Oscillatory universes in loop quantum cosmology and initial conditions for inflation

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Positively curved oscillatory universes are studied within the context of loop quantum cosmology subject to a consistent semiclassical treatment. The semiclassical effects are reformulated in terms of an effective phantom fluid with a variable equation of state. In cosmologies sourced by a massless scalar field, these effects lead to a universe that undergoes ever-repeating cycles of expansion and contraction. The presence of a self-interaction potential for the field breaks the symmetry of the cycles and can enable the oscillations to establish the initial conditions for successful slow-roll inflation, even when the field is initially at the minimum of its potential with a small kinetic energy. The displacement of the field from its minimum is enhanced for lower and more natural values of the parameter that sets the effective quantum gravity scale. For sufficiently small values of this parameter, the universe can enter a stage of eternal self-reproduction.

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I. INTRODUCTION

Cyclic/oscillatory universes have a long history in cosmology [1]. Originally, one of their main attractions was that initial conditions could in principle be avoided. Closer consideration of such models, however, revealed severe difficulties in their construction within the context of general relativity (GR). Apart from entropy constraints, which restrict the number of bounces in the past, the central difficulty is that when treated classically any bounce would be singular, thereby resulting in the breakdown of GR. Recent developments in M-theory inspired brane world models have renewed interest in cyclic/oscillatory universes [2], although problems still remain in such models in developing a successful treatment of the bounce. An oscillating universe that ultimately undergoes inflationary expansion after a finite number of cycles has also been investigated [3]. However, a physical mechanism for inducing the bounces was not employed in this model.

Our aim here is to study oscillatory universes within the context of loop quantum cosmology (LQC) which is the application of loop quantum gravity (LQG) to an homogeneous minisuperspace environment. LQG is at present the main background independent and nonperturbative candidate for a quantum theory of gravity (see for example [4,5]). This approach provides a (discrete) description of high-energy dynamics in the form of a difference equation. An important consequence of this discretization is the removal of the initial singularity [6]. As the universe expands and its volume increases, it enters an intermediate semiclassical phase in which the evolution equations take a continuous form but include modifications due to nonperturbative quantization effects [7]. It has recently been shown that a collapsing positively curved Friedmann-Robertson-Walker (FRW) universe containing a massive scalar field can undergo a nonsingular bounce within the semiclassical region [8]. Other studies have shown that for a flat universe semiclassical LQC effects increase the parameter space of initial conditions for successful inflation, and this behavior is found to be robust to ambiguities in the theory [9,10].

We study the behavior of a positively curved universe sourced by a massless scalar field and show that the combined effects of the LQC corrections and the curvature enable the universe to undergo ever-repeating cycles. The presence of a self-interaction potential for the field breaks the symmetry of the cycles and can establish the conditions for inflation, even if the field is initially located in the minimum of its potential. The viability of this mechanism is studied by identifying the constraints that need to be satisfied for a consistent semiclassical treatment.

II. EFFECTIVE FIELD EQUATIONS IN LOOP QUANTUM COSMOLOGY

We consider positively curved FRW cosmologies sourced by a scalar field ϕ with self-interaction potential $V(\phi)$. The semiclassical phase of LQC arises when the scale factor lies in the range $a_i < a < a_*$, where $a_i \equiv \sqrt{\gamma} l_{\rm Pl}$, $a_* \equiv \sqrt{\gamma j/3} l_{\rm Pl}$, $\gamma = \ln 2/\sqrt{3}\pi \approx 0.13$ and j is a quantization parameter which must take half integer values. Below the scale a_i, the discrete nature of spacetime is important, whereas the standard classical cosmology is recovered above a_*. The parameter j therefore sets the effective quantum gravity scale. The modified Friedmann equation is given by

