

**Moose models with vanishing  $S$  parameter**

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In the linear moose framework, which naturally emerges in deconstruction models, we show that there is a unique solution for the vanishing of the  $S$  parameter at the lowest order in the weak interactions. We consider an effective gauge theory based on  $K$   $SU(2)$  gauge groups,  $K + 1$  chiral fields, and electroweak groups  $SU(2)_L$  and  $U(1)_Y$  at the ends of the chain of the moose.  $S$  vanishes when a link in the moose chain is cut. As a consequence one has to introduce a dynamical nonlocal field connecting the two ends of the moose. Then the model acquires an additional custodial symmetry which protects this result. We examine also the possibility of a strong suppression of  $S$  through an exponential behavior of the link couplings as suggested by the Randall Sundrum metric.

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**I. INTRODUCTION**

Even after the very precise measurements made at LEPI, LEP II, SLC, and TEVATRON, the problem of the nature of the electroweak symmetry breaking remains to be unveiled. In particular, the Higgs particle has not been observed.

An approach to the problem of electroweak symmetry breaking is offered by technicolor (TC) theories where the Higgs particle is realized as a composite state of strongly interacting fermions, the techniquarks. However, the TC solution suffers the drawback arising from the electroweak precision measurements. These difficulties, especially the ones coming from the experiments at the  $Z$  pole, can be summarized in a single observable. This quantity is the so-called  $S$  parameter [1,2] or the related  $\epsilon_3 = g^2 S / (16\pi)$  [3]. The experimental value of  $\epsilon_3$  is of the order of  $10^{-3}$  [4], whereas the value expected in TC theories is naturally an order of magnitude bigger. There are two other important quantities which parameterize the electroweak observables at the  $Z$ -pole,  $\epsilon_1$  and  $\epsilon_2$  [3] (the parameters  $T$  and  $U$  in the notations of Ref. [1,2]). Contrarily to  $\epsilon_3$  these two parameters can be made generally small due to the custodial symmetry  $SU(2)$ , which is typically present in the TC models. As far as  $\epsilon_3$  is concerned an enhanced symmetry  $SU(2) \otimes SU(2)$  is necessary to make it small [5]. It turns out that producing this symmetry is quite difficult in TC theories.

A possible solution to the problem of  $\epsilon_3$  was proposed in Refs. [6,7] (see also Ref. [8]). This was realized in terms of an effective TC theory of nonlinear  $\sigma$ -model scalars and massive gauge fields. The model contains three nonlinear  $SU(2)$  fields and two  $SU(2)$  gauge groups (before introducing the electroweak gauge interactions). The physical spectrum consists of three massless scalar fields (the Goldstone bosons giving mass to the gauge vector particles) and two triplets of massive vector fields degenerate in mass and couplings. This model, named the degenerate BESS model (D-BESS), has an enhanced

custodial symmetry such to allow  $\epsilon_3 = 0$  at the lowest order in the electroweak interactions.

A more general case with  $n + 1$  gauge groups  $SU(2)$  and  $n + 2$  nonlinear  $\sigma$ -model scalar fields was studied in Ref. [9]. This model has the same content of fields and symmetries of the open linear moose [10–13] but a more general Lagrangian. In fact, in the linear moose models the scalar fields interact only with their nearest neighborhood gauge groups along the chain. Therefore, a linear moose looks the same as a linear lattice with lattice sites represented by the gauge groups and links by the scalar fields. This structure is particularly interesting and it is the basis of the “deconstruction” models [10–13]. Its continuum limit leads to a five-dimensional gauge theory. It is also possible to start with a five-dimensional theory, discretize (or deconstruct) the fifth dimension, and obtain a linear moose.

The typical value of  $\epsilon_3$  obtained in the linear moose models is of the same order of magnitude as in the TC theories. However, in this class of models we have an example, the D-BESS model, giving  $\epsilon_3 = 0$  (at the lowest order in weak interactions). Then it seems natural to investigate the possible solutions to  $\epsilon_3 = 0$  within the moose models. We have indeed found a general solution, which turns out to be a simple generalization of the mechanism present in D-BESS.

In Section II we introduce the notations and the main constitutive elements of a linear moose model such to describe the electroweak symmetry breaking in a minimal way. In particular, this requires that after the gauge fields have acquired mass only three massless scalar fields (the ones giving masses to  $W$  and  $Z$ ) should remain in the spectrum, and also that the fermions couple to the electroweak gauge fields in the standard way. In this case there is no contribution to  $\epsilon_3$  from fermions.

In Section III we make use of the analysis of Ref. [2] to get a general expression for  $\epsilon_3$ . The result can be written in a very compact form in terms of a particular matrix element (the one between the ends of the moose) of  $M_2^{-2}$ ,

where  $M_2$  is the quadratic mass matrix of the gauge bosons. We express also this matrix element in terms of the decay coupling constants (or link couplings) of the scalar fields. As a by-product we get the result that  $\epsilon_3$  is a semipositive definite expression (see also Refs. [14–16]).

In Section IV we investigate the possible models with  $\epsilon_3 = 0$ . We show that the unique solution corresponds to have a single vanishing decay coupling constant (or, if more, they should go to zero in an independent way). However, letting two or more couplings decrease to zero in a correlated way leads to a nonvanishing  $\epsilon_3$ . This solution corresponds to cut a link and to disconnect the linear moose in two parts. By choosing this option one needs to introduce an additional scalar field in order to have the right number of degrees of freedom to give masses to  $Z$  and  $W$ . This new dynamical field is provided by a nonlocal field (in lattice space), connecting the two ends of the original moose. Therefore, at the lowest order in the weak interactions, the original moose splits in three disconnected parts producing an enhancement of the custodial symmetry from  $SU(2)$  to  $SU(2) \otimes SU(2)$  and leading to a vanishing  $\epsilon_3$ . We have also examined the case of a linear moose with a reflection symmetry with respect to the ends of the moose. It is again possible to have  $\epsilon_3 = 0$  but only for an even number of gauge groups. The original D-BESS model corresponds exactly to this latter case with two gauge groups. Another relevant aspect of cutting a link is that the Goldstone bosons related to the weak symmetry breaking are associated only with the nonlocal field. As a consequence the unitarity properties of these models are the same as in the Higgs-less Standard Model.

In Section V we give a detailed description of the D-BESS model showing its relation to the linear moose case with a cut.

A value of  $\epsilon_3$  strongly suppressed is equally acceptable as the case  $\epsilon_3 = 0$ . Therefore, in Section VI, we examine the possibility of substituting the cut of a link with a strong suppression of the corresponding coupling. In particular, we have examined the possibility of an exponential law for the couplings. In this way, the decay constant at one of the ends of the moose is exponentially suppressed with respect to all the others. As a result  $\epsilon_3$  is strongly suppressed in agreement with our findings in Section IV. We have also examined a powerlike behavior of the couplings with similar results. By requiring reflection symmetry in both the previous cases we have shown that the suppression is present only for an even number of gauge groups. At the end of this Section we have studied the continuum limit of the linear moose with exponential law, which corresponds to a five-dimensional gauge theory with a Randall Sundrum metric [17]. Again, we find a suppression, although not as large as in the discrete case.

In Section VII we study the possibility of extending the linear moose to a planar one. In particular, we show that

no loops are allowed on the plane and that, by a convenient redefinition of the gauge couplings, the expression for  $\epsilon_3$  is the same as in the linear case.

Conclusions are given in Section VIII.

The Appendix A is devoted to the explicit calculation of  $\epsilon_3$  for the linear and the planar moose. In Appendix B we prove the main result of Section IV.

