Effective Lagrangian approach to nuclear $\mu^- - e^-$ conversion and the role of vector mesons

Amand Faessler,[1] Th. Gutsche,[1] Sergey Kovalenko,[2] V. E. Lyubovitskij,[1,*] Ivan Schmidt,[2] and F. Šimkovic[1,†]

[1]*Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

[2]*Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile*

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We study nuclear $\mu^- - e^-$ conversion in the general framework of an effective Lagrangian approach without referring to any specific realization of the physics beyond the standard model (SM) responsible for lepton flavor violation ($\not{L}_f$). We examine the impact of a specific hadronization prescription on the analysis of new physics in nuclear $\mu^- - e^-$ conversion and stress the importance of vector meson exchange between lepton and nucleon currents. A new issue of this mechanism is the presence of the strange quark vector current contribution induced by the ϕ meson. This allows us to extract new limits on the $\not{L}_f$ lepton-quark effective couplings from the existing experimental data.

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I. INTRODUCTION

The discovery of neutrino oscillations has established the fact that Nature does not respect lepton flavor conservation contrary to the expectations within the Standard Model (SM). However, this is so far the only experimental observation of the violation of this conservation law. On the other hand, once $\not{L}_f$ is proved, it is natural to expect its manifestations in the charged lepton sector as well. The smallness of neutrino mass square differences, Δm^2, observed in neutrino oscillation experiments makes the neutrino induced $\not{L}_f$ effects (tree-level exchange, loops, boxes) in the processes with charged leptons extremely suppressed as $(\Delta m^2)^2/M_W^4$, pushing them far beyond the reach of experimental searches[1]. However, the charged lepton processes may receive another $\not{L}_f$ contributions from the physics beyond the SM attributed to a certain high-energy $\not{L}_f$ scale Λ_{LFV}. Thus, searching for the lepton flavor violation (LFV) in reactions with charged leptons becomes challenging both from the experimental and theoretical points of view.

Muon-to-electron ($\mu^- - e^-$) conversion in nuclei

$$\mu^- + (A, Z) \longrightarrow e^- + (A, Z)^* \tag{1}$$

is commonly recognized as one of the most promising probes of lepton flavor violation in the charged lepton sector and of related physics beyond the SM (for reviews, see [2–4]). This is, in particular, due to the very high sensitivity of the experiments dedicated to search for this process.

At present there is one running experiment, SINDRUM II [5], and two planned ones, MECO [6,7] and PRIME [8]. The SINDRUM II experiment at PSI [5] with 48Ti as stopping target has established the best upper bound on the branching ratio [5]

$$R_{\mu e}^{Ti} = \frac{\Gamma(\mu^- + ^{48} Ti \to e^- + ^{48} Ti)}{\Gamma(\mu^- + ^{48} Ti \to \nu_\mu + ^{48} Sc)} \leq 6.1 \times 10^{-13}, \quad (90\%\mathrm{C.L.}). \tag{2}$$

The MECO experiment with ^{27}Al is going to start soon at Brookhaven [7]. The sensitivity of this experiment is expected to reach the level of

$$R_{\mu e}^{\mathrm{Al}} = \frac{\Gamma(\mu^- + ^{27}\mathrm{Al} \to e^- + ^{27}\mathrm{Al})}{\Gamma(\mu^- + ^{27}\mathrm{Al} \to \nu_\mu + ^{27}\mathrm{Mg})} \leq 2 \times 10^{-17} \tag{3}$$

The PSI experiment is also running with the very heavy nucleus 197Au aiming to improve the previous limit by the same experiment [5,9] up to

$$R_{\mu e}^{\mathrm{Au}} = \frac{\Gamma(\mu^- + ^{197}\mathrm{Au} \to e^- + ^{197}\mathrm{Au})}{\Gamma(\mu^- + ^{197}\mathrm{Au} \to \nu_\mu + ^{197}\mathrm{Pt})} \leq 6 \times 10^{-13} \tag{4}$$

The proposed new experiment PRIME (Tokyo) [8] is going to utilize 48Ti as stopping target with an expected sensitivity of

$$R_{\mu e}^{\mathrm{Ti}} = \frac{\Gamma(\mu^- + ^{48}\mathrm{Ti} \to e^- + ^{48}\mathrm{Ti})}{\Gamma(\mu^- + ^{48}\mathrm{Ti} \to \nu_\mu + ^{48}\mathrm{Sc})} \leq 10^{-18}. \tag{5}$$

These experimental bounds would allow to set stringent limits on the mechanisms of $\mu^- - e^-$conversion and the underlying theories of $\not{L}_f$. In the literature various mechanisms beyond the SM have been studied (see [2–4] and references therein). They can be classified as photonic and non-photonic, that is with and without photon exchange between the lepton and nuclear vertices, respectively. These two categories of mechanisms differ significantly from each other in various respects. In fact, they receive different contributions from the new physics and also require different treatments of the effects of the

*On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia

†On leave of absence from Department of Nuclear Physics, Comenius University, Mlynská dolina F1, SK–842 15 Bratislava, Slovakia

[1]This conclusion is valid for the conventional three light neutrino mixing scenarios. The incorporation of sterile neutrinos may significantly reduce the suppression factor [1].

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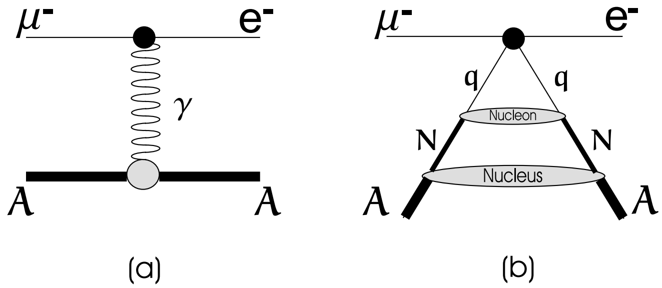

FIG. 1. (a) Photonic (long-distance) and (b) non-photonic (short-distance) mechanisms to the nuclear $\mu^- - e^-$ conversion.

nucleon and the nuclear structure. Latter aspect is, in particular, attributed to the fact that the two mechanisms operate at different distances and, therefore, involve different details of the nucleon and nuclear structure.

