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Extended holographic dark energy

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The idea of relating the infrared and ultraviolet cutoffs is applied to the Brans-Dicke theory of gravitation. We find that the Hubble scale or the particle horizon as the infrared cutoff will not give accelerating expansion. The dynamical cosmological constant with the event horizon as the infrared cutoff is a viable dark energy model.

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The type Ia supernova (SN Ia) observations suggest that the expansion of our universe is accelerating and dark energy contributes 2/3 to the critical density of the present universe [1,2]. SN Ia observations also provide the evidence of a decelerated universe in the recent past with the transition redshift $z_{q=0} \sim 0.5$ [3,4]. The cosmic background microwave observations support a spatially flat universe as predicted by the inflationary models [5,6]. The simplest candidate of dark energy is the cosmological constant. However, the unusually small value of the cosmological constant leads to the search for dynamical dark energy models [7,8]. For a review of dark energy models, see, for example, Ref. [8] and references therein. Cohen, Kaplan, and Nelson proposed that, for any state in the Hilbert space with energy E, the corresponding Schwarzschild radius $R_s \sim E$ is less than the infrared (IR) cutoff L [9]. Therefore, the maximum entropy is $S_{\rm BH}^{3/4}$. Under this assumption, a relationship between the ultraviolet cutoff and the infrared cutoff is derived, i.e., $8\pi G L^3 \rho_\Lambda/3 \le L$ [9,10]. So the holographic cosmological constant is

$$\rho_\Lambda = 3(8\pi G L^2)^{-1}. \tag{1}$$

Hsu found that the holographic cosmological constant model based on the Hubble scale as IR cutoff will not give an accelerating universe [11]. Li showed that the holographic dark energy model based on event horizon gave an accelerating universe; this model was also found to be consistent with current observations [12,13].

Einstein's theory of gravity may not describe gravity at very high energy. The simplest alternative to general relativity is Brans-Dicke scalar-tensor theory. The recent interest in scalar-tensor theories of gravity arises from inflationary cosmology, supergravity, and superstring theory. The dilaton field appears naturally in the low energy effective bosonic string theory. Scalar degree of freedom arises also upon compactification of higher dimensions. In this paper, we apply the holographic dark energy idea to Brans-Dicke cosmology.

The Brans-Dicke Lagrangian in the Jordan frame is given by

$$\mathcal{L}_{\rm BD} = \frac{\sqrt{-g}}{16\pi}\left[\phi R - \omega g^{\mu\nu}\frac{\partial_\mu\phi\partial_\nu\phi}{\phi}\right] - \mathcal{L}_m(\psi, g_{\mu\nu}). \tag{2}$$

In the Jordan frame, the matter minimally couples to the metric and there is no interaction between the scalar field ϕ and the matter field ψ. Here we work on the Jordan frame so that test particles follow geodesic motion. The gravitational part of the above Lagrangian (2) is conformal invariant under the conformal transformations

$$\gamma_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega = \phi^\lambda \qquad \left(\lambda \neq \frac{1}{2}\right),$$
$$\sigma = \phi^{1-2\lambda}, \qquad \bar{\omega} = \frac{\omega - 6\lambda(\lambda-1)}{(2\lambda-1)^2}.$$

Note that the matter Lagrangian $\mathcal{L}_m(\psi, g_{\mu\nu})$ in Eq. (2) is not conformal invariant under the above conformal transformations. For the case $\lambda = 1/2$, we make the following transformations:

$$\gamma_{\mu\nu} = e^{\alpha\sigma} g_{\mu\nu}, \tag{3}$$

$$\phi = \frac{8\pi}{\kappa^2} e^{\alpha\sigma}, \tag{4}$$

where $\kappa^2 = 8\pi G$, $\alpha = \beta\kappa$, and $\beta^2 = 2/(2\omega + 3)$. Remember that the Jordan-Brans-Dicke Lagrangian is not invariant under the above transformations (3) and (4). The homogeneous and isotropic Friedmann-Robertson-Walker (FRW) space-time metric is

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega\right]. \tag{5}$$

Based on the flat FRW metric and the perfect fluid $T_m^{\mu\nu} = (\rho + p)U^\mu U^\nu + p g^{\mu\nu}$ as the matter source, we can get the evolution equations of the universe from the action (2):

$$H^2 + H\frac{\dot\phi}{\phi} - \frac{\omega}{6}\left(\frac{\dot\phi}{\phi}\right)^2 = \frac{8\pi}{3\phi}\rho, \tag{6}$$

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$$\ddot{\phi} + 3H\dot{\phi} = 4\pi\beta^2(\rho - 3p), \tag{7}$$

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{8}$$

For ordinary pressureless dust matter, $p_m = 0$, we have $\rho_m a^3 = \rho_{m0} a_0^3$; here subscript 0 means the current value. During matter dominated epoch, we can get power-law solutions to Eqs. (6) and (7):

$$a(t) = a_0 t^p, \qquad \phi(t) = \phi_0 t^q, \tag{9}$$

where

$$p = \frac{2 + 2\omega}{4 + 3\omega}, \qquad q = \frac{2}{4 + 3\omega}, \tag{10}$$

and $[q(q-1) + 3pq]\phi_0 = 4\pi\beta^2\rho_{m0}$. We set $t_0 = 1$.

