Space-time geometry and thermodynamic properties of a self-gravitating ball of fluid in phase transition

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A numerical solution of Einstein field equations for a spherical symmetric and stationary system of identical and autogravitating particles in phase transition is presented. The fluid possesses a perfect fluid energy-momentum tensor, and the internal interactions of the system are represented by a van der Walls-like equation of state, able to describe a first order phase transition of the type gas-liquid. We find that the space-time curvature, the radial component of the metric, and the pressure and density show discontinuities in their radial derivatives in the phase coexistence region. This region is found to be a spherical surface concentric with the star, and the system can be thought of as a foliation of acronal, concentric and isobaric surfaces in which the coexistence of phases occurs in only one of these surfaces. This kind of system can be used to represent a star with a high energy density core and low energy density mantle in hydrodynamic equilibrium.

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I. INTRODUCTION

In relativistic astrophysics and pure general relativity, the concept of "phase transition" is usually associated with very different scenarios. In general relativity we have the spherically symmetric collapse of a massless scalar field, or Yang-Mills field, that leads to a formation of a black hole [1,2]. In the astrophysical context we have first order phase transition in neutron-star models, where we found matter condensation from neutron matter to pion-condensed matter and quark matter [3–5]. Also we can have phase transitions in the core of a rotating neutron star [6]. In the last few years, the phase transition phenomenon has become important in relativistic astrophysics since we can have emission of gravitational waves [7,8] associated with this transition. Observational evidence of this phenomenon can be attained [9] with the present laser interferometer gravitational wave-observatory (LIGO). The LIGO-I maximum distance for detection of gravitational waves is about 6.4 Mpc, well beyond the Andromeda Galaxy (M31), whereas the next generation LIGO-II detectors will probably see phase transition events at distances 2 times longer [8].

The model studied in this work is indirectly related to the topics mentioned above. Our aim is to study the behavior of the thermodynamic properties and the space-time structure in a simple albeit important astrophysical object, a self-gravitating ball of fluid performing a generic first order phase transition. In particular, the Riemann-Christoffel curvature tensor is studied in some detail for the space-time associated the different regions of the matter that presents a first order phase transition.

To model this system in the context of General Relativity, we used the TOV equations (Tolman-Oppenheimer-Volkoff) [10,11]. The TOV equations represent the Einstein field equations for a stationary, spherical symmetric system with a perfect fluid energy-momentum tensor and a local equation of state.

The most simple and general equation of state to model a phase transition in a classical system of particles is the van der Walls equation of state together with the Maxwell energy balance construction. The Maxwell energy balance construction has been studied in a rigorous treatment in systems with two-body interaction potential proving the equal-area rule at the transition region in the equation of state [12]. Also, the nonanalyticity of the van der Walls equation of state together with the Maxwell energy balance construction appears to be generic for first order phase transitions originated in microscopic short range forces [13,14].

To solve the TOV equations one needs a smooth equation of state. In order to overcome the fact that in a first order phase transition, we will have a nonanalytic equation of state, we shall perform several smoothing interpolations. Afterwards we show that the real situation can be obtained as a limit that is independent of the used interpolations.

The article is organized as follows, in Sec. II we present the basic equations that model our system, in particular, we study the TOV equations, and present a van der Walls-like equation of state that models a phase transition. In Sec. III, we analyze the special case of

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constant pressure. This case needs to be treated in a different form because, strictly, the TOV equations are not valid in this case. In Sec. IV, we present the behavior of the thermodynamic variables, the metric components, the Kretschmann curvature invariant, and the components of the Riemann-Christoffel curvature tensor inside the star undergoing a phase transition. Finally, in Sec. V we summarized our results.

