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Compton scattering in the presence of Lorentz and *CPT* violation

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We examine the process of Compton scattering, in the presence of a Lorentz- and *CPT*-violating modification to the structure of the electron. We calculate the complete tree-level contribution to the cross section; our result is valid to all orders in the Lorentz-violating parameter. We find a cross section that differs qualitatively from the Klein-Nishina result at small frequencies, and we also encounter a previously undescribed complication that will arise in the calculation of many Lorentz-violation cross sections: The Lorentz violation breaks the spin degeneracy of the external states, so we cannot use a closure relation to calculate the unpolarized cross section.

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Recently, there has been a great deal of interest in the possibility of there existing small *CPT*- and Lorentz-violating corrections to the standard model [1–4]. Such small corrections might arise from larger violations of Lorentz symmetry occurring at the Planck scale. The most general possible Lorentz-violating effective field theory has been described in detail, and its renormalizability has been studied. These results open the way for a wide variety of experiments that could test for the existence of Lorentz violation.

There are already many experimental constraints on Lorentz-violating corrections to the standard model. The tests have included studies of matter-antimatter asymmetries for trapped charged particles [5–8] and bound state systems [9,10], frequency standard comparisons [11–13], measurements of neutral meson oscillations [14–16], polarization measurements on the light from distant galaxies [17,18], and many others. However, although there have been a number of kinematical analyses of the astrophysical consequences of Lorentz violation in particle scattering [19–24], there has as yet been very little investigation into the possible effects of Lorentz-violating dynamics in laboratory scattering experiments [25–27]. In this paper, we shall examine some of those effects.

We shall examine the process of Compton scattering, in the presence of a particular Lorentz- and *CPT*-violating modification of the electron sector. The study of Compton scattering has historically been very important to the development of quantum mechanics and quantum field theory [28–30] and in the future might provide an important test of Lorentz violation.

The calculation of scattering cross sections in a Lorentz-violating theory involves a number of subtleties that are not present in the standard, Lorentz-invariant case. Different reference frames are no longer necessarily equivalent, and the correct definition of the particle flux becomes potentially ambiguous. However, with appropriate care, meaningful cross sections can be found, and a general theory for their calculation is given in [26]. Our analysis will also reveal an additional complication, not discussed in [26], that may arise in Lorentz-violating scattering processes. The various spin states of the scattered particles may have differing energies, and this can affect the velocity and phase space factors that appear in the cross section. As a result, it may become impossible to calculate an unpolarized cross section by the usual means.

The Lagrange density for our theory is

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m - eA\!\!\!/ - b\!\!\!/\gamma_5)\psi. \quad (1)$$

The action includes only the single Lorentz-violating coefficient b^μ. This is the simplest perturbatively nontrivial form of Lorentz violation that can exist in the electron sector. The domain of validity of this Lagrange density extends all the way up the Planck scale [3].

Considering a theory with only a b term would not be reasonable for calculations beyond tree level; other Lorentz-violating terms would be radiatively generated at one-loop order [4]. We shall therefore consider only tree-level effects. However, although we shall only be working to leading order in the electromagnetic coupling e^2, our results will be correct to all orders in b.

In general, the spacetime direction of b is arbitrary. However, we shall choose b to be purely timelike, $b^\mu = (B, \vec{0})$, in the laboratory frame. It is a common practice to suppose that any Lorentz-violating coefficients have vanishing spatial components; this practice arises from the observation that the Universe shows a very high degree of isotropy in the rest frame of the cosmic microwave background. In this case, considering only a timelike b will also substantially simplify our b-exact analysis of the theory.

Since our Lorentz-violating Lagrange density (1) involves no changes to the electrons' kinetic term, and there are no additional time derivatives not present in the Lorentz-invariant theory, the electrons may be quantized without any changes to the spinor representation [3,26]. The exact electron propagator may be read off directly from the Lagrange density; it is

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$$S(l) = \frac{i}{\not{l} - m - \not{b}\gamma_5}. \tag{2}$$

We may rationalize this expression and obtain [31,32]

$$S(l) = i\frac{(\not{l} + m - \not{b}\gamma_5)(l^2 - m^2 - b^2 + [\not{l}, \not{b}]\gamma_5)}{(l^2 - m^2 - b^2)^2 + 4[l^2b^2 - (l \cdot b)^2]}. \tag{3}$$

This modification of the propagator represents one of the ways in which the presence of b will affect the theory.