Long-distance photonic mechanisms (Fig. 1(a)) are mediated by virtual photon exchange between the nucleus and the $\mu - e$ lepton current. They suggest that the $\mu^- - e^-$ conversion occurs in the lepton-flavor non-diagonal electromagnetic vertex which is presumably induced by non-standard model physics at the loop level. The hadronic vertex is characterized in this case by ordinary electromagnetic nuclear form factors. Contributions to $\mu^- - e^-$ conversion via virtual photon exchange exist in all models which allow $\mu \to e\gamma$ decay. On the other hand, short-distance non-photonic mechanisms (Fig. 1(b)) are described by the effective $\not{L}_f$ 4-fermion quark-lepton interactions which may appear after integrating out heavy intermediate states (W, Z, Higgs bosons, supersymmetric particles etc.).

In this paper we concentrate on the non-photonic mechanisms of $\mu^- - e^-$ conversion. The generic $\not{L}_f$ effects of physics beyond the SM are parameterized by an effective Lagrangian with all possible 4-fermion quark-lepton interactions consistent with Lorentz covariance and gauge symmetry. We pay special attention to the hadronization of the quark currents of this Lagrangian focusing on its special realization when quarks are embedded into meson fields. Previously, in Ref. [10], we have shown that this realization is especially relevant for vector interactions which receive an appreciable contribution from vector meson exchange. This result uncovers the important role of vector mesons in the analysis of new physics in $\mu^- - e^-$conversion. Below we present a detailed discussion of the vector meson exchange mechanism of $\mu^- - e^-$nuclear conversion and examine some basic assumptions underlying the choice of the effective Lagrangian of $\not{L}_f$ meson-lepton interactions.

II. GENERAL FRAMEWORK

We start with the 4-fermion effective Lagrangian describing the non-photonic $\mu^- - e^-$ conversion at the quark level. The most general form of this Lagrangian has been derived in Ref. [11]. Here we present only those terms which contribute to the coherent $\mu^- - e^-$ conversion:

$$\mathcal{L}_{eff}^{lq} = \frac{1}{\Lambda_{LFV}^2}[(\eta_{VV}^q j_\mu^V + \eta_{AV}^q j_\mu^A)J_q^{V\mu} + (\eta_{SS}^q j^S + \eta_{PS}^q j^P)J_q^S], \tag{6}$$

where Λ_{LFV} is the characteristic high energy scale of lepton flavor violation attributed to new physics. The summation runs over all the quark species $q = \{u, d, s, c, b, t\}$. Lepton and quark currents are defined as:

$$j_\mu^V = \bar{e}\gamma_\mu\mu, \qquad j_\mu^A = \bar{e}\gamma_\mu\gamma_5\mu, \qquad j^S = \bar{e}\mu, \qquad j^P = \bar{e}\gamma_5\mu, \qquad J_q^{V\mu} = \bar{q}\gamma^\mu q, \qquad J_q^S = \bar{q}q. \tag{7}$$

The $\not{L}_f$parameters η^q in Eq. (6) depend on a concrete $\not{L}_f$ model.

The next step is the derivation of a Lagrangian in terms of effective nucleon fields which is equivalent to the quark level Lagrangian (6). First, we write down the lepton-nucleon $\not{L}_f$ Lagrangian of the coherent $\mu^- - e^-$ conversion in a general Lorentz covariant form with the isospin structure of the $\mu^- - e^-$transition operator [11]:

$$\mathcal{L}_{eff}^{lN} = \frac{1}{\Lambda_{LFV}^2}[j_\mu^a(\alpha_{aV}^{(0)}J^{V\mu(0)} + \alpha_{aV}^{(3)}J^{V\mu(3)}) + j^b(\alpha_{bS}^{(0)}J^{S(0)} + \alpha_{bS}^{(3)}J^{S(3)})], \tag{8}$$

where the summation runs over the double indices $a = V, A$ and $b = S, P$. The isoscalar $J^{(0)}$ and isovector $J^{(3)}$ nucleon currents are defined as

$$J^{V\mu(k)} = \bar{N}\gamma^\mu\tau^k N, \qquad J^{S(k)} = \bar{N}\tau^k N, \tag{9}$$

where N is the nucleon isospin doublet, $k = 0, 3$ and $\tau_0 \equiv \hat{I}$.

This Lagrangian is supposed to be generated by the one of Eq. (6) and, therefore, must correspond to the same order $1/\Lambda_{LFV}^2$ in inverse powers of the $\not{L}_f$ scale. The Lagrangian (8) is the basis for the derivation of the nuclear transition operators.

Now one needs to relate the lepton-nucleon $\not{L}_f$ parameters α in Eq. (8) to the more fundamental lepton-quark $\not{L}_f$ parameters η in Eq. (6). This implies a certain hadronization prescription which specifies the way in which the effect of quarks is simulated by hadrons. In the absence of a true theory of hadronization we rely on some reasonable assumptions and models.

There are basically two possibilities for the hadronization mechanism. The first one is a direct embedding of the quark currents into the nucleon (Fig. 2(a)), which we call direct nucleon mechanism (DNM). The second possibility is a two stage process (Fig. 2(b)). First, the quark currents are embedded into the interpolating meson fields which then interact with the nucleon currents. We call this

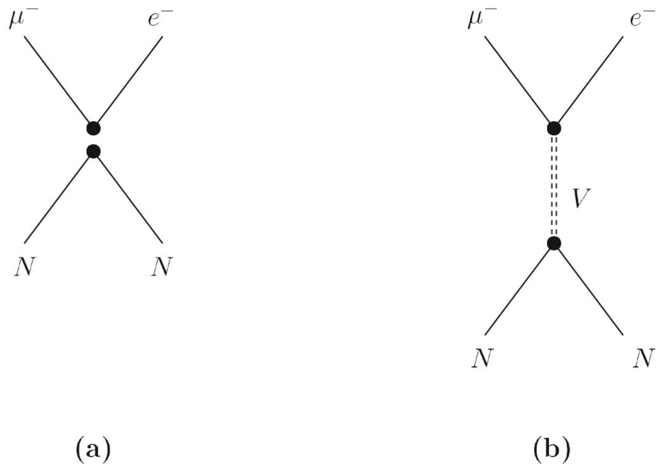

FIG. 2. Diagrams contributing to the nuclear $\mu^- - e^-$ conversion: direct nucleon mechanism (a) and meson-exchange mechanism (b).

possibility meson-exchange mechanism (MEM). In general one expects all the mechanisms to contribute to the coupling constants α in Eq. (8). However, at present the relative amplitudes of each mechanism are unknown. In view of this problem one may try to understand the importance of a specific mechanism, assuming for simplicity, that only this mechanism is operative and estimating its contribution to the process in question. We follow this procedure for the case of $\mu^- - e^-$ conversion and consider separately the contributions of the direct nucleon mechanism $\alpha_{[N]}$ and the meson-exchange one $\alpha_{[MN]}$ to the couplings of the Lagrangian (8).

