New mechanism for the top-bottom mass hierarchy

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We propose a mechanism to generate hierarchy between masses of the top and bottom quarks without fine-tuning of the Yukawa coupling constants in the context of the two Higgs doublet model (THDM). In the THDM with a discrete symmetry, there exists the vacuum where only the top quark receives the mass of the order of the electroweak symmetry breaking scale $v(\simeq 246$ GeV), while the bottom quark remains massless. By introducing a small soft-breaking parameter m_3^2 of the discrete symmetry, the bottom quark perturbatively acquires a nonzero mass. We show a model in which the small $m_3^2[\sim v^2/(4\pi)^2]$ is generated by the dynamics above the cutoff scale of the THDM. The ratio $\tan\beta$ of the two vacuum expectation values is necessarily very large, i.e., $\tan\beta \sim m_t/m_b$. We also find a salient relation, $1/\tan\beta \simeq m_3^2/m_H^2$, where m_H is the mass of the extra CP-even Higgs boson. Our scenario yields some specific features that can be tested in future collider experiments.

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I. INTRODUCTION

The measured quark mass spectrum shows a specific feature. Only the top quark has the mass of the order of the electroweak symmetry breaking (EWSB) scale $v[= (\sqrt{2}G_F)^{-1/2} \simeq 246$ GeV], while masses of the other quarks are much smaller. The top quark mass is 174 GeV$(\simeq v/\sqrt{2})$, while the bottom quark, the second heaviest, has the mass of 4.2 GeV$(\ll v)$ [1]. In the Standard Model (SM), however, the unique Higgs doublet field $\Phi_{\rm SM}$ is responsible for the EWSB and gives masses of all quarks via the Yukawa interactions, i.e., $m_f \simeq y_f\langle\Phi_{\rm SM}\rangle$ with $\langle\Phi_{\rm SM}\rangle = (0, v/\sqrt{2})^T$. Therefore, the observed mass spectrum is obtained only by assuming unnatural hierarchy among the Yukawa coupling constants y_f. For instance, the hierarchy $y_b/y_t \simeq 1/40$ must be required for the top and bottom quarks. Nevertheless, no explanation for such fine-tuning is given in the SM.

In this paper, we propose an alternative scenario in which the quark mass spectrum is reproduced without fine-tuning in magnitude of the Yukawa coupling constants. We study the hierarchy between m_t and m_b under the assumption of $y_t \sim y_b \sim \mathcal{O}(1)$. In order to realize $m_b/m_t \sim 1/40$ in a natural way, we consider the two Higgs doublet model (THDM) with Φ_1 and Φ_2, imposing the discrete Z_2 symmetry [2] under the transformation

$$\Phi_1 \to -\Phi_1, \quad \Phi_2 \to +\Phi_2 \tag{1}$$

as well as

$$\begin{pmatrix} t \\ b \end{pmatrix}_L \to +\begin{pmatrix} t \\ b \end{pmatrix}_L, \quad t_R \to +t_R, \quad b_R \to -b_R. \tag{2}$$

Because of the Z_2 symmetry, only Φ_1 couples to the bottom quark while Φ_2 does to the top quark. The hierarchy $m_t \gg m_b$ is then equivalent to $v_2 \gg v_1$, where $\langle\Phi_{1,2}\rangle = (0, v_{1,2}/\sqrt{2})^T$. We note that there exists the vacuum with $v_1 = 0$ and $v_2 = v$ when the Z_2 symmetry is exact. A nonzero value of $v_1(\ll v_2)$ is induced as a perturbation of a small soft-breaking parameter m_3^2 for the Z_2 symmetry. The small $m_3^2[\sim v^2/(4\pi)^2]$ is generated by the dynamics above the cutoff scale of the THDM. We find a salient relation, $1/\tan\beta \equiv v_1/v_2 \simeq m_3^2/m_H^2 \ll 1$, where m_H is the mass of the extra CP-even Higgs boson. Consequently, we obtain $m_b/m_t \ll 1$. This scenario is extended to include the first two generation quarks.