## II. A LINEAR MOOSE MODEL FOR THE ELECTROWEAK SYMMETRY BREAKING

Following the idea of the dimensional deconstruction [10–13] and the hidden gauge symmetry approach applied to the strong interactions [16,18–21] and to the electroweak symmetry breaking [9,16,22], we consider  $K + 1$  nonlinear  $\sigma$ -model scalar fields  $\Sigma_i$ ,  $i = 1, \dots, K + 1$ ,  $K$  gauge groups,  $G_i$ ,  $i = 1, \dots, K$  and a global symmetry  $G_L \otimes G_R$ . Since the aim of this paper is to investigate a minimal model of electroweak symmetry breaking, we will assume  $G_i = SU(2)$ ,  $G_L \otimes G_R = SU(2)_L \otimes SU(2)_R$ . The Standard Model (S-M) gauge group  $SU(2)_L \times U(1)_Y$  is obtained by gauging a subgroup of  $G_L \otimes G_R$ . The  $\Sigma_i$  fields can be parameterized as  $\Sigma_i = \exp[i/(2f_i)\vec{\tau}_i \cdot \vec{\tau}]$  where  $\vec{\tau}$  are the Pauli matrices and  $f_i$  are  $K + 1$  constants that we will call link couplings.

The transformation properties of the fields are

$$\begin{aligned} \Sigma_1 &\rightarrow L \Sigma_1 U_1^\dagger, & \Sigma_i &\rightarrow U_{i-1} \Sigma_i U_i^\dagger, & i &= 2, \dots, K, \\ \Sigma_{K+1} &\rightarrow U_K \Sigma_{K+1} R^\dagger, \end{aligned} \quad (1)$$

with  $U_i \in G_i$ ,  $i = 1, \dots, K$ ,  $L \in G_L$ ,  $R \in G_R$ .

The Lagrangian is given by

$$\mathcal{L} = \sum_{i=1}^{K+1} f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2} \sum_{i=1}^K \text{Tr}[(F_{\mu\nu}^i)^2], \quad (2)$$

with the covariant derivatives defined as follows:

$$\begin{aligned} D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 + i \Sigma_1 g_1 A_\mu^1, \\ D_\mu \Sigma_i &= \partial_\mu \Sigma_i - i g_{i-1} A_\mu^{i-1} \Sigma_i + i \Sigma_i g_i A_\mu^i, \\ & i = 2, \dots, K, \\ D_\mu \Sigma_{K+1} &= \partial_\mu \Sigma_{K+1} - i g_K A_\mu^K \Sigma_{K+1}, \end{aligned} \quad (3)$$

where  $A_\mu^i$  and  $g_i$  are the gauge fields and gauge coupling constants associated with the groups  $G_i$ ,  $i = 1, \dots, K$ .

The model described by the Lagrangian (2) is represented in Fig. 1. Notice that the field defined as

$$U = \Sigma_1 \Sigma_2 \cdots \Sigma_{K+1} \quad (4)$$

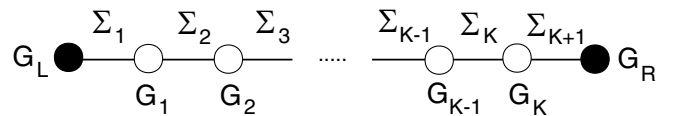


FIG. 1. The linear moose model.

is the usual chiral field: in fact, it transforms as  $U \rightarrow LUR^\dagger$  and it is invariant under the  $G_i$  transformations.

The mass matrix of the gauge fields can be obtained by choosing  $\Sigma_i = I$  in Eq. (2). We find

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= \sum_{i=1}^{K+1} f_i^2 \text{Tr}[(g_{i-1}A_\mu^i - g_iA_\mu^i)^2] \\ &\equiv \frac{1}{2} \sum_{i,j=1}^{K+1} (M_2)_{ij} A_\mu^i A_\mu^j, \end{aligned} \quad (5)$$

with

$$\begin{aligned} (M_2)_{ij} &= g_i^2(f_i^2 + f_{i+1}^2)\delta_{i,j} - g_i g_{i+1} f_{i+1}^2 \delta_{i,j-1} \\ &\quad - g_j g_{j+1} f_{j+1}^2 \delta_{i,j+1}. \end{aligned} \quad (6)$$

The squared mass matrix can be diagonalized through an orthogonal transformation  $S$ . By calling  $\tilde{A}_\mu^n$ ,  $n = 1, \dots, K$  the mass eigenstates, and  $m_n^2$  the squared mass eigenvalues, we have

$$A_\mu^i = \sum_{n=1}^K S_n^i \tilde{A}_\mu^n, \quad (7)$$

and

$$S_m^i (M_2)_{ij} S_n^j = m_n^2 \delta_{m,n}. \quad (8)$$

We will assume  $m_n \neq 0$ , otherwise the model describes an unphysical situation.

The vector meson decay constants are defined in terms of the matrix elements of the vector and axial vector currents between the vacuum and the one vector meson state, i.e.

$$\begin{aligned} \langle 0 | J_{V\mu}^a | \tilde{A}_b^n(p, \epsilon) \rangle &= g_{nV} \delta^{ab} \epsilon_\mu, \\ \langle 0 | J_{A\mu}^a | \tilde{A}_b^n(p, \epsilon) \rangle &= g_{nA} \delta^{ab} \epsilon_\mu, \end{aligned} \quad (9)$$

where  $|\tilde{A}_b^n(p, \epsilon)\rangle$  is the  $b$  component of the single particle state of the  $n$ -vector boson with polarization  $\epsilon_\mu$ . Notice that the vector and axial vector currents are defined as the conserved currents associated with the global symmetry  $G_L \otimes G_R$  acting at the ends of the moose. Therefore, the vector meson decay constants can be very easily obtained by considering the contribution of the vector mesons to the canonical currents. Notice that only the scalar fields  $\Sigma_1$  and  $\Sigma_{K+1}$  transform under the vector and axial transformations according to

$$\begin{aligned} \text{vector: } \Sigma_1 &\rightarrow T \Sigma_1, & \Sigma_{K+1} &\rightarrow \Sigma_{K+1} T^\dagger, \\ \text{axial: } \Sigma_1 &\rightarrow V \Sigma_1, & \Sigma_{K+1} &\rightarrow \Sigma_{K+1} V. \end{aligned} \quad (10)$$

Then, the contributions of the vector mesons to the conserved vector and axial vector currents are

$$\begin{aligned} J_{V\mu}^a |_{\text{vector mesons}} &= f_1^2 g_1 A_\mu^{1a} + f_{K+1}^2 g_K A_\mu^{Ka}, \\ J_{A\mu}^a |_{\text{vector mesons}} &= f_1^2 g_1 A_\mu^{1a} - f_{K+1}^2 g_K A_\mu^{Ka}. \end{aligned} \quad (11)$$

It follows

$$\begin{aligned} g_{nV} &= f_1^2 g_1 S_n^1 + f_{K+1}^2 g_K S_n^K, \\ g_{nA} &= f_1^2 g_1 S_n^1 - f_{K+1}^2 g_K S_n^K. \end{aligned} \quad (12)$$

### III. DETERMINATION OF $\epsilon_3$

To compute the new physics contribution to the electroweak parameter  $\epsilon_3$  [3] we will make use of the dispersive representation given in Refs. [1,2] for the related parameter  $S$  ( $\epsilon_3 = g^2 S / (16\pi)$ ), where  $g$  is the  $SU(2)_L$  gauge coupling)

$$\epsilon_3 = -\frac{g^2}{4\pi} \int_0^\infty \frac{ds}{s^2} \text{Im}[\Pi_{VV}(s) - \Pi_{AA}(s)], \quad (13)$$

where  $\Pi_{VV}(AA)$  is the current-current correlator

$$\begin{aligned} \int d^4x e^{-iq \cdot x} \langle J_{V(A)}^\mu J_{V(A)}^\nu \rangle &= i g^{\mu\nu} \Pi_{VV(AA)}(q^2) \\ &\quad + (q^\mu q^\nu \text{ terms}). \end{aligned} \quad (14)$$