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi l_{\rm Pl}^2}{3}\left[\frac{1}{2}\frac{\dot{\phi}^2}{D} + V(\phi)\right] - \frac{1}{a^2}, \qquad (1)$$

where the quantum correction factor $D(q)$ is defined by

$$D(q) = \left(\frac{8}{77}\right)^6 q^{3/2}\{7[(q+1)^{11/4} - |q-1|^{11/4}] - 11q[(q+1)^{7/4} - \mathrm{sgn}(q-1)|q-1|^{7/4}]\}^6 \quad (2)$$

with $q \equiv (a/a_*)^2$. Equation (2) represents an approximate expression for the eigenvalues of the inverse volume operator [7]. As the universe evolves through the semiclassical phase, this function varies as $D \propto a^{15}$ for $a \ll a_*$, has a global maximum at $a \approx a_*$ and falls monotonically to $D = 1$ for $a > a_*$.

The scalar field equation has the form

$$\ddot{\phi} = -3H\left(1 - \frac{1}{3}\frac{d\ln D}{d\ln a}\right)\dot{\phi} - DV', \quad (3)$$

where a prime denotes differentiation with respect to the scalar field. Differentiating Eq. (1) and substituting Eq. (3) then implies that

$$\dot{H} = -\frac{4\pi l_{\rm Pl}^2 \dot{\phi}^2}{D}\left(1 - \frac{1}{6}\frac{d\ln D}{d\ln a}\right) + \frac{1}{a^2}. \quad (4)$$

Equations (1) and (3) can be written in the standard form of the Einstein field equations sourced by a perfect fluid:

$$H^2 = \frac{8\pi l_{\rm Pl}^2}{3}\rho_{\rm eff} - \frac{1}{a^2}, \quad (5)$$

$$\dot{\rho}_{\rm eff} = -3H(\rho_{\rm eff} + p_{\rm eff}), \quad (6)$$

where

$$\rho_{\rm eff} \equiv \frac{1}{2}\frac{\dot{\phi}^2}{D} + V, \quad (7)$$

$$p_{\rm eff} \equiv \frac{1}{2}\frac{\dot{\phi}^2}{D}\left(1 - \frac{1}{3}\frac{d\ln D}{d\ln a}\right) - V, \quad (8)$$

define the effective energy density and pressure of the fluid, respectively. The effective equation of state, $w \equiv p_{\rm eff}/\rho_{\rm eff}$, is given by

$$w = -1 + \frac{2\dot{\phi}^2}{\dot{\phi}^2 + 2DV}\left(1 - \frac{1}{6}\frac{d\ln D}{d\ln a}\right). \quad (9)$$

In this picture the effects of the LQC corrections on the cosmic dynamics are parametrized entirely in terms of the equation of state. When $d\ln D/d\ln a > 6$, the fluid represents "phantom" matter ($w < -1$) that violates the null energy condition ($\rho_{\rm eff} + p_{\rm eff} \geq 0$) and, numerically, this is equivalent to $a < a_{\rm ph} \equiv 0.914 a_*$. This condition depends only on the quantization parameter j and is independent of the potential. Since, within the context of GR, a violation of the null energy condition can lead to a nonsingular bounce, this provides an alternative description of how LQC effects can result in a bouncing cosmology when the universe contracts below a critical size: the decreasing energy density of the phantom fluid during the collapsing phase is eventually balanced by the growing curvature term in the Friedmann equation.

In the following Sections, we develop this picture further for the case of massless and self-interacting scalar fields, respectively.

III. OSCILLATIONS WITH A MASSLESS SCALAR FIELD

The field equation (3) for a massless scalar field ($V = 0$) admits the first integral:

$$\dot{\phi} = \dot{\phi}_{\rm init}\left(\frac{a_{\rm init}}{a}\right)^3\left(\frac{D}{D_{\rm init}}\right), \quad (10)$$

where $D_{\rm init} = D(a_{\rm init}/a_*)$ and a subscript "init" denotes initial values. Without loss of generality we choose initial conditions such that $a_i < a_{\rm init} < a_{\rm ph}$, $\dot{\phi}_{\rm init} > 0$ and $H_{\rm init} = 0$. The subsequent cosmic dynamics can then be divided into four phases (Fig. 1).