We find that the extra Higgs bosons almost decouple with the weak gauge bosons in our model. Moreover, the extra Higgs bosons as well as the SM-like one turn out to have masses of the order of v. The Higgs bosons with such masses are expected to be discovered at the CERN LHC because of the large value of $\tan\beta$ [3]. The characteristics of our scenario can further be tested by precision measurement at future linear colliders (LC's) [4].

II. MINIMAL MODEL

The Lagrangian of the THDM with the softly-broken Z_2 symmetry is described as

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_Y - V, \tag{3}$$

where $\mathcal{L}_{\rm kin}$ and $\mathcal{L}_Y$ are the kinetic and Yukawa interaction terms, respectively. The Higgs potential V is given by

$$\begin{aligned} V = {} & m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - [m_3^2\Phi_2^\dagger\Phi_1 + ({\rm H.c.})] \\ & + \lambda_1|\Phi_1|^4 + \lambda_2|\Phi_2|^4 + 2\lambda_3|\Phi_1|^2|\Phi_2|^2 \\ & + 2\lambda_4|\Phi_1^\dagger\Phi_2|^2 + [\lambda_5(\Phi_2^\dagger\Phi_1)^2 + ({\rm H.c.})], \end{aligned} \tag{4}$$

where m_1^2, m_2^2 and λ_1 to λ_4 are real, while m_3^2 and λ_5 are complex. The Higgs doublet fields Φ_i $(i = 1, 2)$ with hypercharge $Y = 1/2$ are parameterized by

$$\Phi_i = \begin{bmatrix} \phi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + ia_i) \end{bmatrix}, \tag{5}$$

where the vacuum expectation values (VEV's) v_i ($i = 1, 2$) satisfy $v_1^2 + v_2^2 = v^2$. The mass matrices for the Higgs bosons are diagonalized by mixing angles α and β [5]. We then obtain five physical scalar states, h and H (CP-even), A (CP-odd), and $H^\pm$ (charged), as well as three Nambu-Goldstone bosons, ϕ^0 and $\phi^\pm$.

We consider only the top and bottom quarks among fermions at first. We discuss the extension for the other quarks later on. In order to describe the assumption of $y_t \simeq y_b$, we introduce the global $SU(2)_R$ symmetry [6,7], in addition to the $SU(2)_L$ gauge symmetry:

$$q_{L,R} \to q'_{L,R} = U_{L,R} q_{L,R}, \tag{6}$$

$$M_{21} \to M'_{21} = U_L M_{21} U_R^\dagger, \tag{7}$$

where $q_{L,R} \equiv (t_{L,R}, b_{L,R})$ and $U_{L,R} \in SU(2)_{L,R}$, respectively. The 2×2 matrix M_{21} is defined by

$$M_{21} \equiv (\tilde{\Phi}_2, \Phi_1), \quad \text{with} \quad \tilde{\Phi}_2 = i\tau_2 \Phi_2^*. \tag{8}$$

The Z_2 symmetry can be expressed in terms of $q_{L,R}$ and M_{21} by

$$q_L \to q'_L = q_L, \qquad q_R \to q'_R = \tau_3 q_R, \tag{9}$$

$$M_{21} \to M'_{21} = M_{21}\tau_3. \tag{10}$$

The Yukawa interaction then is written as

$$\mathcal{L}_Y = -y\bar{q}_L M_{21} q_R + (\text{H.c.}), \tag{11}$$

with $y \equiv y_t = y_b$. We also set

$$\lambda_1 = \lambda_2 = \lambda_3 (\equiv \lambda) \tag{12}$$

in Eq. (4) to realize the $SU(2)_R$ symmetry in quartic interactions. The Higgs potential then is expressed by

