Cosmic attractors and gauge hierarchy

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We suggest a new cosmological scenario which naturally guarantees the smallness of scalar masses and vacuum expectation values , without invoking supersymmetry or any other (nongravitationally coupled) new physics at low energies. In our framework, the scalar masses undergo discrete jumps due to nucleation of closed branes during (eternal) inflation. The crucial point is that the step size of variation decreases in the direction of decreasing scalar mass. This scenario yields exponentially large domains with a distribution of scalar masses, which is sharply peaked around a hierarchically small value of the mass. This value is the ''attractor point'' of the cosmological evolution.

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I. GENERAL IDEA

The radiative instability of scalar masses is the key point of the gauge hierarchy problem. In the effective 4D field theory, the scalar masses are quadratically sensitive to the ultraviolet cutoff. The only known exceptions to this rule are Goldstone bosons. This fact is hard to reconcile with the observed smallness of the weak scale, relative to the Planck mass $M_p \sim 10^{19}$ GeV. So far supersymmetry is the only known symmetry that renders masses of elementary scalars radiatively stable. The scalar masses are controlled by supersymmetry breaking scale. Given the fact that we do not understand the origin of this scale, supersymmetry *per se* does not really explain the origin of the weak scale but rather makes the gauge hierarchy technically natural.

In view of the above, it is crucial to explore other possible mechanisms of scalar mass stabilization. In the present paper we suggest an alternative mechanism that can guarantee zero or very small scalar masses [and vacuum expectation values (VEVs)] without invoking supersymmetry or any other nongravitationally coupled new physics at low energies.

In our scenario, a small scalar mass is selected with probability one during the cosmological evolution. This selection works as follows. We construct a simple framework in which scalar masses (and VEVs) undergo discrete variations due to nucleation of closed domain wall bubbles (branes) during inflation. Values of the scalar mass on different sides of the wall differ by a finite step. The bubbles expand exponentially fast and create domains of a new vacuum with a new value of the scalar mass. New bubbles are created within the old, and the scalar mass changes further. Since inflation is known to be eternal [1,2], the process of wall nucleation continues forever, populating the Universe with exponentially large domains having different values of the scalar mass. However, not all the values of the scalar mass (VEV) are equally probable. In our model, in the absence of gravitational backreaction, the probability is sharply peaked around zero because the step $\Delta\phi$ decreases towards small values of the VEV ϕ faster than ϕ itself. That is,

$$\Delta\phi/\phi \propto \phi^n, \tag{1}$$

where $n > 0$ is some power. As a result, the density of states diverges for small VEV (mass) of ϕ.

Thus, in the first approximation, the probability distribution for ϕ has an infinitely sharp peak at $\phi = 0$. We will show, however, that infrared effects, such as the Gibbons-Hawking temperature and quantum fluctuation of ϕ during inflation, can shift the most probable value of the scalar mass (and VEV) away from zero to a small value and round off the maximum of the peak.

II. COSMIC ATTRACTORS

To introduce our mechanism, we use a simple toy model. The main ingredients are: (1) a scalar field ϕ; (2) domain walls (branes) charged under an antisymmetric three-form field $A_{\alpha\beta\gamma}$ with the field strength $F_{\alpha\beta\gamma\delta} = F\epsilon_{\alpha\beta\gamma\delta}$. These objects are engaged in the following interrelation. The branes are sources for the three-form field. The value of the brane charge is determined by the VEV of ϕ. The VEV of ϕ is in turn determined by the three-form field strength F.

These couplings result in the following dynamics. Nucleation of a closed brane changes the value of F. The step of change (the brane charge) is determined by ϕ. We construct the model so that an increase in F decreases ϕ, which in turn decreases the charge of new branes that can be nucleated. Decrease of the brane charge diminishes the minimal step of change in F. As a result, the subsequent decrease of ϕ requires more steps, and their number diverges towards small values of ϕ.

