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Direct measurement of the scattered light effect on the sensitivity in TAMA300

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Laser interferometer gravitational wave detectors need vacuum tubes through which the laser beams pass. The light scattered from the arm cavity mirrors will make multiple reflections from the inside wall of the polished tube back onto the mirrors causing phase noise on the interferometer output beam. The TAMA300 has two 300 m length arms enclosed by vacuum tubes. By vibrating one of the tubes of the TAMA300, we directly observed the effect of the scattered light on the displacement sensitivity. It was found that a tube vibration amplitude of 5.6 μm at 776.5 Hz increased the mirror displacement noise by 1.2×10^{-17} m. This noise level is consistent with the calculated noise due to the scattered light effect.

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I. INTRODUCTION

Noise due to scattered light from the surface of the beam tubes is one of the noise sources in an interferometric gravitational wave detector. Although a large number of theoretical studies have been made on this noise [1], few experimental studies have been reported. We successfully confirmed the noise due to scattered light using a 100 m scale interferometer for the first time.

TAMA300, a Japanese antenna for detection of gravitational waves, uses a 300 m Fabry-Perot–type Michelson interferometer to detect tiny changes of the differential length between two orthogonal arms. The aim of this project is to develop advanced techniques needed for a future km-sized interferometer and to detect gravitational waves that may occur by chance within our local group of galaxies. The instrument sensitivity reached $\tilde{h} = 5 \times 10^{-21}/\sqrt{\mathrm{Hz}}$ in 2000 [2].

The beam from a Nd:YAG laser with a wavelength of 1.06 μm passes through two 300 m length beam tubes of 400 mm diameter. The inside surface of the stainless steel (SS) tubes is treated with an electrochemical buffing (ECB) process in order to minimize outgassing [3]. The treated surfaces had roughness less than 0.4 μm. The reflection coefficients of a metal surface for s- and p-polarized beams R_s and R_p are expressed, respectively, using the incident angle θ.

$$R_s(\theta) = \frac{\cos(\theta) - n\cos(\theta')}{\cos(\theta) + n\cos(\theta')}, \qquad (1)$$

$$R_p(\theta) = \frac{\cos(\theta') - n\cos(\theta)}{cos(\theta') + n\cos(\theta)}, \qquad (2)$$

$$\sin(\theta') = \frac{1}{n}\sin(\theta), \qquad (3)$$

where n is the refractive index of the material. The measured refractive index of the SS-ECB surface with the Nd:YAG laser beam was $n = 2.5 - 4.1i$ [4]. Using this value, the calculated reflectivity of the beam tube is almost 1 for large incident angle $\theta \simeq \pi/2$ rad. Although the scattered light is low enough to obtain the present sensitivity, the noise caused by the scattered light was confirmed directly using TAMA300 in order to design the future km-sized interferometer like LCGT [5]. In this paper, we describe a direct measurement of the scattered light effect on the sensitivity by vibrating the entire surface of the beam tube in TAMA300.

II. MODELING OF SCATTERED LIGHT NOISE

The geometry of the scattering path is shown schematically in Fig. 1.

The main beam, which is resonant on the axis of the cavity consisting of two mirrors, has a power of P_0. The light power scattered from the near mirror and reflected only once from the beam tube surface toward the far mirror is proportional to the scattering properties of the mirror surface described by the bidirectional reflection distribution function BRDF (ϕ), to the magnitude of the incident light power, to the reflectivity of the beam tube

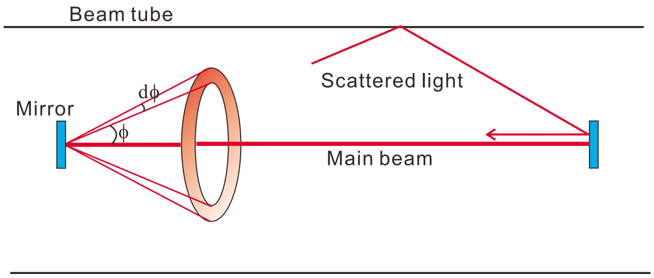

FIG. 1 (color online). Geometry of the scattered light path from the arm cavity mirrors. A part of the light reflected from the beam tube surface recombines to the main beam.