II. STELLAR MODEL FOR A SYSTEM IN PHASE TRANSITION

We consider a static spherical symmetric space-time described by the line element

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (1)$$

where $\nu = \nu(r)$ and $\lambda = \lambda(r)$. The matter is represented by the perfect fluid energy-momentum tensor

$$T_{\alpha\beta} = \rho u_\alpha u_\beta + \frac{p}{c^2}(u_\alpha u_\beta - g_{\alpha\beta}), \qquad (2)$$

where ρ is the energy density, u^α is the four-velocity, c is the speed of light, and p is the pressure measured in the local rest frame. We also assume that the pressure and the energy density are related by an equation of state of the form $p = p(\rho)$.

The Einstein field equations,

$$R_{\alpha\beta} = -\zeta\left(T_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} T\right), \qquad (3)$$

where $\zeta = \frac{8\pi G}{c^2}$ and G is the gravitational constant, can be written with the help of (1) and (2) as a set of three equations,

$$\begin{aligned} e^{-\lambda}\left[-\frac{1}{2}\nu'' + \frac{1}{4}\lambda'\nu' - \frac{1}{4}(\nu')^2 - \frac{1}{r}\nu'\right] &= -\zeta\left(\frac{\rho}{2} + \frac{3}{2}\frac{p}{c^2}\right), \\ e^{-\lambda}\left[\frac{1}{2}\nu'' - \frac{1}{4}\lambda'\nu' + \frac{1}{4}(\nu')^2 - \frac{1}{r}\lambda'\right] &= -\zeta\left(\frac{\rho}{2} - \frac{1}{2}\frac{p}{c^2}\right), \\ e^{-\lambda}\left[\frac{1}{r^2} + \frac{1}{2r}(\nu' - \lambda')\right] - \frac{1}{r^2} &= -\zeta\left(\frac{\rho}{2} - \frac{1}{2}\frac{p}{c^2}\right), \end{aligned} \qquad (4)$$

in which $'$ denotes differentiation with respect to the coordinate r. To write the equations above in a simpler form we use the function $m(r)$, defined by

$$m(r) = \frac{r}{2}(1 - e^{-\lambda}). \qquad (5)$$

$m(r) = GM(r)/c^2$ is the geometric mass which has dimensions of distance and $M(r)$ is the quantity of mass inside a sphere of radius r. With this change of variable, we obtain the system of equations known as the TOV equations,

$$p = p(\rho), \qquad (6)$$

$$m'(r) = \frac{4\pi G}{c^2} r^2 \rho, \qquad (7)$$

$$p' = -\frac{(\rho + \frac{p}{c^2})(m + \frac{4\pi G}{c^4} r^3 p)c^2}{r(r - 2m)}, \qquad (8)$$

$$e^{-\lambda} = 1 - \frac{2m(r)}{r}, \qquad (9)$$

$$\nu' = -\frac{2p'}{\rho c^2 + p}. \qquad (10)$$

The first three equations allow us to solve for the geometric mass $m(r)$, the pressure $p(r)$, and the energy density $\rho(r)$, i.e., the thermodynamic quantities of the system. The last two equations define the space-time geometry through the functions $\lambda(r)$ and $\nu(r)$ present in the metric.

The equation of state (6) depends on the particle interactions of the system. An equation of state that allows a phase transition is a van der Walls-like equation of state of the form

$$p = \frac{kT\rho}{1 - b\rho} - a\rho^2, \qquad (11)$$

where a and b are characteristic parameters of a particular physical system. To solve the TOV equations using the equation of state (11), we have to specify the type of particle considered through the parameters a and b. Using the law of the corresponding states is possible to obtain an equation of state which is independent of these parameters, in other words, valid for any "real gas." This equation of state in the so-called reduced quantities (p_r, T_r, ρ_r) is

$$p_r = \frac{8T_r\rho_r}{3 - \rho_r} - 3\rho_r^2, \qquad (12)$$

with p_r, T_r and ρ_r defined by the adimensional relations

$$p_r = \frac{p}{p_c}, \qquad \rho_r = \frac{\rho}{\rho_c}, \qquad T_r = \frac{T}{T_c}, \qquad (13)$$