Let us discuss this dynamics in more detail. The action of a free three-form field in 4D can be written as

$$\int_{3+1} F^2. \tag{2}$$

It is invariant under gauge transformations

$$A_{\alpha\beta\gamma} \to A_{\alpha\beta\gamma} + \partial_{[\alpha} B_{\beta\gamma]}, \tag{3}$$

where B is a two-form. Because of this gauge freedom, F contains no propagating degrees of freedom. The solution to the equations of motion is an arbitrary constant value of the field strength,

$$F = \text{const}. \tag{4}$$

The situation changes in the presence of 2-branes, or domain walls, which may act as sources for A due to the following coupling:

$$q \int_{2+1} A, \tag{5}$$

where the integral is taken over the $2 + 1$-dimensional world volume and q is the brane charge. The role of such branes can be played by the field theoretic solitonic domain walls [3] (see Appendix A), or by fundamental branes of some sort. Their precise origin is unimportant for the present discussion. The change of F across the wall is given by

$$\Delta F = q. \tag{6}$$

Thus, F can undergo discrete variations due to nucleation of closed branes [4].

This mechanism can be used to induce spatial variations of the field ϕ, by coupling it to the F-form. We *do not* require that the Lagrangian contains any small scale (such as the supersymmetry breaking scale). We allow the (renormalized) potential of ϕ to be the most general function, including all possible interactions with the four-form field F,

$$V(\phi) = \left(-m^2 + \frac{F^2}{M_p^2} + \cdots\right)|\phi|^2 + \left(1 + \frac{F^2}{M_p^4} + \cdots\right)|\phi|^4 + \cdots. \tag{7}$$

The dimensionless coefficients are not shown explicitly and are assumed to be of order one. The couplings linear in F are suppressed by parity symmetry. The couplings in (7) effectively convert the mass and the VEV of ϕ into functions of the four-form field strength, e.g.,

$$\phi^2 \sim (m^2 - F^2/M_p^2). \tag{8}$$

We shall assume that the F-independent part of the mass m^2 takes its natural value, $m^2 \sim M_P^2$. For definiteness, we shall assume the sign of this contribution to be negative and the sign of the F-dependent contribution to be positive. Then, the F-dependent contribution will lead to a partial cancellation of the effective mass. This mass will take different values in different parts of the Universe due to nucleation of branes charged under F.

To ensure that the brane charge q is suppressed at small values of ϕ, we require that the system is invariant under a Z_{2N} symmetry, which acts on ϕ as

$$\phi \to e^{i(\pi/N)}\phi, \tag{9}$$

and at the same time changes branes into antibranes and vice versa (leaving the three-form A invariant).[1] The coupling of the three-form to the branes (5) should then be replaced by

$$\int_{2+1} \frac{\phi^N}{M_p^{N-2}} A. \tag{10}$$

Note that for a nonconstant ϕ, the above coupling is not gauge invariant, and extra nonlocal terms have to be added to the action to restore the gauge invariance. A detailed discussion is given in Appendix A. These additional terms are particularly important for understanding of the radiation of A-waves from the branes that occurs if the background value of ϕ changes in time (see Appendix B). We have shown in [3] that a nonlocal action of this type can arise from a local field theory in a two-dimensional toy model after integration over some massless fermions. It is not clear whether or not this mechanism can be extended to $4D$, and more generally, whether or not the required type of action can be obtained in effective field theory. We leave this question for future investigation.

In the context of brane nucleation, however, the additional terms in (10) are unimportant. In the regime of interest to us here (small ϕ), each act of brane nucleation changes ϕ by a very small amount, so ϕ can be regarded as nearly constant, and additional terms are negligible.

The magnitude of the F-step between neighboring domains is set by ϕ,

$$\Delta F \propto \phi^N, \tag{11}$$

and so is the change of the VEV of ϕ,

$$\Delta\phi^2 \propto \phi^N. \tag{12}$$

This is the key point of our mechanism. With every step that decreases the VEV of ϕ, we create a region in the Universe where the brane charges are smaller. This allows for finer and finer adjustment of the ϕ-VEV, accompanied by further decrease of the brane charges. Thus, the step size of the field F (and therefore of ϕ) decreases and the "level density" grows towards smaller values of ϕ, and if the process is not for some reason terminated, the total number of levels diverges.

