev. D **61**, 065004 (2000); D. Boyanovsky, H. J. de Vega, D.-S. Lee, Y. J. Ng, and S.-Y. Wang, *ibid.* **59**, 105001 (1999).