In the present paper we concentrate on the meson-exchange mechanism and compare its contribution to $\mu^- - e^-$conversion with that of the direct nucleon mechanism which we shortly review in the next section following Ref. [11].

III. DIRECT NUCLEON MECHANISM

As mentioned before, this mechanism relies on a direct embedding of the quark currents of the Lagrangian (6) into the corresponding nucleon currents. This hadronization prescription directly leads to the nucleon level Lagrangian in Eq. (8) describing a contact type $\mu^- - e^-$ conversion as shown in Fig. 2(a).

Now we relate the coefficients α in Eq. (8) with the "fundamental" $\not{L}_f$ parameters η of the quark level Lagrangian (6). Towards this end we apply the on-mass-shell matching condition [12]

$$\langle e^- N|\mathcal{L}^q_{eff}|\mu^- N\rangle \approx \langle e^- N|\mathcal{L}^N_{eff}|\mu^- N\rangle, \tag{10}$$

in terms of the matrix elements of the Lagrangians (6) and (8) between the initial and final states of $\mu^- - e^-$conversion at the nucleon level.

In order to solve this equation in Ref. [11] various relations for the matrix elements of quark operators between nucleon states were used

$$\langle N|\bar{q}\Gamma_K q|N\rangle = G_K^{(q,N)}\bar{N}\Gamma_K N, \tag{11}$$

with $q = \{u, d, s\}$, $N = \{p, n\}$. Since the maximum momentum transfer in $\mu - e$ conversion is much smaller than the typical scale of nucleon structure one can safely neglect the $\mathbf{q}^2$-dependence of the nucleon form factors $G_K^{(q,N)}$.

Isospin symmetry requires that

$$G_K^{(u,p)} = G_K^{(d,n)} \equiv G_K^u, \qquad G_K^{(d,p)} = G_K^{(u,n)} \equiv G_K^d, \tag{12}$$
$$G_K^{(s,p)} = G_K^{(s,n)} \equiv G_K^s, \qquad G_K^{(h,p)} = G_K^{(h,n)} \equiv G_K^h.$$

Here $h = c, b, t$ are the heavy quarks.

The vector quark currents in the non-relativistic limit result in the quark number operators and, thus, the vector form factors at $q^2 = 0$ are equal to the total number of the corresponding species of quarks in the nucleon. Therefore,

$$G_V^u = 2, \qquad G_V^d = 1, \qquad G_V^s = 0, \qquad G_V^h = 0. \tag{13}$$

Because of the last two equalities s, c, b, t quarks do not contribute to the couplings of the vector nucleon current in Eq. (8).

Now solving Eq. (10) with the help of Eqs. (11)-(13) one can express the coefficients α of the nucleon level Lagrangian (8) in terms of the generic $\not{L}_f$ parameters η of the quark level effective Lagrangian Eq. (6). Here we present only the results of Ref. [11] relevant for our analysis which are the couplings of the vector nucleon currents in Eq. (8):

$$\begin{aligned} \alpha^{(3)}_{aV[N]} &= \frac{1}{2}(\eta^u_{aV} - \eta^d_{aV})(G_V^u - G_V^d), \\ \alpha^{(0)}_{aV[N]} &= \frac{1}{2}(\eta^u_{aV} + \eta^d_{aV})(G_V^u + G_V^d), \end{aligned} \tag{14}$$

where $a = V, A$.

Concluding this section we stress the fact that s, c, b, t quarks do not contribute to the couplings of the vector nucleon current in Eq. (14). In the next section we will show that the vector meson exchange may drastically modify this situation and introduce the contribution of strange quarks into these couplings.

IV. VECTOR MESON CONTRIBUTION

Now let us turn to the contributions of the meson-exchange mechanism to the couplings α of the lepton-nucleon Lagrangian (8). The mesons which can contribute to this mechanism are the unflavored vector and scalar ones. Since the case of the scalar meson candidate $f_0(600)$ is still quite uncertain [13] we do not study its contribution. Thus, we are left with the vector mesons. The lightest of them, giving the dominant contributions, are the isovector $\rho(770)$ and the two isoscalar $\omega(782)$, $\phi(1020)$ mesons. In our analysis we adopt ideal singlet-octet mixing corresponding to the following quark content of the ω and ϕ mesons [13]:

$$\omega = (u\bar{u} + d\bar{d})/\sqrt{2},\ \phi = -s\bar{s}. \tag{15}$$

We estimate the vector meson contribution to the lepton-nucleon Lagrangian (8) in two ways. First we adopt a model independent effective Lagrangian approach and then we present a more restrictive approach based on a simplified model of hadronization. The latter case results in a significant suppression of the vector meson contribution. A comparison of both approaches allows us to get an idea of the impact of the hadronization procedure on the new physics contribution to $\mu^- - e^-$conversion and the reliability of the limits on the corresponding parameters derived from the experimental data.