Phase I.—The universe is effectively sourced by phantom matter since $a < a_{\rm ph}$. This drives an epoch of superinflationary expansion ($\dot{H} > 0$) until $d\ln D/d\ln a = 6$. The scalar field accelerates during this phase.

Phase II.—The expansion rate now slows down. The scalar field continues to accelerate until the logarithmic slope of the quantum correction function has fallen to $d\ln D/d\ln a = 3$. For $a > a_*$, the energy density of the field, $\rho_\phi \propto 1/a^6$, falls more rapidly than the curvature and the expansion eventually reaches a turnaround.

Phase III.—Equation (3) implies that the scalar field begins to accelerate immediately after the turnaround

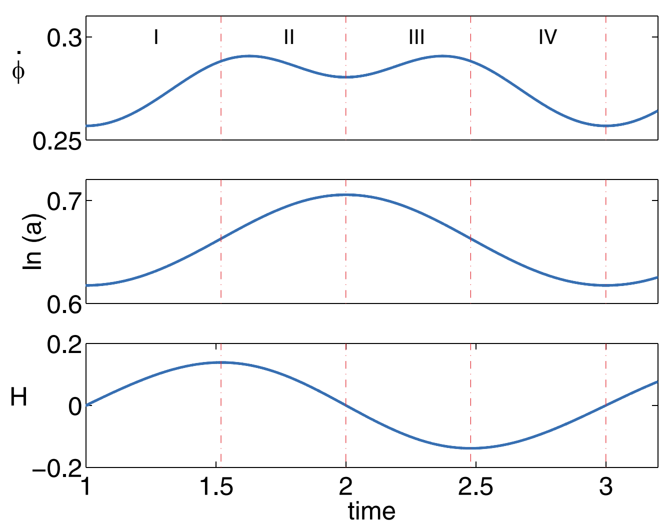

FIG. 1 (color online). Time evolution of the scalar field velocity $\dot{\phi}$, the logarithmic scale factor and the Hubble parameter when the potential $V = 0$, $j = 100$, $a_{\rm init}/a_* = 0.9$ and $H_i = 0$. Axes are labeled in Planck units.

and continues to do so until the universe has collapsed to the point where $d\ln D/d\ln a = 3$ once more. Shortly afterwards, the universe contracts below the critical scale $a_{\rm ph}$, at which point the Hubble parameter begins to increase.

Phase IV.—Substituting Eq. (10) into Eq. (7) implies that the effective energy density varies as $\rho_{\rm eff} = \dot{\phi}^2/(2D) \propto D/a^6$. It is therefore decreasing during this phase, since $d\ln D/d\ln a > 6$, whereas the curvature is growing. Hence, a scale is reached where the Hubble parameter vanishes instantaneously. Equations (1) and (4) imply that the second time derivative of the scale factor is positive during this phase and the universe therefore undergoes a nonsingular bounce into a new phase of superinflationary expansion (phase I).

The effective equation of state (9) is independent of the field's kinetic energy when $V = 0$. This implies that identical cycles are repeated indefinitely into the future (and the past), as summarized in Fig. 2(a). During the collapsing phases, the field retraces the trajectory it mapped out during the expanding phases. Oscillatory behavior is therefore possible in the absence of an interaction potential. Furthermore, Eq. (10) implies that the field's kinetic energy never vanishes during the cycle since the scale factor remains finite. The value of the field therefore increases monotonically with time.