However, before we can investigate how the modified propagator $S(l)$ affects the dynamics of the scattering, we must examine the effects of the Lorentz violation on the kinematics. The coefficient b will affect the structure of the theory's asymptotic states. The photon states are, of course, unaffected, but the incoming and outgoing spinors will be significantly modified. We must solve the free momentum-space Dirac equation, with the b term included, to determine the propagation modes of the electrons. Since the matrix γ_5 features prominently in the theory, it is natural to use the Weyl chiral representation for the Dirac matrices:

$$\gamma^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}, \qquad \gamma_5 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4}$$

For an electron mode with energy E and three-momentum $\vec{p} = p_3\hat{z}$, with $p_3 \geq 0$, the Dirac equation may be reduced to

$$\begin{bmatrix} E + B + p_3\sigma^3 & -m \\ -m & E - B - p_3\sigma^3 \end{bmatrix} u(p) = 0. \tag{5}$$

If the electron has spin $\frac{s}{2}$ along the z axis, then we may replace $\sigma^3 \rightarrow s$. The eigenvalue condition for E then becomes

$$E^2 = m^2 + (sp_3 + B)^2 = m^2 + (s|\vec{p}| + B)^2, \tag{6}$$

and the spinor is

$$u^s(p) = \begin{bmatrix} \sqrt{\sqrt{m^2 + (sp_3 + B)^2} - (sp_3 + B)}\,\xi_s \\ \sqrt{\sqrt{m^2 + (sp_3 + B)^2} + (sp_3 + B)}\,\xi_s \end{bmatrix}, \tag{7}$$

where the ξ_s are basis spinors quantized in the z direction. This solution may easily be generalized to describe electrons with arbitrary three-momentum, so long as the spin is quantized along the direction of the motion. Our spinors satisfy the conventional normalization conditions $\bar{u}^{s'}(p)u^s(p) = 2m\delta^{ss'}$ and $u^{s'\dagger}(p)u^s(p) = 2E(p)\delta^{ss'}$. Note that even though there is no breaking of rotation invariance, the energy depends upon the spin direction, through the helicity s.

Just as in the Lorentz-invariant case, a great deal can be learned about the scattering simply from an analysis of the energy-momentum relation (6). Let us consider an experiment in which the initial electron has vanishing three-momentum—$p = (E, \vec{0}) = (\sqrt{m^2 + B^2}, \vec{0})$; then the incoming spinor is

$$u_i^s(p) = \begin{bmatrix} \sqrt{\sqrt{m^2 + B^2} - B}\,\xi_s \\ \sqrt{\sqrt{m^2 + B^2} + B}\,\xi_s \end{bmatrix}. \tag{8}$$

However, even though the electron's three-momentum vanishes, it is not really stationary; because of the Lorentz violation, the group velocity for a wave packet centered around $\vec{p} = \vec{0}$ is nonvanishing, and we must account for this velocity in the definition of the electron flux. In general, Lorentz-violating effects could also cause the velocity and three-momentum of the electron to point in different directions, but in this case, the two quantities are always collinear.

The almost stationary electron is struck by a photon with momentum $k^\mu = (\omega, \omega\hat{z})$. (This is a reasonable setup for a low- or medium-energy experiment.) The photon is scattered through an angle θ and has outgoing momentum $k'^\mu = (\omega', \omega'\sin\theta, 0, \omega'\cos\theta)$. The scattered electron has three-momentum $\vec{p}'$, and the corresponding adjoint spinor is

$$\bar{u}_f^{s'}(p') = \begin{bmatrix} \sqrt{\sqrt{m^2 + (s'|\vec{p}'| + B)^2} + (s'|\vec{p}'| + B)}\,\xi'^*_{s'} \\ \sqrt{\sqrt{m^2 + (s'|\vec{p}'| + B)^2} - (s'|\vec{p}'| + B)}\,\xi'^*_{s'} \end{bmatrix}^T, \tag{9}$$

where $\frac{s'}{2}$ is the spin, quantized along the $\vec{p}'$ direction. For the sake of brevity, we define $C = s'|\vec{p}'| + B$. The energy of the scattered electron is $E' = \sqrt{m^2 + C^2}$.

