Early reionization by cosmic strings reexamined

Levon Pogosian and Alexander Vilenkin
Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155, USA
(Received 17 June 2004; published 20 September 2004)

Measurements of the Cosmic Microwave Background (CMB) temperature anisotropy and the temperature-polarization cross correlation by Wilkinson Microwave Anistropy Probe (WMAP) suggest a reionization redshift of $z \sim 17 \pm 5$. On the other hand, observations of high redshift galaxies indicate a presence of a significant fraction of neutral hydrogen at redshift $z \sim 6 - 7$. We show that cosmic strings with tensions well within, but not far from, current observation bounds could cause early star formation at a level sufficient to explain the high reionization redshift.

DOI: 10.1103/PhysRevD.70.063523 PACS numbers: 98.80.Cq

Most of the baryonic matter in the present day universe is in the form of ionized plasma. Yet we know that the universe was neutral immediately after the recombination. Therefore, sometime between now and redshift $z \sim 10^3$ the universe was reionized. The detailed history of reionization is a mystery and has become a particularly hot topic of discussion following the release of Wilkinson Microwave Anistropy Probe's (WMAP) first year data. The WMAP team has reported an optical depth to the last scattering $\tau = 0.17 \pm 0.06$, which for the instantaneous and homogeneous reionization implies a redshift of $z \sim 17 \pm 5$ [1,2]. The precise value of the reionization redshift inferred from Cosmic Microwave Background (CMB) data depends on the assumed ionization history and a more conservative interpretation of WMAP's result is that the measured optical depth to last scattering is consistent with a significant ionized fraction at redshifts $z \sim 11 - 30$ [2].

The commonly accepted general picture of reionization is that it was caused by first stars and quasars emitting high energy photons. In order for the universe to be significantly ionized by redshift $z_r \sim 20$ a sufficiently large fraction of matter would need to be collapsed to form stars prior to z_r. It remains to be seen if collapse of Gaussian initial inhomogeneities can produce the required number of reionizing stars formed by such early time [3].

To add to the puzzle, the WMAP value of the reionization redshift is significantly higher than the value of $z_r \sim 6 - 7$ which one would infer from measurements of Lyman-α absorption in the spectra of high redshift galaxies (e.g., see [4]). This could mean that the ionized fraction evolved nonmonotonically with redshift and involved processes more complex than previously thought [5–9].

In this paper we investigate the possibility that early reionization was triggered by stars formed in the wakes of moving cosmic strings. The idea that strings could cause reionization has been discussed in a number of papers [10–12]. Refs. [10,11] were written over a decade ago and used cosmological parameters that were significantly different from the ones we know now. Also, at the time those papers were written, cosmic strings were considered a prime candidate to explain the formation of large scale structure and were generally attributed a higher value of mass per unit length then currently allowed by observations. In Ref. [12] stings are represented by a spectrum of linear perturbations. It is not clear to us why this should give an adequate description, since we know that the inhomogeneities induced by strings on scales relevant to reionization are highly nonlinear in character [1]. Here, we reinvestigate the reionization by strings in the light of the current data, and with nonlinear string dynamics fully taken into account.

Another motivation for this study is the recent realization that cosmic strings (or D-branes) are naturally produced in many brane inflation scenarios, motivated by String/M Theory [14–18]. In these theories strings can move and interact in extra dimensions, in addition to the observed three spatial dimensions. In particular, while appearing to intersect in our three dimensions, they may actually miss each other in the extra dimension(s). Hence, the effective intercommutation rate of these strings will generally be lower than one. As a consequence, one would expect more strings per horizon in these theories, with more small-scale structure accumulating on the strings [17]. This can have interesting observational consequences which must be investigated.