A. Model-Independent Approach

First, we derive the $\not{L}_f$ lepton-meson effective Lagrangian in terms of the interpolating ρ^0, ω and ϕ fields retaining all the interactions consistent with Lorentz and electromagnetic gauge invariance. It can be written as:

$$\begin{aligned}\mathcal{L}^{lV}_{eff} = \frac{\Lambda_H^2}{\Lambda_{LFV}^2}\Big[&\{(\xi^\rho_V j^V_\mu + \xi^\rho_A j^A_\mu)\rho^{0\mu} + (\xi^\omega_V j^V_\mu + \xi^\omega_A j^A_\mu)\omega^\mu \\ &+(\xi^\phi_V j^V_\mu + \xi^\phi_A j^A_\mu)\phi^\mu\} + \frac{1}{\Lambda_H^2}\{\xi^{\rho(2)}_V j^V_\mu \partial^\mu\partial^\nu\rho^0_\nu \\ &+ \ldots\} + \ldots\Big],\end{aligned} \tag{16}$$

with the unknown dimensionless coefficients ξ to be determined from the hadronization prescription. Since this Lagrangian is supposed to be generated by the quark-lepton Lagrangian (6) all its terms have the same suppression Λ_{LFV}^{-2} with respect to the large $\not{L}_f$scale Λ_{LFV}. Another scale in the problem is the hadronic scale $\Lambda_H \sim 1$ GeV which adjusts the physical dimensions of the terms in Eq. (16). Typical momenta involved in $\mu^- - e^-$ conversion are $q \sim m_\mu$, where m_μ is the muon mass. Thus, from naive dimensional counting one expects that the contribution of the derivative terms to $\mu^- - e^-$ conversion is suppressed by a factor $(m_\mu/\Lambda_H)^2 \sim 10^{-2}$ in comparison to the contribution of the non-derivative terms. Therefore, at this step in Eq. (16) we retain only the dominant non-derivative terms. However, it is worth noting that a true hadronization theory, yet non-existing, may forbid such terms so that the expansion in Eq. (16) starts from the derivative terms. Later on we demonstrate how it happens in a particular model.

In order to relate the parameters ξ of the Lagrangian (16) with the "fundamental" parameters η of the quark-lepton Lagrangian (6) we use, as in the previous section, an approximate method based on the standard on-mass-shell matching condition [12] which in this case takes the form

$$\langle\mu^+e^-|\mathcal{L}^{lq}_{eff}|V\rangle \approx \langle\mu^+e^-|\mathcal{L}^{lV}_{eff}|V\rangle, \tag{17}$$

with $|V = \rho, \omega, \phi\rangle$ corresponding to vector meson states on their mass-shell. We solve equation (17) using the well-known quark current matrix elements

$$\begin{aligned}\langle 0|\bar{u}\gamma_\mu u|\rho^0(p,\epsilon)\rangle &= -\langle 0|\bar{d}\gamma_\mu d|\rho^0(p,\epsilon)\rangle = m_\rho^2 f_\rho \epsilon_\mu(p),\\ \langle 0|\bar{u}\gamma_\mu u|\omega(p,\epsilon)\rangle &= \langle 0|\bar{d}\gamma_\mu d|\omega(p,\epsilon)\rangle = 3m_\omega^2 f_\omega \epsilon_\mu(p),\\ \langle 0|\bar{s}\gamma_\mu s|\phi(p,\epsilon)\rangle &= -3m_\phi^2 f_\phi \epsilon_\mu(p).\end{aligned} \tag{18}$$

Here p, m_V and ϵ_μ are the vector-meson four-momentum, mass and the polarization state vector, respectively. The quark operators in Eq. (18) are taken at $x = 0$. The coupling constants f_V are determined from the $V \to e^+e^-$ decay width:

$$\Gamma(V \to e^+e^-) = \frac{4\pi}{3}\alpha^2 f_V^2 m_V, \tag{19}$$

where α is the fine-structure constant. The current central values of the meson couplings f_V and masses m_V are [13]:

$$\begin{gathered} f_\rho = 0.2, \qquad f_\omega = 0.059, \qquad f_\phi = 0.074,\\ m_\rho = 771.1\ \text{MeV}, \qquad m_\omega = 782.57\ \text{MeV},\\ m_\phi = 1019.456\ \text{MeV}.\end{gathered} \tag{20}$$

Solving Eq. (17) with the help of Eqs. (18), we obtain the desired expressions for the coefficients ξ of the lepton-meson Lagrangian (16) in terms of generic $\not{L}_f$ parameters η of the lepton-quark effective Lagrangian Eq. (6):

$$\begin{aligned}\xi^\rho_a &= \left(\frac{m_\rho}{\Lambda_H}\right)^2 f_\rho(\eta^u_{aV} - \eta^d_{aV}),\\ \xi^\omega_a &= 3\left(\frac{m_\omega}{\Lambda_H}\right)^2 f_\omega(\eta^u_{aV} + \eta^d_{aV}),\\ \xi^\phi_a &= -3\left(\frac{m_\phi}{\Lambda_H}\right)^2 f_\phi \eta^s_{aV},\end{aligned} \tag{21}$$

where $a = V, A$.

Now we derive the vector meson exchange contributions to the couplings of the effective Lagrangian in Eq. (8) expressed in terms of nucleon fields. To this end we introduce the effective Lagrangian describing the interaction of nucleons with vector mesons [14–16]:

$$\mathcal{L}_{VN} = \frac{1}{2}\bar{N}\gamma^\mu[g_{\rho NN}\vec{\rho}_\mu\vec{\tau} + g_{\omega NN}\omega_\mu + g_{\phi NN}\phi_\mu]N. \tag{22}$$

In this Lagrangian we neglected the derivative terms, irrelevant for coherent $\mu^- - e^-$ conversion. For the meson-nucleon couplings g_{VNN} we use numerical values taken from an updated dispersive analysis [15,17]

$$\begin{gathered} g_{\rho NN} = 4.0, \qquad g_{\omega NN} = 41.8,\\ g^{(1)}_{\phi NN} = -18.3, \qquad g^{(2)}_{\phi NN} = -0.24.\end{gathered} \tag{23}$$