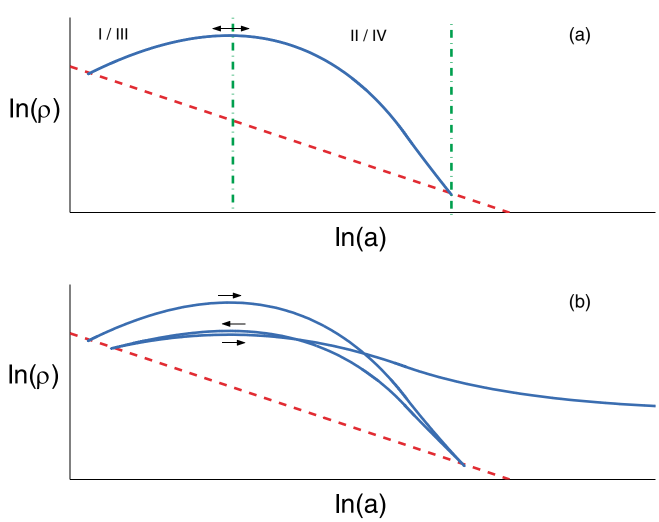

FIG. 2 (color online). (a) Schematically illustrating the logarithmic variation of the effective energy density of a massless scalar field (solid line) and the curvature term in the Friedmann equation (dashed line). The universe oscillates indefinitely between the intersection points of the two lines. (b) Schematically illustrating the effects of introducing a self-interaction potential for the field. The cycles are eventually broken as the potential becomes dynamically significant, thereby resulting in slow-roll inflation. In both figures, the slope of the trajectories is given by $d\ln\rho_{\rm eff}/d\ln a = -3[1 + w(a)]$.

IV. SELF-INTERACTING SCALAR FIELD

We now consider how self-interactions of the scalar field modify this cyclic dynamics. We make very weak assumptions about the potential, specifying only that it has a global minimum at $V_{\rm min}(0) = 0$ and is a positive-definite and monotonically varying function when $\phi \neq 0$ such that $V'' > 0$. We suppose the field is initially located at the minimum of its potential with the same initial conditions as those considered in Sec. III.

The qualitative dynamics of the universe is illustrated in Fig. 2(b). In general, the path of the field in the $\{\ln\rho_{\rm eff}, \ln a\}$ plane is determined by the variation of the effective equation of state (9). The gradient of the trajectory is given by $d\ln\rho_{\rm eff}/d\ln a = -3[1 + w(a)]$ and there is a turning point whenever the scale factor passes through $a_{\rm ph}$, regardless of the form of the potential.

Two factors determine how the equation of state changes during each cycle and these can be parametrized by defining the quantities $\mathcal{B} = [(6 - d\ln D/d\ln a)/6]$ and $C \equiv 2\dot{\phi}^2/[\dot{\phi}^2 + 2DV]$, respectively. For a massless field, $C = 2$, and the deviation of the equation of state away from the value $w = -1$ is determined by the magnitude of $\mathcal{B}$. The effects of the potential are parametrized by C. Introducing a potential energy into the system necessarily implies $C < 2$ and it follows that for a given value of the scale factor, the equation of state is closer to -1 than in the massless case.

Since the field moves monotonically up the potential, the gradient term DV' in the scalar field equation (3) is more significant in a given cycle relative to the previous one. This implies that the scalar field gains kinetic energy less rapidly during phases I and III and loses kinetic energy more rapidly during phases II and IV. Consequently, the value of C at the end of phase I of a given cycle is smaller than the value it had at the corresponding point of the previous cycle. In other words, the rate of change of the trajectory's gradient around $a_{\rm ph}$ becomes progressively smaller with each successive cycle.

A massless field begins accelerating immediately after the turnaround and bounce have been attained. However, Eq. (3) implies that $\ddot{\phi} = -DV' < 0$ at the instant when the Hubble parameter vanishes, so the field does not accelerate immediately after the turnaround (bounce); there is a short delay during which C continues to decrease. The trajectory of the field immediately before the turnaround (bounce) is therefore slightly steeper than the trajectory immediately after, i.e., the trajectory for phase III (I) lies *below* that of phase II (IV). Moreover, because the field does not acquire as much kinetic energy during phase III as it lost during phase II, the value of C (for a given value of the scale factor) is smaller during phase III than it was during phase II. Thus, the phase III part of the trajectory always lies below that of phase II and, similarly, the trajectory of phase I lies below that of the phase IV trajectory of the previous cycle. The potential therefore breaks the symmetric cycles of the massless field and the effective energy density at the end of phases I and III falls with successive cycles.