$$\begin{aligned} V(M_{21}) = {} & \frac{1}{2} m^2 \mathrm{tr}(M_{21}^\dagger M_{21}) - \frac{1}{2}\Delta_{12} \mathrm{tr}(M_{21}^\dagger M_{21}\tau_3) \\ & - [m_3^2 \det M_{21} + (\text{H.c.})] + \lambda[\mathrm{tr}(M_{21}^\dagger M_{21})]^2 \\ & + 2\lambda_4 \det(M_{21}^\dagger M_{21}) + [\lambda_5 (\det M_{21})^2 + (\text{H.c.})], \end{aligned} \tag{13}$$

where $m^2 = m_1^2 + m_2^2$ and $\Delta_{12} = m_1^2 - m_2^2$. The Z_2 symmetry is softly broken by the mass term of m_3^2. A nonzero value of Δ_{12} measures the soft breaking of the global $SU(2)_R$ symmetry. In order to evade explicit CP violation, we choose the phases in m_3^2 and λ_5 to be zero.

We have introduced the global $SU(2)_R$ symmetry only for the description of $y_t = y_b$ in terms of a symmetry. The Higgs potential also becomes simple since this symmetry requires the relation (12). Our main results, however, turn out to be unchanged even when this relation is relaxed to some extent. Cases without $SU(2)_R$ as well as those with CP violation will be discussed in details elsewhere [8].

Let us consider the effective potential $V(\langle M_{21}\rangle)$ to study the vacuum structure. By using $SU(2)_L$ and $U(1)_Y$, the VEV's in the THDM can be generally parameterized as

$$\langle M_{21} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 & v_E \\ 0 & v_1 + i v_A \end{pmatrix}. \tag{14}$$

Spontaneous breakdown of $U(1)_{EM}$ and the CP symmetry occurs if $v_E \neq 0$ and $v_1 v_A \neq 0$, respectively. We can easily show that the spontaneous $U(1)_{EM}$ breaking cannot occur at the tree level in our model. The conditions for CP conservation are studied in Ref. [9]. The effective potential is bounded from below by the requirement of the vacuum stability [10], which leads to

$$\lambda > 0, \qquad 2\lambda + \lambda_4 - |\lambda_5| > 0. \tag{15}$$

We investigate details of the vacuum structure of our model in the tree-level approximation. We first study the case with $m_3^2 = 0$ where the discrete Z_2 symmetry is exact. We next include effects of $m_3^2 \neq 0$.

For $m_3^2 = 0$, the effective potential $V(\langle M_{21}\rangle)$ is given by

$$\begin{aligned} V(\langle M_{21}\rangle) = {} & \frac{m_1^2}{2}(v_1^2 + v_A^2) + \frac{m_2^2}{2} v_2^2 + \frac{\lambda}{4}(v_1^2 + v_A^2 + v_2^2) \\ & + \frac{\lambda_4}{2}(v_1^2 + v_A^2) v_2^2 + \frac{\lambda_5}{2}(v_1^2 - v_A^2) v_2^2, \end{aligned} \tag{16}$$

where we used Eq. (14) with $v_E = 0$. The VEV's, v_1, v_2, and v_A, are determined by the stationary conditions $\partial V(\langle M_{21}\rangle)/\partial v_i = 0$, ($i = 1, 2, A$). Since spontaneous CP violation does not occur for $m_3^2 = 0$, three types of the nontrivial vacuum are possible [10]:

(a) $v_1 = v_A = 0, \quad v_2 \neq 0$,
(b) $v_1 v_2 \neq 0, \quad v_A = 0$,
(c) $v_A v_2 \neq 0, \quad v_1 = 0$.