For instance, imagine that we start in a domain where $F \sim 1$ and $\phi^2 = \xi < 1$ in Planck units. In these units, the value of the brane charge in that domain is

$$q_0 = \xi^{N/2}. \tag{13}$$

For the sake of definiteness, let us assume that $\xi \sim 0.1$ or

[1] Alternatively, we could require that $A \to -A$, while branes are unaffected. In fact, the system is invariant under both assignments, independently.

so. After the first step of brane nucleation, the change in F is

$$\Delta F = q_0 \sim \xi^{N/2} \tag{14}$$

and the VEV of ϕ is partially cancelled to

$$\phi^2 \to (\xi - \xi^{N/2}). \tag{15}$$

Then it will take approximately $n = 1/\xi^{(N/2)-1}$ steps to cancel ϕ^2 to $\phi^2 \sim \xi^2$. At this point the brane charge becomes

$$q_{\rm new} \sim \xi^N, \tag{16}$$

and now it will take $n \sim 1/\xi^{N-2}$ steps to cancel ϕ to

$$\phi^2 \sim \xi^3, \tag{17}$$

and so on. In general, the number of steps required to cancel ϕ^2 to an accuracy ξ^k is

$$(\text{number of steps}) \sim \xi^{Nk/2}. \tag{18}$$

All the allowed values of ϕ near $\phi = 0$ have nearly identical vacuum energies, and the corresponding regions will therefore occupy equal fractions of the volume in the post-inflationary universe. The corresponding prior probability for ϕ is then simply proportional to the density of states,

$$\mathcal{P}_*(\phi)d\phi \propto d\phi/\phi^N. \tag{19}$$

Thus, regions with zero mass and VEV of ϕ are maximally probable. This hierarchy attractor provides a dynamical mechanism for explaining a zero mass of an interacting scalar without need for supersymmetry.

We note that although the wall charge vanishes as $\phi \to 0$, the wall tension remains large, $\sigma \sim M_p^3$, and in the limit the walls become simply domain walls separating degenerate vacua. Nucleation of such walls is suppressed by a huge factor [5] $\sim \exp(-\pi M_p^2/H^2)$, where H is the expansion rate during inflation. This, however, does not change our conclusions, since eternal inflation provides unlimited time for the distribution (19) to establish.

In order to use this "attractor" mechanism for solving the gauge hierarchy problem, we have to overcome the fact that the attractor point is at *exactly* zero mass and VEV of ϕ. In the following section we will show that curvature corrections to the potential $V(\phi)$ generally shift the attractor point away from zero to a small value of ϕ.

III. SMALL HIGGS MASS FROM QUANTUM FLUCTUATIONS

In the above analysis we have ignored the effects of the gravitational backreaction on the Higgs mass. One possible source of this backreaction is a nonminimal coupling to the curvature,

$$|\phi|^2 R. \tag{20}$$

This will create an additional contribution to the Higgs mass during inflation,

$$\Delta m^2_{\rm curvature} \sim H^2. \tag{21}$$

Even in the absence of such coupling, ϕ will get a thermal-type contribution to its mass due to de Sitter quantum fluctuations. This effect is analogous to that of a thermal bath at temperature $T_{GH} \sim H$ (Gibbons-Hawking temperature [6]). In the domains where the Higgs VEV drops below H, the corresponding contribution to the Higgs mass is

$$\Delta m^2_{GH} \sim H^2. \tag{22}$$

(For $\phi \gg H$, the fields interacting with ϕ get masses greater than T_{GH} and do not contribute to the Higgs potential.)