where $(p_c, \rho_c, T_c) = (\frac{a}{27b^2}, \frac{1}{3b}, \frac{8a}{27kb})$ are the values of the pressure, energy density and temperature in which the first and second derivative of (12) become zero simultaneously. Also $0 \leq \rho_r < 3$. In Fig. 1 we show a typical van der Walls-like isothermal for a system in phase transition (gas-liquid) in reduced coordinates. We know that the van der Walls-like equation of state well describes the regions of gas and liquid but in the region of coexistence it presents physical inconsistencies, e.g., in the transition region we do not have dynamical equilibrium between the phases. Therefore, we modify the equation of state (12) using the Maxwell energy balance construction [15]. The Maxwell energy balance construction allows us to calculate the pressure of coexistence p_0 and the coexistence

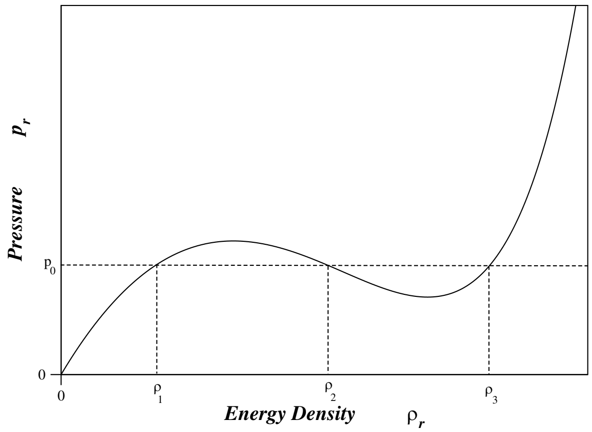

FIG. 1. A typical van der Walls isothermal for a system in gas-liquid phase transition using reduced quantities. The coexistence pressure p_0 and the coexistence region $\rho_1 \leq \rho_r \leq \rho_3$ is calculated using the Maxwell construction.

energy density interval $[\rho_1, \rho_3]$ through the system of equations

$$\int_{\rho_1}^{\rho_3} p_r(\rho_r)d\rho_r = p_0(\rho_3 - \rho_1), \tag{14}$$

$$\frac{8T_r\rho_1}{3-\rho_1} - 3\rho_1^2 = p_0 = \frac{8T_r\rho_3}{3-\rho_3} - 3\rho_3^2.$$

First we fix a temperature value T_r and then we solve the system of Eqs. (14) (e.g., with the Newton-Raphson method for nonlinear systems). Once p_0 is found, we can say that the state equations for our system in phase transition between gas and liquid is of the form

$$p_r = \begin{cases} \frac{8T_r\rho_r}{3-\rho_r} - 3\rho_r^2 & 0 \leq \rho_r \leq \rho_1 \quad \text{(gas)} \\ p_0 & \rho_1 \leq \rho_r \leq \rho_3 \quad \text{(coexistence)} \\ \frac{8T_r\rho_r}{3-\rho_r} - 3\rho_r^2 & \rho_3 \leq \rho_r < 3 \quad \text{(liquid)} \end{cases}. \tag{15}$$

We note that the equation of state (15) is not of class C^2 because its derivatives have discontinuities in $\rho = \rho_1$ and $\rho = \rho_3$. So, the TOV equations can not be solved around these energy densities. Furthermore, in the coexistence region, the equation of state satisfies the equation $p_r' = 0$ that brings no physical solutions, e.g., negative masses [see Eq. (8)]. For this reason, the case of constant pressure has to be studied independently. The problem of smoothness mentioned above can be solved using an equation of state of class C^2 or smoother instead of (15). This is done redefining the equation of state (15) in the region of coexistence using a virial-like equation to make, at least, the joints in ρ_1 and ρ_3 of class C^2. The line $p_r =$ constant in the coexistence region can be approximated by a straight line with a small slope angle (α). In this way, our equation of state for the system becomes