We may now derive a generalization of Compton's wavelength shift relation, $1 - \cos\theta = m(1/\omega' - 1/\omega)$. From three-momentum conservation, we have that $\omega'\sin\theta = |\vec{p}'|\sin\Theta$ and $\omega - \omega'\cos\theta = |\vec{p}'|\cos\Theta$, where Θ is the angle through which the electron is scattered. (More precisely, Θ is the angle describing the orientation of the momentum. If $|\vec{p}'|$ is small enough that $|\vec{p}'| + sB < 0$, then the three-momentum and the group velocity are oriented in opposite directions, so the correct scattering angle is $\pi - \Theta$.) Taken together, the equations for Θ give us

$$\Theta = \tan^{-1}\frac{\sin\theta}{\frac{\omega}{\omega'} - \cos\theta} \tag{10}$$

and

$$|\vec{p}'|^2 = \omega^2 + (\omega')^2 - 2\omega\omega'\cos\theta; \tag{11}$$

these equations do not depend upon B. Using (11), the energy conservation condition becomes

$$\omega + \sqrt{m^2 + B^2} = \omega' + \sqrt{m^2 + [s'\sqrt{\omega^2 + (\omega')^2 - 2\omega\omega'\cos\theta} + B]^2}. \quad (12)$$

By repeatedly squaring the equation (12), we can arrive at a quadratic equation for $(1 - \cos\theta)$:

$$\omega^2(\omega')^2(1-\cos\theta)^2 - 2\omega\omega'[(\omega-\omega')\times \sqrt{m^2+B^2} + B^2](1-\cos\theta) + m^2(\omega-\omega')^2 = 0. \quad (13)$$

Note that the s' dependence of the energy has vanished from this expression.

In the Lorentz-invariant case, $B = 0$, Eq. (13) is a perfect square, with only one solution for $(1 - \cos\theta)$. For $B \neq 0$, (13) has two solutions. The correct one may be identified by noting that forward scattering ($\theta = 0$) must correspond to $\omega = \omega'$. We then see that

$$1-\cos\theta = \frac{1}{\omega\omega'}[(\omega-\omega')\sqrt{m^2+B^2} + B^2 - |B|\sqrt{(\omega-\omega')^2 + B^2 + 2(\omega-\omega')\sqrt{m^2+B^2}}]. \quad (14)$$

This relation represents the modification of the Compton effect caused by the presence of the Lorentz-violating coefficient b. If we then expand (14) to first order in B, we find

$$1-\cos\theta \approx m\left(\frac{1}{\omega'} - \frac{1}{\omega}\right)\left[1 - \frac{|B|}{m}\sqrt{1 + \frac{2m}{(\omega-\omega')}}\right]. \quad (15)$$

Alternatively, to obtain the $\mathcal{O}(B)$ correction to the Compton wavelength shift, we may replace $\omega - \omega'$ in (15) with the $B = 0$ expression

$$\omega - \omega' = \omega\frac{1-\cos\theta}{\frac{m}{\omega} + (1-\cos\theta)}. \quad (16)$$

This gives $1/\omega' - 1/\omega$ as a function of $|B|$, ω, and $(1 - \cos\theta)$. So the relations (10), (11), and (14) are sufficient to express all the kinematically constrained variables in the problem in terms of a single quantity—either the scattered photon's energy ω' or the scattering angle θ.