In Brans-Dicke theory, the scalar field ϕ takes the role of $1/G$, so we propose to modify the holographic dark energy Eq. (1) as

$$\rho_\Lambda = \frac{3\phi}{8\pi L^2}. \tag{11}$$

Now let us consider the dark energy dominated universe. First, we choose $L = H^{-1}$. Substituting the relation to Eq. (11), we can get the solution to Eq. (6),

$$\frac{\phi}{\phi_0} = \left(\frac{a}{a_0}\right)^{6/\omega}. \tag{12}$$

Combining Eqs. (11) and (12) with Eqs. (7) and (8), we can get the following power-law solutions:

$$a(t) = a_0 t^{\omega/(4\omega+6)}, \tag{13}$$

$$\phi(t) = \phi_0 t^{3/(2\omega+3)}, \tag{14}$$

$$\rho_\Lambda = \frac{3\omega^2}{8\pi(4\omega + 6)^2}\phi_0\left(\frac{a}{a_0}\right)^{-2(4\omega+3)/\omega}. \tag{15}$$

To get accelerating expansion, we must require $-2 < \omega < 0$. Even though the low energy effective theory of the string theory can lead to $\omega = -1$, the current classical experimental constraints on ω is $\omega > 500$, so the choice of Hubble scale as the IR cutoff cannot give an accelerating universe. Next, we choose the particle horizon as the IR cutoff. The particle horizon was proposed by Fischler and Susskind to apply the holographic principle to cosmology [14]. In Ref. [15], it was also shown that the holographic principle by using the particle horizon was applicable in Brans-Dicke cosmology. With the choice of particle horizon, we get

$$L = R_{\rm H} = a(t)\int_0^t \frac{d\tilde{t}}{a(\tilde{t})}, \tag{16}$$

$$\rho_\Lambda = \frac{3\phi}{8\pi R_{\rm H}^2}. \tag{17}$$

Substitute this holographic dark energy into Eqs. (6)–(8) and look for power-law solutions $a(t) = a_0 t^p$ and $\phi(t) = \phi_0 t^q$. From these power-law solutions, we get the particle horizon $R_{\rm H} = t/(1-p)$. Substituting these solutions to Eqs. (6)–(8), we get

$$p(q + 2) = \frac{\omega}{6}q^2 + 1, \tag{18}$$

$$(2\omega + 3)pq(3p + q - 1) = (p - 1)^2(12p + q - 2). \tag{19}$$

The solutions to Eqs. (18) and (19) are $p \sim 1/2$ and $q \sim 4/(2\omega + 3)$. Therefore, the expansion of the universe is not accelerating. In Table I we list some numerical solutions of p and q. From the power-law solutions, it is easy to see that an accelerating expansion requires $p > 1$. However, the particle horizon gives $p < 1$. Therefore, the choice of particle horizon as the IR cutoff does not give an accelerating expansion. Finally, we consider the event horizon as the IR cutoff.

TABLE I. The values of p and q for different ω.

ω	10	50	100	500	600	800	1000
p	0.483	0.496	0.498	0.4997	0.4997	0.4998	0.4998
q	0.148	0.037	0.019	0.004	0.0033	0.0025	0.002

$$L = R_{\rm h} = a(t)\int_t^\infty \frac{d\tilde{t}}{a(\tilde{t})}, \tag{20}$$

$$\rho_\Lambda = \frac{3\phi}{8\pi R_{\rm h}^2}. \tag{21}$$

To solve Eqs. (6)–(8) with $\rho = \rho_\Lambda$, we assume that $\phi/\phi_0 = (a/a_0)^\alpha$. Substituting this relation into Eq. (6), we get

$$\frac{H}{H_0} = \left(\frac{a}{a_0}\right)^{c-1},$$

where $c = \sqrt{1 + \alpha - \omega\alpha^2/6}$. So

$$\rho_\Lambda = \frac{3\phi_0 H_0^2}{8\pi}c^2\left(\frac{a}{a_0}\right)^{2c+\alpha-2}.$$

Combining this solution with Eqs. (7) and (8), we get the following equation for α:

$$(2\omega + 3)\alpha(\alpha + c + 2) = 3c^2(\alpha + 2c + 2). \tag{22}$$

TABLE II. The values of c and α for different ω.

ω	10	50	100	500	600	800	1000
c	1.064	1.013	1.007	1.001	1.001	1.0008	1.0007
α	0.196	0.0398	0.01996	0.004	0.0033	0.0025	0.002

So $\alpha \sim 4/(2\omega + 3)$ and $c \sim 1$. The numerical solutions to Eq. (22) for different ω are shown in Table II. Therefore, the event horizon gives an accelerating universe. In fact, this result is expected, because Brans-Dicke cosmology becomes standard cosmology when $\omega \rightarrow \infty$. We know that in standard cosmology the Hubble scale and the particle horizon do not provide the holographic dark energy, but the event horizon gives the holographic dark energy which drives the accelerating expansion of our universe. Therefore, the event horizon as the IR cutoff should provide the extended holographic dark energy in Brans-Dicke cosmology.

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