At this point the following comment is in order: The relatively large value of the ϕNN coupling $g^{(1)}_{\phi NN} = -18.3$ in Eq. (23) has been derived in Ref. [15] on the basis of the assumption on the "maximal" violation of the Okuba-Zweig-Iizuka rule. It was also stressed in Ref. [15] that this value corresponds to the upper limit for the ϕNN coupling which parameterizes the full spectral function in the mass region of $\sim$1 GeV within the ϕ pole dominance approximation. The inclusion of other contributions such as the $\pi\rho$ continuum leads to a significant reduction of the $g_{\phi NN}$ coupling [17]. The detailed analysis of various meson and baryon cloud contributions to the vector ϕNN coupling results in the value $g^{(2)}_{\phi NN} = -0.24$ [17]. For completeness we consider both values of $g_{\phi NN}$ coupling presented in Eq. (23) in our numerical analysis. Regarding these two values of the $g_{\phi NN}$ coupling the following comments are in order. The "maximal" value of the $g_{\phi NN}$ coupling is compatible with SU(3) symmetry prediction. Substituting the values of the $g_{\rho NN}$, $g_{\omega NN}$ and $g^{(1)}_{\phi NN}$ constants from Eq. (23) into the SU(3) relation [18]

$$g_{\phi NN} = g_{\rho NN}(\sqrt{3}/\cos\theta_V) - g_{\omega NN}\tan\theta_V, \tag{24}$$

we estimate the $\omega - \phi$ mixing angle to be $\theta_V = 32.4^{\rm o}$. This value is quite close to the ideal angle $\theta_V^I = 35.3^{\rm o}$ and to the experimental one $\theta_V^{\rm exp} \simeq 39^{\rm o}$, and therefore, is consistent with our initial assumption on the quark content of the ω and ϕ mesons (15). It is instructive to estimate the SU(3) symmetry prediction for the coupling $g_{\phi NN}$ for the angles θ_V^I and $\theta_V^{\rm exp}$. Substituting their values to Eq. (24) with $g_{\rho NN} = 4.0$ and $g_{\omega NN} = 41.8$ one finds the corresponding values of the ϕ-nucleon coupling

$$\theta_V^I: \quad g_{\phi NN} = -21.1, \qquad \theta_V^{\rm exp}: \quad g_{\phi NN} = -24.9, \tag{25}$$

which are larger than $g^{(1)}_{\phi NN} = -18.3$ in Eq. (23).

On the other hand the value $g^{(2)}_{\phi NN} = -0.24$ results in a strong violation of SU(3) symmetry and Eq. (24) is no longer valid giving a very small estimate for $\theta_V \simeq 9.9^{\rm o}$. This result looks controversial in view of successful predictions of the approximate SU(3) symmetry for the masses of ω and ϕ mesons with the mixing angle close to its ideal value. We assume that the true value of $g_{\phi NN}$ lies in the interval $-18.3 \le g_{\phi NN} \le -0.24$.

The vector meson-exchange contribution to the nucleon-lepton effective Lagrangian (8) arises in second order in the Lagrangian $\mathcal{L}^{lV}_{eff} + \mathcal{L}_{VN}$ and corresponds to the diagram in Fig. 2(b). We estimate this contribution only for the coherent $\mu^- - e^-$ conversion process. In this case we disregard all the derivative terms of nucleon and lepton fields. Neglecting the kinetic energy of the final nucleus, the muon binding energy and the electron mass, the square momentum transfer q^2 to the nucleus has a constant value $q^2 \approx -m_\mu^2$. In this approximation the vector meson propagators convert to δ-functions leading to effective lepton-nucleon contact type operators. Comparing them with the corresponding terms in the Lagrangian (8), we obtain for the vector meson-exchange contribution to the coupling constants:

$$\begin{aligned} \alpha^{(3)}_{aV[MN]} &= -\beta_\rho(\eta^u_{aV} - \eta^d_{aV}), \\ \alpha^{(0)}_{aV[MN]} &= -\beta_\omega(\eta^u_{aV} + \eta^d_{aV}) - \beta_\phi \eta^s_{aV}, \end{aligned} \tag{26}$$

with $a = V, A$ and the coefficients

$$\begin{aligned} \beta_\rho &= \frac{1}{2}\frac{g_{\rho NN} f_\rho m_\rho^2}{m_\rho^2 + m_\mu^2}, \qquad \beta_\omega = \frac{3}{2}\frac{g_{\omega NN} f_\omega m_\omega^2}{m_\omega^2 + m_\mu^2}, \\ \beta_\phi &= -\frac{3}{2}\frac{g_{\phi NN} f_\phi m_\phi^2}{m_\phi^2 + m_\mu^2}. \end{aligned} \tag{27}$$

Substituting the values of the meson coupling constants and masses from Eqs. (20) and (23), and including the two different options for the $g_{\phi NN}$ constant, we obtain for these coefficients

$$\begin{aligned} \beta_\rho &= 0.39, \qquad \beta_\omega = 3.63, \\ \beta^{(1)}_\phi &= 2.0, \qquad \beta^{(2)}_\phi = 0.03. \end{aligned} \tag{28}$$

A new issue of the vector meson contribution (26) is the presence of the strange quark vector current contribution associated with the LFV parameter η^s_{aV}, absent in the direct nucleon mechanism as it follows from Eq. (14). This opens up the possibility of deriving new limits on this parameter from the experimental data on $\mu^- - e^-$ conversion. Another surprising result is that the contribution (26) of the meson-exchange mechanism is comparable to the contribution (14) of the direct nucleon mechanism.

B. Simplified Model of Hadronization

Now, instead of constructing the $\not{L}_f$ lepton-meson Lagrangian in a general phenomenological way, only requiring its consistency with basic symmetries, as done in the previous subsection, we present a simple model which allows us to derive this Lagrangian within certain assumptions. The model assumes that the vector mesons fields manifest themselves in the interactions with leptons only indirectly via their interactions with quarks. Thus, the model Lagrangian consists of two terms

$$\mathcal{L}^{qVl}_{eff} = \mathcal{L}^{lq}_{eff} + \mathcal{L}^{qV} \tag{29}$$

where the first term is the $\not{L}_f$ lepton-quark Lagrangian (6) and the second one represents the vector meson-quark interaction Lagrangian which we write down in a general isospin invariant form as

$$\begin{aligned} \mathcal{L}^{qV} = \frac{g_{\rho qq}}{2}\bar{q}\gamma^\mu\vec{\tau}q\vec{\rho}_\mu + \frac{g_{\omega qq}}{2}\bar{q}\gamma^\mu q\omega_\mu \\ + \frac{g_{\phi qq}}{2}\bar{s}\gamma^\mu s\phi_\mu. \end{aligned} \tag{30}$$