We have implicitly assumed that the kinetic energy of the field never vanishes during phase II. However, the potential becomes progressively more important with each completed cycle and this assumption must necessarily break down after a finite number of cycles have been completed. Since the effects of the quantum correction function are negligible once the universe has expanded beyond a_*, the energy density of the field during phase II redshifts more rapidly than the curvature term if the strong energy condition is satisfied, i.e., if $\dot{\phi}^2 > V$. As the field moves monotonically up the potential, it becomes progressively harder to maintain this condition. Eventually, therefore, a cycle is reached where this condition is violated during phase II. This leads inevitably to an epoch of slow-roll inflationary expansion, as the field slows down, reaches a point of maximum displacement and moves back down the potential [11]. The transition into a potential-driven, slow-roll inflationary epoch is shown in Figs. 2(b) and 3.

Thus far, we have discussed the cyclic dynamics for a given value of the quantization parameter, j. In spatially flat models, the maximum value attained by the field as it moves up the potential increases for higher values of j, since the universe undergoes greater expansion before the end of the superinflationary, semiclassical phase. For positively curved models, on the other hand, the field moves farther up the potential before the onset of slow-roll inflation for *smaller* values of j. This behavior can be understood qualitatively by making some simplifying assumptions. Let us assume that the transition from phase I to II occurs at $a = a_*$; that $D \propto a^{15}$ for $a < a_*$; that $D = 1$ for $a > a_*$; and that the potential becomes dynamically significant only during the last cycle before slow-roll inflation. At this level of approximation, the field's kinetic energy varies as $\dot{\phi}^2|_{a<a_*} \propto a^{24}$ and $\dot{\phi}^2|_{a>a_*} \propto a^{-6}$, respectively, so $\dot{\phi}_*^2 \propto a_*^{24} \propto j^{12}$ at the transition. At the turnaround $(a = a_t)$, $\dot{\phi}_t^2 = \dot{\phi}_*^2 (a_*/a_t)^6$, but since $\dot{\phi}_t^2 \approx 1/a_t^2$, it follows that $\dot{\phi}_t^2 \propto j^{-15/2}$. Hence, the kinetic energy of the field at the turnaround is higher for lower values of j and, consequently, slow-roll inflation occurs ($V > \dot{\phi}^2$) when the field is further from the minimum of the potential.

To summarize, the cyclic nature of positively curved cosmologies within LQC can in principle establish the conditions for slow-roll inflation even if the inflaton is initially located in the minimum of its potential. We now proceed to consider the parameter space of a viable model of inflation.