To complete our discussion of the kinematics, we must determine the flux normalization and phase space factors that appear in the differential cross section. These factors account for the properties of the initial and final states, respectively. The flux normalization factor in the cross section is $1/F$, where

$$F = N_\gamma N_e|\vec{v}_\gamma - \vec{v}_e|. \quad (17)$$

N_γ and N_e are the photon and electron beam densities, while $\vec{v}_\gamma$ and $\vec{v}_e$ are the corresponding velocities in the laboratory frame. All the particles obey conventional normalization conditions, so $N_\gamma = 2\omega$ and $N_e = 2E = 2\sqrt{m^2 + B^2}$. The photon velocity is clearly $\vec{v}_\gamma = \hat{z}$, and the group velocity for the electron wave packet is

$$\vec{v}_e = \frac{sB}{E}\hat{z}. \quad (18)$$

The impact of the outgoing states is more subtle. The phase space integral is [26]

$$\int d\Pi = \int \frac{d^3k'}{(2\pi)^3}\frac{1}{2\omega'}\frac{d^3p'}{(2\pi)^3}\frac{1}{E'}(2\pi)^4\delta^4(k'+p'-k-p) \quad (19)$$

$$= \frac{1}{16\pi^2}\int(\omega')^2 d\omega' d\Omega \frac{1}{\omega' E'}\delta(\omega' + E' - \omega - E). \quad (20)$$

We use the δ function to perform the ω' integration; this gives us a factor of

$$\frac{1}{|\frac{\partial}{\partial\omega'}(\omega' + E' - \omega - E)|} = \frac{1}{1 + (1 + \frac{s'B}{|\vec{p}'|})\frac{\omega' - \omega\cos\theta}{E'}}. \quad (21)$$

Therefore, the phase space factor is given by

$$\int d\Pi = \frac{1}{16\pi^2}\int d\Omega \frac{\omega'}{E' + (1 + \frac{s'B}{|\vec{p}'|})(\omega' - \omega\cos\theta)}. \quad (22)$$

Equations (18) and (22) depend explicitly upon s and s', so the impact velocity and the available phase space are not independent of the electron's spin. This is an important observation, because it affects the way in which we must calculate the cross section. In high-energy physics experiments, one frequently measures only unpolarized cross sections. Moreover, the unpolarized formulas are often especially simple in form and easy to obtain, thanks to Casimir's trick of using the closure relation for the Dirac spinors to perform the spin sum. However, in order to use this trick, the velocity and phase space factors must be independent of the incoming and outgoing polarizations. In the situation we are considering, Lorentz violation has broken the spin degeneracy of the electron states' energies. We therefore cannot use Casimir's method to sum over the spin states. Instead, we shall calculate the cross section using a basis of explicit polarization states for all the incoming and outgoing particles.

This completes our discussion of the Compton scattering kinematics, and we now turn our attention to the details of the dynamics. The scattering matrix element $\mathcal{M}$ is given by

$$i\mathcal{M} = \bar{u}_f^{s'}(p')(-ie\gamma^\mu)\epsilon_\mu'^*(k')S(p+k)(-ie\gamma^\nu)\epsilon_\nu(k)u_i^s(p) + \bar{u}_f^{s'}(p')(-ie\gamma^\nu)\epsilon_\nu(k)S(p-k') \times(-ie\gamma^\mu)\epsilon_\mu'^*(k')u_i^s(p), \quad (23)$$

where $\epsilon(k)$ and $\epsilon'(k')$ are the polarization vectors for the external photons. Although we have restricted the electron's spin to be quantized along its direction of motion, we may allow the photon polarization basis to remain arbitrary, since there is no tree-level Lorentz violation in the electromagnetic sector.

The propagators appearing in (23) have arguments of the form $l^\mu = (\sqrt{m^2+B^2}+\Omega, \Omega\hat{u})$, where $\Omega = \omega$ or $-\omega'$, and $\hat{u}$ is a unit three-vector. So the denominator of (3) reduces to the extremely simple form

$$(l^2 - m^2 - b^2)^2 + 4[l^2 b^2 - (l\cdot b)^2] = 4\Omega^2 m^2. \quad (24)$$

Using our explicit representation (4) of the gamma matrices, the entire propagator then becomes

$$S(l) = \frac{i}{2\Omega m^2}\left\{\begin{bmatrix} mE & (\Omega+E)(E-B) \\ (\Omega+E)(E+B) & mE \end{bmatrix} + \begin{bmatrix} mB & (B-\Omega)(E-B) \\ (B+\Omega)(E+B) & mB \end{bmatrix}\sigma_{\hat{u}}\right\}, \quad (25)$$

where $\sigma_{\hat{u}} = \vec{\sigma}\cdot\hat{u}$ is the Pauli spin matrix corresponding to the direction $\hat{u}$.