In Fig. 1, the area (I) corresponds to the vacuum (a), while the areas (II) and (III) do to the vacua (b) and (c), respectively. Because of the vacuum stability conditions (15), there does not exist the stable vacuum out of the

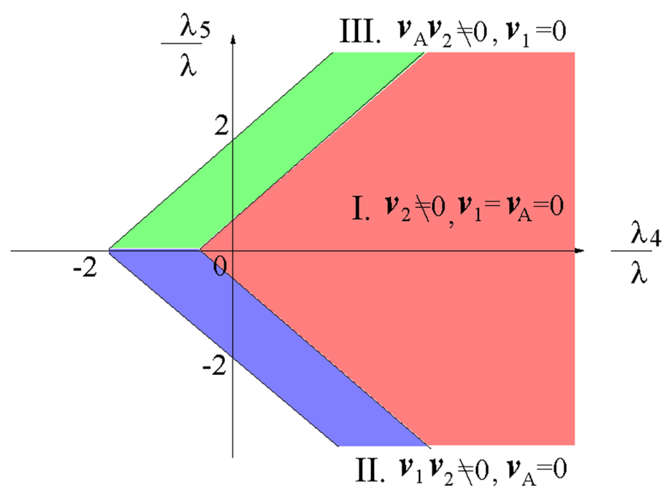

FIG. 1 (color online). Vacuum structure for $m_3^2 = 0$ and $m_2^2 < -|m_1^2|$.

three areas. Performing the transformation $\Phi_1 \to e^{i\pi/2}\Phi_1$ to the nontrivial vacuum (b), we obtain the vacuum (c). The transformation corresponds to $\lambda_5 \to -\lambda_5$ in the Higgs potential with $m_3^2 = 0$. The area (III) is thus the mirror image of the area (II).

In order to realize $m_b/m_t \ll 1$ without fine-tuning of Yukawa couplings, we choose the vacuum (a) which leads to

$$m_t = \frac{1}{\sqrt{2}} y v, \qquad m_b = 0, \tag{17}$$

because of $v_2 = v$. Although the bottom quark may receive a small mass even in the vacuum (b), the parameters of the Higgs potential must be very close to the boundary between the areas of (I) and (II). This is fine-tuning in a sense, so that we avoid such a case. The vacuum (a) for $m_3^2 = 0$ is realized when[1]

$$m_2^2 < -|m_1^2|, \qquad -\frac{\lambda_4}{\lambda} - \frac{\Delta_{12}}{-m_2^2} < \frac{\lambda_5}{\lambda} < \frac{\lambda_4}{\lambda} + \frac{\Delta_{12}}{-m_2^2}, \tag{18}$$

$$\text{or} \quad m_1^2 \geq -m_2^2 > 0. \tag{19}$$

Only the doublet Φ_2 is responsible for the EWSB in the vacuum (a). The doublet fields Φ_1 and Φ_2 do not mix for $m_3^2 = 0$ because of the remaining Z_2 symmetry after the EWSB, $\Phi_1 \to -\Phi_1$. The mass formulae of the physical Higgs bosons are

$$m_h^2 = 2\lambda v^2, \tag{20}$$

$$m_{H^\pm}^2 = \Delta_{12}, \tag{21}$$

$$m_H^2 = \Delta_{12} + (\lambda_4 + \lambda_5)v^2, \tag{22}$$

$$m_A^2 = \Delta_{12} + (\lambda_4 - \lambda_5)v^2. \tag{23}$$

When $\Delta_{12} = 0$, the charged Higgs bosons become the extra Nambu-Goldstone bosons associated with the breaking of the exact $SU(2)_R$ symmetry.

We now switch on a *small* soft-breaking parameter $m_3^2 (\ll v^2)$ of the discrete Z_2 symmetry. We do not consider the possibility of spontaneous *CP* violation[2]. A nonzero v_1 is necessarily induced for $m_3^2 \neq 0$ from the stationary condition. As a perturbation from the vacuum (a) with $m_3^2 = 0$, we consequently obtain

$$\frac{v_1}{v_2}\left(\equiv \frac{1}{\tan\beta}\right) = \frac{m_3^2}{m_H^2}\left\{1 + \mathcal{O}\left(\frac{m_3^4}{v^4}\right)\right\}, \tag{24}$$

where we used the tree-level mass formula in Eq. (22). Because of $v_1^2 + v_2^2 = v^2$, the expression for v_2 is slightly modified to $v_2 = v[1 - \mathcal{O}(m_3^4/v^4)]$ from $v_2 = v$. The masses of the top and bottom quarks are given by