It should be noted that the  $\epsilon_3$  parameter is evaluated with reference to the S-M, and therefore the corresponding contributions should be subtracted. For instance, the contribution of the pion pole to  $\Pi_{AA}$ , that is of the Goldstone particles giving mass to the  $W$  and  $Z$  gauge bosons, does not appear in  $\epsilon_3$ . In the model described by the Lagrangian (2) all the new physics contribution comes from the new vector bosons (we are assuming the standard couplings for the fermions to  $SU(2)_L \times U(1)_Y$ ). Therefore, from

$$\text{Im} \Pi_{VV(AA)} = -\pi \sum_{V_n, A_n} g_{nV, nA}^2 \delta(s - m_n^2), \quad (15)$$

we get

$$\epsilon_3 = \frac{g^2}{4} \sum_n \left( \frac{g_{nV}^2}{m_n^4} - \frac{g_{nA}^2}{m_n^4} \right). \quad (16)$$

Substituting the expressions (12) for the decay vector couplings, we find

$$\epsilon_3 = g^2 g_1 g_K f_1^2 f_{K+1}^2 \sum_n \frac{S_n^1 S_n^K}{m_n^4} = g^2 g_1 g_K f_1^2 f_{K+1}^2 (M_2^{-2})_{1K}. \quad (17)$$

In Appendix A we have derived the following explicit expression for  $\epsilon_3$ , valid for a generic linear moose model (the same result has been obtained in [16]):

$$\epsilon_3 = g^2 \sum_{i=1}^K \frac{(1 - y_i) y_i}{g_i^2}, \quad (18)$$

where we have introduced the following notations,

$$y_i = \sum_{j=1}^i x_j, \quad x_i = \frac{f_i^2}{f_i}, \quad i = 1, \dots, K+1, \quad (19)$$

with

$$\frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}. \quad (20)$$

Therefore,  $\sum_{i=1}^{K+1} x_i = 1$ .

From Eq. (18), it follows that for an open moose one always has

$$\epsilon_3 \geq 0, \quad (21)$$

since  $0 \leq y_i \leq 1$ ,  $i = 1, \dots, K + 1$ . The positivity of  $\epsilon_3$  is a simple consequence of the positivity of all the matrix elements of  $M_2^{-1}$ . This can be proved by using the decomposition of  $M_2$  [see Eq. (6)] in triangular matrices. The positivity of  $\epsilon_3$  was already noted [14] for the warped five-dimensional models (whose deconstruction generates linear moose models) and for the deconstructed QCD [15].

$$\epsilon_3 = \frac{g^2}{g_1^2 g_2^2 g_3^2} f_1^2 f_2^2 f_3^2 f_4^2 \frac{(f_1^2 f_2^2 + f_2^2 f_3^2 + f_1^2 f_3^2) g_1^2 g_2^2 + (f_1^2 + f_2^2)(f_3^2 + f_4^2) g_1^2 g_3^2 + (f_3^2 f_4^2 + f_2^2 f_3^2 + f_2^2 f_4^2) g_2^2 g_3^2}{(f_1^2 f_2^2 f_3^2 + f_1^2 f_3^2 f_4^2 + f_2^2 f_3^2 f_4^2 + f_1^2 f_2^2 f_4^2)}. \quad (24)$$

#### IV. CUTTING A LINK

Is it possible to get  $\epsilon_3 = 0$  at the lowest order in the weak interactions? This can be realized by noticing that if one of the  $f_i$ , with  $i = 2, \dots, K$ , vanishes, the mass matrix  $M_2$  is block-diagonal. The case  $f_1 = 0$  or  $f_{K+1} = 0$  implies the vanishing of  $\epsilon_3$  in a trivial way due to Eq. (17) and the fact that the matrix  $M_2$  is not singular under these hypotheses. This general result can be explicitly verified for  $K = 2$  and  $K = 3$  [see Eqs. (23) and (24)]. We will refer to this situation as ‘‘cutting a link.’’ In such a case,  $M_2^{-2}$  is also block-diagonal, implying the vanishing of  $\epsilon_3$ . This can also be derived from the explicit expression (18). Let us choose  $f_m = 0$ , then  $x_i = \delta_{i,m}$  and  $y_i = \sum_{j=1}^i \delta_{j,m} = \theta_{i,m}$ , where we have defined the discrete step function

$$\theta_{i,j} = \begin{cases} 1 & \text{for } i \geq j, \\ 0 & \text{for } i < j. \end{cases} \quad (25)$$

Then we obtain

$$\epsilon_3 = g^2 \sum_{i=1}^K \frac{(1 - \theta_{i,m}) \theta_{i,m}}{g_i^2} = 0. \quad (26)$$

However, cutting a link corresponds to lose one scalar multiplet, which is necessary to give masses to the gauge bosons of the standard  $SU(2)_L \times U(1)_Y$ . We can solve this problem by adding a term to the Lagrangian of the linear moose given by

$$f_0^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U], \quad (27)$$

where  $U$  is the chiral field given in Eq. (4) and  $f_0$  is a new parameter related to the Fermi scale.

Furthermore, if all the  $f_i$  and the gauge couplings  $g_i$  are of the same order of magnitude, the typical size for  $\epsilon_3$  is

$$\epsilon_3 \sim \frac{g^2}{g_i^2}. \quad (22)$$

However, since the experimental value of  $\epsilon_3$  is of the order  $10^{-3}$  [4], in order to get a realistic model, one should have strongly coupled vector bosons  $A_\mu^i$ .

As a simple example, let us consider the case  $K = 2$ . The result for  $\epsilon_3$  is

$$\epsilon_3 = \frac{g^2}{g_1^2 g_2^2} f_1^2 f_2^2 f_3^2 \frac{(f_1^2 + f_2^2) g_1^2 + (f_2^2 + f_3^2) g_2^2}{(f_1^2 f_2^2 + f_1^2 f_3^2 + f_2^2 f_3^2)}. \quad (23)$$

Analogously for  $K = 3$  we obtain

Correspondingly, there is an enhancement of the symmetry from  $G_L \otimes G_R \otimes \prod_{i=1}^K G_i$  to  $G_L \otimes G_R \otimes \tilde{G}_L \otimes \tilde{G}_R \otimes \prod_{i=1}^K G_i$ , where  $\tilde{G}_{L(R)}$  is a copy of  $G_{L(R)}$  and  $U$  transforms as

$$U \rightarrow \tilde{L} U \tilde{R}^\dagger, \quad (28)$$

with  $\tilde{L}(\tilde{R}) \in \tilde{G}_{L(R)}$ . The Lagrangian for the model, with the  $m$  link cut, is given by

$$\begin{aligned} \mathcal{L} = & f_0^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U] + \sum_{i=1}^{m-1} f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] \\ & + \sum_{i=m+1}^{K+1} f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2} \sum_{i=1}^K \text{Tr}[(F_{\mu\nu}^i)^2]. \end{aligned} \quad (29)$$

As already mentioned, this Lagrangian has an enhanced symmetry with respect to the Lagrangian (2) since the global symmetry  $\tilde{G}_L \otimes \tilde{G}_R$  under which the kinetic term for the field  $U$  is invariant does not coincide with the symmetry  $G_L \otimes G_R$  acting upon the scalar fields  $\Sigma_1$  and  $\Sigma_{K+1}$ . These two global symmetries are to be identified only after the gauging of the electroweak symmetry. The model corresponding to the Lagrangian (29) is shown in Fig. 2. Before the weak gauging, we have three disconnected chains and this is the reason why the symmetry gets enhanced. Clearly the main difference with respect to the linear moose model is the fact that a link is cut and the invariant term containing the scalar field  $\Sigma_m$  is substituted by the invariant involving the field  $U$  coupling the two ends of the chain. Cutting a link implies that, in the unitary gauge, the gauge fields  $A_\mu^i$  become massive by

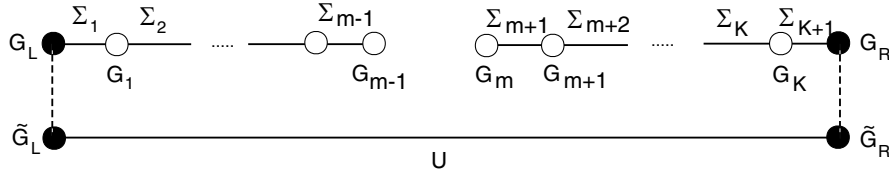


FIG. 2. Graphic representation of the linear moose model with the  $m$  link cut described by the Lagrangian (29). The dashed lines represent the identification of the global symmetry groups after weak gauging.

eating the  $\Sigma_i$  fields, while the  $U$  field contains the Goldstone bosons which give masses to the standard gauge bosons once the gauge group  $SU(2)_L \times U(1)_Y$  is switched on. This additional term does not contribute to  $\epsilon_3$  because the  $U$  field does not couple to the gauge fields  $A_\mu^i$ ,  $i = 1, \dots, K$ ; as a consequence, the gauge boson mass matrix  $M_2$  remains unchanged.