We now need to evaluate $u_f^{s'}\gamma^\alpha S(l)\gamma^\beta u_i^s$. The products of Pauli matrices that will arise may be simplified by noting that, since the polarization vectors $\epsilon(k)$ and $\epsilon'(k')$ are purely spacelike, α and β will take on only spacelike values. Setting $\alpha = j$ and $\beta = k$, we see that we need only evaluate the combinations

$$\sigma^j\sigma^k = \delta^{jk} + i\epsilon^{jkl}\sigma^l, \quad (26)$$

$$\sigma^j\sigma^l\sigma^k = \delta^{jl}\sigma^k + \delta^{kl}\sigma^k - \delta^{jk}\sigma^l - i\epsilon^{jkl}. \quad (27)$$

These expressions are to be contracted with $\vec{\epsilon}(k)$, $\vec{\epsilon}'^*(k')$, and $\hat{u}$.

Ultimately, there are four terms in the matrix element, since there are two diagrams in (23) and two terms in the propagator (25). We shall evaluate each term individually. We begin with the contribution $\mathcal{M}_1$ coming from the first terms in both (23) and (25). If we contract (26) with the external vectors and use the identity

$$m = \sqrt{\sqrt{m^2+B^2}+B}\sqrt{\sqrt{m^2+B^2}-B}, \quad (28)$$

we find the final expression

$$\mathcal{M}_1 = \frac{-e^2}{2\omega m^2}\Big\{\sqrt{\sqrt{m^2+C^2}+C}\sqrt{\sqrt{m^2+B^2}+B} \times[(\omega+2B)\sqrt{m^2+B^2}+\omega B] + \sqrt{\sqrt{m^2+C^2}-C}\sqrt{\sqrt{m^2+B^2}-B}[(\omega-2B) \times\sqrt{m^2+B^2}-\omega B]\Big\}[(\vec{\epsilon}'^*\cdot\vec{\epsilon})(\xi_{s'}'^\dagger\xi_s) + i(\vec{\epsilon}'^*\times\vec{\epsilon})\cdot(\xi_{s'}'^\dagger\vec{\sigma}\xi_s)]. \quad (29)$$

The second contribution from the same Feynman diagram is similar:

$$\mathcal{M}_2 = \frac{-e^2}{2\omega m^2}\Big\{\sqrt{\sqrt{m^2+C^2}+C}\sqrt{\sqrt{m^2+B^2}+B} \times[(\omega+2B)B+\omega\sqrt{m^2+B^2}] + \sqrt{\sqrt{m^2+C^2}-C}\sqrt{\sqrt{m^2+B^2}-B}[(\omega-2B)B - \omega\sqrt{m^2+B^2}]\Big\}[(\vec{\epsilon}'^*)_3\vec{\epsilon}\cdot(\xi_{s'}'^\dagger\vec{\sigma}\xi_s) - i(\vec{\epsilon}'^*\times\vec{\epsilon})_3 \times(\xi_{s'}'^\dagger\xi_s) - (\vec{\epsilon}'^*\cdot\vec{\epsilon})(\xi_{s'}'^\dagger\sigma^3\xi_s)]. \quad (30)$$