$$p_r = \begin{cases} \frac{8T_r\rho_r}{3-\rho_r} - 3\rho_r^2 & 0 \leq \rho_r \leq \rho_1 - \chi \\ \sum_{n=0}^{5} a_n\rho_r^n & \rho_1 - \chi \leq \rho_r \leq \rho_1 + \chi \\ \sum_{n=0}^{1} b_n\rho_r^n & \rho_1 + \chi \leq \rho_r \leq \rho_3 - \chi \\ \sum_{n=0}^{5} c_n\rho_r^n & \rho_3 - \chi \leq \rho_r \leq \rho_3 + \chi \\ \frac{8T_r\rho_r}{3-\rho_r} - 3\rho_r^2 & \rho_3 + \chi \leq \rho_r < 3 \end{cases}, \tag{16}$$

where a_n, b_n, c_n are constants to be determined, and 2χ is the interval for the coupling polynomials. In our case we define the coefficients b_n so that the straight line passes through the point p_0 at the middle of the interval $[\rho_1 + \chi, \rho_3 - \chi]$. In this way

$$b_0 = p_0 - \left(\frac{\rho_3 + \rho_1}{\rho_3 - \rho_1 - 2\chi}\right)\xi, \qquad b_1 = \frac{2\xi}{\rho_3 - \rho_1 - 2\chi}, \tag{17}$$

where $\xi \ll 1$ is directly associated with the angle $\alpha[\tan\alpha = 2\xi/(\rho_3 - \rho_1 - 2\chi)]$ of the straight line in the transition region. The polynomial coefficients a_n and c_n are calculated using the six boundary conditions given by the continuity of the pressure, its first derivative, and its second derivatives at the junctions. We found that our C^2 class curve defined in this way is a good approximation for the case of a gas-liquid transition. The analysis of the case with $p =$ constant will be discussed in Sec. .III

For generality's sake, we want the system of Eqs. (6)–(10) to be independent on the characteristic parameters of the gas. We introduce new variables $\hat{r}$ and $\hat{F}$ defined as

$$r = \Lambda\hat{r}, \qquad m = \Lambda\hat{F}\hat{r}^2, \tag{18}$$

where the quantity Λ is a scale factor. Furthermore we fix our units by choosing

$$\frac{a\rho_c}{c^2} = 1, \qquad \frac{4\pi\rho_c G}{c^2}\Lambda^2 \equiv 1. \tag{19}$$

Substituting these expressions in Eqs. (6), (7) and (8), the internal structure of the star undergoing a phase transition is found solving the system of equations

$$\frac{\partial\rho_r}{\partial\hat{r}} = -\frac{1}{(1 - 2\hat{r}\hat{F})}\left[\hat{F} + \frac{\hat{r}p_r}{3}\right][3\rho_r + p_r]\left[\frac{\partial p_r}{\partial\rho_r}\right]^{-1}$$
$$\frac{\partial\hat{F}}{\partial\hat{r}} = \rho_r - 2\frac{\hat{F}}{\hat{r}}, \tag{20}$$

where for each region of the star we have to use the isothermal corresponding to the temperature that allows a phase transition.

III. ANALYSIS OF THE CASE OF CONSTANT PRESSURE

To perform the integration of the TOV equations through the discontinuities of the coexistence region, we use the class C^2 equation of state (16) with the constants of the coupling polynomials calculated using the six boundary conditions mentioned in Sec. II. The coexistence region is modeled, in first approximation, by a straight line with a small slope angle ($\alpha \approx 10^{-3}$). Then we let the angle α go to zero to assure that the abrupt change presented in the energy density figure (and other figures that depend on the energy density) are indeed vertical lines. The same result can be obtained in a different way. Knowing the pressure and energy densities interval for the coexistence region (e.g., from the Maxwell construction) one can solve the TOV equation independently for the liquid phase as well as for the gas phase. Then the end points are joined with a straight line. With this second method we never integrate the TOV equations through the discontinuities. Both methods give the same results, we use the first one because in our opinion the results appear in a more transparent way.