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surface, and to the magnitude of the solid angle into which the light is scattered. The power ratio of the scattered light incident on the far mirror was calculated for a single reflection at the middle of the beam tube. In this expression, $R(\phi_i)$ is the reflectivity of the beam tube wall, and ϕ_i is the average scattering angle related to the incident angle θ by equation $\phi_i = \pi/2 - \theta$.

$$\frac{P}{P_0} = R(\phi_i) \int_{\phi_{\rm min}}^{\phi_{\rm max}} {\rm BRDF}(\phi) 2\pi\phi d\phi. \tag{4}$$

The BRDF of the TAMA mirrors is not known. We used the typical BRDF of mirrors in the Laser Interferometer Gravitational Wave Observatory [6].

$$\text{B RDF}(\phi) = \frac{1000}{(1 + 5.302 \times 10^8 \phi^2)^{1.55}}. \tag{5}$$

The minimum and maximum scattering angles are determined by the radius of the beam tube $r_{\rm tube}$, the cavity length L, the average scattering angle, and the length of the reflection region of the tube wall ΔL, as follows:

$$\phi_{\rm min} = \frac{2r_{\rm tube} - 2\sin(\phi_i)\Delta L}{L}, \tag{6}$$

$$\phi_{\rm max} = \frac{2r_{\rm tube} + 2\sin(\phi_i)\Delta L}{L}. \tag{7}$$

The incidence angle at the far mirror, which is also the average scattering angle from the near mirror, is

$$\phi_i = \frac{2r_{\rm tube}}{L}. \tag{8}$$

Next, the scattered light incident on the far mirror rescatters within the diffraction angle of the main beam and combines coherently, causing phase noise. The diffraction angle of the main beam $\phi_{\rm main}$ is proportional to the wavelength λ and inversely proportional to the radius of the beam waist $\omega_0 = \sqrt{L\lambda/2\pi}$.

$$\phi_{\rm main} = \frac{\lambda}{\pi\omega_0}. \tag{9}$$

Finally, the fraction of the light power scattered into the interferometer beam is

$$\frac{P_{\rm scat}}{P_0} = \frac{P}{P_0} \int_{\phi_i - \phi_{\rm main}}^{\phi_i + \phi_{\rm main}} {\rm BRDF}(\phi) 2\pi\phi d\phi. \tag{10}$$

Since the path difference traveled by the scattered light is much greater than a wavelength of light, it is difficult to know the static phase difference $\Delta\Phi_{\rm scat}$ of the scattered electric field vector $E_{\rm scat}$ in Fig. 2.

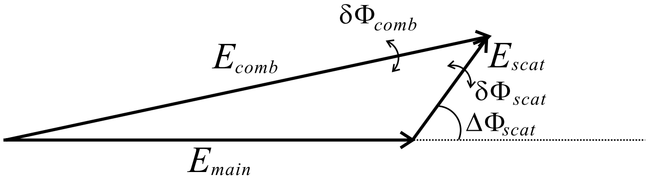

FIG. 2. Relation between the main beam $E_{\rm main}$ and the scattered light $E_{\rm scat}$. The static phase difference $\Delta\Phi_{\rm scat}$ is unreliable.

However, the maximum phase fluctuation $\delta\Phi_{\rm comb}$ of the combined noisy electric field vector $E_{\rm comb}$ occurs when $\Delta\Phi_{\rm scat}$ is equal to zero or π. Both mirrors scatter light into the interferometer beam. Then

$$\delta\Phi_{\rm comb} = 2\frac{E_{\rm scat}}{E_{\rm main}} \delta\Phi_{\rm scat}. \tag{11}$$

The square of $E_{\rm scat}/E_{\rm main}$ corresponds to the scattered power ratio $P_{\rm scat}/P_0$ exactly. The optical path length of the scattered light—as it traverses between the near mirror, the reflecting surface at the midpoint of the beam tube, and the far mirror—fluctuates by the peak amount $\delta\ell$ when the beam tube undergoes a peak radial displacement δz.