To analyze the effect of these contributions on our attractor mechanism, we shall first consider a simplified picture where the expansion rate H remains nearly constant during inflation. (This situation is realized in some models of hybrid inflation [7].) Because of the mass corrections (21), (22), which we shall assume to be positive, the Higgs VEV during inflation will not be given by (8), but will rather be shifted to

$$\phi^2_{\rm inflationary} \sim (m^2 - F^2/M_p^2 - \Delta m^2), \tag{23}$$

where $\Delta m^2 = \Delta m^2_{\rm curvature} + \Delta m^2_{GH} \sim H^2$. Now, the discussion in the preceding section indicates that ϕ will be driven not to the point where its post-inflationary VEV (8) vanishes, but rather to the point where its *inflationary* VEV vanishes. That is, most of the space in the Universe will be occupied by domains where $\phi^2_{\rm inflationary} \sim 0$.

Now, it should be noted that light scalar fields with masses $m \lesssim H$ are subject to large quantum fluctuations during inflation. Assuming the minimum of the potential is at $\phi = \phi_0$, the characteristic amplitude of the fluctuations $\delta\phi$ is generally given by $[V(\phi_0 + \delta\phi) - V(\phi_0)] \sim H^4$ [8]. In our case, if the mass of ϕ is driven to zero, then $V(\phi)$ is reduced to the quartic term in (7), and we have $\delta\phi \sim H$. This means that the field ϕ can be driven to zero only with an accuracy $\sim\mathcal{O}(H)$,

$$\phi^2_{\rm inflationary} \sim [m^2 - F^2/M_p^2 - \mathcal{O}(H^2)] \sim H^2. \tag{24}$$

After the end of inflation, the gravitational ($\sim H^2$) contribution to the mass vanishes, and the VEV is shifted to

$$\phi^2_{\rm today} \sim (m^2 - F^2/M_p^2) \sim H^2. \tag{25}$$

Thus, the post-inflationary Higgs mass will not be exactly zero, but will rather be comparable to the inflationary Hubble parameter. Because of the quantum fluctuations, the infinite peak in the probability distribution (19) will be smeared, and the distribution will be nearly flat, $\mathcal{P}_*(\phi) \approx$ const, for $|\phi| \lesssim H$. At larger values of ϕ, the probability suppression is at least as strong as in Eq. (19). It can be even stronger, due to the effect of differential expansion. Larger expectation values of ϕ correspond to smaller values of F, resulting in a smaller vacuum energy (both because the potential (7) is more negative, and because the F-field energy density, $\rho_F \sim F^2$, is smaller). As a result, the inflationary expansion rate is lower, which can lead to an exponential suppression at large ϕ [9].

In order to solve the hierarchy problem, we require that the peak of the probability distribution is at $\phi \sim 1$ TeV. Then the Hubble expansion rate during inflation must be $H \sim 1$ TeV and the corresponding vacuum energy density $\sim(10^{11}$ GeV$)^4$. This is a constraint that our mechanism imposes on the inflationary scenario.

We now discuss the dynamics of the model in some more detail. Quantum fluctuations of ϕ occur on length and time scales $\delta l \sim \delta t \sim H^{-1}$. Fluctuations at the locations of domain walls cause fluctuations of the wall charge q, which in turn cause variation of the F-form in the adjacent domains. This variation propagates in the form of waves (see Appendix B) from the walls to the interior of the domains. The wavelengths of these waves are stretched by the exponential expansion of the Universe, and as a result, the form field will vary on an exponentially large scale in the domain interiors. Shorter waves, emitted near the end of inflation, have not travelled far away from the walls. The sizes of the domains are huge compared to the present horizon, and it will take an exponentially long time for the shorter waves to propagate well into domain interiors.

After the end of inflation, the Higgs mass takes its zero-temperature value and the Higgs rolls away to its new minimum. At this point, the charge of the walls and the values of F-form in the adjacent regions change, but once again, this change propagates in the form of waves and is confined to the neighborhood of the walls. Moreover, the hierarchy is not destabilized even in regions affected by the change. The change of F triggered by the wall is proportional to the final charge of the wall, which in attractor domains is set by today's value of the Higgs field $\sim$100 GeV. Thus, even after the new value of the field strength is established, the change with respect to the inflationary value will be $\Delta F \sim 10^{-17N} M_p^2$. Already for $N = 2$, the corresponding change in ϕ is $\Delta\phi \sim 100$ GeV, which does not upset the hierarchy. For $N > 2$, the backreaction on ϕ is negligible.