It is also worth noting that the enhanced symmetry acts as a custodial symmetry and this explains why the parameter  $\epsilon_3$  is vanishing [5].

Of course the enhanced symmetry is broken by the weak gauging and corrections to  $\epsilon_3$  of order  $\alpha(M_Z/M)^2$ , where  $M$  is the mass scale of the new vector bosons (see Refs. [6,7]), are expected.

In the linear moose model described by the Lagrangian (2), one has the possibility of making  $\epsilon_3$  small by choosing one  $f_i$  much smaller than the other ones: an explicit calculation will be presented in Section VI. However, in this case there is no additional symmetry which protects the result.

In both cases, the parameters  $\epsilon_1$  and  $\epsilon_2$  are zero, at the lowest order in the weak interactions, because of the presence of the usual  $SU(2)_{L+R}$  custodial symmetry.

In the Appendix B of [9] it was already shown that in the case of  $G_i = SU(2)$  for  $i = 1, \dots, K$  and  $G_{L(R)} = SU(2)_{L(R)}$  one exactly gets  $\epsilon_1 = 0$  and, requiring the decoupling of the gauge fields  $A_\mu^i$ , the parameter  $f_0$  in Eq. (29) satisfies  $f_0^2 = (\sqrt{2}G_F)^{-1}$ .

Concerning the fermions, if we assume the usual representation assignments with respect to  $SU(2)_L \times U(1)_Y$ , mass terms can be generated by Yukawa couplings to the  $U$  field. In this case, fermion couplings to  $W$  and  $Z$  are the standard ones if we neglect the effect of the mixing with the additional vector bosons. Of course it would be possible to add new couplings of the fermions to the gauge bosons [23]. These new couplings would modify  $\epsilon_3$  but, in order to get the necessary cancellation to fulfill the electroweak constraints, one would need a fine tuning of the parameters (as an example, see the BESS model corresponding to  $K = 1$ , Ref. [22]).

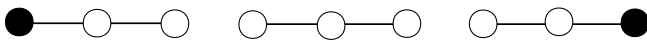


FIG. 3. For  $K$  odd, putting one of the  $f_i$  to zero in a reflection invariant model, one is left with a string containing more vector fields than scalars.

Up to now we have not required the reflection invariance with respect to the ends of the moose. If we do require invariance, we get the following relations among the couplings

$$f_i = f_{K+2-i}, \quad g_i = g_{K+1-i}. \quad (30)$$

If  $K$  is odd, we have an even number of scalar fields and, putting one link coupling  $f_i$  to zero, implies that two links are cut (the two connected by the reflection symmetry). This leads to an unphysical situation, since a multiplet of vector fields remains massless. This is illustrated in Fig. 3. The original string is broken in three pieces with the central one containing more vector fields than scalar ones. As a consequence, there are massless vector fields in the spectrum of the theory. In this case, the matrix  $M_2$  is singular and Eq. (17) is not applicable as it stands. However, as we shall see,  $\epsilon_3$  can be defined through a limiting procedure.

The situation is different if  $K$  is even, since in this case we can cut the central link, and  $K$  remains as depicted in Fig. 4. That is, we are left with two strings disconnected, each of them with a gauge field at one end point. Therefore, for  $K = 2N$  the Lagrangian is given by the formula (29) with  $m = N + 1$  with the field  $U$  expressed as

$$U = \Sigma_1 \cdots \Sigma_{N+1} \cdots \Sigma_{2N+1}. \quad (31)$$

Another interesting point is that, due to the reflection invariance, the mass matrices of the two disconnected strings containing the  $\Sigma$  fields are equal. Therefore, there is complete degeneracy between vector and axial vector resonances. The models so obtained can be considered as a generalization of the D-BESS model, corresponding to  $N = 1$ , as shown in Section V.

As a general result, it is possible to build a model with  $\epsilon_3 = 0$  and an extra custodial symmetry even without requiring the reflection invariance.

Finally, let us mention the unitarity bounds. In general, for a cut linear moose model the longitudinal components of the electroweak gauge bosons are only coupled to the  $U$  field. As a consequence, the corresponding scattering



FIG. 4. For  $K$  even, cutting the central link we are left with two strings, each of them ending with a gauge field.

amplitudes violate partial wave unitarity at the same energy scale as in the Higgs-less S-M. Therefore, the violation of unitarity is not postponed to higher scales as in the five-dimensional Higgs-less model, which, however, seem to be difficult to be reconciled with the precision electroweak measurements unless one includes brane kinetic terms [14,24,25].

It follows that the moose models with a cut link will be valid up to energies of the order of 2 TeV (roughly the unitarity bound on the Higgs-less S-M). On the other hand, the unitarity limit can be overcome by extending the effective theory through an UV completion. Then the relevant question is rather if the result  $\epsilon_3 = 0$  holds true also at the level of the extended theory. Though we have no general answer to this problem, it is possible that, at least in some UV completion, the answer could be positive. In fact, the question was investigated in the context of the D-BESS model in [26]. In this paper a particular UV completion was considered and it was shown that  $\epsilon_3 = 0$  (still at the lowest order in the weak interactions) persists at one-loop level.

## V. THE D-BESS MODEL

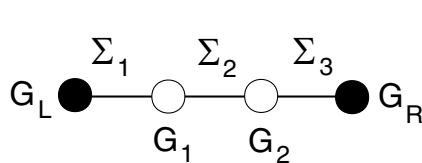
From the general formalism developed in the previous Sections, assuming  $K = 2$  and reflection invariance, one can easily derive the Lagrangian of Ref. [9], describing new vector and axial vector gauge bosons in the Higgs-less S-M, in two particular cases. Let us recall that, requiring gauge invariance and symmetry under reflection, the most general invariant Lagrangian is

$$\mathcal{L} = -\frac{1}{4}v^2[a_1 I_1 + a_2 I_2 + a_3 I_3 + a_4 I_4] - \frac{1}{2} \times \sum_{i=1}^2 \text{Tr}[(F_{\mu\nu}^i)^2] \quad (32)$$

with

$$\begin{aligned} I_1 &= \text{Tr}[(V_1 - V_2 - V_3)^2], & I_2 &= \text{Tr}[(V_1 + V_3)^2], \\ I_3 &= \text{Tr}[(V_1 - V_3)^2], & I_4 &= \text{Tr}[V_2^2], \end{aligned} \quad (33)$$

where



$$\begin{aligned} V_1^\mu &= \Sigma_1^\dagger D^\mu \Sigma_1, & V_2^\mu &= \Sigma_2 D^\mu \Sigma_2^\dagger, \\ V_3^\mu &= \Sigma_2 (\Sigma_3 D^\mu \Sigma_3^\dagger) \Sigma_2^\dagger, \end{aligned} \quad (34)$$

and

$$\begin{aligned} D_\mu \Sigma_1 &= \partial_\mu \Sigma_1 + i \Sigma_1 g_1 A_{1\mu}^1, \\ D_\mu \Sigma_2 &= \partial_\mu \Sigma_2 - i g_1 A_{1\mu}^1 \Sigma_2 + i \Sigma_2 g_2 A_{2\mu}^2, \\ D_\mu \Sigma_3 &= \partial_\mu \Sigma_3 - i g_2 A_{2\mu}^2 \Sigma_3. \end{aligned} \quad (35)$$

The invariance under reflections implies

$$\Sigma_3 \leftrightarrow \Sigma_1^\dagger, \quad \Sigma_2 \leftrightarrow \Sigma_2^\dagger, \quad A_\mu^1 \leftrightarrow A_\mu^2, \quad g_1 = g_2, \quad (36)$$

where  $A_\mu^1$  and  $A_\mu^2$  are the gauge fields related to the gauge groups  $G_1$  and  $G_2$ , respectively.