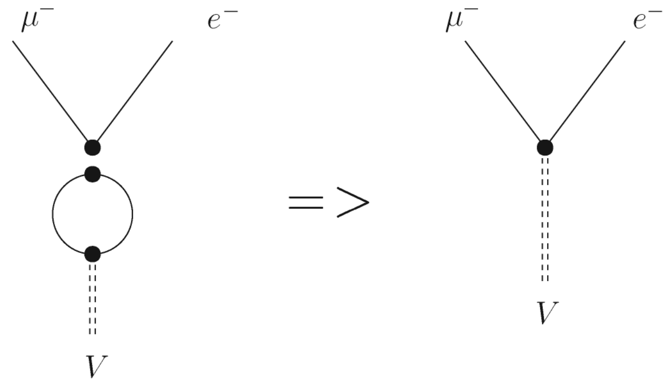

FIG. 3. Generation of the $\not{L}_f$ lepton-meson effective Lagrangian in the simplified model of hadronization.

Here q is the quark isodoublet. The vector meson-quark couplings g_{Vqq} are free phenomenological parameters which, however, do not appear explicitly in the final results.

The Lagrangian (29) generates the $\not{L}_f$ lepton-meson effective Lagrangian in second order of perturbation theory via the vector-vector quark loops in Fig. 3. All the quark loops are logarithmically divergent and transversal. Their Lorentz structure can be written in momentum space as

$$\Pi^{\rm QL}_{\mu\nu}(q) \sim (g_{\mu\nu}q^2 - q_\mu q_\nu). \tag{31}$$

This property immediately follows from the observation that these loops are nothing but the quark contribution to the vacuum polarization operator of the photon, which is transversal. In coordinate space this structure is reproduced at the first order in the effective lepton-meson Lagrangian with the transversal double derivative operator $(g_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)$ acting on meson and/or lepton fields. We write down this Lagrangian in an equivalent but more conventional form[2]

$$\begin{aligned}\mathcal{L}^{lV}_{eff} = -\frac{1}{2\Lambda^2_{LFV}}[(\tilde{\xi}^\rho_V j^V_{\mu\nu} + \tilde{\xi}^\rho_A j^A_{\mu\nu})\mathcal{R}^{0\mu\nu} + (\tilde{\xi}^\omega_V j^V_{\mu\nu} \\ + \tilde{\xi}^\omega_A j^A_{\mu\nu})\Omega^{\mu\nu} + (\tilde{\xi}^\phi_V j^V_{\mu\nu} + \tilde{\xi}^\phi_A j^A_{\mu\nu})\Phi^{\mu\nu}]\end{aligned} \tag{32}$$

in terms of the vector meson and lepton current stress tensors [16]:

$$\begin{gathered}\mathcal{R}^{0\mu\nu} = \partial^\mu\rho^{0\nu} - \partial^\nu\rho^{0\mu}, \qquad \Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu, \\ \Phi^{\mu\nu} = \partial^\mu\phi^\nu - \partial^\nu\phi^\mu, \qquad j^V_{\mu\nu} = \partial_\mu j^V_\nu - \partial_\nu j^V_\mu, \\ j^A_{\mu\nu} = \partial_\mu j^A_\nu - \partial_\nu j^A_\mu.\end{gathered} \tag{33}$$

Thus, this model does not allow non-derivative terms in the lepton-meson Lagrangian. The relations between the couplings $\tilde{\xi}^\alpha_a$ and the "fundamental" couplings η^q of the lepton-quark Lagrangian (6) cannot be established in this simplified model without additional assumptions. In particular, this is because of the divergence of the quark loops. However, we do not intend to employ this model for detailed calculations. Our goal is to illustrate the situation which may happen in a true, yet non-existing, theory of hadronization when the non-derivative terms in the effective lepton-meson Lagrangian are prohibited contrary to the general phenomenological treatment allowing such terms in Eq. (16).

Let us study the impact of such a situation on $\mu^- - e^-$ conversion. To this end we repeat the analysis of the previous section for the derivative Lagrangian in Eq. (32). In the following we do not refer to any hadronization model which may be the basis of that Lagrangian, treating the couplings $\tilde{\xi}^\alpha$ as free parameters. We relate them to the $\not{L}_f$ parameters η of the quark-lepton Lagrangian in Eq. (6), using the on-mass-shell matching condition (17) and relations (18). The result is:

$$\begin{gathered}\tilde{\xi}^\rho_a = f_\rho(\eta^u_{aV} - \eta^d_{aV}), \qquad \tilde{\xi}^\omega_a = 3f_\omega(\eta^u_{aV} + \eta^d_{aV}), \\ \tilde{\xi}^\phi_a = -3f_\phi\eta^s_{aV}.\end{gathered} \tag{34}$$

Note, that the vector meson-quark couplings g_{Vqq} introduced in Eq. (30) do not appear in these formulas explicitly, being absorbed, alone with the divergent quark loops, by the couplings f_V which are experimentally measurable quantities (19) with the values given in Eq. (20).