$$\delta\ell = 4\frac{r_{\rm tube}}{L}\delta z. \tag{12}$$

The average radial displacement for the half-period of the sinusoidal axial mode between the tube supports is approximately $1/\sqrt{2}$ smaller than the peak amplitude, so the mirror displacement noise is correspondingly smaller. The round-trip phase fluctuation $\delta\Phi_{\rm scat}$ is proportional to this optical path fluctuation. Therefore the maximum average mirror displacement noise δx caused by the scattered light is

$$\delta x = \sqrt{\frac{P_{\rm scat}}{P_0}} \cdot \delta\ell. \tag{13}$$

III. EXPERIMENTAL SETUP

The beam tube was vibrated by a shaker vertically. To excite the maximum displacement of the scattered light with one reflection from the surface of the beam tube, the shaker was put between the frame for the gate valve to separate the tube at the midpoint of the arm and the nearest tube support 2.7 m distance from the frame. Thus, the length of the reflection region was 2.7 m. Two different frequencies, 776.5 and 788.5 Hz, were used to shake the tube. The tube surface has mechanical resonances at these frequencies. According to a modal analysis of the tube surface, the mode shape for the 776.5 Hz mode has four lobes in the radial direction, as shown in Fig. 3.

In the axial direction, the mode has a half-cycle of approximately 2.7 m, with nodes at the quasirigid supports 2.7 m apart. The higher mode at 788.5 Hz also has four lobes in the radial direction; but the mode, which does not have nodes at the supports, extends approxi-

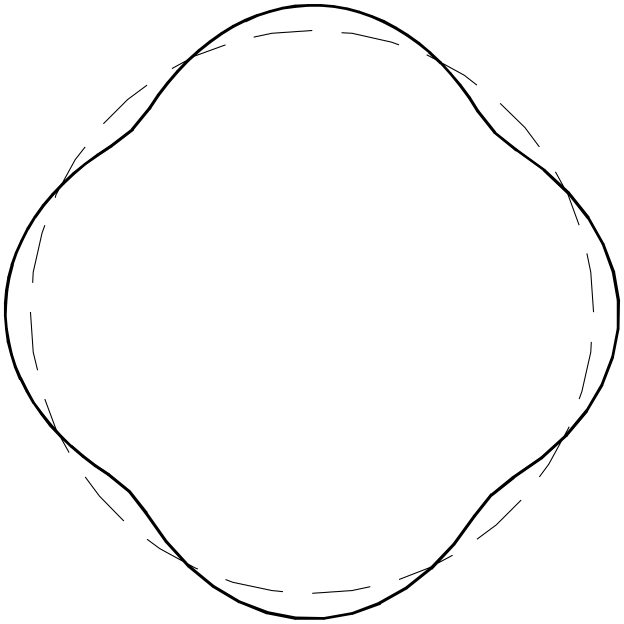

FIG. 3. Mode shape of the tube surface. The mode has four lobes in the radial direction.

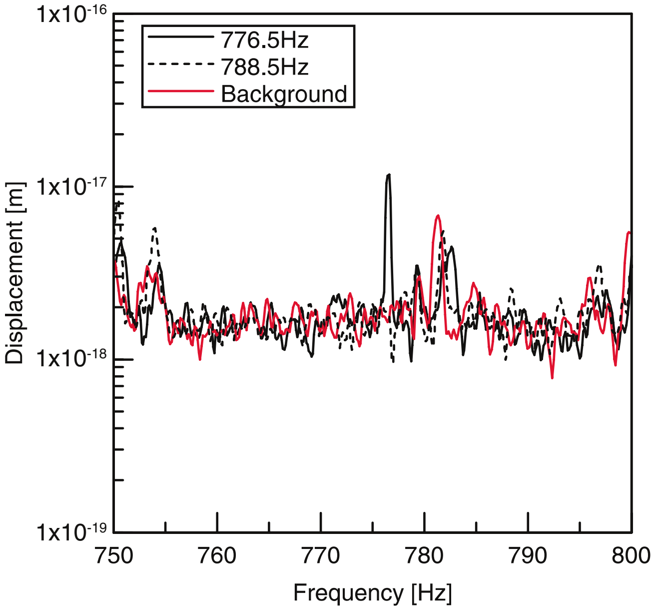

FIG. 4 (color online). Observed excess signal due to the beam tube excitation using two different frequencies, 776.5 and 788.5 Hz.

mately one and one-half cycles between the tube supports in the axial direction and is probably highly attenuated by the quasirigid supports.