$$m_t \simeq \frac{1}{\sqrt{2}} y v, \qquad m_b = \frac{1}{\sqrt{2}} y v_1, \tag{25}$$

so that the bottom quark finally obtain the small mass. The mass hierarchy of m_t and m_b then is deduced from Eqs. (24) and (25) without fine-tuning of the Yukawa coupling constants, i.e., $m_t/m_b = \tan\beta$. With nonzero m_3^2 the Higgs doublets Φ_1 and Φ_2 do mix. The mixing angle $\beta - \alpha$ is expressed as

$$\sin(\beta - \alpha) = 1 - \left(\frac{m_H^2 - m_{H^\pm}^2}{m_H^2 - m_h^2}\right)^2 \frac{2}{\tan^2\beta} + \mathcal{O}\left(\frac{m_3^6}{v^6}\right), \tag{26}$$

where Eqs. (20)–(23) and $\tan\beta \gg 1$ are used. From Eqs. (20) and (26), the property of the *CP*-even Higgs boson h is similar to the SM one. We note that Higgs boson masses in Eqs. (20)–(23) receive corrections of $\mathcal{O}(m_3^4/v^4)$. These corrections, however, do not affect the expressions in Eqs. (24) and (26).

Let us estimate the typical size of the masses of the extra Higgs bosons. The value of $\tan\beta$ is fixed by $\tan\beta = m_t/m_b \sim 40$. On the other hand, the small value of $m_3^2 (\ll v^2)$ can be interpreted as $m_3^2 \simeq v^2/(4\pi)^2$. In the next section, we shall present a concrete model in which such a small $m_3^2[\simeq v^2/(4\pi)^2]$ is radiatively induced by the dynamics above the cutoff scale of the THDM. From Eq. (24), the mass of H is expressed as

$$m_H^2 \simeq m_3^2 \tan\beta. \tag{27}$$

Therefore, the size of m_H is at most of the order of v. Furthermore, the masses of A and $H^\pm$ are also the same order because of the relations

$$m_{H^\pm}^2 = m_H^2 - (\lambda_4 + \lambda_5)v^2, \qquad m_A^2 = m_H^2 - 2\lambda_5 v^2, \tag{28}$$

which are obtained from Eqs. (21)–(23).

We have found Eqs. (24) and (26), assuming the softly-broken $SU(2)_R$ symmetry, i.e., $\lambda_1 = \lambda_2 = \lambda_3 (= \lambda)$. We now give comments on the case with $\lambda_1 \neq \lambda_2 \neq \lambda_3$, relaxing the $SU(2)_R$ symmetry. First, it can be shown that Eq. (24) does not change. Second, although Eq. (26) is slightly modified, the essential result of $\sin(\beta - \alpha) = 1 - \mathcal{O}(\tan^{-2}\beta)$ still holds. Finally, the masses of the extra Higgs bosons remain $\mathcal{O}(v)$ even for $\lambda_1 \neq \lambda_2 \neq \lambda_3$, because Eq. (28) turns out to be unchanged as well.

III. A MECHANISM FOR SMALL m_3^2

We discuss an example where the small m_3^2 is generated radiatively in the low-energy scale. Let us consider a model with a complex scalar field S which is a $SU(2)_L$ singlet without $U(1)_Y$ charge. The Lagrangian is given by

[1]There are the three vacua for $m_2^2 < -|m_1^2|$ as depicted in Fig. 1, while the vacua (b) and (c) are squeezed out for the region (19).

[2]This subject will be addressed in Ref. [8].