To determine the behavior of the energy density in the coexistence region and the radial interval in which the transition occurs in the limit $\alpha \rightarrow 0$, we define a straight line passing through the points (ρ_i, p_0) and $(\rho_f, p_0 + \epsilon)$, where we see that for a small $\epsilon[\epsilon = (\rho_f - \rho_i)\tan(\alpha)]$ a good approximation of the line $p_r = p_0 =$ constant is found. We choose arbitrary values for the energy density

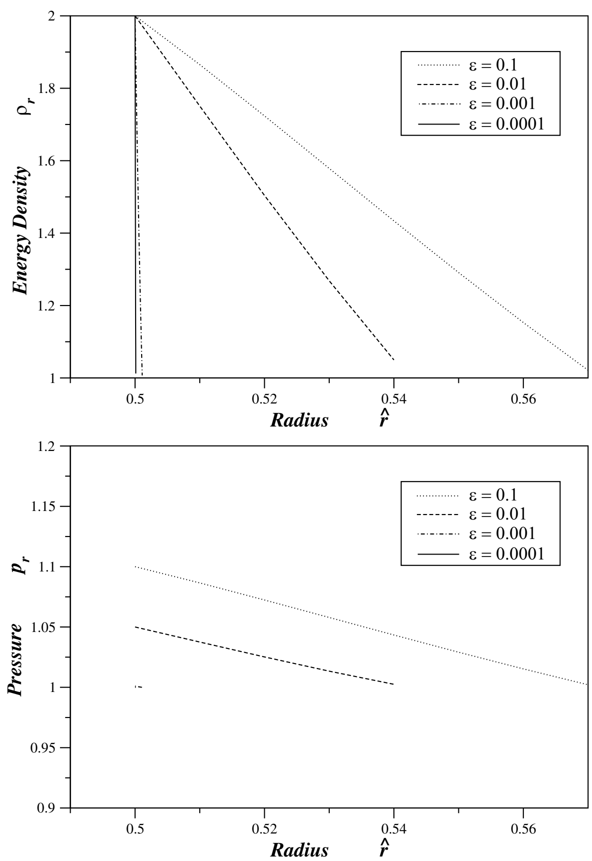

FIG. 2. Analysis of the energy density and pressure profiles. We see from the graphs that in the limit when $\epsilon \rightarrow 0$ ($p_r =$ constant), the energy density profile tends to a vertical line at a fixed radius while the radial domain of the pressure profile becomes a point.

interval $(\rho_i, \rho_f) = (1, 2)$ and pressure $p_0 = 1$, with these values the equation of state in the coexistence region is of the form

$$p_r = 1 + \epsilon(\rho_r - 1) \qquad 1 \leq \rho_r \leq 2. \tag{21}$$

When $\epsilon \rightarrow 0$ we obtain the case of constant pressure $p_r = 1$. In Fig. 2 we solved numerically the TOV Eqs. (20) with the equation of state (21) for different values of ϵ. In the limit $\epsilon \rightarrow 0$ we can affirm that the transition occurs only in a concentric spherical surface of the star. This means that in the limit of $\epsilon \rightarrow 0$ for a given equation of state of the type (16) with (21), the TOV system brings us a solution of a star model that consists in a foliation of spherical, isobaric, and acronal surfaces. This is in accordance with the stationary hypothesis made for the model, because if the transition takes place in a radial interval instead of a fixed radius, the different gravitational forces at a different radius will make a nonstationary system.