Let us now consider the problem in more quantitative detail. The approximate fraction of total baryonic matter that needs to be in stars in order to ionize the rest of the gas can be roughly estimated by accounting for the following main effects. The ionization energy of hydrogen is 13.6 eV, while nuclear fusion in stars produces ~7 MeV per hydrogen atom. Not all photons emitted by stars have energies above 13.6 eV and each hydrogen atom can recombine more than once. The number of ionizing photons produced per baryon depends on the mass and the composition of the star which produced them. Massive,

[1]This is confirmed by numerical studies of cosmic string wakes [13]

metal-free stars (the so-called Population III start) can produce roughly 10^5 ionizing photons per baryon. Locally observed stars, which generically have a much lower mass and some metal content, can produce ~4000 ionizing photons per baryon [5]. Only a small fraction of the gas in the halo will have time to form Population III stars, since those will soon explode as supernovae and prevent the formation of other supermassive stars. Hence, star formation regions quickly become dominated by low-mass stars. The fraction of matter that would need to be in stars in order to reionize the entire universe can roughly be estimated [5,19,20] to be

$$f_{\rm stars} \sim 10^{-3} - 10^{-4}(\eta/10), \tag{1}$$

where η (typically >1 for $z > 7$) is the number of ionizing photons needed per hydrogen atom. In (1) the lower number corresponds to metal-free stars. However, the metallicity is likely to be increased rather quickly as the first stars explode as supernovae, hence we are going to use a more conservative estimate of $f_{\rm stars}$, corresponding to the higher number in (1).

What if WMAP's estimates are true and the universe was indeed reionized around redshift $z \sim 20$? Assuming that cosmic structures grew by gravitational instability from initial Gaussian density fluctuations, one could apply the so-called extended version of the Press-Schechter model [21] and estimate the fraction of matter that collapsed and formed stars prior to $z \sim 20$. Some analytical and numerical work estimating that fraction has recently been done in [3]. The fraction of matter in stars is related to the quantity $F_G(M_{\rm min})$, which is the fraction of matter in halos of mass $M_{\rm min}$ or greater, where $M_{\rm min}$ is the minimum mass that a halo must have in order to form a galaxy. For $M_{\rm min} \approx 7 \times 10^5 M_\odot$ Barkana and Loeb find $F_G(M_{\rm min}) \approx 4 \times 10^{-5}$ [3], which is roughly consistent with the needed fraction given in Eq. (1). However, the precise estimates of f using this method and the use of Press-Schecher itself at such high redshift is still a matter of some debate.

We are going to suggest that cosmic strings could provide the fraction which is just as large and hence play a significant role in the early reionization. We will discuss current observational bounds on cosmic strings later in the paper, but only mention now that the strings we consider have tensions that are consistent with the most recent observations of CMB and large scale structure (LSS) and, therefore, do not contribute appreciably to structure formation on large scales.

During the radiation and matter dominated eras the string network evolves according to a scaling solution [22–24] which on sufficiently large scales can be described by two length scales. The first scale, $\xi(t)$, is the coherence length of strings, i. e., the distance beyond which directions along the string are uncorrelated. The second scale, $L(t)$, is the average interstring separation. Scaling implies that both length scales grow in proportion to the horizon. Cosmic string simulations indicate that $\xi(t) \sim t$, while

$$L(t) = \gamma t, \tag{2}$$

with $\gamma \approx 0.8$ in the matter era [22,23]. The so-called one-scale model [25,26], in which the two length scales are taken to be the same, has been quite successful in describing the general properties of cosmic string networks inferred from numerical simulations. These simulations have assumed that cosmic strings would reconnect on every intersection. It is of interest to us, however, to also consider the case when the reconnection probability is less than one. Then, because of the straightening of wiggles on subhorizon scales due to the expansion of the universe, one would still expect $\xi(t) \sim t$, but the string density would increase, therefore reducing the interstring distance. Hence, smaller intercommutation probabilities imply smaller γ.

Numerical simulations show that long strings possess significant amounts of small-scale structure in the form of kinks and wiggles on scales much smaller than horizon. To an observer who cannot resolve this structure, the string will appear to be smooth, but with a larger effective mass per unit length $\tilde{\mu}$ and a smaller effective tension $\tilde{T}$. An unperturbed string (with $\mu = T$) exerts no gravitational force on nearby particles. In contrast, a wiggly string with $\tilde{\mu} > \tilde{T}$ attracts particles like a massive rod. The effective equation of state of a wiggly string is $\tilde{\mu}\tilde{T} = \mu^2$ [27,28] and the velocity boost given by a moving wiggly string to nearby matter is [29,30]

$$u_i = 4\pi G\tilde{\mu} v_s \gamma_s + \frac{2\pi G(\tilde{\mu} - \tilde{T})}{v_s \gamma_s}, \tag{3}$$

where v_s is the string velocity and $\gamma_s = (1 - v_s^2)^{-1/2}$.