We can now select two particular cases:

(1) *The linear moose model.* By choosing

$$a_1 = 0, \quad a_2 = a_3, \quad (37)$$

we have

$$\mathcal{L} = \sum_{i=1}^3 f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2} \sum_{i=1}^2 \text{Tr}[(F_{\mu\nu}^i)^2], \quad (38)$$

with

$$f_1^2 = f_3^2 = \frac{1}{2} a_2 v^2, \quad f_2^2 = \frac{1}{4} a_4 v^2. \quad (39)$$

This is indeed the Lagrangian for a linear moose with three links and two gauge fields with reflection invariance (see Refs. [21,27]). The corresponding diagram is shown in the left panel of Fig. 5.

(2) *The D-BESS model* [6,7]. This corresponds to the choice

$$a_4 = 0, \quad a_2 = a_3, \quad (40)$$

giving

$$\begin{aligned} \mathcal{L}_{\text{D-BESS}} &= f^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U] \\ &+ f_1^2 (\text{Tr}[D_\mu \Sigma_1^\dagger D^\mu \Sigma_1] \\ &+ \text{Tr}[D_\mu \Sigma_3^\dagger D^\mu \Sigma_3]), \end{aligned} \quad (41)$$

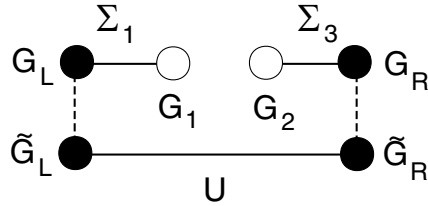


FIG. 5. The left panel gives a graphic representation of the Lagrangian (32) for  $a_1 = 0, a_2 = a_3$ . The right panel gives a graphic representation of the D-BESS model Lagrangian (41). The dashed lines represent the identification of the global symmetry groups after the electroweak gauging.

with

$$f^2 = \frac{1}{4} a_1 v^2, \quad f_1^2 = \frac{1}{2} a_2 v^2, \quad (42)$$

and

$$U = \Sigma_1 \Sigma_2 \Sigma_3. \quad (43)$$

The diagram corresponding to the previous Lagrangian is shown in the right panel of Fig. 5. Before the electroweak gauging, we have three disconnected chains and this is the reason why the symmetry  $SU(2)_L \otimes \prod_{i=1}^2 SU(2)_i \otimes SU(2)_R$  gets enhanced to  $[SU(2) \otimes SU(2)]^3$ .

We have shown in [6] that in order to have a vanishing parameter  $\epsilon_3$ , or  $S$ , at the lowest order in the weak interactions,  $a_4 = 0$  is necessary. This is equivalent to eliminating from the Lagrangian the term corresponding to the central link. The requirement  $a_2 = a_3$  implies degeneracy between vector and axial vector gauge bosons. Since the contribution of the vector and of the axial vector particles are of opposite sign, one gets exactly  $\epsilon_3 = S = 0$  at the leading order. However, as we have already noted in Section III, it is possible to build a model with  $\epsilon_3 = 0$  and an extra custodial symmetry even without requiring the reflection invariance. In other words, the degeneracy of vector and axial vector resonances is not necessary to ensure  $\epsilon_3 = 0$ .

## VI. SEWING THE CUT

We have shown in Appendix B that, in order to get a vanishing  $\epsilon_3$ , the necessary and sufficient condition is that one and only one of the link couplings  $f_i$  is zero. As a consequence, by requiring reflection invariance,  $\epsilon_3 = 0$  can be achieved only if  $K$  is even. On this basis, it is easy to see how a suppression of  $\epsilon_3$  (a smoother situation with respect to the vanishing) can be realized. In fact, it will be enough to require that a link is suppressed with respect to all the others. In this case, however, it is not necessary to consider the additional dynamical degree of freedom given by the chiral field  $U$ , and therefore there is no additional custodial symmetry for  $\epsilon_3$ .

A simple model grasping the main features is obtained by assuming an exponential law for the link couplings  $f_i$ , and equal gauge couplings

$$f_i = \bar{f} e^{c(i-1)}, \quad g_i = \bar{g}, \quad i = 1, \dots, K+1. \quad (44)$$

Here  $\bar{f}$  is an overall scale not playing any role in the dimensionless quantity  $\epsilon_3$ . By contrast, the relevant parameter is  $c$  since it controls the amount of suppression. By using Eqs. (18)–(20), we easily obtain

$$f^2 = \bar{f}^2 e^{cK} \frac{\sinh c}{\sinh c(K+1)}, \quad (45)$$

$$x_i = e^{-2ci} e^{c(K+2)} \frac{\sinh c}{\sinh c(K+1)}, \quad (46)$$

and

$$\epsilon_3 = \frac{1}{4} \left( \frac{g}{\bar{g}} \right)^2 \frac{\sinh[2c(K+1)] - (K+1) \sinh 2c}{\sinh 2c \sinh^2[c(K+1)]}. \quad (47)$$

For increasing  $c$ , the first link  $f_1$  is more and more suppressed with respect to the other links. In fact, for large  $c$  we get

$$\epsilon_3 \sim \left( \frac{g}{\bar{g}} \right)^2 e^{-2c}. \quad (48)$$

Therefore, the suppression factor is about  $2 \times 10^{-2}$  for  $c \approx 2$ . It is interesting to look at the behavior of the variables  $x_i$  vs  $i$  for fixed  $c$ . This is plotted in Fig. 6.

From Fig. 6 we see that for  $c = 2$  we are practically in the ideal situation  $x_1 = 1$  and  $x_i = 0$  for  $i \neq 1$ . In Fig. 7 we show the suppression factor in  $\epsilon_3$  as a function of  $c$ . We see that, in agreement with the analytical result,  $\epsilon_3$  does not depend on  $K$  as soon as  $c \approx 2$ .

We have also considered the case of reflection symmetry by assuming the link couplings  $f_i$  of the form

$$f_i = \hat{f} \cosh \left[ c \left( 1 + \frac{K}{2} - i \right) \right], \quad (49)$$

with  $\hat{f} = \bar{f} [\hat{f} = \bar{f} / \cosh(c/2)]$  for  $K$  even (odd). With this choice, the central link couplings are equal to  $\bar{f}$ . From Fig. 6 the difference appears clearly between  $K$  even and

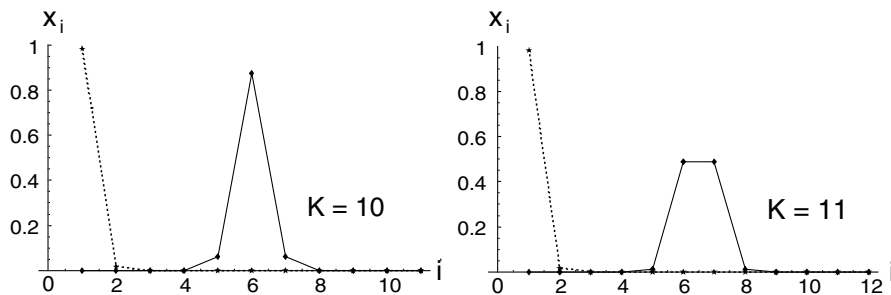


FIG. 6. The behavior of  $x_i = f^2/f_i^2$  vs  $i$  for  $c = 2$ ,  $K = 10$  (left panel) and  $K = 11$  (right panel). The dotted (continuous) lines correspond to the choice of link couplings  $f_i$  without reflection symmetry, given in Eq. (44) [with reflection symmetry, given in Eq. (49)].