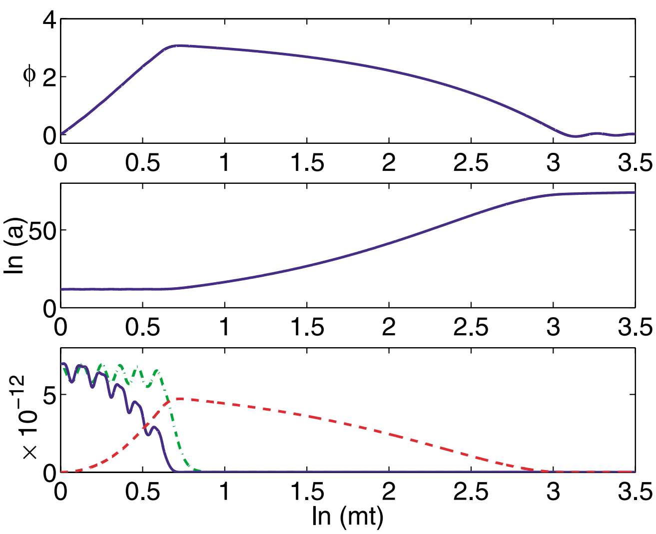

FIG. 3 (color online). Illustrating the time evolution of the scalar field ϕ (top panel) and the logarithmic scale factor (middle panel) for a quadratic potential $V = m^2\phi^2/2$. The bottom panel illustrates how the effective kinetic energy $\dot{\phi}^2/2D$ (solid line) and potential energy (dashed line) of the field vary compared to the curvature term (dot-dashed line). The cycles end and slow-roll inflation commences at the point when the potential begins to dominate the kinetic energy. Initial conditions are chosen such that $\phi_{\rm init} = H_{\rm init} = 0$ and $a_{\rm init}/a_* = 0.9$, with $m = 10^{-6} l_{\rm Pl}^{-1}$ and $j = 5 \times 10^{11}$ (see the text for details). The axes are labeled in Planck units.

V. PARAMETER SPACE OF A VIABLE MODEL

A number of constraints must be satisfied by any successful inflationary scenario of the type outlined above. In particular, the field must be sufficiently displaced from the minimum of its potential at the end of the oscillatory phase for the horizon problem to be solved. Typically this requires at least 60 e-folds of accelerated expansion. Self-consistency of the semiclassical analysis also requires that (a) H^2 must be non-negative; (b) the Hubble length must be larger than the limiting value of the scale factor, $|H| a_i < 1$; and (c) the scale factor at the bounce must exceed a_i. Imposing constraint (b) is equivalent to requiring that energy scales during the classical regime do not exceed the Planck scale [9].

Constraint (a) necessarily implies that the initial kinetic energy of the field is bounded from below by the Friedmann equation (1):

$$\dot{\phi}^2_{\rm init} > \frac{6D_{\rm init}}{8\pi l_{\rm Pl}^2 a_{\rm init}^2}. \tag{11}$$

On the other hand, constraint (b) results in an upper bound on the field's kinetic energy. Equation (4) implies that $|H|$ is maximized when $d\ln D/d\ln a \approx 6$ and substituting Eq. (10) into the Friedmann equation (1) then implies that $|H_{\max}| a_i < 1$ for

$$\dot{\phi}^2_{\rm init} < \frac{3}{4\pi l_{\rm Pl}^2}\left(\frac{a_{\rm ph}}{a_*}\right)^6 \left(\frac{a_*}{a_{\rm init}}\right)^6 \frac{D_{\rm init}^2}{D_{\rm ph}}\left(\frac{1}{\gamma l_{\rm Pl}^2} + \frac{1}{a_{\rm ph}^2}\right), \tag{12}$$

where $D_{\rm ph} = D(a_{\rm ph}/a_*)$. The discussion leading to

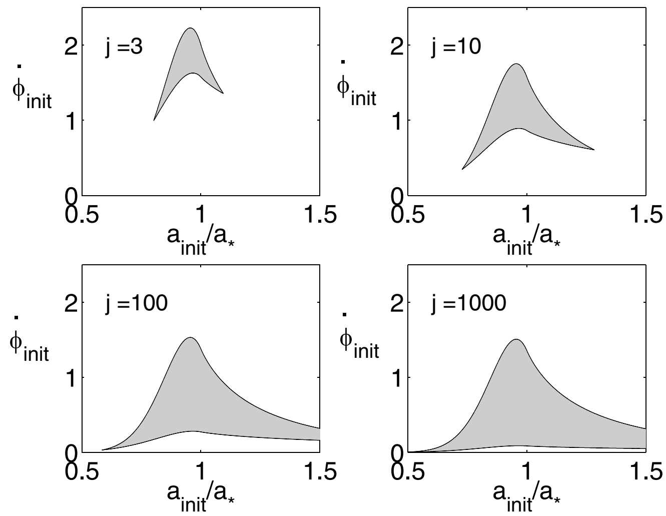

FIG. 4. Constraints on the initial velocity of the field $|\dot{\phi}|_{\rm init}$ for a given initial value of the scale factor, $a_{\rm init}/a_*$, for different values of the quantization parameter, j. The axes are labeled in Planck units and the shaded areas represent the regions where constraints (11) and (12) are satisfied. For all values of j, the area of the shaded region is finite, and the points of intersection occur farther from $a_{\rm init}/a_* \approx 1$ as j is increased. Note that for $a_{\rm init} > a_*$, the universe is initially in a contracting phase and subsequently bounces, after which the behavior discussed in the text is followed.