Let us now consider a wake [31] formed behind a string segment of length $\xi(t) \sim t$ that travelled with a speed v_s at some early time t_i for a period of time $\sim t_i$. For now, let us assume that $t_i \gtrsim t_{eq}$, where t_{eq} is the time of radiation-matter equality. There would be wakes formed during the radiation era as well, and we will discuss them separately later in the paper. The length $l_w(z)$, the width $w_w(z)$ and the thickness $d_w(z)$ of the wake will evolve with redshift as [32–34]

$$l_w \sim t_i \frac{z_i}{z}, \qquad w_w \sim v_s t_i \frac{z_i}{z}, \qquad d_w \sim u_i t_i (\frac{z_i}{z})^2, \tag{4}$$

as long as

$$z > z_i u_i / v_s. \tag{5}$$

At smaller redshifts the wake thickness d_w becomes comparable to the width w_w and the wake takes on a shape of a cylinder whose diameter grows as $z^{-3/2}$. We find that the condition (5) is satisfied for all wakes that have a chance to play a role in the early reionization. Numerical simulations [23] show that average string velocities on scales comparable to the horizon are of order $v_s \sim 0.15$. The effective mass per unit length of these strings is $\tilde{\mu} \approx 1.6\mu$, which implies that the second term in Eq. (3) will dominate[2]:

$$u_i \approx \frac{2\pi G\mu\alpha}{v_s}, \tag{6}$$

where we have defined $\alpha \equiv (\tilde{\mu} - \tilde{T})/\mu$.

As the universe expands, the fraction of matter in the wakes grows in proportion to the scale factor. For wakes formed at redshift z_i, this fraction is

$$f_w(z, z_i) \sim \frac{l_w w_w d_w}{\gamma^2 t_i^3 (z_i/z)^3} \sim 2\pi G\mu\alpha\gamma^{-2}\frac{z_i}{z}, \tag{7}$$

where we have used Eq. (2) for the average interstring distance. To make quantitative estimates easier, let us rewrite this using some characteristic values for the parameters:

$$f_w(z, z_i) \sim 10^{-3}\gamma^{-2}\left(\frac{20}{z}\right)\left(\frac{z_i}{z_{eq}}\right)\left(\frac{G\mu\alpha}{10^{-6}}\right), \tag{8}$$

where we have used $z_{eq} \approx 3400$. Whether any part of this fraction collapses into luminous objects depends on the values of z_i, z and $G\mu\alpha$.

What are the conditions leading to the formation of luminous objects within a wake? As it grows, the wake will fragment (due to the wiggliness of the string that produced the wake and due to intersections with smaller wakes produced at earlier times) into chunks of size comparable to the wake thickness d_w [11,35]. These fragments form the Cold Dark Matter halos into which the baryons will fall after decoupling from photons. Generally, one would expect the baryonic gas inside the wake to collapse when its Jeans length becomes smaller than the thickness of the wake. The Jeans length is defined as

$$L_J(z) = c_s(z)\left(\frac{\pi}{G\rho_{bw}(z)}\right)^{1/2}, \tag{9}$$

where $c_s(z)$ and $\rho_{bw}(z)$ are, respectively, the speed of sound and the energy density of baryons inside the wake. The infall velocity grows with time as [34]

$$u(z) \approx \frac{2}{5}u_i\left(\frac{z_i}{z}\right)^{1/2}. \tag{10}$$

After a certain time the infall becomes supersonic and shocks develop on either side of the baryonic wake[3]. The compression of the gas within the shock will heat it to a temperature [36]

$$T_{\rm shock} \approx \frac{3}{16}m_H u(z)^2, \tag{11}$$

where m_H is the hydrogen mass. This implies that the baryonic speed of sound inside the wake, $c_s \sim \sqrt{2T_{\rm shock}/m_H}$, is comparable to $u(z)$:

$$c_s(z) \approx 0.6u(z). \tag{12}$$

The density of baryons inside the shock is enhanced by a factor of 4 compared to the background baryon density. In addition, if the gas is able to cool to some equilibrium temperature $T_{\rm cool}$, its density would be enhanced by an additional factor $T_{\rm shock}/T_{\rm cool}$. Taking these effects into account, the baryon density in the wake can be written as

$$\rho_{bw}(z) \approx \frac{4X_b}{6\pi G t_i^2}\frac{T_{\rm shock}}{T_{\rm cool}}\left(\frac{z}{z_i}\right)^3, \tag{13}$$

where $X_b \equiv \Omega_b/\Omega_M$ is the fraction of matter in baryons. Substituting Eqs. (10), (12), and (13) into (9) gives

$$L_J(z) \sim X_b^{-1/2}\left(\frac{T_{\rm cool}}{T_{\rm shock}}\right)^{1/2} u_i t_i\left(\frac{z_i}{z}\right)^2. \tag{14}$$

The collapse condition $d_w(z) \geq L_J(z)$ then leads to

$$X_b^{-1/2}\left(\frac{T_{\rm cool}}{T_{\rm shock}}\right)^{1/2} \lesssim 1, \tag{15}$$

or,

$$\frac{T_{\rm shock}}{T_{\rm cool}} \gtrsim 6, \tag{16}$$

where we have used $X_b \approx 0.16$. From Eqs. (11), (10), and (6), it follows that

$$T_{\rm shock} \approx \frac{3\pi^2 m_H}{25}\left(\frac{G\mu\alpha}{v_s}\right)^2\frac{z_i}{z} \approx 600\ {\rm K}\frac{z_i}{z}\left(\frac{G\mu\alpha}{10^{-6}}\right)^2\left(\frac{0.15}{v_s}\right)^2, \tag{17}$$

The lowest temperature to which the gas can cool depends on its metal abundance. The metal-free primordial gas can cool via atomic transitions of hydrogen and helium down to 10^4 K, while molecular hydrogen can cool the gas down to 200 K [37]. Once the gas is enriched with metals, it could in principle cool down to the CMB temperature. Since the metallicity is likely to increase quickly, as discussed earlier, we are going to adopt

$$T_{\rm cool}(z) = T_{cmb}(z) = 2.726(1 + z)\ {\rm K}. \tag{18}$$

The collapse condition in Eq. (16) can now be written as

[2]Strings with lower intercommutation probabilities are expected to accumulated even more small-scale structure due to the suppression of loop production [17].

[3]One could allow for the possibility for the Jeans condition to be satisfied before the infall into the wake becomes supersonic. However, the condition for supersonic infall, $c_s \lesssim u(z)$, and the Jeans condition, roughly $c_s t \lesssim d_w(z)$, are essentially the same inequality. Hence, it is sufficient to consider only the wakes that form shocks.

$$\frac{z_i^{(m)}}{z^2} \gtrsim 0.03\left(\frac{10^{-6}}{G\mu\alpha}\right)^2\left(\frac{v_s}{0.15}\right)^2, \tag{19}$$

where the superscript (m) on z_i denotes matter era. This, in turn, implies a constraint on the value of $G\mu$ (using $z_i^{(m)} \leq z_{eq} \sim 3400$):

$$G\mu \gtrsim 0.6 \times 10^{-7}\alpha^{-1}\left(\frac{z}{20}\right)\left(\frac{v_s}{0.15}\right). \tag{20}$$

Formation of gaseous objects can only occur after the recombination, because of the Compton drag. We are mainly interested in the formation at much later redshifts, with inequality (20) providing the necessary condition.