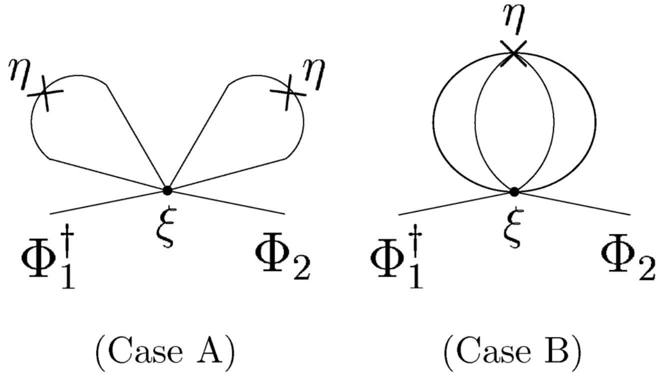

FIG. 2. Feynman diagrams to induce a small m_3^2.

$$\mathcal{L} = \mathcal{L}_{\rm kin} - V_\Phi - V_S - V_{\not Z_2}, \tag{29}$$

where $\mathcal{L}_{\rm kin}$ represents the kinetic term and V_Φ is the Z_2 symmetric part[3] of the THDM potential (4) with $m_3^2 = 0$. The potential V_S for the complex scalar S and the interaction term $V_{\not Z_2}$ between S and $\Phi_{1,2}$ are given by

$$V_S = M_S^2 S^\dagger S + \kappa(S^\dagger S)^2 + V_{Z_{2n}}, \qquad M_S^2 > 0, \tag{30}$$

with

$$V_{Z_{2n}} = \frac{\eta}{\Lambda^{2n-4}}(S^{2n} + {\rm H.c.}), \qquad \eta \sim \mathcal{O}(1), \tag{31}$$

and

$$V_{\not Z_2} = \frac{\xi}{\Lambda^{2\ell-2}}(S^{2\ell}\Phi_1^\dagger\Phi_2 + {\rm H.c.}), \qquad \xi \sim \mathcal{O}(1), \tag{32}$$

respectively. In Eqs. (31) and (32), Λ denotes the cutoff scale of the model. We now set $n = 1$ (case A) or $n = \ell$ (case B) with $\ell \geq 1$. We note that V_S has the Z_{2n} symmetry under $S \to e^{i(\pi/n)}S$, while V_Φ is Z_2 invariant under the transformation $\Phi_1 \to -\Phi_1, \Phi_2 \to +\Phi_2$. The interaction term (32) explicitly breaks both Z_{2n} and Z_2. Some invariant terms under Z_{2n} and Z_2 are not explicitly included here, as they are irrelevant to our conclusion.

Supposing that $M_S(\sim\Lambda)$ is much larger than the EWSB scale, we integrate out the field S and thereby obtain the THDM with the softly-broken Z_2 symmetry ($m_3^2 \neq 0$) as the low-energy effective theory. From the Feynman diagrams depicted in Fig. 2, we estimate

$$m_3^2 \sim \xi\eta^\ell \frac{1}{(4\pi)^{2\ell}} M_S^2, \qquad \text{for case A}, \tag{33}$$

$$m_3^2 \sim \xi\eta \frac{1}{(4\pi)^{2(2\ell-1)}} M_S^2, \qquad \text{for case B}. \tag{34}$$

For example, we can obtain $m_3^2 \sim v^2/(4\pi)^2$ for $\ell = 2$, if we take the cutoff $M_S = 4\pi v$ for Case A or $M_S = (4\pi)^2 v$ for Case B. For $\ell = 2$, we do not need higher dimensional operators except for the V_{Z_2} breaking term $\not Z_2$.

We may consider other possibilities to obtain small m_3^2 values based on many ideas such as Topcolor instanton [11] and large extra dimensions [12]. Also useful is a model which provides effectively $(\Phi_1^\dagger\Phi_2)^3$ with a coefficient $\sim\mathcal{O}(1)$ while prohibits the hard breaking terms of the Z_2 symmetry such as $(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2)$.

[3]We here concentrate on the mechanism to induce m_3^2, assuming that the Z_2 invariant part V_Φ comes from some other dynamics.

IV. QUARK MASS MATRICES

We discuss the extension of our model incorporating first two generation quarks. Can we reproduce the observed quark mass spectrum and the Kobayashi-Maskawa (KM) matrix?