The other two contributions to $\mathcal{M}$ are also similar in form. They differ from $\mathcal{M}_1$ and $\mathcal{M}_2$ in three ways: We must make the replacement $\omega \to -\omega'$, reverse the signs of the cross product terms (because the order of γ^μ and γ^ν has been switched), and change $\hat{u}$ from $\hat{z}$ to $\hat{k}' = (\sin\theta, 0, \cos\theta)$. The resulting contributions are

$$\mathcal{M}_3 = \frac{-e^2}{2\omega' m^2}\Big\{\sqrt{\sqrt{m^2+C^2}+C}\sqrt{\sqrt{m^2+B^2}+B} \times[(\omega'-2B)\sqrt{m^2+B^2}+\omega' B] + \sqrt{\sqrt{m^2+C^2}-C}\sqrt{\sqrt{m^2+B^2}-B}[(\omega'+2B) \times\sqrt{m^2+B^2}-\omega' B]\Big\}[(\vec{\epsilon}'^*\cdot\vec{\epsilon})(\xi_{s'}'^\dagger\xi_s) - i(\vec{\epsilon}'^*\times\vec{\epsilon})\cdot(\xi_{s'}'^\dagger\vec{\sigma}\xi_s)], \quad (31)$$

$$\mathcal{M}_4 = \frac{-e^2}{2\omega' m^2}\Big\{\sqrt{\sqrt{m^2+C^2}+C}\sqrt{\sqrt{m^2+B^2}+B} \times[(\omega'-2B)B+\omega'\sqrt{m^2+B^2}] + \sqrt{\sqrt{m^2+C^2}-C}\sqrt{\sqrt{m^2+B^2}-B}[(\omega'+2B)B - \omega'\sqrt{m^2+B^2}]\Big\}[(\vec{\epsilon}\cdot\hat{k}')\vec{\epsilon}'^*\cdot(\xi_{s'}'^\dagger\vec{\sigma}\xi_s) + i(\vec{\epsilon}'^*\times\vec{\epsilon})\cdot\hat{k}'(\xi_{s'}'^\dagger\xi_s) - (\vec{\epsilon}'^*\cdot\vec{\epsilon})\hat{k}'\cdot(\xi_{s'}'^\dagger\vec{\sigma}\xi_s)]. \quad (32)$$

An analysis of the general properties of the cross section is difficult. The expression for $\mathcal{M}$ takes its simplest form when the incoming and outgoing photons are both polarized along the y direction: $\vec{\epsilon} = \vec{\epsilon}' = \hat{y}$. Then only the terms containing $\vec{\epsilon}'^{*} \cdot \vec{\epsilon}$ are nonzero. Even with this simplification, however, the expression for the cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \frac{\omega'}{\omega(\sqrt{m^2 + B^2} - sB)[E' + (1 + \frac{s'B}{|\vec{p}'|})(\omega' - \omega\cos\theta)]} |\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \mathcal{M}_4|^2 \tag{33}$$

is rather unwieldy. Nor does extracting only the $\mathcal{O}(B)$ part of $d\sigma/d\Omega$ simplify things very much. However, we may still evaluate the cross section numerically. Figure 1 shows the $\mathcal{O}(B)$ contribution to the quantity $\Delta(d\sigma/d\Omega) = (d\sigma/d\Omega - d\sigma/d\Omega|_{B=0})$, for one particular combination of polarization states—$\vec{\epsilon} = \vec{\epsilon}' = \hat{y}$ and $s = s' = 1$—and four representative values of ω. For other choices of spin states, the magnitudes of the deviations are comparable. The scaling of $\Delta(d\sigma/d\Omega)$ with θ and ω is primarily controlled by the magnitude the $d\sigma/d\Omega$ itself. The cross section decreases with increasing ω, and, for the particular choice of polarizations used in Fig. 1, it also decreases with increasing θ.

We would also like to show that the cross section (33) has qualitative properties that distinguish it strongly from the usual Klein-Nishina formula. We shall therefore consider a special limit—that of near-vanishing photon energy—in which (33) is completely dominated by the Lorentz-violating contributions. Specifically, we consider the case of $\omega \ll B^2/m \ll m$. Since $|B|$ is expected to be small, this regime may not be accessible in a laboratory setting; however, our results might still be testable astrophysically. In this limit, we may set $C \approx B$ and $\omega \approx \omega'$ and neglect B^2 compared with m^2. Then the matrix element becomes

$$\mathcal{M} \approx -i\frac{4e^2B^2}{\omega m}(\vec{\epsilon}'^{*} \times \vec{\epsilon}) \cdot (\xi_{s'}'^{\dagger}\vec{\sigma}\xi_s). \tag{34}$$