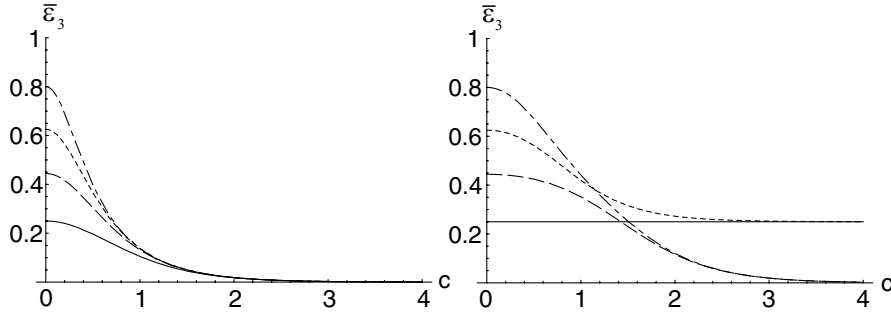


FIG. 7. The behavior of  $\bar{\epsilon}_3 = \epsilon_3 / (g/\tilde{g})^2$  vs  $c$  for different values of  $K$ , on the left (right) panel for the choice of link couplings  $f_i$  without reflection symmetry, given in Eq. (44) [with reflection symmetry, given in Eq. (49)]. The continuous lines correspond to  $K = 1$ , the dashed lines to  $K = 2$ , the dotted lines to  $K = 3$  and the dash-dotted lines to  $K = 4$ .

$K$  odd. In particular, for  $K$  odd there are two central  $x_i$  much bigger than the others. Therefore, in agreement with the discussion in Appendix B, we expect no suppression factor. In fact, for large  $c$  the limiting value of the two central  $x_i$  is 0.5 and

$$\epsilon_3 \rightarrow \frac{1}{4} \left( \frac{g}{\tilde{g}} \right)^2. \quad (50)$$

The numerical results for  $\epsilon_3$  vs  $c$  are given in Fig. 7. We see that the suppression factor is operating only for  $K$  even.

We have also analyzed the case of a powerlike behavior of the link couplings

$$f_i = \bar{f} i^c, \quad (51)$$

with the related reflection invariant case.

The results are similar to the exponential case. In order to have a suppression factor of the order  $2 \times 10^{-2}$  we need  $c \approx 3$  for the nonsymmetric case. On the other hand, when reflection invariance is required, we have a similar suppression only for  $K$  even and  $c \approx 7.5$ . The last result follows from the  $x_i$  distribution, which is broader around the central links. Also in this case there is no suppression for  $K$  odd and  $\epsilon_3$  goes to the limiting value given in Eq. (50).

It is interesting to observe that for any of the previous choices of  $f_i$  we have

$$\lim_{c \rightarrow 0} f_i = \bar{f}. \quad (52)$$

Therefore, from our general expression for  $\epsilon_3$  [see Eq. (18)], as well as from Eq. (47) for  $c \rightarrow 0$ ,

$$\epsilon_3 = \frac{1}{6} \left( \frac{g}{\tilde{g}} \right)^2 \frac{K(K+2)}{K+1}, \quad (53)$$

which coincides with the result of Refs. [16,28].

Another interesting aspect is the continuum limit. It is known that the discretization of a gauge theory Lagrangian in a 4 + 1 dimensional space-time along the fifth dimension (the segment of length  $\pi R$ ) gives

rise to a linear moose chiral Lagrangian after a suitable identification of the gauge and link couplings [10–13].

For the case of equal couplings  $f_i$  we take

$$K \rightarrow \infty, \quad a \rightarrow 0, \quad Ka \rightarrow \pi R, \quad (54)$$

where  $a$  is the lattice size. We find

$$\epsilon_3 \rightarrow \frac{1}{6} \left( \frac{g}{\tilde{g}} \right)^2 K. \quad (55)$$

By introducing the gauge coupling in five dimensions by the relation

$$g_5^2 = a \tilde{g}^2, \quad (56)$$

we get

$$\epsilon_3 \rightarrow \frac{1}{6} \left( \frac{g}{g_5} \right)^2 \pi R, \quad (57)$$

in agreement with Ref. [28].

The discretization of a five-dimensional gauge theory has also been considered for the warped metric case [29–31]. This corresponds to a linear moose with link couplings given by Eq. (44) with

$$c = \frac{\pi k R}{K}. \quad (58)$$

This exponential behavior of  $f_i$  corresponds to the Randall Sundrum metric [17]. Then  $c \rightarrow 0$  for  $K \rightarrow \infty$  and  $a(i-1) \rightarrow y$  where  $y \in [0, \pi R]$  is the coordinate along the fifth dimension. That is,

$$f_i = \bar{f} e^{ka(i-1)} \rightarrow f(y) = \bar{f} e^{ky}. \quad (59)$$

Therefore, in the warped case we find, from Eq. (47),

$$\epsilon_3 = \frac{1}{4k} \frac{e^{4k\pi R} - 4k\pi R e^{2k\pi R} - 1}{(1 - e^{2k\pi R})^2} \left( \frac{g}{g_5} \right)^2. \quad (60)$$

For large values of  $k\pi R$  we get

$$\epsilon_3 \rightarrow \frac{1}{4k} \left( \frac{g}{g_5} \right)^2. \quad (61)$$

Assuming  $k \sim M_{Pl}/10$ ,  $R \sim 10^2 M_{Pl}^{-1}$  and  $g_5^2 = \pi R g_4^2$ ,



where  $g_4$  is the gauge coupling obtained after dimensional reduction of the fifth dimension, it follows  $\epsilon_3 \sim 0.008g^2/g_4^2$ . In the reflection invariant case, we obtain  $\epsilon_3 \sim 0.016g^2/g_4^2$ . Therefore, also in the continuum limit we get a suppression factor, although not as large as in the discrete case.

## VII. FURTHER EXTENSIONS: THE PLANAR MOOSE

Possible generalizations of the linear moose are obtained by extending the moose graph in the plane. A realistic model for the electroweak symmetry breaking must contain only three independent scalar fields (the Goldstone bosons necessary to provide the masses to the electroweak gauge bosons) and will have additional massive vector gauge bosons. Then we can immediately show that the only possible diagrams are the ones with zero loops. In fact, a moose diagram is like a Feynman diagram with lines corresponding to links and vertices corresponding to gauge groups. Therefore, by introducing the following notation:

$$\begin{aligned} E &= \text{number of external links,} \\ I &= \text{number of internal links,} \\ V_\ell &= \text{number of gauge groups with } \ell \text{ links,} \\ L &= \text{number of loops,} \\ S &= \text{number of remaining Goldstone multiplets,} \end{aligned} \quad (62)$$

we have

$$L = I - \left( \sum_\ell V_\ell - 1 \right), \quad (63)$$

$$S = I + E - \sum_\ell V_\ell. \quad (64)$$

By using Eqs. (63) and (64) we get

$$L = S - (E - 1). \quad (65)$$

In the models considered in this paper, we have associated a global symmetry with the external links. Therefore, we need at least two external links ( $E = 2$ ) in order to get the right weak phenomenology. This, together with the requirement of one scalar multiplet ( $S = 1$ ), implies that the number of loops must be equal to zero.

Avoiding loops, the way to generalize the linear moose to a planar one is to attach a string to each of the groups  $G_i$  as illustrated in Fig. 8.