Fig. 2(b) has shown that the maximum value of the square of the Hubble parameter during a given cycle is smaller than that of the previous cycle. Thus, if condition (12) is satisfied in the first cycle, where the potential is negligible, $|H|a_i < 1$ during all subsequent cycles.

Figure 4 illustrates the region of parameter space where constraints (11) and (12) are simultaneously satisfied. These constraints are independent of the potential. Increasing the value of j reduces the upper and lower bounds on the initial kinetic energy but widens the range of allowed values of $a_{\rm init}/a_*$. A further consequence of Fig. 4 is that for a given value of j there exists an upper limit to the size of the universe at the turnaround that is consistent with the semiclassical dynamics. In other words, if the universe is too large at the turnaround, the kinetic energy of the scalar field will exceed the Planck scale before the universe has contracted below a_*. This implies, in particular, that if the inflaton did not decay at the end of inflation, future cycles after the slow-roll inflationary epoch could not be realized within this semiclassical framework.

Constraint (c) is not necessarily satisfied if the bounce occurs at progressively smaller values of the scale factor. It is necessary, therefore, to consider this constraint for each specific model. We have considered a quadratic potential $V(\phi) = m^2\phi^2/2$, where the mass of the field is set by the Cosmic Background Explorer normalization to be $m = 10^{-6} l_{\rm Pl}^{-1}$. We have verified numerically that constraints (a)–(c) remain satisfied for all initial values of the parameters contained within the shaded regions of Fig. 4 and, furthermore, that the maximum value attained by the field is sufficient to solve the horizon problem, i.e., $\phi_{\rm max} > 3 l_{\rm Pl}^{-1}$. Indeed, for the initial condition $a_{\rm init} = 0.9 a_*$ considered in Fig. 3, sufficient inflation is possible for $j \leq 5 \times 10^{11}$. Numerical integration indicates that successful slow-roll inflation is also possible for other potentials such as quartic and hyperbolic models.

VI. CONCLUSION

We have shown that loop quantum gravity corrections to the Friedmann equation in a positively curved universe enable a scalar field to move up its potential due to a series of contracting and expanding phases in the cosmic dynamics. The oscillatory nature of such models can be described by reformulating the semiclassical dynamics in terms of an effective phantom fluid. In general, the field is able to move farther from the potential minimum for lower and more natural values of the quantization ambiguity parameter j. This is important because for sufficiently small j, the field may move far enough up the potential before the cycles are broken for the universe to enter a stage of eternal self-reproduction [12]. Eternal inflation occurs if $3V'^2 < 128\pi l_{\rm Pl} V^3$ and for a quadratic potential this implies that $\phi > 1/2 m^{1/2} l_{\rm Pl}^3 \approx 500 l_{\rm Pl}^{-1}$ [12]. Numerical simulations indicate, for example, that this condition is satisfied for $j < 2 \times 10^7$ when $a_{\rm init} = 0.9 a_*$. We conclude, therefore, that even if the field is at the minimum of its potential, a wide range of initial conditions leads to successful (and eternal) inflation. Since the assumptions made about the form of the potential were weak, this mechanism should be very generic.

Finally, the oscillatory dynamics also arises when the minimum of the potential is at $V_{\rm min} < 0$. In positively curved universes, the same mechanism enables the field to work its way out of the negative region and continue up the potential until the cycles are broken, as discussed in Sec. IV. For spatially flat universes, the picture is similar although the oscillations come to end as soon as the field reaches a positive region of the potential. This is particularly interesting given that negative potentials are known to arise in string/M-theory compactifications (e.g., [13]). It would be interesting to explore these possibilities further.

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