Let us finally discuss how the above scenario is modified in more generic models of inflation, in which the value of H fluctuates during eternal inflation and then gradually decreases during a prolonged slow-roll period. To be specific, we shall assume that inflation is of the "new" type, with expansion rate $H_{\max} \ll M_p$ at the maximum of the potential and $H_{\min}$ at the end of inflation. Another important parameter is the borderline value H_* between the regimes of eternal inflation and slow roll. In the course of eternal inflation, the bubble walls will be exposed to H in the range $H_* \lesssim H \lesssim H_{\max}$. Since the bubble nucleation rate is so low, a typical geodesic will go through the whole range many times between successive nucleations. This suggests that the accuracy with which the attractor mechanism can drive $\phi_{\text{inflationary}}$ to zero cannot exceed $\sim H_{\max}$. The slow-roll period is relatively short and therefore affects only the immediate vicinity of the bubble walls. Thus, in order to solve the hierarchy problem, we have to require $H_{max} \sim 1$ TeV.

IV. CONCLUSIONS

It is usually assumed that the solution to the hierarchy problem requires introduction of some new physics at low energies. In the present paper we have provided a counterexample to this statement. We have suggested a novel cosmological selection mechanism in which small scalar masses and VEVs become attractors during the cosmological evolution.

The key idea is that (1) the scalar mass is dynamically promoted to a stochastic variable that undergoes discrete jumps during eternal inflation; (2) the size of the minimal step is a continuous function of the "jumping" scalar VEV ϕ. That is, the order parameter in question controls its own steps. As a result the probability distribution is sharply peaked around a small value, for which the step vanishes. We call such a value an attractor. The post-inflationary value of the scalar mass is determined by the mismatch of masses during and after inflation, due to the Gibbons-Hawking temperature $T_{GH} \sim H$, and by the magnitude of quantum fluctuations of ϕ, also $\sim H$. Thus, the observed value of the Higgs mass in our scenario is determined not by ultraviolet physics, but rather by the maximal inflationary expansion rate $H_{\max}$. Our scenario requires that $H_{\max} \sim 1$ TeV. It is obvious that this constraint is not in any respect analogous to the conventional approaches invoking supersymmetry or some other new physics around TeV.

Our model involves nonlocal couplings of the form field F, and we have emphasized that it is not presently clear how this kind of coupling can be obtained in the low-energy effective theory. We note, however, that this coupling does not seem to have any of the pathologies usually associated with nonlocal interactions. Another unusual feature of our model is that it has solutions describing waves of the field F propagating at the speed of light. This suggests the presence of massless degrees of freedom, which can potentially lead to testable predictions. These issues need further investigation.

Finally, let us note that the attractor mechanism can be combined with the usual anthropic approach to solving the cosmological constant problem [10], if one is willing to introduce more three-form fields. The main technical problem in this approach has been to guarantee a sufficiently small step of vacuum energy variation. With our attractor mechanism this is trivially achieved, as it generates branes with tiny charges. Anthropic selection can also be used to explain the observed Higgs VEV in models of inflation with $H_{\max} \gg 1$ TeV.

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APPENDIX A: BRANES WITH CHARGES SUPPRESSED BY SYMMETRIES

In this Appendix we shall give a more detailed discussion of branes and domain walls with field-dependent three-form charges. Below Planck energies, such branes can be treated as fundamental objects or be explicitly constructed as field theoretic solitonic domain walls, as it was suggested in [3]. The dynamics of three-form fields in $3+1$ dimensions is in many respects analogous to the $(1+1)$-dimensional electrodynamics, with electrically charged particles playing the role of "branes." So we shall first review our mechanism in a simplified 2-dimensional example, and then generalize to $(3+1)$ dimensions. For simplicity, we shall work in Planck units, and will set all the mass scales equal to one.