For simplicity we take all the strings of equal length, with  $N - 1$  links, but as we shall see, the result can be immediately extended to strings of different length. As shown in Fig. 8, we introduce gauge groups  $G_{(i,j)}$  with  $i = 1, \dots, K$  and  $j = 1, \dots, N$ , and associated fields  $A_{(i,j)}$  with corresponding gauge couplings  $g_{(i,j)}$ . However, it is

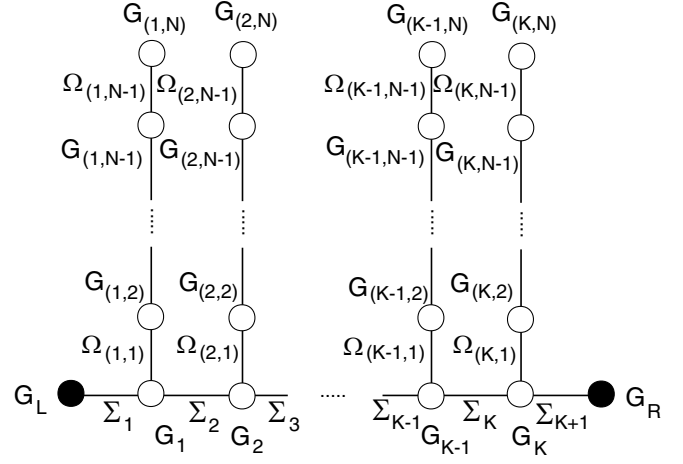


FIG. 8. The planar moose.

convenient to identify explicitly the gauge fields and couplings on the original string

$$A_{(i,1)} = A_i, \quad g_{(i,1)} = g_i, \quad i = 1, \dots, K. \quad (66)$$

In addition to the scalar fields  $\Sigma_i$ ,  $i = 1, \dots, K + 1$  linking the gauge fields  $A_i$ , we also have new scalar fields  $\Omega_{(i,j)}$ ,  $i = 1, \dots, K$ ,  $j = 1, \dots, N - 1$  linking the gauge fields along the vertical direction. We introduce the following notation

$$B_i = g_i A_i - g_{i+1} A_{i+1}, \quad i = 0, \dots, K, \quad (67)$$

with the boundary condition

$$g_0 = g_{K+1} = 0. \quad (68)$$

Notice that the  $K + 1$  fields  $B_i$  are not independent, since

$$\sum_{i=0}^K B_i = 0. \quad (69)$$

We also define

$$\begin{aligned} V_{(i,j)} &= g_{(i,j)} A_{(i,j)} - g_{(i,j+1)} A_{(i,j+1)}, \quad i = 1, \dots, K, \\ & \quad j = 1, \dots, N - 1. \end{aligned} \quad (70)$$

Then the vector boson mass term will be given by

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} \sum_{i=1}^{K+1} f_i^2 B_{i-1}^2 + \frac{1}{2} \sum_{i=1}^K \sum_{j=1}^{N-1} h_{(i,j)}^2 V_{(i,j)}^2, \quad (71)$$

where  $h_{(i,j)}$  are new  $K \times (N - 1)$  link couplings. As shown in Appendix A, the result for  $\epsilon_3$  is

$$\epsilon_3 = g^2 \sum_{i=1}^K \frac{y_i (1 - y_i)}{\bar{g}_i^2}, \quad (72)$$

where

$$\frac{1}{\bar{g}_i^2} = \sum_{j=1}^N \frac{1}{g_{(i,j)}^2}. \quad (73)$$

We see that the only effect of attaching a string at any of the initial groups  $G_i$  is simply to define a new gauge coupling according to Eq. (73).

As in the linear moose, we can consider the continuum limit. Let us introduce a lattice size  $b$  along the vertical direction and take the limit

$$b \rightarrow 0, \quad N \rightarrow \infty, \quad bN \rightarrow \pi R'. \quad (74)$$

By defining a six-dimensional gauge coupling as

$$\frac{ab}{g_6^2} = \frac{1}{\tilde{g}^2}, \quad (75)$$

we get

$$\epsilon_3 = \frac{1}{6} \left( \frac{g}{g_6} \right)^2 \pi^2 R R', \quad (76)$$

for the constant case  $f_i = f$ , while for exponential  $f_i$  [see Eq. (44)]

$$\epsilon_3 = \frac{1}{4} \left( \frac{g}{g_6} \right)^2 \frac{\pi R'}{k}. \quad (77)$$

Notice that we do not get a suppression factor from the vertical links, since the result for  $\epsilon_3$  does not depend on these variables.

## VIII. CONCLUSIONS

Models with replicas of gauge groups have been recently considered because they appear in the deconstruction of five-dimensional gauge models, which have been used to describe the electroweak breaking without the Higgs [14,28,32–34]. The four-dimensional description is based on the linear moose Lagrangians that were already proposed in technicolor and composite Higgs models [35]. In general, these models satisfy the constraints arising from the parameters  $T$  and  $U$  (or  $\epsilon_1$  and  $\epsilon_2$ ) due to the presence of a custodial SU(2) symmetry. However, such models generally give a correction of order  $\mathcal{O}(1)$  ( $\mathcal{O}(10^{-2})$ ) to the parameter  $S$  ( $\epsilon_3$ ). In this paper we have considered a linear moose based on replicas of SU(2) gauge groups and with the electroweak gauge groups SU(2)<sub>L</sub> and U(1)<sub>Y</sub> at the two ends of the moose string. After having obtained a general expression for the parameter  $\epsilon_3$ , we have shown that a unique solution exists which guarantees  $\epsilon_3 = 0$ . The corresponding model has an additional custodial symmetry which protects this result. It is obtained from the linear moose when a link is cut and a nonlocal field connecting the two ends of the moose is included. This solution is a generalization of the simplest case, corresponding to the degenerate BESS model. It contains additional vector resonances; however, their contribution to the  $S$  parameter is zero at the leading order in the electroweak couplings. At the same order, the new resonances do not couple to the longitudinal  $W$  and  $Z$ . As a consequence, the breaking of partial wave uni-

arity is expected to happen at the same scale as in the Higgsless S-M and it is not postponed to higher scales.

We have also shown that it is possible to control the size of  $\epsilon_3$  by taking one of the link couplings that is much smaller than the other ones.

A generalization to the planar case has also been investigated: we have shown that no loops are allowed in the moose graph and that with a convenient redefinition of the gauge couplings the result for  $S$  is the same as for the linear moose case.

## ACKNOWLEDGMENTS

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## APPENDIX A: EVALUATION OF $\epsilon_3$

In this Appendix we will obtain explicit expressions for  $\epsilon_3$ , both for the linear moose and for its planar generalization considered in Section VII. We start evaluating the inverse of the vector boson mass matrix  $M_2$ . Actually, for  $\epsilon_3$  we need only the elements  $(M_2^{-1})_{li}$  and  $(M_2^{-1})_{Ki}$ , where the index  $i$  runs over all the gauge fields. A technique to determine  $M_2^{-1}$  is to add to the mass term a source,

$$\mathcal{L}_{\text{mass+source}} = \frac{1}{2} A^T M_2 A - JA. \quad (A1)$$

Evaluating from this Lagrangian the equations of motion and solving for the  $A$ 's in terms of the sources  $J$ 's we find

$$A = M_2^{-1} J, \quad (A2)$$

from which we can read the relevant matrix elements.

### 1. The linear moose

The mass term in Eq. (5) can be written as

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} \sum_{i=1}^{K+1} f_i^2 B_{i-1}^2. \quad (A3)$$

The variables  $B_i$  are the analogue of canonical momenta in the discrete case and are given by

$$B_i = g_i A_i - g_{i+1} A_{i+1}, \quad i = 0, \dots, K, \quad (A4)$$

with the boundary condition  $g_0 = g_{K+1} = 0$ . Notice that the  $K + 1$  fields  $B_i$  are not independent, since

$$\sum_{i=0}^K B_i = 0. \quad (A5)$$

Therefore, we solve the equations of motion (A2), which involve three nearest neighborhoods by solving first in the  $B_i$  and then inverting the relation between the  $B_i$  and the fields  $A_j$ . These equations involve only first

neighborhoods. This is the analogue of converting a second order differential equation in a pair of first order equations.