So far, we have only discussed wakes formed *after* t_{eq}. Let us now consider wakes formed at some time t_i during the radiation era. For these wakes the gravitational instability only sets in at $t \sim t_{eq}$. At earlier times, the surface density of the wakes decreases as $t^{-1/2}$, while the fraction of dark matter accreted onto all wakes formed within a Hubble time of t_i remains roughly constant [32]. The infall velocity into these wakes will decrease with time as $(1 + z)$ until the matter starts to dominate. Therefore, for a wake formed at some $t_i < t_{eq}$, the infall velocity at $t > t_{eq}$ will be given by

$$\tilde{u}(z) \approx \frac{2}{5}u_i\left(\frac{z_{eq}}{z}\right)^{1/2}\left(\frac{z_{eq}}{z_i}\right). \tag{21}$$

Consequently, the temperature inside the shocks that would form in such wakes at $t > t_{\rm dec}$ would be given by

$$\tilde{T}_{\rm shock}(z, z_i) \approx \frac{3\pi^2 m_H}{25}\left(\frac{G\mu\alpha}{v_s}\right)^2 \frac{z_{eq}^3}{z z_i^2} \tag{22}$$

The necessary condition given by Eq. (16) will now become

$$z_i^{(r)} \lesssim 55000\left(\frac{20}{z}\right)\left(\frac{G\mu\alpha}{10^{-6}}\right)\left(\frac{0.15}{v_s}\right). \tag{23}$$

Again, we can use $z_i^{(r)} > z_{eq}$ to rewrite this as a lower bound on the value of $G\mu$, which leads to the same constraint as in Eq. (20).

Ultimately, we want to find the fraction of matter in all wakes, formed before and after t_{eq}, that satisfy the collapse condition in Eq. (16). This total fraction can be expressed as $f_w(z, z_i)$ from Eq. (8) evaluated at $z_i = z_{eq}$ and multiplied by a factor a_w. This factor can be written as a sum of the contributions from wakes formed before and after t_{eq}:

$$a_w = a_w^{(r)} + a_w^{(m)}. \tag{24}$$

Because the fraction of matter in wakes formed within each Hubble time (defined as the time required for the horizon to double in size) remains roughly constant during the radiation era, $a_w^{(r)}$ is simply equal to the number of Hubble times between z_{eq} and $z_i^{(r)}$, which is bounded by inequality (23). Namely,

$$a_w^{(r)} \approx \log_2\left(\frac{z_i^{(r)}}{z_{eq}}\right)^2 \lesssim 8 + 2\log_2\left[\left(\frac{20}{z}\right)\left(\frac{G\mu\alpha}{10^{-6}}\right)\left(\frac{0.15}{v_s}\right)\right]. \tag{25}$$

The fraction of matter in wakes formed after t_{eq} decreases with the redshift as z_i^{-1}. Therefore, the dependence of $a_w^{(m)}$ on the number of Hubble times between z_{eq} and $z_i^{(m)}$ is sufficiently weak and we can simply set $a_w^{(m)} \sim 1$. Hence, the total fraction of matter in the wakes that satisfy the collapse condition can be written as

$$f_w(z) \sim 10^{-3}\gamma^{-2}a_w\left(\frac{20}{z}\right)\left(\frac{G\mu\alpha}{10^{-6}}\right), \tag{26}$$

where

$$a_w \approx 9 + 2\log_2\left[\left(\frac{20}{z}\right)\left(\frac{G\mu\alpha}{10^{-6}}\right)\left(\frac{0.15}{v_s}\right)\right]. \tag{27}$$

Note that a_w decreases with a decrease in $G\mu$ or an increase in z, e. g., $a_w \sim 10$ for $G\mu\alpha \sim 10^{-6}$ and $z \sim 20$ yet $a_w \sim 1$ for $G\mu\alpha \sim 10^{-7}$ and $z \sim 30$.