As explained in the previous section the vector meson exchange generates the lepton-nucleon Lagrangian (8) in second order in $\mathcal{L}^{lV}_{eff} + \mathcal{L}_{VN}$ shown in Fig. 2b. The corresponding couplings are given by the expressions:

$$\begin{aligned}\alpha^{(3)}_{aV[MN]} &= -\tilde{\beta}_\rho(\eta^u_{aV} - \eta^d_{aV}), \\ \alpha^{(0)}_{aV[MN]} &= -\tilde{\beta}_\omega(\eta^u_{aV} + \eta^d_{aV}) - \tilde{\beta}_\phi\eta^s_{aV},\end{aligned} \tag{35}$$

where the coefficients $\tilde{\beta}_V$ are:

$$\begin{gathered}\tilde{\beta}_\rho = \frac{1}{2}\frac{g_{\rho NN}f_\rho m^2_\mu}{m^2_\rho + m^2_\mu}, \qquad \tilde{\beta}_\omega = \frac{3}{2}\frac{g_{\omega NN}f_\omega m^2_\mu}{m^2_\omega + m^2_\mu}, \\ \tilde{\beta}_\phi = -\frac{3}{2}\frac{g_{\phi NN}f_\phi m^2_\mu}{m^2_\phi + m^2_\mu}.\end{gathered} \tag{36}$$

Using the numerical values of the vector meson masses and coupling constants from Eqs. (20) and (23) we obtain

$$\begin{aligned}\tilde{\beta}_\rho &= 7.5\times 10^{-3}, \qquad \tilde{\beta}_\omega = 6.6\times 10^{-3}, \\ \tilde{\beta}^{(1)}_\phi &= 2.2\times 10^{-3}, \qquad \tilde{\beta}^{(2)}_\phi = 2.9\times 10^{-5}.\end{aligned} \tag{37}$$

Naturally, in the case of the double derivative lepton-meson Lagrangian (32) the $\not{L}_f$ couplings α of the corresponding lepton-nucleon Lagrangian (8) are much smaller, by a factor $\tilde{\beta}_V/\beta_V = (m_\mu/m_V)^2 \sim 10^{-2}$, than in the previously analyzed non-derivative case (26).

[2]There are other equivalent forms of this Lagrangian suitable for description of chiral invariant interactions of vector mesons with pseudoscalar mesons and the electromagnetic field [16,19].

This demonstrates that the hadronization prescription may have a strong impact on the $\not{L}_f$ new physics contribution to $\mu^- - e^-$ conversion.

V. CONSTRAINTS ON $\not{L}_f$ PARAMETERS FROM MUON-ELECTRON CONVERSION

Starting from the Lagrangian (8) it is straightforward to derive the formula for the total $\mu - e$ conversion branching ratio [20]. In the present paper we focus on the coherent process, i.e. ground state to ground state transitions, which is the dominant channel of $\mu - e$ conversion exhausting, for the majority of experimentally interesting nuclei, more than 90% of the total $\mu^- - e^-$branching ratio [21]. To leading order of the nonrelativistic reduction the coherent $\mu - e$ conversion branching ratio takes the form [2,20]

$$R^{coh}_{\mu e^-} = \frac{\mathcal{Q}}{2\pi\Lambda^4_{LFV}} \frac{p_e E_e(\mathcal{M}_p + \mathcal{M}_n)^2}{\Gamma(\mu^- \to capture)}, \tag{38}$$

where p_e, E_e are 3-momentum and energy of the outgoing electron. The nuclear transition matrix elements $\mathcal{M}_{p,n}$ in Eq. (38), for the case of a ground state to ground state $\mu^- - e^-$ transition, are defined as

$$\mathcal{M}_{p,n} = 4\pi \int j_0(p_e r)\Phi_\mu(r)\rho_{p,n}(r)r^2 dr, \tag{39}$$

where j_0 is the zero-order spherical Bessel function. The quantities $\rho_{p,n}(r)$ are the spherically symmetric proton (p) and neutron (n) nuclear densities normalized to the atomic number Z and neutron number N of the nucleus, respectively. $\Phi_\mu(r)$ is the space dependent part of the muon wave function. The factor $\mathcal{Q}$ in Eq. (38) has the form [11]

$$\begin{aligned}\mathcal{Q} &= |\alpha^{(0)}_{VV} + \alpha^{(3)}_{VV}\epsilon|^2 + |\alpha^{(0)}_{AV} + \alpha^{(3)}_{AV}\epsilon|^2 + |\alpha^{(0)}_{SS} + \alpha^{(3)}_{SS}\epsilon|^2 \\ &+|\alpha^{(0)}_{PS} + \alpha^{(3)}_{PS}\epsilon|^2 + 2\mathrm{Re}\{(\alpha^{(0)}_{VV} + \alpha^{(3)}_{VV}\epsilon)(\alpha^{(0)}_{SS} \\ &+\alpha^{(3)}_{SS}\epsilon)^* + (\alpha^{(0)}_{AV} + \alpha^{(3)}_{AV}\epsilon)(\alpha^{(0)}_{PS} + \alpha^{(3)}_{PS}\epsilon)^*\}.\end{aligned} \tag{40}$$

in terms of the parameters of the lepton-nucleon effective Lagrangian (8) and the nuclear structure factor

$$\epsilon = (\mathcal{M}_p - \mathcal{M}_n)/(\mathcal{M}_p + \mathcal{M}_n). \tag{41}$$

The nuclear matrix elements $\mathcal{M}_{p,n}$, defined in Eq. (39), have been numerically calculated in Refs. [11,22] for the nuclear targets ^{27}Al, ^{48}Ti and ^{197}Au, using proton densities ρ_p from Ref. [23] and neutron densities ρ_n from Ref. [24] whenever possible. The muon wave function $\Phi_\mu(r)$ has been obtained by solving the Schröndinger equation with the Coulomb potential produced by the densities $\rho_{p,n}$, taking into account the vacuum polarization corrections [22]. The results for $\mathcal{M}_{p,n}$ corresponding to the nuclei Al, Ti and Au are given in Table I where we also show the muon binding energy ϵ_b and the experimental total rates $\Gamma(\mu^- \to capture)$ of the ordinary muon capture reaction [25].

As follows from Table I, the parameter ϵ in Eq. (40) is small $\epsilon \sim 10^{-1}$ and, therefore, the contribution of isovector $\alpha^{(3)}\epsilon$ terms can be neglected except for a very special domain in the $\not{L}_f$parameter space where $\alpha^{(0)} \leq \alpha^{(3)}\epsilon$. For this reason the role of the isovector ρ-meson exchange in $\mu^- - e^-$ conversion is expected to be unimportant except for this special case.