Two acceleration sensors were placed on the tube surface. One was at a point as close as possible to the shaking point. The other was at a point near the interferometer input test mass mirror to investigate the noise caused by vibrating the mirror directly. The amplitude of displacement at the shaking point was 5.6 μm for 776.5 Hz and 4.7 μm for 788.5 Hz, respectively. Additionally we put a microphone in front of the door into the 300 m tunnel for the beam tube to investigate effects due to the sound caused by shaking the tube.

IV. RESULT AND DISCUSSION

We found excess signal at the tube excitation frequencies in the displacement spectrum of the interferometer (Fig. 4).

The peak value of the signal due to the 776.5 Hz excitation was 1.2×10^{-17} m compared with the background level of 0.16×10^{-17} m. The other peak value due to the 788.5 Hz excitation was 0.25×10^{-17} m compared with the background level of 0.13×10^{-17} m. By comparing the outputs of the two acceleration sensors, we estimated that the displacement noise from the direct vibration of the interferometer mirrors was less than 0.1×10^{-17} m for 776.5 Hz and less than 0.01×10^{-17} m for 788.5 Hz, respectively. Therefore these effects were negligible. The door into the 300 m tunnel is effective in eliminating the sound due to shaking the tube. With the 776.5 Hz excitation, we could not find any difference in the excess signal in the interferometer with the door open or closed. There was a small increase of the excess signal due to the sound of the shaking for the 788.5 Hz excitation case. By comparing the microphone outputs, we estimated that the displacement noise caused by the sound of shaking the tube was 0.2×10^{-17} m. Since the observed excess signal due to the 788.5 Hz tube vibration was comparable to the quadratic sum of the estimated sound and background displacement noise levels, we cannot say that the excess signal is caused by the scattered light in this case.

Using parameters for TAMA: $r_{\mathrm{tube}} = 0.2$ m and $L = 300$ m, and an effective reflection surface (2 m length) at the midpoint of the 300 m beam tube corresponding to the scattering angle range; $\phi_{\mathrm{min}} - \phi_{\mathrm{max}}$, the calculated value for the average scattered light displacement noise is $\delta x = 1.3 \times 10^{-16}$ m. This calculation assumed the static phase difference $\Delta\Phi_{\mathrm{scat}}$ is optimal, the axial length of the reflection surface extended across one-half-period of the mode in the axial direction, and the tube surface had uniform radial motion. A smaller value would have been calculated if the actual value of $\Delta\Phi_{\mathrm{scat}}$ had been known.

The actual, symmetric radial mode shape shown in Fig. 3 will not cause a net change in the scattered light path length because the increase in path length from the surface peaks will be exactly offset by a decrease in path length at the surface valleys. However, it is likely that there will be a slight optical path fluctuation because the mode shape is probably slightly nonuniform, both in the radial and axial directions, and the scattered light cone is not centered precisely on the tube axis.

A 10% nonuniformity in the tube mode shape would account for the observed excess signal of 1.2×10^{-17} m with the 776.5 Hz excitation being caused by the scattered light in the beam tube. It is likely that the scattered light displacement noise caused by the 788.5 Hz excitation is much smaller because of the attenuation of the surface amplitude by the quasirigid tube supports.

A recent estimation of the target sensitivity in LCGT requires a radiation pressure noise limited strain $\tilde{h} = 1 \times 10^{-23}/\sqrt{\mathrm{Hz}}$ at 30 Hz [7,8]. This corresponds to horizontal displacement noise $\delta\tilde{x} = 3 \times 10^{-20}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$. Using parameters for LCGT: $\delta\tilde{z} = 1 \times 10^{-11}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$, $r_{\mathrm{tube}} = 0.45$ m, $L = 3000$ m, and 0.15 m in the radius of the mirror, the calculated displacement noise for a single reflection from the tube wall at the middle portion of the beam tube was $\delta\tilde{x} = 1.1 \times 10^{-21}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$. The tube vibration amplitude was assumed to be the same as the typical seismic motion in the Kamioka mine. The calculation indicates that with a polished tube the displacement noise due to scattered light reflecting from the tube wall will have a safety factor of 28 smaller than the requirement. However, if the seismic motion is excited by some instruments (vacuum pumps, cryostats, compressors, etc.), the large safety factor would be lost easily.

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