Under the discrete symmetry [2], two types of Yukawa interactions are possible in the THDM, so-called Model I and Model II [5]. The flavor changing neutral current then does not appear at the tree level [2]. Obviously Model I is inconsistent with our scenario, so that we here study Model II,

$$-\mathcal{L}_Y = \sum_{i,j=1}^{3}[Y_D^{ij}\bar{q}_L^{(i)}\Phi_1 D_R^{(j)} + Y_U^{ij}\bar{q}_L^{(i)}\tilde{\Phi}_2 U_R^{(j)}] + ({\rm H.c.}), \tag{35}$$

where $q_L^{(i)}$ is the left-handed quark doublet of the ith generation, and $D_R^{(i)} = (d_R, s_R, b_R)^T$ and $U_R^{(i)} = (u_R, c_R, t_R)^T$. We then assume that matrices of the Yukawa coupling take the following forms,

$$Y_U^{ij} \sim Y_D^{ij} \sim y\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \qquad y \sim \mathcal{O}(1), \tag{36}$$

which lead to $m_t \gg m_c, m_u$ and $m_b \gg m_s, m_d$, and the KM matrix becomes approximately diagonal. We can numerically reproduce the data for the mass spectrum and the KM matrix [1], allowing fluctuations of the Yukawa coupling constants,

$$Y_U^{ij} = y\epsilon_{ij}^U, Y_D^{ij} = y\epsilon_{ij}^D, \quad \text{with} \quad 0.5 < |\epsilon_{ij}^{U,D}| < 1.5. \tag{37}$$

Three comments are in order: (a) Although we can avoid hierarchy among Yukawa couplings, subtle cancellation among the $\mathcal{O}(1)$ mass-matrix elements is required to obtain masses of light quarks. (b) We may adopt Model III [13] to our scenario, if the flavor changing neutral current is suppressed by some mechanism. (c) It is possible to apply our scenario to the lepton sector. The τ lepton then receives the small mass due to the similar mechanism to the bottom quark. At the same time, however, the Dirac mass of the tau neutrino could be produced around m_t. To explain the tiny (Majorana) mass of the tau neutrino, additional mechanism such as the seesaw [14] might be helpful.

V. SUMMARY AND DISCUSSIONS

We have proposed the mechanism to explain the mass hierarchy between the top and bottom quarks without fine-tuning, starting from the vacuum with $(v_1, v_2) = (0, v)$. Such a vacuum can exist when the Z_2 symmetry is exact. The observed mass spectrum $m_t \gg m_b \neq 0$ is realized via the small soft-breaking parameter m_3^2 for the Z_2 symmetry. We have presented the model in which a small m_3^2 is induced from the underlying physics above the cutoff scale of the THDM.

The phenomenological implication is as follows. The size of $\tan\beta$ corresponds to the ratio $m_t/m_b \sim 40$. We have found the relation $m_H^2 \simeq m_3^2 \tan\beta \sim \mathcal{O}(v^2)$. Therefore, the masses of the extra Higgs bosons H, A and $H^\pm$ are expected to be $\mathcal{O}(v)$. The THDM with such parameters is constrained by the theoretical considerations [15,16] as well as the available data. When $m_{H^\pm} \simeq m_H$, or $m_{H^\pm} \simeq m_A$, our model can satisfy the constraint from the LEP precision data [1]. The mass of the charged Higgs boson in our scenario may not conflict with the $b \to s\gamma$ result [17]. The doublet Φ_2 is mainly responsible for the EWSB, so that we obtain $\sin(\beta - \alpha) \simeq 1$ in a good approximation.

In addition to the SM-like Higgs boson h, all the extra Higgs bosons in our model are expected to be discovered at the LHC. Our prediction of $\sin(\beta - \alpha) \simeq 1$ can also be confirmed at the LHC and LC's. Our scenario may further be tested by measuring the hhh coupling at future LC's [18]. More detailed phenomenological analysis will be done elsewhere [8].

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