With these matrix elements we find, for the combination of the dimensionless vector nucleon couplings $\alpha^{(0)}_{aV}$ ($a = A, V$) and the characteristic $\not{L}_f$ scale Λ_{LFV} in the effective lepton-nucleon Lagrangian (8) the following limit

$$\alpha^{(0)}_{aV}\left(\frac{1\ \mathrm{GeV}}{\Lambda_{LFV}}\right)^2 \leq 1.2 \times 10^{-12} B(\mathrm{exp}). \tag{42}$$

Here the factor $B(A)$ depends on the nuclear matrix element $\mathcal{M}(A) = \mathcal{M}_n(A) + \mathcal{M}_p(A)$ of the target nucleus A used in an experiment setting the upper limit $R^A_{\mu e}(\mathrm{exp})$ on the branching ratio of $\mu^- - e^-$conversion $R^A_{\mu e} \leq R^A_{\mu e}(\mathrm{exp})$. This factor has the form

$$B(A) = \frac{\mathcal{M}(A)}{\mathcal{M}(^{48}\mathrm{Ti})}\left(\frac{R^A_{\mu e}(\mathrm{exp})}{1.2 \times 10^{-12}}\right)^{1/2} \tag{43}$$

and allows one to translate the limits in Eq. (42) to the limits of a specific experiment. For the running and forthcoming experiments discussed in the introduction this factor takes the values

$$\begin{aligned}&\mathrm{Eq.}(2){:}B(^{48}Ti) = 1; \qquad \mathrm{Eq.}(3){:}B(^{27}Al) = 7.3 \times 10^{-3}; \\ &\mathrm{Eq.}(4){:}B(^{197}Au) = 0.57; \qquad \mathrm{Eq.}(5){:}B(^{48}Ti) \sim 10^{-3}.\end{aligned} \tag{44}$$

From the limit in Eq. (42) one can deduce the individual limits on different terms entering in the expressions for the coefficients $\alpha^{(0)}_{aV}$ in the direct nucleon and meson-exchange mechanisms, assuming that significant cancellations (unnatural fine-tuning) between different terms are absent. In this way we derive constraints for the η parameters of the quark-lepton Lagrangian (6) for the meson-exchange mechanism (MEM). We present these limits in Table II only for the case of the model independent approach based on the Lagrangian in Eq. (16). For the parameter $|\eta^s_{aV}|$ we derive the limits for the two different cases of $g_{\phi NN}$ coupling, given in Eq. (23). In

TABLE I. Transition nuclear matrix elements $\mathcal{M}_{p,n}$ (in $fm^{-3/2}$) of Eq. (39) and other useful quantities (see the text).

Nucleus	$p_e(fm^{-1})$	$\epsilon_b(MeV)$	$\Gamma_{\mu c}(\times 10^6 s^{-1})$	$\mathcal{M}_p$	$\mathcal{M}_n$
^{27}Al	0.531	-0.470	0.71	0.047	0.045
^{48}Ti	0.529	-1.264	2.60	0.104	0.127
^{197}Au	0.485	-9.938	13.07	0.395	0.516

TABLE II. Upper bounds on the $\not{L}_f$ parameters inferred from the SINDRUM II data on ^{48}Ti [Eq. (2)] corresponding to the direct mechanism (DNM) and the meson exchange mechanism (MEM). The subscript notation is $a = V, A$. The value in square brackets refers to the $g_{\phi NN}{}^{(1)}$ value of ϕNN coupling presented in Eq. (23). The experimental factor $B(A)$ is defined in Eqs. (43) and (44).

Parameter	DNM	MEM
$\lvert\eta_{aV}^{u,d}\rvert(1\text{ GeV}/\Lambda_{LFV})^2$	$8.0 \times 10^{-13} B(A)$	$3.3 \times 10^{-13} B(A)$
$\lvert\eta_{aV}^{s}\rvert(1\text{ GeV}/\Lambda_{LFV})^2$	no limits	$4.0 \times 10^{-11} B(A)$ $[6.0 \times 10^{-13} B(A)]$

Table II we also show for comparison the limits corresponding to the direct nucleon mechanism (DNM) derived in Ref. [11]. With the values of the experimental factor $B(A)$ given in Eqs. (43) and (44) one can translate the limits in Table II to the case of a particular experiment.

The limits presented in Table II show the importance of the vector meson exchange mechanism to $\mu^- - e^-$ conversion. The MEM leads to the new experimental upper bound on the previously unconstrained strange quark $\not{L}_f$ parameter η_{aV}^s. Also, the MEM limits on the $\not{L}_f$ parameters $\eta_{aV}^{u,d}$ are by a factor ~ 2 better than the DNM limits.

The limits in Table II represent a general outcome of the $\mu^- - e^-$conversion experiments for the $\not{L}_f$ physics. Here we do not show the limits for the scalar quark current coefficients η_{PS}^q of the Lagrangian in Eq. (6), which can be found in Ref. [11]. The limits in Table 2 can be easily translated into limits on the parameters of specific $\not{L}_f$ model predicting the $\mu^- - e^-$conversion. This is achieved by adjusting the quark level effective Lagrangian of the model to the form of Eq. (6) and by identifying the effective parameters $\eta^{(q)}$ with expressions in terms of model parameters. Then, assuming absence of cancellation between different terms in these expressions, one can extract the upper bounds on the model parameters.

SUMMARY

We studied nuclear $\mu^- - e^-$ conversion in a general framework based on an effective Lagrangian without referring to any specific realization of the physics beyond the standard model responsible for lepton flavor violation. We demonstrated that the vector meson-exchange contribution to this process is significant. A new issue of the meson-exchange mechanism in comparison to the previously studied direct nucleon mechanism is the presence of the strange quark vector current contribution induced by the ϕ meson. This allowed us to extract new limits on the $\not{L}_f$ lepton-quark effective couplings from the existing experimental data. To our best knowledge these limits have not yet been discussed in the literature. We also presented a simplified model of hadronization which leads to the derivative lepton-meson effective Lagrangian. The model results in a suppression, by a factor of $\sim 10^{-2}$, of the vector meson contribution to $\mu^- - e^-$ conversion in comparison to the general phenomenological treatment. This demonstrates the impact of a hadronization prescription on the analysis of new physics in processes involving hadrons and nuclei. In the literature it is a common point to mention the uncertainties which come from the nuclear structure models. We stress that the uncertainties arising at the preceding level dealing with the hadronization could be comparable or even larger than the nuclear uncertainties.

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