The equations of motion can be written in the following form

$$-f_i^2 B_{i-1} + f_{i+1}^2 B_i = L_i, \quad i = 1, \dots, K, \quad (\text{A6})$$

where we have redefined the sources as

$$L_i = \frac{J_i}{g_i}. \quad (\text{A7})$$

We can solve for all the  $B_i$ ,  $i = 1, \dots, K$  in terms of  $B_0$  finding

$$B_i = \frac{1}{f_{i+1}^2} \left( \sum_{j=1}^i L_j + f_1^2 B_0 \right), \quad i = 1, \dots, K. \quad (\text{A8})$$

It is convenient to introduce the following variables:

$$\frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}, \quad x_i = \frac{f^2}{f_i^2}, \quad i = 1, \dots, K+1, \quad (\text{A9})$$

$$y_i = \sum_{j=1}^i x_j, \quad z_i = \sum_{j=i+1}^{K+1} x_j, \quad (\text{A10})$$

with the properties

$$y_i + z_i = 1, \quad y_1 = x_1, \quad z_K = x_{K+1}. \quad (\text{A11})$$

By summing the Eq. (A8) over  $i$  from one to  $K$  and using Eq. (A5), we get a relation for  $B_0$  which can be easily solved obtaining

$$B_0 = -\frac{x_1}{f^2} \sum_{i=1}^K z_i L_i, \quad (\text{A12})$$

and

$$B_i = \frac{x_{i+1}}{f^2} \left( \sum_{j=1}^i y_j L_j - \sum_{j=i+1}^K z_j L_j \right). \quad (\text{A13})$$

By using the discrete step function given in Eq. (25), we can write

$$B_i = \frac{x_{i+1}}{f^2} \sum_{j=1}^K (\theta_{i,j} y_j - \theta_{j,i+1} z_j) L_j, \quad i = 0, \dots, K. \quad (\text{A14})$$

Notice that this equation also holds for  $i = 0$ , due to the properties of the discrete  $\theta$ -function. Further, we need to reexpress the fields  $A_i$  in terms of the  $B_i$ . We find

$$A_i = \frac{1}{g_i} \sum_{j=1}^K \theta_{j,i} B_j. \quad (\text{A15})$$

Using Eq. (A14) we obtain

$$(M_2^{-1})_{1i} = \frac{x_1 z_i}{g_1 g_i f^2}, \quad (M_2^{-1})_{iK} = \frac{x_{K+1} y_i}{g_K g_i f^2}. \quad (\text{A16})$$

Therefore, from the expression (17) we get

$$\epsilon_3 = g^2 \sum_{i=1}^K \frac{z_i y_i}{g_i^2} = g^2 \sum_{i=1}^K \frac{(1-y_i) y_i}{g_i^2}. \quad (\text{A17})$$

## 2. The planar moose

Starting from the mass term of the planar case, Eq. (71), we get the following set of equations of motion by differentiating with respect to  $A_{(i,j)}$

$$-f_i^2 B_{i-1} + f_{i+1}^2 B_i + h_{(i,1)}^2 V_{(i,1)} = \frac{J_i}{g_i} \equiv L_i, \quad i = 1, \dots, K, \quad (\text{A18})$$

$$-h_{(i,j-1)}^2 V_{(i,j-1)} + h_{(i,j)}^2 V_{(i,j)} = \frac{J_{(i,j)}}{g_{(i,j)}} \equiv L_{(i,j)}, \quad i = 1, \dots, K, \quad j = 2, \dots, N. \quad (\text{A19})$$

It is also convenient to introduce

$$L_{(i,1)} = L_i. \quad (\text{A20})$$

The solution of Eq. (A19) is

$$V_{(i,j)} = -\frac{1}{h_{(i,j)}^2} \sum_{m=j+1}^N L_{(i,m)}. \quad (\text{A21})$$

Inserting this result inside Eq. (A18) we obtain

$$-f_i^2 B_{i-1} + f_{i+1}^2 B_i = \tilde{L}_i, \quad (\text{A22})$$

with

$$\tilde{L}_i = \sum_{j=1}^N L_{(i,j)} = \sum_{j=1}^N \frac{J_{(i,j)}}{g_{(i,j)}}. \quad (\text{A23})$$

These equations are the same as for the linear moose with the substitution  $L_i \rightarrow \tilde{L}_i$ . Therefore, we immediately get

$$(M_2^{-1})_{1,(i,j)} = \frac{x_1 z_i}{f^2 g_1 g_{(i,j)}}, \quad (M_2^{-1})_{K,(i,j)} = \frac{x_{K+1} y_i}{f^2 g_K g_{(i,j)}}, \quad (\text{A24})$$

where the variables  $x_i$ ,  $y_i$ ,  $z_i$  and  $f^2$  are the same as for the linear moose. Therefore, the result is

$$\epsilon_3 = g^2 \sum_{i=1}^K \frac{y_i (1-y_i)}{\tilde{g}_i^2}, \quad (\text{A25})$$

where

$$\frac{1}{\tilde{g}_i^2} = \sum_{j=1}^N \frac{1}{g_{(i,j)}^2}. \quad (\text{A26})$$

**APPENDIX B: SOLUTIONS TO  $\epsilon_3 = 0$** 

We want to prove the following statement:  $\epsilon_3 = 0$  if and only if one or more  $f_i$  are sent to zero in an independent way.

We start noticing that, due to the condition  $\sum_{i=1}^{K+1} x_i = 1$ , all the  $y_i$ , defined in Eq. (19) are such that  $0 \leq y_i \leq 1$ . Since  $\epsilon_3$  is made from positive terms, each of them proportional to  $0 \leq y_i(1 - y_i) \leq 1/4$ , in order to get a vanishing  $\epsilon_3$  we need  $y_i(1 - y_i) = 0$  for all values of  $i$ . This implies  $y_i = 0$  or  $y_i = 1$  for all  $i$ . However, since  $x_i = y_i - y_{i-1}$ , the same properties must hold true for the quantities  $x_i$ , that is  $x_i = 0$  and  $x_i = 1$  for all values of  $i$ . Also, we have already shown that  $\epsilon_3 = 0$  if one of the  $f_i$  is chosen to vanish [see Eq. (26)]. Therefore, if we send to zero several link couplings  $f_i$ , we get  $\epsilon_3 = 0$  *a fortiori*.

Let us now show that if we send more than one  $f_i$  to zero in a correlated way, then  $\epsilon_3 \neq 0$ . Assume that  $p$  of the constants  $f_i$  go to zero with the variable  $\eta$  in a simultaneous way

$$f_i^2 = c_i \eta, \quad c_i > 0, \quad i \in \mathcal{P}. \quad (\text{B1})$$

Here  $i$  takes  $p$  values in the subset  $\mathcal{P}$  of the set  $(1, \dots, K + 1)$ . Notice that the assumption of correlation implies that the coefficients  $c_i$  are strictly positive. In the limit  $\eta \rightarrow 0$  we immediately get

$$\frac{1}{f^2} \approx \frac{1}{\eta} \sum_{i \in \mathcal{P}} \frac{1}{c_i}. \quad (\text{B2})$$

It follows

$$x_i = \begin{cases} \frac{1}{c_i} \frac{1}{\sum_{j \in \mathcal{P}} \frac{1}{c_j}} & i \in \mathcal{P}, \\ 0 & i \notin \mathcal{P}. \end{cases} \quad (\text{B3})$$

Unless the set  $\mathcal{P}$  contains a single element, we have

$$x_i < 1, \quad i \in \mathcal{P}. \quad (\text{B4})$$

As a consequence, some of the  $y_i$  are neither zero nor one and  $\epsilon_3$  is not vanishing.

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