IV. SOLUTION OF TOV EQUATIONS FOR A SYSTEM IN PHASE TRANSITION

A. Thermodynamic properties of the star

For didactic purposes we choose the isothermal with reduced temperature $T_r = 0.9$ and parameters ($\chi = 0.005$, $\xi = 0.00005$) to study the thermodynamic properties and the space-time behavior. It can be verified that all the isotherms that allow a phase transition have the same qualitative behavior. With the values of the coexistence pressure $p_0 = 0.583$ and the energy density coexistence interval $(\rho_1, \rho_3) = (0.350, 1.623)$ found from (14), we can solve numerically the system of Eqs. (20). The inner boundary condition can be set recalling that $m(r)$ is proportional to the mass inside the sphere of radius r, $m(r = 0) = 0$. So, the inner boundary condition for the new variables is $(\hat{r}, \rho_0, \hat{F}_0) = (0, 2, 0)$, which represents a liquid core of high energy density, and the value $\rho_0 = 2$ is set arbitrarily. In Fig. 3, we show the numerical solution of the TOV equations for the thermodynamic quantities in the star with phase transition. The phase transition shows up like a sudden break in the energy density at $r \approx 0.6\Lambda$. Looking at the energy density plot we can think of the high dense region with $r \lesssim 0.6\Lambda$ as a liquid core and the region with small density $r \gtrsim 0.6\Lambda$ as a gas mantle. The pressure has a discontinuity in its radial derivative at $r \approx 0.6\Lambda$ and presents the expected behavior, i.e., high and low pressures appear in the high energy density (core) and low energy density (mantle) regions, respectively. The geometric mass increases rapidly in the liquid region and slowly in the gas region, it also presents a discontinuity in its radial derivative at the same radius of the star. We note that the energy density and the pressure go to zero for large values of $\hat{r}$, so in principle we can match continuously to a Schwarzschild free-space solution, as was done in neutron-star models [16], or to an exterior dust fluid (with zero pressure).

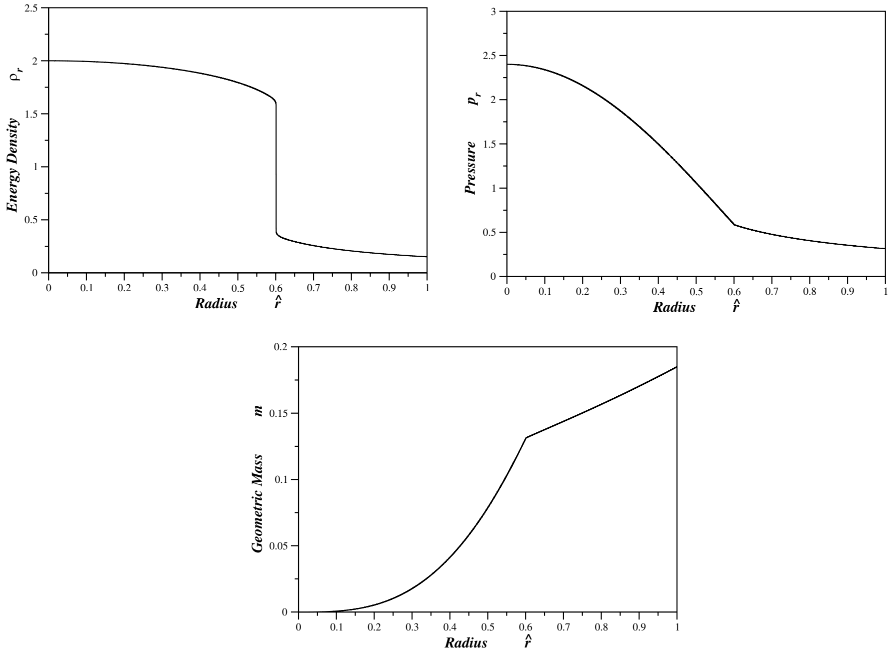

FIG. 3. Energy density, pressure, and geometric mass profiles for the interior of a star in phase transition. The discontinuities in their radial derivatives at $\hat{r} \approx 0.6$ represent the change of phases from a liquid core (high energy density) to a gas mantle (low energy density).

B. Space-time geometry in a star in phase transition

The metric components in the star are found through the functions (ν, λ) from Eqs. (9) and (10). The integration constant from Eq. (10) can be obtained by imposing that the component g_{00} at a distant radius has to be equal to the Schwarzschild exterior solution

$$g_{00}(r_s) = 1 - \frac{2m(r_s)}{r_s}, \tag{22}$$

where r_s is the radius of the star. The component g_{11} is calculated directly from Eq. (9).