The condition in Eq. (16) does not say anything about the ability of the gas to cool. Only some fraction $f_{\rm s}$ of the total number of baryons satisfying this condition will actually be able to cool and form stars. At present, there is no good theory of this fraction. However, one could assume, based on the current ratio of the average mass density in stars to the total baryon density, that this fraction is of order 10% [38]. Finally, we can estimate the fraction of matter in stars, formed in the wakes behind cosmic strings, as

$$f_{\rm w} \sim 10^{-4}\gamma^{-2}a_w\left(\frac{20}{z}\right)\left(\frac{f_{\rm s}}{0.1}\right)\left(\frac{G\mu\alpha}{10^{-6}}\right) \tag{28}$$

This, together with Eq. (1) and the WMAP's upper bound on the reionization redshift ($z_r \lesssim 30$), can be used to put an approximate upper bound on the value of $G\mu$:

$$G\mu \lesssim 10^{-6}\left(\frac{\gamma^2\alpha^{-1}a_w^{-1}}{0.1}\right)\left(\frac{\eta}{10}\right)\left(\frac{0.1}{f_{\rm s}}\right), \tag{29}$$

which is roughly an order of magnitude higher than the upper bound reported in [12]. Inequalities (29) and (20) define the approximate range of values of $G\mu$ for which strings may play a role in the early reionization. Interestingly, this range is quite narrow—only an order of magnitude wide. However, this range also depends on parameters α and γ, which could be significantly different from their canonical values if, e. g., strings were D-branes formed at the end of brane inflation [14–18].

Current CMB and LSS data imply that $G\mu \lesssim 10^{-6}$ [16,39,40][4]. This bound, however, also depends on the

[4]These constraints only apply to local cosmic strings considered in this paper. For recent constraints on global strings see [41].

number of strings within a horizon or, more generally, on the scaling parameter γ. When this dependence is taken into account, the constraint based on CMB and LSS power spectra becomes

$$G\mu \lesssim 10^{-6}\gamma. \tag{30}$$

Substituting this into Eq. (28) gives

$$f_{\mathrm{w}} \lesssim 10^{-3}\left(\frac{\gamma^{-1}a_w\alpha}{10}\right)\left(\frac{20}{z}\right)\left(\frac{f_{\mathrm{s}}}{0.1}\right), \tag{31}$$

which is of the same order of magnitude as the fraction needed to reionize the universe.

It may be worth mentioning that recently there has been a detection of galaxy lensing that looks very much like a lensing by a cosmic string [42]. It remains to be confirmed, but assuming it is a cosmic string, the implied value of the string tension would be $G\mu \sim 4 \times 10^{-7}$. For this value of $G\mu$, and assuming that $\alpha \approx 1$, from Eq. (27) it follows that $a_w \approx 7$ at $z \sim 20$. Recent analysis of the WMAP and Sloan Digital Sky Survey data [40] shows that data may actually prefer a nonzero contribution from strings, corresponding to $G\mu \sim 4 \times 10^{-7}\gamma$. The combination of these two measurements would imply $\gamma \sim 1$, which for $z \sim 20$, would give

$$f_{\mathrm{w}} \approx 3 \times 10^{-4}\left(\frac{f_{\mathrm{s}}}{0.1}\right), \tag{32}$$

which is close to what would be required for the reionization by redshift $z \sim 20$.

Our analysis was based on the expectation that the metallicity of the gas in the wakes will quickly reach the level sufficient to prevent the formation of heavy metal-free stars (the so-called Population III stars). If, however, a significant number of Population III stars managed to form prior to metal enrichment, then our results would be modified. In particular, a smaller fraction of matter in the wakes would satisfy the collapse condition (16) due to the higher value of T_{cool} for a metal-free gas. On the other hand, a smaller value of f_{rmstar} would be required to reionize the universe. In combination, this is likely to reduce the upper bound on $G\mu$ (Eq. (29)) and increase the threshold value in Eq. (20), thus, narrowing the range of string parameters relevant for the reionization. If a sizable fraction of matter was ionized by Population III stars in the wakes it could lead to a nonmonotonic reionization similar to the two-step reionization models of [5,43].

In conclusion, our analysis has shown that cosmic strings can play a role in early reionization, provided that their mass per unit length is within the range given by inequalities (20) and (29). This range is consistent with, and may even be somewhat favored by the current data.

We thank Avi Loeb for very helpful comments upon reading the draft of this paper and Ken Olum for useful discussions.