This is the dominant contribution to $\mathcal{M}$ unless $B = 0$ or the cross product is near vanishing. To determine a specific cross section, let us take $\vec{\epsilon} = \hat{y}$ and have $\vec{\epsilon}' = (\cos\theta, 0, -\sin\theta)$ lie in the xz plane. Since the scattered electron has a nearly vanishing three-momentum, we may take the quantization axis for the outgoing spin to be along the z direction. The phase space factor approaches its usual Lorentz-invariant form in this limit, so the cross section is

$$\frac{d\sigma}{d\Omega} \approx \frac{e^4}{4\pi^2} \frac{B^4}{\omega^2 m^4}\left(\frac{1 + ss'}{2}\cos^2\theta + \frac{1 - ss'}{2}\sin^2\theta\right), \tag{35}$$

which grows rapidly as $\omega \to 0$. This is in sharp contrast with the behavior of the Lorentz-invariant expression, which approaches the frequency-independent Thomson result as $\omega \to 0$. Although the Thomson cross section is sometimes held to be a "universal" consequence of gauge invariance [33–36], derivations of this fact rely on additional assumptions, such as Lorentz symmetry, regularity of the scattering amplitude at $\omega = 0$, or the electron propagator having a specialized form. Each of these assumptions is violated in this instance. Moreover, our nonperturbative analysis was necessary for the correct understanding of this limit. Since $B^2/m\omega$ is a large parameter, terms with arbitrarily high powers of B could potentially contribute to the cross section. Without a nonperturbative argument, we could not know what power of B would be most important.

We see that the presence of Lorentz violation can change the structure of the Compton scattering cross section in a significant way. Our calculations have been exact to all orders in b. Moreover, in addition to determining the cross section for this particular process, we have also noted a general property of Lorentz-violating scattering; when the Lorentz violation breaks the spin degeneracy of the energy-momentum relations for the external particles, Casimir's trick for performing polarization sums may not work, because the velocity and phase space factors in the cross section may depend upon the particles' spins. Our results show the feasibility

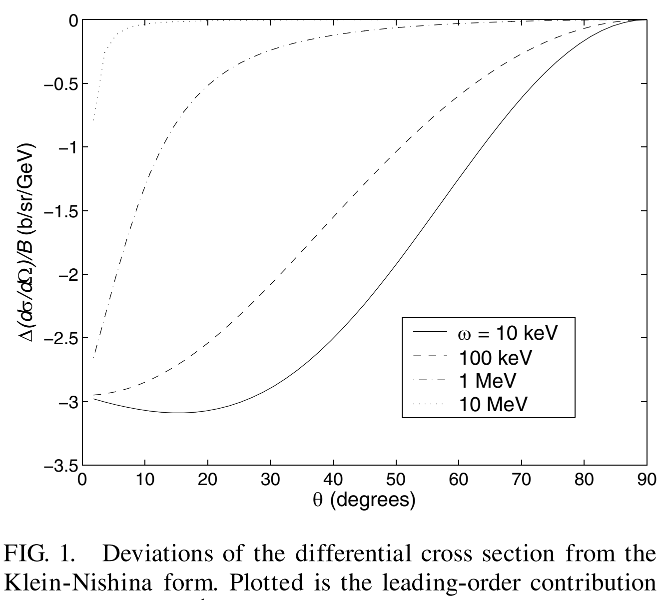

FIG. 1. Deviations of the differential cross section from the Klein-Nishina form. Plotted is the leading-order contribution to $\Delta(d\sigma/d\Omega) \cdot B^{-1}$, for $\vec{\epsilon} = \vec{\epsilon}' = \hat{y}$ and $s = s' = 1$. Although B is given in units of GeV for convenience, we would not expect these linearized results to be valid unless $B \ll m$ and $B \ll \omega$.

of scattering calculations for specific models of Lorentz violation, and this work further demonstrates that such calculations may even be performed nonperturbatively. As such, this represents a major advance in the theory of Lorentz-violating physics.

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