These components are depicted in Fig. 4. We note in this figure that the component g_{00} of the metric does not present discontinuities in the phase transition region, as found in the radial derivative of the g_{11} component at $r \approx 0.6\Lambda$. This motivates us to think that, in general, the phase transition could be manifested only in the spatial part of the metric.

C. Curvature associated with the phase transition

To study the space-time curvature in a system with phase transition, we use the Kretschmann invariant ($K = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$). With the help of relations (4), we find

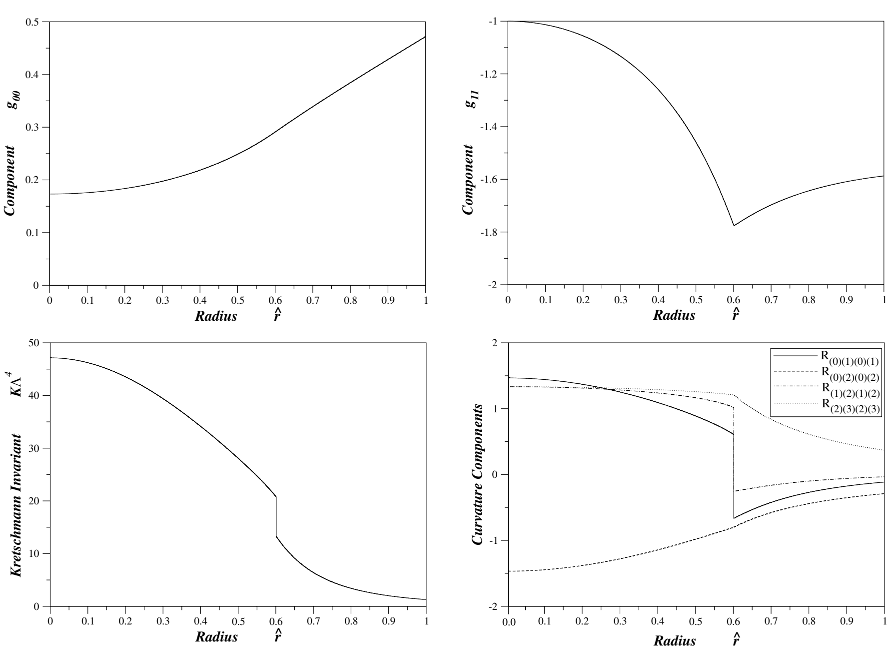

FIG. 4. Behavior of the g_{00} and g_{11} components of the metric, the Kretschmann curvature invariant, and the components of the Riemann-Christoffel tensor in a star in phase transition. We note that the phase transition is manifested only in the spatial component of the metric. The Kretschmann invariant and curvature components $R_{(1)(2)(1)(2)}$ and $R_{(0)(1)(0)(1)}$ experience a sudden change, this can be interpreted as a curvature transition inside the star due to the loss of energy needed for the phase transition to occur.

that the Kretschamnn invariant can be written as

$$K = \frac{4}{\Lambda^4}\left[\left(\rho_r + \frac{p_r}{3} - 2\frac{\hat{F}}{r}\right)^2 + 2\left(\rho_r - \frac{\hat{F}}{r}\right)^2 + 2\left(\frac{p_r}{3} + \frac{\hat{F}}{r}\right)^2 + 4\left(\frac{\hat{F}}{r}\right)^2\right]. \quad (23)$$

In Fig. 4 we see that the Kretschmann invariant presents a similar behavior to the energy density, i.e., highest curvature values in the core of the star and at long distances the curvature tends to zero. The Kretschmann invariant depends on the values of the pressure, energy density, and geometric mass, so it is normal to expect a discontinuity in the coexistence region at the same sphere radius ($r \approx 0.6\Lambda$). For a better understanding of the curvature associated to the phase transition, we project the Riemann tensor along the orthonormal tetrad $\xi^\alpha_{(a)}$, where α is the contravariant tensor index and (a) is a label distinguishing the particular vector. So, we have

$$R_{(a)(b)(c)(d)} = R_{\alpha\beta\gamma\delta}\xi^\alpha_{(a)}\xi^\beta_{(b)}\xi^\gamma_{(c)}\xi^\delta_{(d)}. \quad (24)$$