[1] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003).
[2] A. Koguta *et al.*, Astrophys. J. Suppl. Ser. **148**, 161 (2003).
[3] R. Barkana and A. Loeb (to be published).
[4] A. J. Bunker *et al.* (to be published).
[5] J. S. B. Wyithe and A. Loeb, Astrophys. J. **586**, 693 (2003).
[6] R. Cen, Astrophys. J. **591**, 12 (2003).
[7] J. S. B. Wyithe and A. Loeb, Nature (London) **427**, 815 (2004).
[8] A. Loeb, R. Barkana, and L. Hernquist (to be published).
[9] N. Y. Gnedin Astrophys. J. **610**, 9 (2004).
[10] M. J. Rees, Mon. Not. R. Astron. Soc. **222**, 27 (1986).
[11] T. Hara, P. Mahonen, and S. Miyoshi, Astrophys. J. **412**, 22 (1993).
[12] P. P. Avelino and A. R. Liddle, Mon. Not. R. Astron. Soc. **348**, 105 (2004).
[13] A. Sornborger *et al.*, Astrophys. J. **482**, 22 (1997).
[14] S. Sarangi and S.-H. H. Tye, Phys. Lett. B **536**, 185 (2002).
[15] N. Jones, H. Stoica and S.-H. H. Tye, Phys. Lett. B **563**, 6 2003.
[16] L. Pogosian, S.-H. H. Tye, I. Wasserman, M. Wyman, Phys. Rev. D **68**, 023506 (2003).
[17] G. Dvali and A. Vilenkin, J. Cosmol. Astropart. Phys. **0403**, 010 (2004).
[18] E. J. Copeland, R. C. Myers, J. Polchinski, J. High Energy Phys. 0406, (2004) 013.
[19] V. Bromm, R. P. Kudritzki, and A. Loeb, astro-ph/0007248.
[20] V. Bromm and A. Loeb, Nature (London) **425**, 812 (2003).
[21] W. Press and P. Schechter, Astrophys. J. **187**, 425 (1974).J. R. Bond *et al.*, Astrophys. J. **379**, 440 (1991).
[22] D. P. Bennett and F. R. Bouchet, Phys. Rev. D **41**, 2408 (1990).
[23] B. Allen and E. P. S. Shellard, Phys. Rev. Lett. **64**, 119 (1990).
[24] A. Albrecht and N. Turok, Phys. Rev. D**40**, 973 (1989).
[25] T. W. B. Kibble, Nucl. Phys. B **252**, 227 (1985).
[26] C. J. A. P. Martins and E. P. S. Shellard, Phys. Rev. D **53**, 575 (1996).
[27] B. Carter, Phys. Rev. D **41**, 3869 (1990).
[28] A. Vilenkin, Phys. Rev. D **41**, 3038 (1990).
[29] T. Vachaspati and A. Vilenkin, Phys. Rev. Lett. **67**, 1057 (1991).

[30] D. N. Vollick, Phys. Rev. D **45**, 1884 (1992).
[31] J. Silk and A. Vilenkin, Phys. Rev. Lett. **53**, 1700 (1984).
[32] T. Vachaspati, Phys. Rev. Lett. **57**, 1655 (1986).
[33] A. Stebbins *et al.*, Astrophys. J. **322**, 1 (1992).
[34] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects*, (Cambridge University, Cambridge, MA, 2000).
[35] P. P. Avelino and E. P. S. Shellard, Phys. Rev. D **51**, 369 (1985).
[36] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, (Pergamon, New York, 1987).
[37] R. Barkana and A. Loeb, Phys. Rep. **349**, 125 (2001).
[38] M. Fukugita, C. J. Hogan, and P. J. E. Peebles, Astrophys. J. **503**, 518 (1998).
[39] M. Landriau, E. P. S. Shellard, Phys. Rev. D **69**, 023003 (2004).
[40] L. Pogosian, I. Wasserman and M. Wyman, (to be published); M. Wyman, L. Pogosian, and I. Wasserman, (to be published).
[41] N. Bevis, M. Hindmarsh, and M. Kunz, Phys. Rev. D **70**, 043508 (2004).
[42] M. Sazhin *et al.*, Mon. Not. R. Astron. Soc., **343**, 353 (2003).
[43] R. Cen, Astrophys. J. **591**, 12 (2003).