The action describing a $(1+1)$-dimensional gauge field interacting with pointlike charges can be written as

$$S_{1+1} = \int d^2xF^2 + q\int dx^\mu A_\mu, \tag{A1}$$

where F is the field strength and x^μ is the coordinate of a point charge q. With $q=$ const, the above action is gauge invariant under $A_\mu \to A_\mu + \partial_\mu \omega$. However, we would like to promote the charge q to a function of a scalar field ϕ, that is, $q = \phi^N$. With this substitution, however, the action (A1) is no longer gauge invariant. In order to restore the gauge symmetry, we shall modify it in the following way:

$$S_{1+1} = \int d^2xF^2 + \int \phi^N dx^\mu \Pi_{\mu\nu}A^\nu, \tag{A2}$$

where $\Pi_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}$ is the transverse projector. The modification is only significant for a varying ϕ. For a constant $q = \phi^N$, the actions (A1) and (A2) are equivalent.

The generalization of this action to a $(3+1)$-dimensional model with 2-branes is straightforward:

$$S_{3+1} = \int d^4xF^2 + \int \phi^N d\sigma_{\mu\nu\gamma}\Pi^{\mu\mu'}\Pi^{\nu\nu'}\Pi^{\gamma\gamma'}A_{\mu'\nu'\gamma'}, \tag{A3}$$

where $d\sigma_{\mu\nu\gamma}$ is the world-volume element. This action is manifestly gauge invariant under (3).

An unusual feature of the actions (A2) and (A3) is that the projection operators $\Pi_{\mu\nu}$ are nonlocal. It appears that such operators cannot be obtained from a local field theory by integrating out a finite number of heavy particles. In Ref. [3], the action (A2) was obtained after integrating over massless fermions in a $(1+1)$-dimensional model. However, it is not clear how generic that model is and whether or not it can be extended to higher dimensions. These issues require further study.

As we already mentioned, the branes (or charges) in question can be regarded as fundamental objects or as solitons of the effective field theory. We shall briefly review the latter possibility. Following [3], we assume that in the $4D$ low-energy effective theory F is mixed with a certain phase field a,

$$\frac{q}{2\pi}aF. \tag{A4}$$

The interaction (A4) is invariant under the shift symmetry

$$a \to a + 2\pi, \tag{A5}$$

as well as under gauge transformation of the three-form field $A_{\mu\nu\alpha}$. The equation of motion of the three-form field then demands that the variations of the field strength and a with respect to any particular coordinate must satisfy

$$\Delta F = \frac{q}{2\pi}\Delta a. \tag{A6}$$

Thus, any stable solitonic configuration across which a changes by a finite amount inevitably acts as a source for the three-form field. Such configurations do indeed exist. Since a is a phase, the potential of a must respect the shift symmetry (A5). As long as this is the case, irrespective of the precise form of this potential, due to topological reasons there must be domain wall solutions across which a changes by $\Delta a = 2\pi$. Then, according to (A6), such walls acquire a charge q under A. Thus, the values of the field strength on the two sides of the wall differ by

$$\Delta F = q. \tag{A7}$$

The charge q defines the minimal step by which F can change from one region of the Universe to another.

For our purposes, however, q cannot be truly constant, because it has to depend on ϕ. Also, if we want to think of a as a phase of a certain complex order parameter,

$$X = |X|e^{ia}, \tag{A8}$$

with a VEV around the Planck scale, $|X| \sim 1$, then q will depend on $|X|$ as well. In terms of the fields X and ϕ, the gauge-invariant coupling (A3) can be written as

$$A_{\mu\nu\gamma}\Pi^{\mu\mu'}\Pi^{\nu\nu'}\Pi^{\gamma\gamma'}[i\epsilon_{\mu'\nu'\gamma'\alpha}(X\partial^{\alpha}X^{\dagger} - X^{\dagger}\partial^{\alpha}X)\phi^{N}]. \tag{A9}$$