In our case the orthonormal tetrad is

$$\xi^\alpha_{(0)} = (e^{-\nu/2}, 0, 0, 0), \qquad \xi^\alpha_{(1)} = (0, e^{-\lambda/2}, 0, 0),$$
$$\xi^\alpha_{(2)} = (0, 0, r^{-1}, 0), \qquad \xi^\alpha_{(3)} = (0, 0, 0, [r\sin(\theta)]^{-1}), \quad (25)$$

and we obtain the following non null components of the Riemann tensor:

$$\begin{aligned} R_{(0)(1)(0)(1)} &= \frac{1}{\Lambda^2}\left(\rho_r + \frac{p_r}{3} - \frac{2\hat{F}}{\hat{r}}\right), \\ R_{(0)(3)(0)(3)} &= R_{(0)(2)(0)(2)} = -\frac{1}{\Lambda^2}\left(\frac{\hat{F}}{\hat{r}} + \frac{p_r}{3}\right), \\ R_{(1)(3)(1)(3)} &= R_{(1)(2)(1)(2)} = \frac{1}{\Lambda^2}\left(\rho_r - \frac{\hat{F}}{\hat{r}}\right), \\ R_{(2)(3)(2)(3)} &= \frac{1}{\Lambda^2}\left(\frac{2\hat{F}}{\hat{r}}\right). \end{aligned} \quad (26)$$

These components are plotted in Fig. 4. We see that all the components are functions of the class C^0. We have a discontinuous radial derivative at $r \approx 0.6\Lambda$. In particular, the components $R_{(1)(2)(1)(2)}$ and $R_{(0)(1)(0)(1)}$ present the same abrupt change as the Kretschmann scalar. The Kretschmann scalar can be written as a function of the nonzero components of the Riemann-Christoffel curvature tensor (26). So, we can assure that the abrupt change in the Kretschmann scalar is due to the components $R_{(1)(2)(1)(2)}$ and $R_{(0)(1)(0)(1)}$. These two components depend on the reduced energy density ρ_r. So, we can state that the abrupt change in the Kretschmann scalar can be thought of as a curvature transition in the star due to the loss of energy needed for the phase transition to occur (see the energy density profile from Fig. 3). We note that discontinuities in the scalar curvature are not rare phenomena in nature. One simple example of this discontinuity is provided by a simple star model described in its exterior by the Schwarzschild solution and its interior by the solutions corresponding to a homogeneous sphere [17]. In this case, there exists a discontinuity in the scalar curvature mainly because the energy density has a discontinuity, constant inside the star and zero outside.

V. CONCLUSIONS

We studied a model of a stationary spherical symmetric system in phase transition. The system is a ball of fluid made with identical and autogravitating particles with the energy-momentum tensor of a perfect fluid that obeys a van der Walls-like equation of state. This model appears to be the most simple and general model to describe a stellar object that undergoes a phase transition. The phase transition in the thermodynamic quantities is manifested as an abrupt change in the energy density, and discontinuities in the radial derivatives of the pressure and geometric mass at a fixed radius. This radius divides the star in two regions: one with a high energy density (a liquid core), and the other with a low energy density (a gas mantle). In the space-time geometry of the star, we see an abrupt change in the Kretschmann invariant and some components of the Riemann tensor. These can be thought of as a curvature transition process due to the loss of energy needed for the phase transition to occur. A discontinuity in the radial derivative is also present in the spatial component of the metric but not in its time component, this leads us to think that phase transitions could be manifested only in the spatial part of the metric. For the results obtained in this work, we interpret the stationary system studied as a foliation of concentric and isobaric spherical surfaces where the phase transition occurs in only one of these surfaces. We note that the procedure used in this article can be applied to other models, such as systems with different kind of particles and systems characterized with more than one phase transition.

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