This is the lowest possible $a - A$-mixing operator invariant under gauge (3) and shift (A5) symmetries, and under the Z_{2N} symmetry which acts on the field ϕ as

$$\phi \to e^{i\frac{\pi}{N}}\phi. \tag{A10}$$

The action of this symmetry on the other fields can be defined in two alternative ways. One possibility is to demand

$$a \to -a, \qquad A \to A, \tag{A11}$$

or equivalently, $X \to X^{\dagger}$. Note that the transformation $a \to -a$ replaces solitons by antisolitons and vice versa. An alternative choice would be to demand

$$a \to a, \qquad A \to -A. \tag{A12}$$

Equation (A9) is invariant under both of these choices. At low energies, where ϕ and $|X|$ can be treated as constants, the above coupling reduces to the one of (A4). Note that we are not demanding any approximate or exact $U(1)$ symmetry under the shift $a \to a + \text{const}$. For the existence of the wall, all that we need is that $|X| = 0$ be a maximum of the potential. This suffices to ensure the existence of a stable configuration across which a changes by 2π. The corresponding change of F through the wall will be

$$\Delta F \sim \phi^{N}. \tag{A13}$$

APPENDIX B: *F*-WAVES FROM THE WALLS

In 3 + 1-dimensions, gauge-invariant free three-form fields [just as the (1 + 1)-dimensional electromagnetic field] have no propagating degrees of freedom, so there are no wave solutions. The situation is different in our framework, where brane charges depend on the field ϕ. At the end of inflation, ϕ rolls away from zero, and the brane charges change in time. This change triggers the corresponding change of the three-form field strength, which propagates away from the brane in the form of a shock wave. We shall now discuss this dynamics.

To illustrate the point, we shall restrict ourselves to a (1 + 1)-dimensional example. Generalization to (3 + 1) dimensions is straightforward. Labeling the two space-time coordinates x^{μ} by t and z, we shall consider an isolated static brane located at $z = 0$. The gauge-invariant Lagrangian of interest is

$$L = -\frac{1}{4}F^2 + A_{\mu}\Pi^{\mu 0}[\phi^{N}\delta(z)] + |\partial_{\mu}\phi|^2 - (-m_{\text{eff}}^2|\phi|^2 + |\phi|^4), \tag{B1}$$

where m_{eff} is the effective mass of ϕ after inflation, and we have ignored higher-order terms in the potential of ϕ. When ϕ rolls away from $\phi = 0$, the change of the brane charge induces a backreaction on ϕ. This backreaction is suppressed by the brane charge $\sim\phi^{N}$. Since in the domains of interest, the final VEV of ϕ is very small, the backreaction on ϕ is negligible, and we shall ignore it.

Thus, we shall study the dynamics of a gauge field in the background of a time-dependent charge. This dynamics is governed by the following equation

$$\partial^{\mu}F_{\mu\nu} = \Pi_{\nu 0}[\phi^{N}(t)\delta(z)] = \eta_{\nu 0}\phi^{N}(t)\delta(z) - \int\frac{dp^2}{(2\pi)^2}\frac{p_{\nu}p_0}{p^2}e^{-ipx}\tilde{\phi}_N(p_0), \tag{B2}$$

where p is the two-momentum, and $\tilde{\phi}_N(p_0)$ is the Fourier-transform of $\phi^{N}(t)$. A straightforward integration gives the following two equations

$$\partial_z F_{10} = \frac{1}{2}\partial_z[\phi^{N}(t - z)\theta(z) - \phi^{N}(t + z)\theta(-z)], \tag{B3}$$

$$\partial_t F_{10} = \frac{1}{2}\partial_t[\phi^{N}(t - z)\theta(z) - \phi^{N}(t + z)\theta(-z)], \tag{B4}$$

which are solved by

$$F_{10} = \frac{1}{2}[\phi^{N}(t - z)\theta(z) - \phi^{N}(t + z)\theta(-z)]. \tag{B5}$$

This solution describes waves propagating away from the brane in two opposite directions. From this solution it is also obvious that after the transition is finished and ϕ assumes a constant value ϕ_{today}, the change of F across the brane is given by

$$\Delta F = \phi_{\text{today}}^{N}. \tag{B6}$$

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